

Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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1 Multilevel Multilingual

“This curious world which we inhabit is more wonderful than it is convenient...”
(Thoreau, 1975)

“Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled.” (Schanen, 2021)

1.1 Introduction

Below, I describe the use of [Stata](#) (StataCorp, 2023), [R](#) (Bates et al., 2015; R Core Team, 2023), and [Julia](#) (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models and to visualize data.

All of these software packages can estimate multilevel models and can visualize relationships in the data. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. While it costs money to purchase Stata, the price is often very reasonable for academic and educational use. R is free open source software which is less intuitive, but there are many excellent resources for learning R. There is often a cost associated with purchasing books and other materials for learning R, which sometimes feels like it offsets the fact that R is free. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

While any of these software packages can be used for learning and estimating multilevel models, I will offer my own opinion—based upon 15 years of teaching multilevel models at the doctoral level—that Stata offers the quickest pathway for learning the basic and advanced uses of multilevel models. I also believe the intuitive nature of Stata syntax contributes to accurate and replicable work in this area.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but very intuitive for both multilevel modeling and graphing.
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (<code>ggplot</code>).
Julia	free	steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

💡 Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

💡 Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as *R* commands are usually clearly encased in parentheses `()` or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a multi-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire *Julia* command in an outer set of parentheses `()`.

💡 Running Statistical Packages in Quarto

I used Quarto (<https://quarto.org/>) to create this Appendix. Quarto is a programming and publishing environment that can run multiple programming languages, including Stata, R and Julia, and that can write to multiple output formats including HTML, PDF, and MS Word. To run Stata, I used the `Statamarkdown` library in R to connect Stata to Quarto. Quarto has a built in connection to R, and runs R without issue. To run Julia, I used the `JuliaCall` library in R to connect Quarto to Julia. Of course, each of these programs can be run by itself, if you have them installed on your computer.

1.2 The Data

i Datasets

The examples use the `simulated_multilevel_data.dta` and `simulated_multilevel_longitudinal_data.dta` files. Here is a [direct link](#) to download the cross-sectional data. Here is a [direct link](#) to download the longitudinal data.

Table 1.2: Sample of Simulated Multilevel Data

Table 1.2: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	1	0	3
1	69	2	1.2	1	1	2
1	69	3	1.3	0	1	3
1	69	4	1.4	1	0	0
1	69	5	1.5	1	0	4
1	69	6	1.6	0	1	5

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
3	57.47
1	50.1
2	52.92
5	60.17

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
4	55.05
3	49.81

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable y , independent variables x and z , clustering variable `group`, and a random slope for x . i is the index for the person, while j is the index for the `group`.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \quad (1.1)$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x z || group: x
```

1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)
lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels
fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```


2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI
```

```
tabulate identity
```

```
tabulate intervention
```

Variable	Obs	Mean	Std. dev.	Min	Max
-----+-----					
outcome	3,000	52.43327	6.530996	29.60798	74.83553
warmth	3,000	3.521667	1.888399	0	7
physical_p~t	3,000	2.478667	1.360942	0	5
HDI	3,000	64.76667	17.24562	33	87

hypothetica			
l identity			
group			
variable	Freq.	Percent	Cum.
-----+-----			
0	1,507	50.23	50.23
1	1,493	49.77	100.00
-----+-----			
Total	3,000	100.00	

recieved interventio	n	Freq.	Percent	Cum.
0	1,547	51.57	51.57	
1	1,453	48.43	100.00	
Total	3,000	100.00		

2.1.2 R

```
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`¹ to generate descriptive statistics.

```
df$country <- factor(df$country)

df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)

