

Multilevel Multilingual

Multilevel Models in Stata, R and Julia

Andrew Grogan-Kaylor

2024-05-08

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1 Multilevel Multilingual

“This curious world which we inhabit is more wonderful than it is convenient...”
(Thoreau, 1975)

“Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled.” (Schanen, 2021)

1.1 Introduction

Below, I describe the use of [Stata](#) (StataCorp, 2021), [R](#) (Bates et al., 2015; R Core Team, 2023), and [Julia](#) (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

All of these software packages can estimate multilevel models. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. R is free open source software which is less intuitive, but there are many excellent resources for learning R. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but intuitive for both multilevel modeling and graphing.
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (<code>ggplot</code>).
Julia	free	steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

💡 Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

💡 Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as *R* commands are usually clearly encased in parentheses `()` or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a multi-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire *Julia* command in an outer set of parentheses `()`.

💡 Running Statistical Packages in Quarto

I used Quarto (<https://quarto.org/>) to create this Appendix. Quarto is a programming and publishing environment that can run multiple programming languages, including *Stata*, *R* and *Julia*, and that can write to multiple output formats including HTML, PDF, and MS Word. To run *Stata*, I used the `Statamarkdown` library in *R* to connect *Stata* to Quarto. Quarto has a built in connection to *R*, and runs *R* without issue. To run *Julia*, I used the `JuliaCall` library in *R* to connect Quarto to *Julia*.

Of course, each of these programs can be run by itself, if you have them installed on your computer.

1.2 The Data

The examples use the `simulated_multilevel_data.dta` file from *Multilevel Thinking*. Here is a [direct link](#) to download the data.

Table 1.2: Sample of Simulated Multilevel Data

Table 1.2: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	2	1	3
1	69	2	1.2	2	2	2
1	69	3	1.3	1	2	3
1	69	4	1.4	2	1	0
1	69	5	1.5	2	1	4
1	69	6	1.6	1	2	5

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
3	58.47
1	51.1
2	53.92
5	61.17
4	56.05
3	50.81

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable **y**, independent variables **x** and **z**, clustering variable **group**, and a random slope for **x**. i is the index for the person, while j is the index for the **group**.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \quad (1.1)$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x || group: x
```

1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels

fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```


2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI
```

```
tabulate identity
```

```
tabulate intervention
```

Variable	Obs	Mean	Std. dev.	Min	Max
outcome	3,000	53.43327	6.530996	30.60798	75.83553
warmth	3,000	3.521667	1.888399	0	7
physical_punishment	3,000	2.478667	1.360942	0	5
HDI	3,000	64.76667	17.24562	33	87

hypothetical	1	identity	group	variable	Freq.	Percent	Cum.
	1				1,507	50.23	50.23
	2				1,493	49.77	100.00
	Total				3,000	100.00	

recieved interventio	n	Freq.	Percent	Cum.
1	1,547	51.57	51.57	
2	1,453	48.43	100.00	
Total	3,000	100.00		

2.1.2 R

```
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`¹ to generate descriptive statistics.

```
df$country <- factor(df$country)

df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)

summary(df)
```

```

country      HDI      family      id      identity
1      : 100  Min.   :33.00  Min.   :  1.00  Length:3000  1:1507
2      : 100  1st Qu.:53.00  1st Qu.: 25.75  Class :character 2:1493
3      : 100  Median :70.00  Median : 50.50  Mode  :character
4      : 100  Mean   :64.77  Mean   : 50.50
5      : 100  3rd Qu.:81.00  3rd Qu.: 75.25
6      : 100  Max.   :87.00  Max.   :100.00
(Other):2400
intervention physical_punishment  warmth      outcome
1:1547      Min.   :0.000      Min.   :0.000  Min.   :30.61
2:1453      1st Qu.:2.000      1st Qu.:2.000  1st Qu.:49.02
```

¹`skimr` is an excellent new alternative library for generating descriptive statistics in R.

Median	:2.000	Median	:4.000	Median	:53.45
Mean	:2.479	Mean	:3.522	Mean	:53.43
3rd Qu.	:3.000	3rd Qu.	:5.000	3rd Qu.	:57.86
Max.	:5.000	Max.	:7.000	Max.	:75.84

2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))
@transform!(df, :identity = categorical(:identity))
@transform!(df, :intervention = categorical(:intervention))
```

```
describe(df) # descriptive statistics
```

```
9×7 DataFrame
 Row  variable          mean  min  median  max  nmissing  eltype
      Symbol          Union... Any  Union... Any  Int64      Union
  1  country              1.0    30.0
  2  HDI                 64.7667 33.0  70.0    87.0
  3  family              50.5    1.0  50.5   100.0
  4  id                   1.1    9.99
  5  identity             1.0    2.0
  6  intervention         1.0    2.0
  7  physical_punishment  2.47867 0.0  2.0    5.0
  8  warmth              3.52167 0.0  4.0    7.0
  9  outcome             53.4333 30.608 53.449 75.8355
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                1 column omitted
```

