

Summary for “Elementary Number Theory:
Second Edition by Underwood Dudley”

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2018-12-10

1 Integers

Definition 1.1. a divides b (written $a \mid b$) if and only if there is an integer d such that $ad = b$.

Examples $3 \mid 6$, $15 \mid 60$, $9 \mid 9$, $-4 \mid 16$, and $2 \mid -100$.

Definition 1.2. If a does not divide b , we write $a \nmid b$.

Examples $10 \nmid 5$ and $3 \nmid 7$.

Lemma 1.1. If $d \mid a$ and $d \mid b$, then $d \mid (a + b)$.

Example $2 \mid 4$ and $2 \mid 10$, so $2 \mid 12$.

Lemma 1.2. If $d \mid a_1$, $d \mid a_2$, ... $d \mid a_n$, then $d \mid (c_1a_1 + c_2a_2 + \dots + c_na_n)$ for any integers c_1, c_2, \dots, c_n

Example $2 \cdot 6 + 4 \cdot 9 = 12 + 36 = 48$. Because $3 \mid 6$ and $3 \mid 9$, we conclude that $3 \mid 48$.

Definition 1.3. d is the greatest common divisor of a and b (written $d = (a, b)$) if and only if

- (i) $d \mid a$ and $d \mid b$, and
- (ii) if $c \mid a$ and $c \mid b$, then $c \leq d$

Examples $(2, 6) = 2$ and $(5, 7) = 1$.