Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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2024-04-17

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1 Multilevel Multilingual

> "This curious world which we inhabit is more wonderful than it is convenient…" (Thoreau, 1975)

> "Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled." (Schanen, 2021)

1.1 Introduction

Below, I describe the use of Stata (StataCorp, 2021), R (Bates et al., 2015; R Core Team, 2023), and Julia (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

All of these software packages can estimate multilevel models. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. R is free open source software which is less intuitive, but there are many excellent resources for learning R. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but intuitive for both multilevel modeling and graphing.
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (`ggplot`).
Julia	free	steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (`()`) or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire Julia command in an outer set of parentheses (`()`).

Running Statistical Packages in Quarto

I used Quarto (https://quarto.org/) to create this Appendix. Quarto is a programming and publishing environment that can run multiple programming languages, including Stata, R and Julia, and that can write to multiple output formats including HTML, PDF, and MS Word. To run Stata, I used the `Statamarkdown` library in R to connect Stata to Quarto. Quarto has a built in connection to R, and runs R without issue. To run Julia, I used the `JuliaCall` library in R to connect Quarto to Julia.
Of course, each of these programs can be run by itself, if you have them installed on your computer.

1.2 The Data

The examples use the `simulated_multilevel_data.dta` file from *Multilevel Thinking*. Here is a direct link to download the data.

Table 1.2: Sample of Simulated Multilevel Data

Table 1.2: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	2	1	3
1	69	2	1.2	2	2	2
1	69	3	1.3	1	2	3
1	69	4	1.4	2	1	0
1	69	5	1.5	2	1	4
1	69	6	1.6	1	2	5

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
3	58.47
1	51.1
2	53.92
5	61.17
4	56.05
3	50.81

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable `y`, independent variables `x` and `z`, clustering variable `group`, and a random slope for `x`. i is the index for the person, while j is the index for the `group`.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \tag{1.1}$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x || group: x
```

1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```r
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```julia
using MixedModels

fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI

tabulate identity

tabulate intervention
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     outcome |      3,000    53.43327    6.530996   30.60798   75.83553
      warmth |      3,000    3.521667    1.888399          0          7
physical_p~t |      3,000    2.478667    1.360942          0          5
         HDI |      3,000    64.76667    17.24562         33         87


hypothetica |
 l identity |
      group |
   variable |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      1,507       50.23       50.23
          2 |      1,493       49.77      100.00
------------+-----------------------------------
      Total |      3,000      100.00
   recieved |
interventio |
          n |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      1,547       51.57       51.57
          2 |      1,453       48.43      100.00
------------+-----------------------------------
      Total |      3,000      100.00
```

2.1.2 R

```r
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`[1] to generate descriptive statistics.

```r
df$country <- factor(df$country)

df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)

summary(df)
```

```
    country          HDI            family               id               identity
 1      : 100   Min.   :33.00   Min.   :  1.00   Length:3000        1:1507
 2      : 100   1st Qu.:53.00   1st Qu.: 25.75   Class :character   2:1493
 3      : 100   Median :70.00   Median : 50.50   Mode  :character
 4      : 100   Mean   :64.77   Mean   : 50.50
 5      : 100   3rd Qu.:81.00   3rd Qu.: 75.25
 6      : 100   Max.   :87.00   Max.   :100.00
 (Other):2400
 intervention physical_punishment     warmth          outcome
 1:1547       Min.   :0.000       Min.   :0.000   Min.   :30.61
 2:1453       1st Qu.:2.000       1st Qu.:2.000   1st Qu.:49.02
```

[1] `skimr` is an excellent new alternative library for generating descriptive statistics in R.

