OVM Homework Exercises I (WI3405TU)

Submission deadline : 25th October 2025 (electronically via Brightspace)

Please include all answers clearly in your **PDF report** (LaTeX recommended) and attach your code in the appendix. Also submit the code separately as a .ipynb, .py, or .m file for implementation checking. Unclear solutions or non-working code may lose credit. Note that ZIP files will not be accepted (for security and grading reasons).

1 (2P) Market stock prices :

a). Using the daily close prices (i.e., Column "Close" in the provided CSV file `DailyData - STOCK_US_XNAS_AAPL.csv`[1])

First, plot the price movements over time.

Second, compute the daily return $r_i = \frac{S_i - S_{i-1}}{S_{i-1}}$ and daily logarithmic return $\hat{r}_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$, and create plots to illustrate the patterns of both returns (see Lecture 3).

Third, explain under what circumstances these two types of returns can be considered approximately identical.

b). Statistics : First, plot the histogram of daily logarithmic returns, compute the sample mean $\hat{\mu}$ and variance $\hat{\sigma}^2$, and produce the quantile-quantile plot, i.e., daily logarithmic returns vs. normal distribution $N(\hat{\mu}, \hat{\sigma}^2)$.

Second, repeat the above process for the weekly close prices in the provided CSV file `WeeklyData - STOCK_US_XNAS_AAPL.csv`, and compare the Q–Q plot patterns of weekly and daily returns.

2 (0.5P) Central Limit Theorem.

Reproduce the results on page 14 of Lecture 3 to verify the Central Limit Theorem numerically.

3 (2P). Simulation of stock price paths.

a). We have the discrete price model $S(t_{i+1}) = S(t_i)(1 + \mu\delta t + \sigma\sqrt{\delta t}Z)$, where $Z \sim \mathcal{N}(0,1)$, $T = 1$, $S_0 = 170$, $\sigma = 0.344$, and $\mu = 0.1$.

First, choose the number of time intervals $L = 5$ and plot the price evolution of S_t for $t = i \cdot \Delta t$ with $i = 0, \ldots, L$ and $\Delta t = \frac{T}{L}$.

Second, repeat the simulation by $M = 5000$ times, and store the simulated S_T to make the histogram.

Third, repeat the simulation for increasing $L = 10, 20, 50, 100$ to see how the histogram of S_T changes, compared with the theoretical distribution given by the continuous-time stock price model, i.e., $S_T = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z}$:

b). Use the above discrete model to generate the graphs similar to Figure 8.1 in the book and study the behavior of the sum-of-square increments for asset prices.

4(3.5P). Delta hedging and the Black-Scholes model :

1. The data of Apple stock prices can be downloaded from the website `https://www.marketwatch.com/investing/stock/aapl/` under the "Historical quotes" tab.

a(1.5). We can modify the assumptions of the Black-Scholes model to incorporate dividend payments (for example, S&P 500 index option). In this scenario, dividends are paid out at a constant rate $q > 0$, compounded continuously, causing the stock price to drop by the value of dividend $qS(t_i)\delta t$,

$$\delta S(t_i) = S(t_{i+1}) - S(t_i) = \mu S(t_i)\delta t - \underbrace{qS(t_i)\delta t}_{\text{dividend}} + \sigma S(t_i)\sqrt{\delta t}Y_i. \tag{1}$$

Dividends received from holding one share of stock over Δt are the sum $\sum_i qS(t_i)\delta t = qS\Delta t$. The portfolio Π changes over time interval Δt,

$$\Delta\Pi = \Delta(AS + D) = \Delta(AS) + \Delta D = (A\Delta S + AqS\Delta t) + \Delta D \tag{2}$$

where $\Delta S = S(t + \Delta t) - S(t)$. To value a European option with continuous dividends, please complete **two tasks** : i) Explain the financial meaning of Equation (2) and then derive the corresponding option pricing PDE. ii) Develop a discrete hedging strategy suitable for the above scenario.

b(1.0). Consider hedging a European put option with the following parameters : strike price $E = 170$, stock price $S_0 = 170$, expiry time $T = 1$ year, risk-free interest rate $r = 0.05$, time step $\Delta t = 0.01$, and the stock price follows Geometric Brownian Motion with volatility $\sigma = 0.344$, drift $\mu = 0.1$, dividend yield $q = 0.01$ in Equation 1.

Simulate and plot the delta-hedging process for two cases : in-the-money option and out-of-the-money option (refer to Figure 9.1, Figure 9.2 in the book).

c(1.0). i) What is the financial interpenetration of Equation (9.9) in the book ?

ii) Conduct large-scale tests of the delta hedging (using the setting in Question 4.b) by simulating 1000 discrete asset paths to produce plots for Equation (9.9) and using two different drift values, $\mu = \pm 0.1$ (refer to Figure 9.4 in the book).

iii) Check whether your results depend on the value of drift.

5(2P). Implied volatility :

a(0.5). Employ a root-finding algorithm to reproduce Figure 14.2 in the book, by adding 10 units of currency to each option price. For example, 475 becomes 485, 405 becomes 415, etc.

a(1.0). Acquire real-world option data for a company whose name starts with either your initial or the first letter of your surname. This data should have **different strike prices** and **expiry times**. Create an implied volatility figure similar to Figure 14.2 using this data. Be sure to indicate the source of the option data and provide your dataset in the submission.

Hint : For options on stocks from the US market, the risk-free rate can be derived from the daily US Treasury yield curve rates. For stocks from other markets, you can use a similar approach or select a reasonable estimate, such as $r = 0.01$. To calculate the time to maturity, you may use the Act-365 day count convention, where the year fraction between two dates is computed as the number of days between them divided by 365.

b(0.5). Implied volatility may vary along time to maturity $\tau = T - t$ given a specific strike price. This is so-called term structure of implied volatility. Please make plots and check if the data in Question 5.b contains term structure. At least 4 expiry times should be considered.