# Exercises Solution for "Elementary Number Theory: Second Edition by Underwood Dudley"

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# 1 Integers

**Exercise 1.** Which integers divide zero?

For any integer $a$, $0 \cdot a = 0$. Therefore all integers divide zero.

**Exercise 2.** Show that if $a \mid b$ and $b \mid c$, then $a \mid c$.

From the definition, there are integers $d$ and $e$ such that $b = da$ and $c = eb$. Therefore,

$$\begin{aligned} c &= eb \\ &= eda \\ &= (ed)a \end{aligned}$$

Which means $a \mid c$.

**Exercise 3.** Prove that if $d \mid a$ then $d \mid ca$ for any integer $c$.

**Method 1** From the definition, there is an integer $b$ such that $a = bd$. Therefore $ca = cbd = (cb)d$ which means $d \mid ca$.

**Method 2** We can use Lemma 2 by setting $n = 1$, $a_1 = a$, and $c_1 = c$.

**Exercise 4.** What are $(4, 14)$, $(5, 15)$, and $(6, 16)$?

The positive divisors of 4 are 1, 2, and 4, and the positive divisors of 14 are 1, 2, 7, and 14. Therefore $(4, 14) = 2$.

The positive divisors of 5 are 1 and 5. Likewise for 15 they are 1, 3, 5, and 15. Therefore $(5, 15) = 5$.

The positive divisors of 6 are 1, 2, 3, and 6. For 16 they are 1, 2, 4, 8, and 16. Therefore $(6, 16) = 2$.

**Exercise 5.** What is $(n, 1)$, where $n$ is any positive integer? What is $(n, 0)$?

The only divisor of 1 is 1, and it also divides any positive integer $n$, so $(n, 1) = 1$.

$n \mid n$ and is the largest divisor of $n$. Because $n \mid 0$, $(n, 0) = n$.

**Exercise 6.** If $d$ is a positive integer, what is $(d, nd)$?

The largest divisor of $d$ is $d$ itself. Because $d \mid nd$, $(d, nd) = d$.

**Exercise 7.** What are $q$ and $r$ if $a = 75$ and $b = 24$? If $a = 75$ and $b = 25$?

We can create the set

$$\{75, 75 - 24 = 51, 75 - 2 \cdot 24 = 27, 75 - 3 \cdot 24 = 3\}$$

Therefore $75 = 3 \cdot 24 + 3$ so $q = 3$ and $r = 3$.

Similarly, for the second problem we can create the set

$$\{75, 75 - 25 = 50, 75 - 2 \cdot 25 = 25, 75 - 3 \cdot 25 = 0\}$$

So $q = 3$ and $r = 0$.

**Exercise 8.** Verify that the lemma is true when $a = 16$, $b = 6$, and $q = 2$.

We have the equation $16 = 6 \cdot 2 + 4$ so $r = 4$.
$(16, 6) = 2$, and $(6, 4) = 2$, which is according to the lemma.

**Exercise 9.** Calculate $(343, 280)$ and $(578, 442)$.

For the first problem,

$$\begin{aligned} 343 &= 280 + 63 \\ 280 &= 63 \cdot 4 + 28 \\ 63 &= 28 \cdot 2 + 7 \\ 28 &= 7 \cdot 4 \end{aligned}$$

So $(343, 280) = (280, 63) = (63, 28) = (28, 7) = 7$
For the second problem,

$$\begin{aligned} 578 &= 442 + 136 \\ 442 &= 136 \cdot 3 + 34 \\ 136 &= 34 \cdot 4 \end{aligned}$$

So $(578, 442) = (442, 136) = (136, 34) = 34$