```
 Median :2.000       Median :4.000   Median :53.45
 Mean   :2.479       Mean   :3.522   Mean   :53.43
 3rd Qu.:3.000       3rd Qu.:5.000   3rd Qu.:57.86
 Max.   :5.000       Max.   :7.000   Max.   :75.84
```

2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, Data

df = DataFrame(load("simulated_multilevel_data.dta"))
```

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))

@transform!(df, :identity = categorical(:identity))

@transform!(df, :intervention = categorical(:intervention))
```

```
describe(df) # descriptive statistics
```

```
9×7 DataFrame
 Row  variable             mean     min     median  max      nmissing  eltype
      Symbol               Union…   Any     Union…  Any      Int64     Union

   1  country                       1.0             30.0            0  Union{
   2  HDI                  64.7667  33.0    70.0    87.0            0  Union{
   3  family               50.5     1.0     50.5    100.0           0  Union{
   4  id                            1.1             9.99            0  Union{
   5  identity                      1.0             2.0             0  Union{
   6  intervention                  1.0             2.0             0  Union{
   7  physical_punishment  2.47867  0.0     2.0     5.0             0  Union{
   8  warmth               3.52167  0.0     4.0     7.0             0  Union{
   9  outcome              53.4333  30.608  53.449  75.8355         0  Union{
                                                                    1 column omitted
```

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \tag{3.1}$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{var(u_{0j})}{var(u_{0j}) + var(e_{ij})} \tag{3.2}$$

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9856.1548
Iteration 1:  Log likelihood = -9856.1548
```

```
Computing standard errors ...

Mixed-effects ML regression                           Number of obs    = 3,000
Group variable: country                               Number of groups =    30
                                                      Obs per group:
                                                                   min =   100
                                                                   avg = 100.0
                                                                   max =   100
                                                      Wald chi2(0)     =     .
Log likelihood = -9856.1548                           Prob > chi2      =     .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   53.46757   .3539097   151.08   0.000     52.77392    54.16122
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity            |
                  var(_cons) |   3.348734   .9702594      1.897816    5.908906
-----------------------------+------------------------------------------------
               var(Residual) |   40.88284   1.060908       38.8555    43.01597
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 169.64        Prob >= chibar2 = 0.0000
```

```
estat icc // ICC
```

```
Intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
                     country |   .0757091   .0203761      .0442419    .1265931
------------------------------------------------------------------------------
```

3.2.2 R

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
   Data: df

REML criterion at convergence: 19712.5

Scaled residuals:
     Min       1Q   Median       3Q      Max
-2.97650 -0.68006  0.00936  0.67580  3.03510

Random effects:
 Groups   Name        Variance Std.Dev.
 country  (Intercept)  3.478   1.865
 Residual             40.883   6.394
Number of obs: 3000, groups:  country, 30

Fixed effects:
            Estimate Std. Error t value
(Intercept)    53.47       0.36   148.5
```

```
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient

    Adjusted ICC: 0.078
  Unadjusted ICC: 0.078
```

3.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

```
@transform!(df, :country = categorical(:country))
```

```
m0 = fit(MixedModel,
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + (1 | country)
   logLik   -2 logLik     AIC       AICc        BIC
 -9856.1548 19712.3097 19718.3097 19718.3177 19736.3288

Variance components:
            Column   Variance Std.Dev.
country  (Intercept)   3.34871 1.82995
Residual              40.88285 6.39397
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

               Coef.  Std. Error       z  Pr(>|z|)

