Exercises Solution for "Elementary Number Theory: Second Edition by Underwood Dudley"

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1 Integers

Exercise 1 Which integers divide zero?

For any integer a, $0 \cdot a = 0$. Therefore all integers divide zero.

Exercise 2 Show that if $a \mid b$ and $b \mid c$, then $a \mid c$.

From the definition, there are integers d and e such that $b = da$ and $c = eb$. Therefore,

$$\begin{aligned} c &= eb \\ &= eda \\ &= (ed)a \end{aligned}$$

Which means $a \mid c$.

Exercise 3 Prove that if $d \mid a$ then $d \mid ca$ for any integer c.

Method 1 From the definition, there is an integer b such that $a = bd$. Therefore $ca = cbd = (cb)d$ which means $d \mid ca$.

Method 2 We can use Lemma 2 by setting $n = 1$, $a_1 = a$, and $c_1 = c$.