summary(df)
```

```

country      HDI      family      id      identity
1      : 100  Min.   :33.00  Min.   : 1.00  Length:3000  0:1507
2      : 100  1st Qu.:53.00  1st Qu.: 25.75  Class :character 1:1493
3      : 100  Median :70.00  Median : 50.50  Mode  :character
4      : 100  Mean   :64.77  Mean   : 50.50
5      : 100  3rd Qu.:81.00  3rd Qu.: 75.25
6      : 100  Max.   :87.00  Max.   :100.00
(Other):2400
intervention physical_punishment  warmth      outcome
0:1547      Min.   :0.000      Min.   :0.000  Min.   :29.61
1:1453      1st Qu.:2.000      1st Qu.:2.000  1st Qu.:48.02
```

¹`skimr` is an excellent new alternative library for generating descriptive statistics in R.

Median	:2.000	Median	:4.000	Median	:52.45
Mean	:2.479	Mean	:3.522	Mean	:52.43
3rd Qu.	:3.000	3rd Qu.	:5.000	3rd Qu.	:56.86
Max.	:5.000	Max.	:7.000	Max.	:74.84

2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))
@transform!(df, :identity = categorical(:identity))
@transform!(df, :intervention = categorical(:intervention))
```

```
describe(df) # descriptive statistics
```

```
9×7 DataFrame
 Row  variable          mean  min  median  max  nmissing  eltype
      Symbol          Union... Any  Union... Any  Int64      Union
  1  country              1.0      30.0
  2  HDI                64.7667 33.0  70.0    87.0
  3  family              50.5     1.0  50.5   100.0
  4  id                   1.1     9.99
  5  identity             0.0     1.0
  6  intervention         0.0     1.0
  7  physical_punishment  2.47867 0.0   2.0    5.0
  8  warmth               3.52167 0.0   4.0    7.0
  9  outcome              52.4333 29.608 52.449 74.8355
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                0  Union{
                                1 column omitted
```

2.2 Interpretation

Examining descriptive statistics is an important first step in any analysis. It is important to examine your descriptive statistics first, before skipping ahead to more sophisticated analyses, such as multilevel models.

In examining the descriptive statistics for this data, we get a sense of the data.

- `outcome` has a mean of approximately 52 and ranges from approximately 30 to 75.
- `warmth` and `physical punishment` are both variables that represent the number of times that parents use each of these forms of discipline in a week. The average of the former is about 3.5, while the average of the latter is about 2.5.
- `HDI`, the Human Development Index has an average of about 65, and a wide range.
- `identity` is a categorical variable for a hypothetical identity group, and has values of 0 and 1.
- `intervention` is also a categorical variable, and has values of 0 and 1.

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \quad (3.1)$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{\text{var}(u_{0j})}{\text{var}(u_{0j}) + \text{var}(e_{ij})} \quad (3.2)$$

In a two level multilevel model, the ICC provides a measure of the proportion of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9802.8371

Iteration 1: Log likelihood = -9802.8371

Computing standard errors ...

Mixed-effects ML regression
Group variable: country

Number of obs = 3,000
Number of groups = 30
Obs per group:
 min = 100
 avg = 100.0
 max = 100
Wald chi2(0) = .
Prob > chi2 = .

Log likelihood = -9802.8371

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
_cons	52.43327	.3451217	151.93	0.000	51.75685	53.1097

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Identity				
var(_cons)	3.178658	.9226737	1.799552	5.614658
var(Residual)	39.46106	1.024013	37.50421	41.52

LR test vs. linear model: chibar2(01) = 166.31 Prob >= chibar2 = 0.0000

```
estat icc // ICC
```

Intraclass correlation

Level	ICC	Std. err.	[95% conf. interval]	
country	.0745469	.0201254	.0434963	.1248696

3.2.2 R

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
Data: df
```

```
REML criterion at convergence: 19605.9
```

```
Scaled residuals:
```

Min	1Q	Median	3Q	Max
-3.3844	-0.6655	-0.0086	0.6725	3.6626

```
Random effects:
```

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.302	1.817
	Residual	39.461	6.282

Number of obs: 3000, groups: country, 30

```
Fixed effects:
```

	Estimate	Std. Error	t value
(Intercept)	52.433	0.351	149.4

```
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient
```

```
Adjusted ICC: 0.077
Unadjusted ICC: 0.077
```

3.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,  
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
```

```
df = DataFrame(load("simulated_multilevel_data.dta"))
```

```
@transform!(df, :country = categorical(:country))
```

```
m0 = fit(MixedModel,  
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9802.8371	19605.6742	19611.6742	19611.6822	19629.6933

Variance components:

	Column	Variance	Std.Dev.
country	(Intercept)	3.17863	1.78287
Residual		39.46106	6.28180

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	52.4333	0.345121	151.93	<1e-99