(Intercept)  53.4676    0.353908  151.08    <1e-99
```

```
icc(m0) # ICC
```

```
0.07570852291396266
```

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \tag{4.1}$$

$$\beta_2 \text{physical punishment}_{ij} +$$

$$\beta_3 \text{identity}_{ij} + \beta_4 \text{intervention}_{ij} + \beta_5 \text{HDI}_{ij} +$$

$$u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij}$$

4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 4.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} var(u_{0j}) & 0 \\ 0 & var(u_{1j}) \end{bmatrix} \tag{4.2}$$

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 4.3.

$$\begin{bmatrix} var(u_{0j}) & cov(u_{0j}, u_{1j}) \\ cov(u_{0j}, u_{1j}) & var(u_{1j}) \end{bmatrix} \tag{4.3}$$

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2021).

Table 4.1: Correlated and Uncorrelated Random Effects

<table>
<tr><th>Software</th><th>Uncorrelated Random Effects</th><th>Correlated Random Effects</th></tr>
<tr><td>Stata</td><td>default</td><td>add option: <code>, cov(uns)</code></td></tr>
<tr><td>R</td><td>separate random effects from grouping variable with <code>||</code></td><td>separate random effects from grouping variable with <code>|</code></td></tr>
<tr><td>Julia</td><td>separate terms for each random effect e.g. <code>(1 | group) + (0 + x | group)</code></td><td>separate random effects from grouping variable with <code>|</code>.</td></tr>
</table>

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

4.3 Run Models

4.3.1 Stata

4.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9626.6279
Iteration 1:  Log likelihood =  -9626.607
Iteration 2:  Log likelihood =  -9626.607

Computing standard errors ...

Mixed-effects ML regression                          Number of obs    =  3,000
Group variable: country                              Number of groups =     30
                                                     Obs per group:
                                                                  min =    100
                                                                  avg =  100.0
                                                                  max =    100
                                                     Wald chi2(5)     = 334.14
Log likelihood =  -9626.607                          Prob > chi2      = 0.0000

-------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+----------------------------------------------------------------
             warmth |   .8345368   .0637213    13.10   0.000     .7096453    .9594282
physical_punishment |  -.9916657   .0797906   -12.43   0.000    -1.148052   -.8352791
         2.identity |  -.3004767   .2170295    -1.38   0.166    -.7258466    .1248933
     2.intervention |   .6396427   .2174519     2.94   0.003     .2134448    1.065841
                HDI |   -.003228   .0199257    -0.16   0.871    -.0422817    .0358256
              _cons |   52.99991   1.371257    38.65   0.000      50.3123    55.68753
-------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0227504   .0257784      .0024689    .2096436
                  var(_cons) |   2.963975   .9737647      1.556777    5.643163
-----------------------------+------------------------------------------------
               var(Residual) |   34.97499   .9097109      33.23668    36.80422
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 205.74                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

4.3.2 R

4.3.2.1 Get The Data

```r
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

4.3.2.2 Run The Model

```r
fit1 <- lmer(outcome ~ warmth + physical_punishment +
               identity + intervention + HDI +
               (1 + warmth || country),
             data = df)

summary(fit1)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ warmth + physical_punishment + identity + intervention +
    HDI + ((1 | country) + (0 + warmth | country))
   Data: df

REML criterion at convergence: 19268.8

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9774 -0.6563  0.0187  0.6645  3.6730

Random effects:
 Groups    Name        Variance Std.Dev.
 country   (Intercept)  3.19056 1.786
 country.1 warmth       0.02465 0.157
 Residual              35.01782 5.918
Number of obs: 3000, groups:  country, 30

Fixed effects:
                     Estimate Std. Error t value
(Intercept)         52.672655   1.479571  35.600
warmth               0.834562   0.064252  12.989
```

```
physical_punishment -0.991892   0.079845 -12.423
identity            -0.300350   0.217179  -1.383
intervention         0.639059   0.217603   2.937
HDI                 -0.003395   0.020596  -0.165

Correlation of Fixed Effects:
            (Intr) warmth physc_ idntty intrvn
warmth      -0.121
physcl_pnsh -0.145 -0.003
identity    -0.213 -0.012 -0.003
interventin -0.223  0.034  0.022 -0.018
HDI         -0.902 -0.006  0.009 -0.001  0.000
```

4.3.3 Julia

4.3.3.1 Get The Data

```julia
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

4.3.3.2 Change Country To Categorical

```julia
@transform!(df, :country = categorical(:country))
```

4.3.3.3 Run The Model

```julia
m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
               identity + intervention + HDI +
               (1 | country) +
               (0 + warmth | country)), df)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country) +
   logLik   -2 logLik     AIC       AICc        BIC
 -9626.6070 19253.2140 19271.2140 19271.2742 19325.2713
```