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \quad (3.1)$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{\text{var}(u_{0j})}{\text{var}(u_{0j}) + \text{var}(e_{ij})} \quad (3.2)$$

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...
```

```
Performing gradient-based optimization:
```

```
Iteration 0: Log likelihood = -9856.1548
```

```
Iteration 1: Log likelihood = -9856.1548
```

Computing standard errors ...

Mixed-effects ML regression
Group variable: country

Number of obs = 3,000
Number of groups = 30
Obs per group:
min = 100
avg = 100.0
max = 100
Wald chi2(0) = .
Prob > chi2 = .

Log likelihood = -9856.1548

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
_cons	53.46757	.3539097	151.08	0.000	52.77392	54.16122

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Identity				
var(_cons)	3.348734	.9702594	1.897816	5.908906
var(Residual)	40.88284	1.060908	38.8555	43.01597

LR test vs. linear model: chibar2(01) = 169.64 Prob >= chibar2 = 0.0000

```
estat icc // ICC
```

Intraclass correlation

Level	ICC	Std. err.	[95% conf. interval]	
country	.0757091	.0203761	.0442419	.1265931

3.2.2 R

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
Data: df
```

```
REML criterion at convergence: 19712.5
```

```
Scaled residuals:
```

Min	1Q	Median	3Q	Max
-2.97650	-0.68006	0.00936	0.67580	3.03510

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.478	1.865
	Residual	40.883	6.394

Number of obs: 3000, groups: country, 30

```
Fixed effects:
```

	Estimate	Std. Error	t value
(Intercept)	53.47	0.36	148.5

```
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient
```

```
Adjusted ICC: 0.078
Unadjusted ICC: 0.078
```

3.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,  
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
```

```
df = DataFrame(load("simulated_multilevel_data.dta"))
```

```
@transform!(df, :country = categorical(:country))
```

```
m0 = fit(MixedModel,  
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9856.1548	19712.3097	19718.3097	19718.3177	19736.3288

Variance components:

	Column	Variance	Std.Dev.
country	(Intercept)	3.34871	1.82995
Residual		40.88285	6.39397

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	53.4676	0.353908	151.08	<1e-99

```
icc(m0) # ICC
```

```
0.07570852291396266
```

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\begin{aligned} \text{outcome}_{ij} = & \beta_0 + \beta_1 \text{warmth}_{ij} + \\ & \beta_2 \text{physical punishment}_{ij} + \\ & \beta_3 \text{identity}_{ij} + \beta_4 \text{intervention}_{ij} + \beta_5 \text{HDI}_{ij} + \\ & u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij} \end{aligned} \tag{4.1}$$

4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 4.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} \text{var}(u_{0j}) & 0 \\ 0 & \text{var}(u_{1j}) \end{bmatrix} \tag{4.2}$$

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 4.3.

$$\begin{bmatrix} \text{var}(u_{0j}) & \text{cov}(u_{0j}, u_{1j}) \\ \text{cov}(u_{0j}, u_{1j}) & \text{var}(u_{1j}) \end{bmatrix} \tag{4.3}$$

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2021).

Table 4.1: Correlated and Uncorrelated Random Effects

Software	Uncorrelated Random Effects	Correlated Random Effects
Stata	default	add option: <code>, cov(uns)</code>
R	separate random effects from grouping variable with <code> </code>	separate random effects from grouping variable with <code> </code>
Julia	separate terms for each random effect e.g. <code>(1 group) + (0 + x group)</code>	separate random effects from grouping variable with <code> </code> .