```
icc(m0) # ICC
```

```
0.07454637475695493
```

3.3 Interpretation

In each case, the software finds that nearly 8% of the variation in the outcome is explainable by the clustering of the observations in each country.

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \quad (4.1)$$

$$\beta_2 \text{physical punishment}_{ij} +$$

$$\beta_3 \text{identity}_{ij} + \beta_4 \text{intervention}_{ij} + \beta_5 \text{HDI}_j +$$

$$u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij}$$

4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 4.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} \text{var}(u_{0j}) & 0 \\ 0 & \text{var}(u_{1j}) \end{bmatrix} \quad (4.2)$$

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 4.3.

$$\begin{bmatrix} \text{var}(u_{0j}) & \text{cov}(u_{0j}, u_{1j}) \\ \text{cov}(u_{0j}, u_{1j}) & \text{var}(u_{1j}) \end{bmatrix} \quad (4.3)$$

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2023).

Table 4.1: Correlated and Uncorrelated Random Effects

Software	Uncorrelated Random Effects	Correlated Random Effects
Stata	default	add option: <code>, cov(uns)</code>
R	separate random effects from grouping variable with <code> </code>	separate random effects from grouping variable with <code> </code>
Julia	separate terms for each random effect e.g. <code>(1 group) + (0 + x group)</code>	separate random effects from grouping variable with <code> </code> .

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

4.3 Run Models



Warning

Statistically—as noted in the main text—it is important to be clear on whether independent variables in one’s model are continuous or categorical. *Continuous* variables can be entered straightforwardly into statistical syntax. *Categorical* variables, on the other hand usually require specific attention in Statistical software. In Stata, categorical variables are indicated in a statistical model by prefixing them with an `i..` In R, categorical variables are distinguished by making them into factors e.g. `x <- factor(x)`. In Julia, categorical variables are created by using the `@transform` syntax detailed below.

4.3.1 Stata

4.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9626.6279

Iteration 1: Log likelihood = -9626.607

Iteration 2: Log likelihood = -9626.607

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 3,000

Number of groups = 30

Obs per group:

min = 100

avg = 100.0

max = 100

Wald chi2(5) = 334.14

Prob > chi2 = 0.0000

Log likelihood = -9626.607

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
warmth	.8345368	.0637213	13.10	0.000	.7096453	.9594282
physical_punishment	-.9916657	.0797906	-12.43	0.000	-1.148052	-.8352791
1.identity	-.3004767	.2170295	-1.38	0.166	-.7258466	.1248933
1.intervention	.6396427	.2174519	2.94	0.003	.2134448	1.065841
HDI	-.003228	.0199257	-0.16	0.871	-.0422817	.0358256
_cons	51.99991	1.371257	37.92	0.000	49.3123	54.68753

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Independent				
var(warmth)	.0227504	.0257784	.0024689	.2096436
var(_cons)	2.963975	.9737647	1.556777	5.643163
var(Residual)	34.97499	.9097109	33.23668	36.80422

LR test vs. linear model: $\chi^2(2) = 205.74$

Prob > $\chi^2 = 0.0000$

Note: LR test is conservative and provided only for reference.

4.3.2 R

4.3.2.1 Get The Data

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

4.3.2.2 Change Some Variables To Categorical

```
df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)
```

4.3.2.3 Run The Model

Caution

`lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

Tip

R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

```
library(lme4)

library(lmerTest)
```

```
options(scipen = 999)

fit1 <- lmer(outcome ~ warmth + physical_punishment +
             identity + intervention + HDI +
             (1 + warmth || country),
             data = df)

summary(fit1)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: outcome ~ warmth + physical_punishment + identity + intervention +
      HDI + (1 + warmth || country)
Data: df
```

REML criterion at convergence: 19268.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.9774	-0.6563	0.0186	0.6645	3.6730

Random effects:

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.19120	1.786
country.1	warmth	0.02464	0.157
Residual		35.01779	5.918

Number of obs: 3000, groups: country, 30

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	52.011324	1.414976	30.293141	36.758
warmth	0.834562	0.064250	41.896457	12.989
physical_punishment	-0.991893	0.079845	2968.012381	-12.423
identity1	-0.300354	0.217179	2970.108153	-1.383
intervention1	0.639060	0.217603	2971.186718	2.937
HDI	-0.003394	0.020598	27.592814	-0.165

Pr(>|t|)

(Intercept)	< 0.0000000000000002	***
warmth	0.000000000000000277	***
physical_punishment	< 0.0000000000000002	***
identity1	0.16678	
intervention1	0.00334	**

