Review of Multilevel Models So Far

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1 Graph

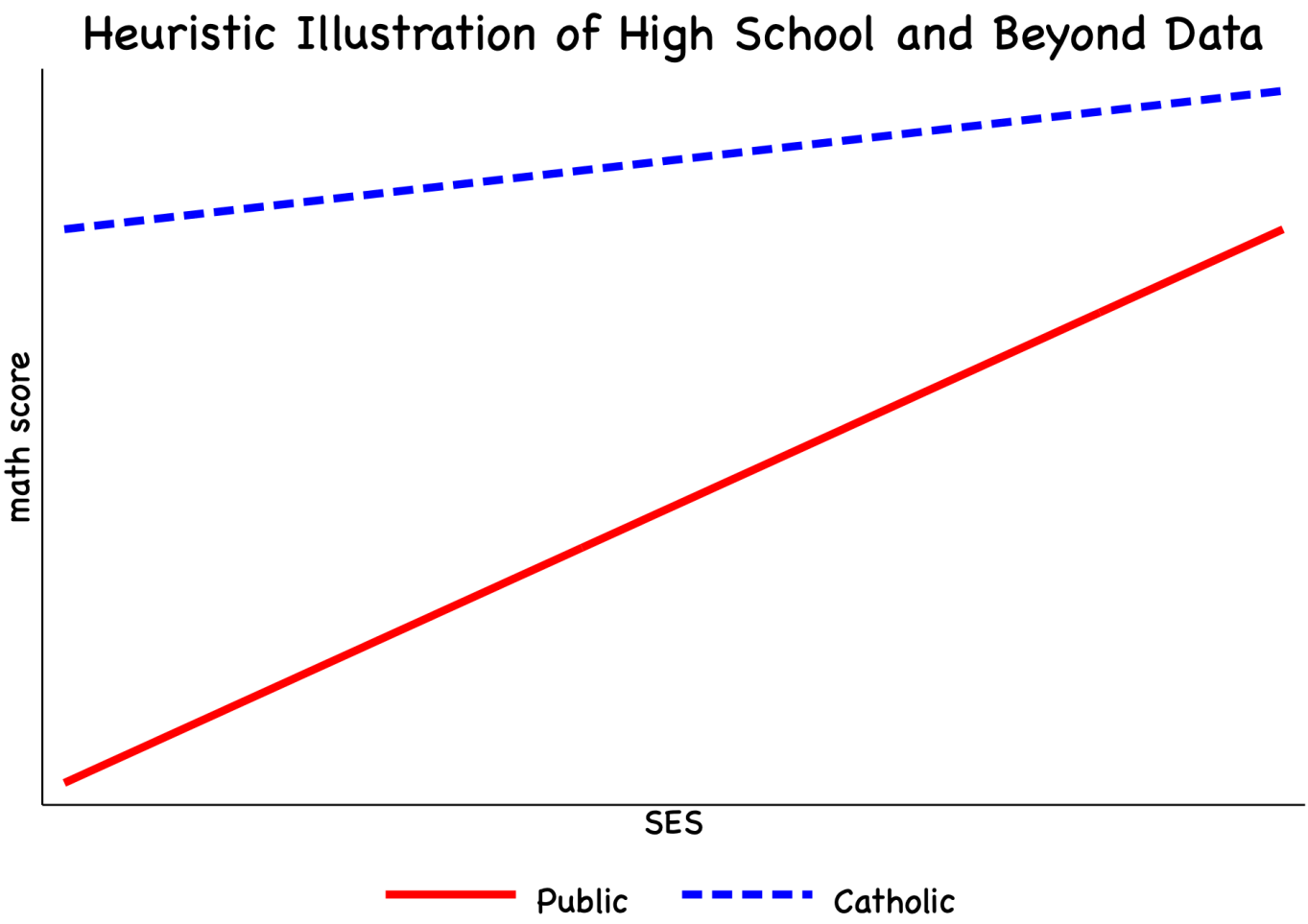

Figure 1: heuristic illustration of HBS data

2 Equation in Multiple Equation Format

$$
\begin{aligned}
y_{ij} &= \beta_{0j} + \beta_{1j}x + e_{ij} \\
\beta_{0j} &= \gamma_{00} + \gamma_{01}\text{sector} + u_{0j} \\
\beta_{1j} &= \gamma_{10} + \gamma_{11}\text{sector} + u_{0j}
\end{aligned}
\tag{1}
$$

Random Effects Are Optional

The random effects in Equation 1 are *optional* in the sense that they may or may not be included. Often only a *random intercept* is included. Inclusion of multiple *random slopes* may lead to non-convergence of the model.

3 Intra-Class Correlation Coefficient

$$\text{icc} = \frac{\text{var}(u_{0j})}{\text{var}(u_{0j}) + \text{var}(e_{ij})} \tag{2}$$

4 How Many Levels?

For the i^{th} student in the j^{th} school in the k^{th} district.

$$\begin{aligned} y_{ijk} &= \pi_{0jk} + \pi_{1jk}a + e_{ijk} \\ &\text{there are p level 1 coefficients} \\ \pi_{pjk} &= \beta_{0pk} + \beta_{1pk} + r_{pjk} \\ &\text{group specific effect for the jth group} \\ &\text{there are q level 2 coefficients} \\ \beta_{pqk} &= \gamma_{0pq} + \gamma_{1pq}w + u_{pqk} \\ &\text{there are p*q level 3 coefficients} \end{aligned} \tag{3}$$

5 A Single Level Equation

Using substitution, we re-write Equation 1.

$$y_{ij} = \gamma_{00} + \gamma_{01}\text{sector} + \gamma_{10}x + \gamma_{11}\text{sector} \times x + e_{ij} + u_{0j} + u_{1j} \times x \tag{4}$$

Let's just call everything a β.

$$y_{ij} = \beta_0 + \beta_1\text{sector} + \beta_2 x + \beta_3\text{sector} \times x + e_{ij} + u_{0j} + u_{1j} \times x \tag{5}$$

x is ses

In the concrete example that I am using x is ses