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

4.3 Run Models

4.3.1 Stata

4.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9626.6279

Iteration 1: Log likelihood = -9626.607

Iteration 2: Log likelihood = -9626.607

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 3,000

Number of groups = 30

Obs per group:

min = 100

avg = 100.0

max = 100

Wald chi2(5) = 334.14

Prob > chi2 = 0.0000

Log likelihood = -9626.607

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
warmth	.8345368	.0637213	13.10	0.000	.7096453	.9594282
physical_punishment	-.9916657	.0797906	-12.43	0.000	-1.148052	-.8352791
2.identity	-.3004767	.2170295	-1.38	0.166	-.7258466	.1248933
2.intervention	.6396427	.2174519	2.94	0.003	.2134448	1.065841
HDI	-.003228	.0199257	-0.16	0.871	-.0422817	.0358256
_cons	52.99991	1.371257	38.65	0.000	50.3123	55.68753

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Independent				
var(warmth)	.0227504	.0257784	.0024689	.2096436
var(_cons)	2.963975	.9737647	1.556777	5.643163
var(Residual)	34.97499	.9097109	33.23668	36.80422

LR test vs. linear model: chi2(2) = 205.74

Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

4.3.2 R

4.3.2.1 Get The Data

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

4.3.2.2 Run The Model

```
fit1 <- lmer(outcome ~ warmth + physical_punishment +
             identity + intervention + HDI +
             (1 + warmth || country),
             data = df)

summary(fit1)
```

Linear mixed model fit by REML ['lmerMod']

Formula: outcome ~ warmth + physical_punishment + identity + intervention +
HDI + ((1 | country) + (0 + warmth | country))
Data: df

REML criterion at convergence: 19268.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.9774	-0.6563	0.0187	0.6645	3.6730

Random effects:

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.19056	1.786
country.1	warmth	0.02465	0.157
Residual		35.01782	5.918

Number of obs: 3000, groups: country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	52.672655	1.479571	35.600
warmth	0.834562	0.064252	12.989

physical_punishment	-0.991892	0.079845	-12.423
identity	-0.300350	0.217179	-1.383
intervention	0.639059	0.217603	2.937
HDI	-0.003395	0.020596	-0.165

Correlation of Fixed Effects:

	(Intr)	warmth	physc_	idntty	intrvn
warmth	-0.121				
physcl_pnsh	-0.145	-0.003			
identity	-0.213	-0.012	-0.003		
interventin	-0.223	0.034	0.022	-0.018	
HDI	-0.902	-0.006	0.009	-0.001	0.000

4.3.3 Julia

4.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

4.3.3.2 Change Country To Categorical

```
@transform!(df, :country = categorical(:country))
```

4.3.3.3 Run The Model

```
m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
    identity + intervention + HDI +
    (1 | country) +
    (0 + warmth | country)), df)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 country)				
logLik	-2 logLik	AIC	AICc	BIC
-9626.6070	19253.2140	19271.2140	19271.2742	19325.2713

Variance components:

	Column	Variance	Std.Dev.	Corr.
country	(Intercept)	2.963849	1.721583	
	warmth	0.022756	0.150852	.
Residual		34.974984	5.913965	

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	52.6608	1.43785	36.62	<1e-99
warmth	0.834537	0.0637228	13.10	<1e-38
physical_punishment	-0.991665	0.0797906	-12.43	<1e-34
identity	-0.300475	0.217029	-1.38	0.1662
intervention	0.639641	0.217452	2.94	0.0033
HDI	-0.0032286	0.0199255	-0.16	0.8713

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

5.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \quad (5.1)$$

$$\beta_4 \text{identity}_{itj} + \beta_5 \text{intervention}_{itj} + \beta_6 \text{HDI}_{itj} +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0i} + v_{1i} \times \text{time}_{itj} + e_{itj}$$

5.3 Run Models

5.3.1 Stata

5.3.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.3.1.2 Run The Model

5.3.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28739.506

Iteration 1: Log likelihood = -28739.506

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 9,000

Number of groups = 30

Obs per group:

min = 300

avg = 300.0

max = 300

Wald chi2(6) = 1119.81

Prob > chi2 = 0.0000

Log likelihood = -28739.506

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
t	.9443446	.0756408	12.48	0.000	.7960914	1.092598
warmth	.9123903	.0430042	21.22	0.000	.8281035	.996677
physical_punishment	-.9881587	.0451732	-21.87	0.000	-1.076696	-.8996209
2.identity	-.1241465	.1242225	-1.00	0.318	-.367618	.1193251
2.intervention	.8575839	.1245179	6.89	0.000	.6135332	1.101635
HDI	-.0025173	.0191696	-0.13	0.896	-.0400891	.0350544
_cons	51.54528	1.304146	39.52	0.000	48.9892	54.10136

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Independent				
var(warmth)	.0229349	.0135353	.0072136	.0729194
var(_cons)	3.0009	.8550708	1.716768	5.245553