```

HDI                                0.87030
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
      (Intr) warmth physc_ idntt1 intrv1
warmth      -0.124
physcl_pnsh -0.149 -0.003
identity1    -0.072 -0.012 -0.003
interventn1 -0.082  0.034  0.022 -0.018
HDI          -0.943 -0.006  0.009 -0.001  0.000

```

4.3.3 Julia

4.3.3.1 Get The Data

```

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))

```

4.3.3.2 Change Some Variables To Categorical

```

@transform!(df, :country = categorical(:country))

@transform!(df, :identity = categorical(:identity))

@transform!(df, :intervention = categorical(:intervention))

```

4.3.3.3 Run The Model

```

m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
                             identity + intervention + HDI +
                             (1 | country) +
                             (0 + warmth | country)), df)

```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9626.6070	19253.2140	19271.2140	19271.2742	19325.2713

Variance components:

	Column	Variance	Std.Dev.	Corr.
country	(Intercept)	2.963849	1.721583	
	warmth	0.022756	0.150852	.
Residual		34.974984	5.913965	

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	51.9999	1.37124	37.92	<1e-99
warmth	0.834537	0.0637228	13.10	<1e-38
physical_punishment	-0.991665	0.0797906	-12.43	<1e-34
identity: 1.0	-0.300475	0.217029	-1.38	0.1662
intervention: 1.0	0.639641	0.217452	2.94	0.0033
HDI	-0.0032286	0.0199255	-0.16	0.8713

4.4 Interpretation

Models suggest that parental warmth is associated with increases in the beneficial outcome, while physical punishment is associated with decreases in the beneficial outcome. The intervention is associated with increases in the outcome. There is insufficient evidence that either identity group or the Human Development Index are associated with the outcome.

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

5.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \quad (5.1)$$

$$\beta_4 \text{identity}_{itj} + \beta_5 \text{intervention}_{itj} + \beta_6 \text{HDI}_j +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0ij} + v_{1ij} \times \text{time}_{itj} + e_{itj}$$

5.3 Growth Trajectories

Remember, following Section 6.4, that in longitudinal multilevel models, the variable for *time* assumes an important role as we are often thinking of a *growth trajectory* over time.

As discussed in Section 6.4, think about a model where *identity* is a (1/0) variable for membership in one of two groups:

$$\text{outcome} = \beta_0 + \beta_t \text{time} + \beta_{\text{identity}} \text{identity} + \beta_{\text{interaction}} \text{identity} \times \text{time} + u_{0i} + e_{it}$$

Then, each identity group has its own intercept and time trajectory:

Table 5.1: Slope and Intercept for Each Group

Group	Intercept	Slope (Time Trajectory)
0	β_0	β_t
1	$\beta_0 + \beta_{\text{identity}}$	$\beta_t + \beta_{\text{interaction}}$

💡 Main Effects and Interactions

Thus, again following Section 6.4, in longitudinal multilevel models, *main effects* modify the *intercept* of the time trajectory, while *interactions with time*, modify the *slope* of the time trajectory. Below, we run models with *main effects only*, then models with *main effects*, and *interactions with time*.

5.4 Run Models

5.4.1 Stata

5.4.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.4.1.2 Run The Model

5.4.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