```
Variance components:
            Column    Variance Std.Dev.   Corr.
country  (Intercept)   2.963849 1.721583
         warmth        0.022756 0.150852   .
Residual              34.974984 5.913965
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

                          Coef.  Std. Error       z  Pr(>|z|)

(Intercept)          52.6608      1.43785     36.62    <1e-99
warmth                0.834537    0.0637228   13.10    <1e-38
physical_punishment  -0.991665    0.0797906  -12.43    <1e-34
identity             -0.300475    0.217029    -1.38    0.1662
intervention          0.639641    0.217452     2.94    0.0033
HDI                  -0.0032286   0.0199255   -0.16    0.8713
```

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

5.2 The Equation

$$\begin{gathered}
\text{outcome}_{itj} = \beta_0 + \beta_1\text{parental warmth}_{itj} + \beta_2\text{physical punishment}_{itj} + \beta_3\text{time}_{itj} + \qquad (5.1)\\
\beta_4\text{identity}_{itj} + \beta_5\text{intervention}_{itj} + \beta_6\text{HDI}_{itj} + \\
u_{0j} + u_{1j} \times \text{parental warmth}_{itj} + \\
v_{0i} + v_{1i} \times \text{time}_{itj} + e_{itj}
\end{gathered}$$

5.3 Run Models

5.3.1 Stata

5.3.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.3.1.2 Run The Model

5.3.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28739.506
Iteration 1:  Log likelihood = -28739.506

Computing standard errors ...

Mixed-effects ML regression                         Number of obs    =   9,000
Group variable: country                             Number of groups =      30
                                                    Obs per group:
                                                                 min =     300
                                                                 avg =   300.0
                                                                 max =     300
                                                    Wald chi2(6)     = 1119.81
Log likelihood = -28739.506                         Prob > chi2      =  0.0000

-------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+----------------------------------------------------------------
                  t |   .9443446   .0756408    12.48   0.000     .7960914    1.092598
             warmth |   .9123903   .0430042    21.22   0.000     .8281035     .996677
physical_punishment |  -.9881587   .0451732   -21.87   0.000    -1.076696   -.8996209
         2.identity |  -.1241465   .1242225    -1.00   0.318     -.367618    .1193251
     2.intervention |   .8575839   .1245179     6.89   0.000     .6135332    1.101635
                HDI |  -.0025173   .0191696    -0.13   0.896    -.0400891    .0350544
              _cons |   51.54528   1.304146    39.52   0.000      48.9892    54.10136
-------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0229349   .0135353      .0072136    .0729194
                  var(_cons) |     3.0009   .8550708      1.716768    5.245553
-----------------------------+------------------------------------------------
```

```
               var(Residual) |   34.31935   .5130963      33.3283    35.33988
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 767.22            Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

5.3.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.identity i.intervention c.HDI) || count
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28738.554
Iteration 1:  Log likelihood = -28738.554

Computing standard errors ...

Mixed-effects ML regression                          Number of obs    =   9,000
Group variable: country                              Number of groups =      30
                                                     Obs per group:
                                                                  min =     300
                                                                  avg =   300.0
                                                                  max =     300
                                                     Wald chi2(11)    = 1122.75
Log likelihood = -28738.554                          Prob > chi2      =  0.0000

----------------------------------------------------------------------------------------
               outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------------+----------------------------------------------------------------
                     t |   .7537359   .3719996     2.03   0.043     .0246302    1.482842
                warmth |   .8198365   .0911059     9.00   0.000     .6412723    .9984008
   physical_punishment |  -1.000348   .1198049    -8.35   0.000    -1.235162   -.7655353
            2.identity |  -.2340191   .3271243    -0.72   0.474     -.875171    .4071328
        2.intervention |   .6597456   .3275877     2.01   0.044     .0176856    1.301806
                   HDI |  -.0005531   .0210866    -0.03   0.979     -.041882    .0407757
                       |
          c.t#c.warmth |   .0463746   .0402459     1.15   0.249    -.0325059    .1252551
                       |
                   c.t#|
 c.physical_punishment |   .0061255   .0551491     0.11   0.912    -.1019647    .1142157
```

```
                        |
         identity#c.t |
                    2  |   .0548965   .1513015     0.36   0.717     -.241649    .3514421
                       |
     intervention#c.t |
                    2  |   .0990704    .151503     0.65   0.513      -.19787    .3960108
                       |
              c.t#c.HDI |  -.0009791   .0043888    -0.22   0.823    -.0095811    .0076229
                       |
                  _cons |   51.92503   1.494157    34.75   0.000     48.99654    54.85352
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0228292   .0135078      .0071588    .0728013
                  var(_cons) |   3.001849   .8552796       1.71738    5.247001
-----------------------------+------------------------------------------------
               var(Residual) |   34.31227   .5129896      33.32141    35.33258
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 767.35                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

5.3.2 R

5.3.2.1 Get The Data