```

var(Residual) | 34.31935 .5130963 33.3283 35.33988
-----
LR test vs. linear model: chi2(2) = 767.22 Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

5.3.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.identity i.intervention c.HDI) || country
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28738.554

Iteration 1: Log likelihood = -28738.554

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 9,000

Number of groups = 30

Obs per group:

min = 300

avg = 300.0

max = 300

Wald chi2(11) = 1122.75

Prob > chi2 = 0.0000

Log likelihood = -28738.554

```

-----
outcome | Coefficient Std. err. z P>|z| [95% conf. interval]
-----+-----
t | .7537359 .3719996 2.03 0.043 .0246302 1.482842
warmth | .8198365 .0911059 9.00 0.000 .6412723 .9984008
physical_punishment | -1.000348 .1198049 -8.35 0.000 -1.235162 -.7655353
2.identity | -.2340191 .3271243 -0.72 0.474 -.875171 .4071328
2.intervention | .6597456 .3275877 2.01 0.044 .0176856 1.301806
HDI | -.0005531 .0210866 -0.03 0.979 -.041882 .0407757
|
c.t#c.warmth | .0463746 .0402459 1.15 0.249 -.0325059 .1252551
|
c.t#|
c.physical_punishment | .0061255 .0551491 0.11 0.912 -.1019647 .1142157

```


identity#c.t							
2		.0548965	.1513015	0.36	0.717	-.241649	.3514421
intervention#c.t							
2		.0990704	.151503	0.65	0.513	-.19787	.3960108
c.t#c.HDI		-.0009791	.0043888	-0.22	0.823	-.0095811	.0076229
_cons		51.92503	1.494157	34.75	0.000	48.99654	54.85352

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
<hr/>					
country: Independent					
var(warmth)		.0228292	.0135078	.0071588	.0728013
var(_cons)		3.001849	.8552796	1.71738	5.247001
<hr/>					
var(Residual)		34.31227	.5129896	33.32141	35.33258

LR test vs. linear model: $\chi^2(2) = 767.35$ Prob > $\chi^2 = 0.0000$

Note: LR test is conservative and provided only for reference.

5.3.2 R

5.3.2.1 Get The Data

```
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

5.3.2.2 Run The Model

5.3.2.2.1 Main Effects Only

```
fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
              identity + intervention + HDI +
              (1 | country/id),
```

```
data = dfL)

summary(fit2A)
```

Linear mixed model fit by REML ['lmerMod']

Formula:

outcome ~ t + warmth + physical_punishment + identity + intervention +
HDI + (1 | country/id)

Data: dfL

REML criterion at convergence: 57022.7

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.6850	-0.6094	-0.0035	0.6133	3.6792

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.438	2.905
country	(Intercept)	3.675	1.917
Residual		26.036	5.103

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	50.6570397	1.4460656	35.031
t	0.9433806	0.0658755	14.321
warmth	0.9140307	0.0379336	24.096
physical_punishment	-1.0087537	0.0497972	-20.257
identity	-0.1319548	0.1517350	-0.870
intervention	0.8591495	0.1520510	5.650
HDI	0.0007909	0.0207656	0.038

Correlation of Fixed Effects:

	(Intr) t	warmth	physc_	idntty	intrvn	
t	-0.090					
warmth	-0.091	-0.002				
physcl_pnsh	-0.091	-0.007	-0.012			
identity	-0.152	0.000	-0.013	-0.003		
interventin	-0.160	0.000	0.039	0.019	-0.018	
HDI	-0.930	0.000	-0.004	0.005	0.000	0.002

5.3.2.2.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +  
                identity + intervention + HDI) +  
                (1 | country/id),  
                data = dfL)  
  
summary(fit2B)
```

Linear mixed model fit by REML ['lmerMod']

Formula:

outcome ~ t * (warmth + physical_punishment + identity + intervention +
 HDI) + (1 | country/id)

Data: dfL

REML criterion at convergence: 57042.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.7118	-0.6092	-0.0024	0.6150	3.6779