```
Iteration 0: Log likelihood = -28523.49
Iteration 1: Log likelihood = -28499.987
Iteration 2: Log likelihood = -28499.737
Iteration 3: Log likelihood = -28499.604
Iteration 4: Log likelihood = -28499.603
```

Computing standard errors ...

Mixed-effects ML regression

Number of obs = 9,000

Grouping information

Group variable		No. of groups	Observations per group		
			Minimum	Average	Maximum
country		30	300	300.0	300
family		3,000	3	3.0	3

Log likelihood = -28499.603

Wald chi2(6) = 1096.15

Prob > chi2 = 0.0000

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
t	.943864	.0658716	14.33	0.000	.814758	1.07297
warmth	.913496	.0423731	21.56	0.000	.8304462	.9965457
physical_punishment	-1.007897	.0497622	-20.25	0.000	-1.105429	-.9103647
1.identity	-.1276926	.1515835	-0.84	0.400	-.4247909	.1694056
1.intervention	.8589966	.1519095	5.65	0.000	.5612596	1.156734
HDI	-.0005657	.0196437	-0.03	0.977	-.0390666	.0379352
_cons	50.46724	1.338318	37.71	0.000	47.84418	53.09029

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Independent				
var(warmth)	.0107584	.0127845	.0010477	.1104715
var(_cons)	3.167089	.9146768	1.798157	5.578187
family: Independent				
var(t)	3.74e-09	7.36e-07	1.4e-176	9.7e+158
var(_cons)	8.387276	.472419	7.510631	9.366243
var(Residual)	26.02733	.4753702	25.11211	26.97592

LR test vs. linear model: chi2(4) = 1247.03

Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

5.4.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.identity i.intervention c.HDI) || count;
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28522.21

Iteration 1: Log likelihood = -28498.685

Iteration 2: Log likelihood = -28498.469

Iteration 3: Log likelihood = -28498.31

Iteration 4: Log likelihood = -28498.309

Computing standard errors ...

Mixed-effects ML regression

Number of obs = 9,000

Grouping information

Group variable		No. of groups	Observations per group		
			Minimum	Average	Maximum
country		30	300	300.0	300
family		3,000	3	3.0	3

Log likelihood = -28498.309

Wald chi2(11) = 1100.25

Prob > chi2 = 0.0000

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
t	.7582075	.326177	2.32	0.020	.1189122	1.397503
warmth	.8170757	.082662	9.88	0.000	.6550611	.9790903
physical_punishment	-1.009031	.1112932	-9.07	0.000	-1.227162	-.7909007
1.identity	-.2387167	.3039964	-0.79	0.432	-.8345387	.3571053
1.intervention	.6607606	.3044503	2.17	0.030	.064049	1.257472
HDI	.0013614	.0210842	0.06	0.949	-.0399628	.0426856

c.t#c.warmth		.0483637	.0356074	1.36	0.174	-.0214255	.1181529
c.t#							
c.physical_punishment		.0005421	.0494355	0.01	0.991	-.0963496	.0974338
identity#c.t							
1		.0554389	.1317444	0.42	0.674	-.2027754	.3136532
intervention#c.t							
1		.0992811	.131925	0.75	0.452	-.1592872	.3578493
c.t#c.HDI		-.0009551	.0038216	-0.25	0.803	-.0084453	.0065352
_cons		50.83632	1.483548	34.27	0.000	47.92862	53.74402

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
-----+					
country: Independent					
var(warmth)		.0106014	.0127458	.0010046	.1118779
var(_cons)		3.170089	.9153354	1.800091	5.582753
-----+					
family: Independent					
var(t)		1.36e-09	2.89e-07	1.7e-190	1.1e+172
var(_cons)		8.391888	.4724105	7.515232	9.370806
-----+					
var(Residual)		26.01583	.4751604	25.10101	26.964

LR test vs. linear model: $\chi^2(4) = 1247.84$ Prob > $\chi^2 = 0.0000$

Note: LR test is conservative and provided only for reference.

5.4.2 R

5.4.2.1 Get The Data

```
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

5.4.2.2 Change Some Variables To Categorical

```
dfL$identity <- factor(dfL$identity)

dfL$intervention <- factor(dfL$intervention)
```