```r
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

5.3.2.2 Run The Model

5.3.2.2.1 Main Effects Only

```r
fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
               identity + intervention + HDI +
               (1 | country/id),
              data = dfL)

summary(fit2A)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: 
outcome ~ t + warmth + physical_punishment + identity + intervention +  
    HDI + (1 | country/id)
   Data: dfL

REML criterion at convergence: 57022.7

Scaled residuals: 
    Min      1Q  Median      3Q     Max 
-3.6850 -0.6094 -0.0035  0.6133  3.6792 

Random effects:
 Groups     Name        Variance Std.Dev.
 id:country (Intercept)  8.438   2.905   
 country    (Intercept)  3.675   1.917   
 Residual               26.036   5.103   
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                      Estimate Std. Error t value
(Intercept)         50.6570397  1.4460656  35.031
t                    0.9433806  0.0658755  14.321
warmth               0.9140307  0.0379336  24.096
physical_punishment -1.0087537  0.0497972 -20.257
identity            -0.1319548  0.1517350  -0.870
intervention         0.8591495  0.1520510   5.650
HDI                  0.0007909  0.0207656   0.038

Correlation of Fixed Effects:
            (Intr) t      warmth physc_ idntty intrvn
t           -0.090                                   
warmth      -0.091 -0.002                            
physcl_pnsh -0.091 -0.007 -0.012                     
identity    -0.152  0.000 -0.013 -0.003              
interventin -0.160  0.000  0.039  0.019 -0.018       
HDI         -0.930  0.000 -0.004  0.005  0.000  0.002
```

5.3.2.2.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
               identity + intervention + HDI) +
               (1 | country/id),
             data = dfL)

summary(fit2B)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula:
outcome ~ t * (warmth + physical_punishment + identity + intervention +
    HDI) + (1 | country/id)
   Data: dfL

REML criterion at convergence: 57042.8

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.7118 -0.6092 -0.0024  0.6150  3.6779

Random effects:
 Groups     Name        Variance Std.Dev.
 id:country (Intercept)  8.436   2.905
 country    (Intercept)  3.675   1.917
 Residual               26.046   5.104
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                        Estimate Std. Error t value
(Intercept)           51.3432052  1.6670196  30.799
t                      0.5994732  0.4199189   1.428
warmth                 0.8170912  0.0805355  10.146
physical_punishment   -1.0097729  0.1113557  -9.068
identity              -0.2446453  0.3041604  -0.804
intervention           0.6604672  0.3046286   2.168
HDI                    0.0026692  0.0221295   0.121
t:warmth               0.0486211  0.0356217   1.365
t:physical_punishment  0.0004964  0.0494590   0.010
t:identity             0.0563140  0.1318043   0.427
t:intervention         0.0995037  0.1319917   0.754
t:HDI                 -0.0009379  0.0038233  -0.245
```