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.436	2.905
country	(Intercept)	3.675	1.917
Residual		26.046	5.104

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	51.3432052	1.6670196	30.799
t	0.5994732	0.4199189	1.428
warmth	0.8170912	0.0805355	10.146
physical_punishment	-1.0097729	0.1113557	-9.068
identity	-0.2446453	0.3041604	-0.804
intervention	0.6604672	0.3046286	2.168
HDI	0.0026692	0.0221295	0.121
t:warmth	0.0486211	0.0356217	1.365
t:physical_punishment	0.0004964	0.0494590	0.010
t:identity	0.0563140	0.1318043	0.427
t:intervention	0.0995037	0.1319917	0.754
t:HDI	-0.0009379	0.0038233	-0.245

Correlation of Fixed Effects:

	(Intr)	t	warmth	physcl_	idntty	intrvn	HDI	t:wrmt	t:phy_
t	-0.504								
warmth	-0.170	0.265							
physcl_pnsh	-0.180	0.285	-0.005						
identity	-0.266	0.397	-0.013	-0.002					
intervntn	-0.279	0.417	0.039	0.019	-0.017				
HDI	-0.861	0.206	-0.007	0.012	-0.001	0.003			
t:warmth	0.151	-0.302	-0.882	0.001	0.011	-0.035	0.006		
t:physcl_pn	0.161	-0.319	0.004	-0.894	-0.001	-0.017	-0.010	-0.003	
t:identity	0.230	-0.458	0.011	0.000	-0.867	0.014	0.001	-0.013	0.002
t:intervntn	0.242	-0.481	-0.035	-0.017	0.014	-0.867	-0.003	0.041	0.019
t:HDI	0.301	-0.596	0.015	-0.027	0.002	-0.007	-0.346	-0.016	0.029
	t:dntt	t:ntrv							
t									
warmth									
physcl_pnsh									
identity									
intervntn									
HDI									
t:warmth									
t:physcl_pn									
t:identity									
t:intervntn		-0.016							
t:HDI		-0.002	0.008						

5.3.3 Julia

5.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.3.3.2 Run The Model

5.3.3.2.1 Change Country To Categorical

```
@transform!(dfL, :country = categorical(:country))
```

5.3.3.2.2 Main Effects Only

```
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                physical_punishment +
                                identity + intervention +
                                HDI +
                                (1 | country) +
                                (0 + warmth | country) +
                                (1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-28499.6031	56999.2062	57021.2062	57021.2356	57099.3610

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.387258	2.896076	
country	(Intercept)	3.166920	1.779584	
	warmth	0.010761	0.103736	.
Residual		26.027344	5.101700	

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	50.7359	1.37201	36.98	<1e-99
t	0.943864	0.0658716	14.33	<1e-45
warmth	0.913496	0.0423741	21.56	<1e-99
physical_punishment	-1.0079	0.0497622	-20.25	<1e-90
identity	-0.127692	0.151583	-0.84	0.3996
intervention	0.858997	0.151909	5.65	<1e-07
HDI	-0.000565959	0.0196433	-0.03	0.9770

5.3.3.2.3 Interactions With Time

```
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
                                physical_punishment +
                                identity + intervention +
                                HDI) +
                                (1 | country) +
                                (0 + warmth | country) +
                                (1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + t & warmth

	logLik	-2 logLik	AIC	AICc	BIC
	-28498.3091	56996.6182	57028.6182	57028.6788	57142.2979

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.391748	2.896851	
country	(Intercept)	3.170040	1.780461	
	warmth	0.010609	0.102999	.

Residual 26.015905 5.100579

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	51.4143	1.60324	32.07	<1e-99
t	0.60349	0.419741	1.44	0.1505
warmth	0.817076	0.0826636	9.88	<1e-22
physical_punishment	-1.00903	0.111293	-9.07	<1e-18
identity	-0.238714	0.303996	-0.79	0.4323
intervention	0.660761	0.30445	2.17	0.0300
HDI	0.00136065	0.0210842	0.06	0.9485
t & warmth	0.0483635	0.0356074	1.36	0.1744
t & physical_punishment	0.0005422	0.0494355	0.01	0.9912
t & identity	0.0554384	0.131745	0.42	0.6739
t & intervention	0.0992809	0.131925	0.75	0.4517
t & HDI	-0.000955067	0.00382162	-0.25	0.8027

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