5.4.2.3 Run The Model

Caution

`lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

Tip

R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

5.4.2.3.1 Main Effects Only

```
library(lme4)

library(lmerTest)

options(scipen = 999)

fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
              identity + intervention + HDI +
              (1 | country/id),
              data = dfL)

summary(fit2A)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [`lmerModLmerTest`]

Formula:

`outcome ~ t + warmth + physical_punishment + identity + intervention +`

HDI + (1 | country/id)
Data: dfL

REML criterion at convergence: 57022.7

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.6850	-0.6094	-0.0035	0.6133	3.6792

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.438	2.905
country	(Intercept)	3.675	1.917
Residual		26.036	5.103

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	50.3842343	1.4139114	29.8246912	35.635
t	0.9433806	0.0658755	5998.3764548	14.321
warmth	0.9140307	0.0379336	4745.3497493	24.096
physical_punishment	-1.0087537	0.0497972	6483.6771808	-20.257
identity1	-0.1319548	0.1517350	2968.7828107	-0.870
intervention1	0.8591494	0.1520510	2971.8111995	5.650
HDI	0.0007909	0.0207656	28.0001855	0.038

	Pr(> t)
(Intercept)	< 0.0000000000000002 ***
t	< 0.0000000000000002 ***
warmth	< 0.0000000000000002 ***
physical_punishment	< 0.0000000000000002 ***
identity1	0.385
intervention1	0.0000000175 ***
HDI	0.970

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr) t	warmth	physc_ idntt1	intrv1
t	-0.092			
warmth	-0.091	-0.002		
physcl_pnsh	-0.092	-0.007	-0.012	
identity1	-0.051	0.000	-0.013	-0.003
interventn1	-0.058	0.000	0.039	0.019

HDI	-0.951	0.000	-0.004	0.005	0.000	0.002
-----	--------	-------	--------	-------	-------	-------

5.4.2.3.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
                    identity + intervention + HDI) +
              (1 | country/id),
              data = dfL)

summary(fit2B)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]

Formula:

outcome ~ t * (warmth + physical_punishment + identity + intervention + HDI) + (1 | country/id)

Data: dfL

REML criterion at convergence: 57042.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.7118	-0.6092	-0.0024	0.6150	3.6779

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.436	2.905
country	(Intercept)	3.675	1.917
Residual		26.046	5.104

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	50.7590272	1.5518360	43.2608620	32.709
t	0.7552909	0.3263028	6176.7440549	2.315
warmth	0.8170912	0.0805355	8274.9995422	10.146
physical_punishment	-1.0097729	0.1113557	8084.6084915	-9.068
identity1	-0.2446453	0.3041604	8695.8966197	-0.804
intervention1	0.6604671	0.3046286	8697.0843430	2.168
HDI	0.0026692	0.0221295	36.1037733	0.121
t:warmth	0.0486211	0.0356217	6404.8723333	1.365

t:physical_punishment	0.0004964	0.0494590	6753.0158441	0.010
t:identity1	0.0563140	0.1318043	5993.4518022	0.427
t:intervention1	0.0995037	0.1319917	5994.1433001	0.754
t:HDI	-0.0009379	0.0038233	5993.9090880	-0.245

Pr(>|t|)

(Intercept)	<0.0000000000000002 ***
t	0.0207 *
warmth	<0.0000000000000002 ***
physical_punishment	<0.0000000000000002 ***
identity1	0.4212
intervention1	0.0302 *
HDI	0.9047
t:warmth	0.1723
t:physical_punishment	0.9920
t:identity1	0.6692
t:intervention1	0.4510
t:HDI	0.8062

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr)	t	warmth	physc_	idntt1	intrv1	HDI	t:wrmt	t:phy_
t	-0.421								
warmth	-0.178	0.331							
physcl_pnsh	-0.190	0.360	-0.005						
identity1	-0.093	0.166	-0.013	-0.002					
interventn1	-0.107	0.192	0.039	0.019	-0.017				
HDI	-0.925	0.264	-0.007	0.012	-0.001	0.003			
t:warmth	0.158	-0.377	-0.882	0.001	0.011	-0.035	0.006		
t:physcl_pn	0.170	-0.402	0.004	-0.894	-0.001	-0.017	-0.010	-0.003	
t:identity1	0.081	-0.192	0.011	0.000	-0.867	0.014	0.001	-0.013	0.002
t:intrvntn1	0.093	-0.222	-0.035	-0.017	0.014	-0.867	-0.003	0.041	0.019
t:HDI	0.322	-0.765	0.015	-0.027	0.002	-0.007	-0.346	-0.016	0.029
	t:dnt1	t:ntr1							

t
warmth
physcl_pnsh
identity1
interventn1
HDI
t:warmth
t:physcl_pn
t:identity1


```
t:intrvntn1 -0.016
t:HDI       -0.002  0.008
```

5.4.3 Julia

5.4.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.4.3.2 Run The Model

5.4.3.2.1 Change Country To Categorical