```
Correlation of Fixed Effects:
            (Intr) t      warmth physc_ idntty intrvn HDI    t:wrmt t:phy_
t           -0.504
warmth      -0.170  0.265
physcl_pnsh -0.180  0.285 -0.005
identity    -0.266  0.397 -0.013 -0.002
interventin -0.279  0.417  0.039  0.019 -0.017
HDI         -0.861  0.206 -0.007  0.012 -0.001  0.003
t:warmth     0.151 -0.302 -0.882  0.001  0.011 -0.035  0.006
t:physcl_pn  0.161 -0.319  0.004 -0.894 -0.001 -0.017 -0.010 -0.003
t:identity   0.230 -0.458  0.011  0.000 -0.867  0.014  0.001 -0.013  0.002
t:intervntn  0.242 -0.481 -0.035 -0.017  0.014 -0.867 -0.003  0.041  0.019
t:HDI        0.301 -0.596  0.015 -0.027  0.002 -0.007 -0.346 -0.016  0.029
            t:dntt t:ntrv
t
warmth
physcl_pnsh
identity
interventin
HDI
t:warmth
t:physcl_pn
t:identity
t:intervntn -0.016
t:HDI       -0.002  0.008
```

5.3.3 Julia

5.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.3.3.2 Run The Model

5.3.3.2.1 Change Country To Categorical

```
@transform!(dfL, :country = categorical(:country))
```

5.3.3.2.2 Main Effects Only

```
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                 physical_punishment +
                                 identity + intervention +
                                 HDI +
                                 (1 | country) +
                                 (0 + warmth | country) +
                                 (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | count
    logLik   -2 logLik      AIC         AICc        BIC
 -28499.6031  56999.2062  57021.2062  57021.2356  57099.3610

Variance components:
            Column    Variance Std.Dev.   Corr.
id       (Intercept)   8.387258 2.896076
country  (Intercept)   3.166920 1.779584
         warmth        0.010761 0.103736   .
Residual              26.027344 5.101700
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                          Coef.  Std. Error       z  Pr(>|z|)

(Intercept)          50.7359        1.37201     36.98    <1e-99
t                     0.943864      0.0658716   14.33    <1e-45
warmth                0.913496      0.0423741   21.56    <1e-99
physical_punishment  -1.0079        0.0497622  -20.25    <1e-90
identity             -0.127692      0.151583    -0.84    0.3996
intervention          0.858997      0.151909     5.65    <1e-07
HDI                  -0.000565959   0.0196433   -0.03    0.9770
```

5.3.3.2.3 Interactions With Time

```
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
                                 physical_punishment +
                                 identity + intervention +
                                   HDI) +
                                 (1 | country) +
                                 (0 + warmth | country) +
                                 (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + t & warmth
    logLik   -2 logLik      AIC         AICc        BIC
 -28498.3091  56996.6182  57028.6182  57028.6788  57142.2979

Variance components:
            Column    Variance Std.Dev.   Corr.
id       (Intercept)   8.391748 2.896851
country  (Intercept)   3.170040 1.780461
         warmth        0.010609 0.102999   .
Residual              26.015905 5.100579
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                                Coef.  Std. Error      z  Pr(>|z|)

(Intercept)              51.4143       1.60324     32.07    <1e-99
t                         0.60349      0.419741     1.44    0.1505
warmth                    0.817076     0.0826636    9.88    <1e-22
physical_punishment      -1.00903      0.111293    -9.07    <1e-18
identity                 -0.238714     0.303996    -0.79    0.4323
intervention              0.660761     0.30445      2.17    0.0300
HDI                       0.00136065   0.0210842    0.06    0.9485
t & warmth                0.0483635    0.0356074    1.36    0.1744
t & physical_punishment   0.0005422    0.0494355    0.01    0.9912
t & identity              0.0554384    0.131745     0.42    0.6739
t & intervention          0.0992809    0.131925     0.75    0.4517
t & HDI                  -0.000955067  0.00382162  -0.25    0.8027
```

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