```
@transform!(dfL, :country = categorical(:country))

@transform!(dfL, :identity = categorical(:identity))

@transform!(dfL, :intervention = categorical(:intervention))
```

5.4.3.2.2 Main Effects Only

```
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                physical_punishment +
                                identity + intervention +
                                HDI +
                                (1 | country) +
                                (0 + warmth | country) +
                                (1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

```
outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | country)
logLik    -2 logLik      AIC      AICc      BIC
-28499.6031 56999.2063 57021.2063 57021.2356 57099.3610
```

Variance components:

Column	Variance	Std.Dev.	Corr.
--------	----------	----------	-------

```

id      (Intercept)    8.387214 2.896069
country (Intercept)    3.167143 1.779647
      warmth          0.010762 0.103739 .
Residual                    26.027363 5.101702
Number of obs: 9000; levels of grouping factors: 3000, 30

```

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	50.4673	1.33833	37.71	<1e-99
t	0.943864	0.0658717	14.33	<1e-45
warmth	0.913496	0.0423744	21.56	<1e-99
physical_punishment	-1.0079	0.0497622	-20.25	<1e-90
identity: 1.0	-0.127692	0.151583	-0.84	0.3996
intervention: 1.0	0.858997	0.151909	5.65	<1e-07
HDI	-0.000566026	0.0196439	-0.03	0.9770

5.4.3.2.3 Interactions With Time

```

m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
      physical_punishment +
      identity + intervention +
      HDI) +
      (1 | country) +
      (0 + warmth | country) +
      (1 | id)), dfL)

```

Linear mixed model fit by maximum likelihood

```

outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + t & warmth
logLik   -2 logLik      AIC      AICc      BIC
-28498.3091 56996.6182 57028.6182 57028.6788 57142.2979

```

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.391746	2.896851	
country	(Intercept)	3.170032	1.780458	
	warmth	0.010609	0.102999	.
Residual		26.015906	5.100579	

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	50.8364	1.48355	34.27	<1e-99
t	0.758209	0.326177	2.32	0.0201
warmth	0.817076	0.0826636	9.88	<1e-22
physical_punishment	-1.00903	0.111293	-9.07	<1e-18
identity: 1.0	-0.238714	0.303996	-0.79	0.4323
intervention: 1.0	0.660761	0.30445	2.17	0.0300
HDI	0.00136065	0.0210842	0.06	0.9485
t & warmth	0.0483635	0.0356074	1.36	0.1744
t & physical_punishment	0.000542203	0.0494355	0.01	0.9912
t & identity: 1.0	0.0554385	0.131745	0.42	0.6739
t & intervention: 1.0	0.0992809	0.131925	0.75	0.4517
t & HDI	-0.000955067	0.00382162	-0.25	0.8027

5.5 Interpretation

The *main effects only model* suggests that time is associated with increases in the outcome. In the main effects model, main effects other than time, indicate whether a particular variable is associated with higher or lower intercepts of the time trajectory, at the beginning of the study time. Warmth is again associated with increases in the outcome, while physical punishment is associated with decreases in the outcome. Identity is again not associated with the outcome, while the intervention is associated with higher levels of the outcome. The Human Development Index is again not associated with the outcome.

The second model adds interactions with time to the first model. Results are largely similar to the prior model. However, here we not only examine whether main effects other than time are associated with higher or lower time trajectories, but also whether particular variables are associated with differences in the slope of the time trajectory. In this case, we find insufficient evidence that any independent variable is associated with changes in the slope of the time trajectory.

Which Interactions To Test?

In this example—for the sake of illustration—I test the interaction of *every* independent variable with time. In many cases, it may make sense to test only one or two interactions of time with particular variables of key interest. Also, after finding, as I did

in this model, that none of the interactions of other independent variables with time are significant, I might report the model with interactions, or might report only the results of the model with only main effects.

It may be illustrative to imagine how we would interpret the results had a particular interaction term been statistically significant. Let us consider one of the interaction terms with the largest coefficient, **intervention#time**. The interaction of the intervention with time is positive. Had this coefficient been statistically significant, it would have indicated that the intervention was associated with more rapid increases in the outcome over time *in addition to* the fact that the intervention is associated with higher initial levels of the outcome.

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