Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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1 Multilevel Multilingual

"This curious world which we inhabit is more wonderful than it is convenient..." (Thoreau, 1975)

1.1 Introduction

Below, I describe the use of Stata, R, and Julia to estimate multilevel models.

• Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

- 1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the /// line continuation characters.
- 2. R is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (()) or continued with + signs.
- 3. Like Stata, Julia expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a + sign. An alternative is to encase the entire Julia command in an outer set of parentheses (()).

1.2 The Data

The examples use the simulated_multilevel_data.dta file from *Multilevel Thinking*. Here is a direct link to download the data.

country	HDI	family	id	group	physical_punishment	warmth	outcome
1	69	1	1.1	2	2	3	59.18
1	69	2	1.2	2	4	0	61.54
1	69	3	1.3	1	4	4	51.87
1	69	4	1.4	2	0	6	51.71
1	69	5	1.5	2	3	2	55.88
1	69	6	1.6	1	5	3	60.78

Table 1.1: Sample of Simulated Multilevel Data

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable y, independent variables x and z, clustering variable group, and a random slope for x. i is the index for the person, while j is the index for the group.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij}$$
(1.1)

1.3.1 Stata

In Stata mixed, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x || group: x
```

1.3.2 R

In R lme4, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia MixedModels, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels
fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use summarize for *continuous* variables, and tabulate for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI
tabulate group
```

Variable	0bs	Mean	Std. dev.	Min	Max
outcome	3,000	53.46757	6.65179	33.39014	76.75101
warmth	3,000	3.524333	1.889956	0	7
physical_p~t	3,000	2.494667	1.380075	0	5
HDI	3,000	64.76667	17.24562	33	87

arbitrary group variable	Freq.	Percent	Cum.
1 2	1,507 1,493	50.23 49.77	50.23
Total	3,000	100.00	

2.1.2 R

```
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")</pre>
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (factor) before using summary¹ to generate descriptive statistics.

```
df$country <- factor(df$country)

df$group <- factor(df$group)

summary(df)</pre>
```

```
country
                    HDI
                                   family
                                                     id
                                                                   group
1
       : 100
               Min.
                      :33.00
                               Min.
                                     : 1.00
                                                Length:3000
                                                                   1:1507
2
       : 100
                               1st Qu.: 25.75
                                                                   2:1493
              1st Qu.:53.00
                                                Class : character
3
       : 100
              Median :70.00
                               Median : 50.50
                                                Mode :character
4
       : 100
               Mean
                      :64.77
                               Mean
                                      : 50.50
5
                               3rd Qu.: 75.25
       : 100
               3rd Qu.:81.00
6
       : 100
                     :87.00
                                      :100.00
               Max.
                               Max.
(Other):2400
physical_punishment
                        warmth
                                       outcome
       :0.000
                           :0.000
Min.
                    Min.
                                    Min.
                                           :33.39
1st Qu.:2.000
                    1st Qu.:2.000
                                    1st Qu.:48.78
Median :3.000
                   Median:4.000
                                    Median :53.64
Mean
     :2.495
                   Mean :3.524
                                    Mean :53.47
3rd Qu.:3.250
                    3rd Qu.:5.000
                                    3rd Qu.:58.06
Max.
      :5.000
                    Max. :7.000
                                    Max.
                                          :76.75
```

2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, Data

df = DataFrame(load("simulated_multilevel_data.dta"))
```

¹skimr is an excellent new alternative library for generating descriptive statistics in R.

Similarly to R, Julia relies on the idea of *variable type*. I use transform to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))
@transform!(df, :group = categorical(:group))
```

```
describe(df) # descriptive statistics
```

8×7 Da	7 DataFrame									
Row	variable	mean	min	median	max	nmissing	eltyp			
	Symbol	Union	Any	Union	Any	Int64	Union			
1	country		1.0		30.0	0	Union			
2	HDI	64.7667	33.0	70.0	87.0	0	Union			
3	family	50.5	1.0	50.5	100.0	0	Union			
4	id		1.1		9.99	0	Union			
5	group		1.0		2.0	0	Union			
6	physical_punishment	2.49467	0.0	3.0	5.0	0	Union			
7	warmth	3.52433	0.0	4.0	7.0	0	Union			
8	outcome	53.4676	33.3901	53.6426	76.751	0	Union			

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$outcome_{ij} = \beta_0 + u_{0j} + e_{ij} \tag{3.1}$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$ICC = \frac{var(u_{0j})}{var(u_{0j}) + var(e_{ij})}$$

$$(3.2)$$

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

Performing EM optimization ...

Performing gradient-based optimization: Iteration 0: Log likelihood = -9856.1548 Iteration 1: Log likelihood = -9856.1548

```
Computing standard errors ...
Mixed-effects ML regression
                                  Number of obs = 3,000
Group variable: country
                                  Number of groups = 30
                                  Obs per group:
                                          min = 100
                                          avg = 100.0
                                          max = 100
                                  Wald chi2(0)
                                  Prob > chi2
Log likelihood = -9856.1548
   outcome | Coefficient Std. err. z P>|z| [95% conf. interval]
______
    _cons | 53.46757 .3539097 151.08 0.000
                                    52.77392
______
 Random-effects parameters | Estimate Std. err. [95% conf. interval]
______
country: Identity
           var(_cons) | 3.348734 .9702594 1.897816 5.908906
         var(Residual) | 40.88284 1.060908
                                    38.8555
LR test vs. linear model: chibar2(01) = 169.64
                                 Prob \geq chibar2 = 0.0000
estat icc // ICC
Intraclass correlation
______
                       ICC Std. err.
              Level |
                                    [95% conf. interval]
-----
             country | .0757091 .0203761 .0442419 .1265931
```

3.2.2 R

```
library(haven)
df <- read_dta("simulated_multilevel_data.dta")</pre>
library(lme4) # estimate multilevel models
fit0 <- lmer(outcome ~ (1 | country),</pre>
            data = df) # unconditional model
summary(fit0)
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
  Data: df
REML criterion at convergence: 19712.5
Scaled residuals:
              1Q Median
     Min
                                ЗQ
                                        Max
-2.97650 -0.68006 0.00936 0.67580 3.03510
Random effects:
 Groups Name Variance Std.Dev.
 country (Intercept) 3.478 1.865
 Residual
                    40.883 6.394
Number of obs: 3000, groups: country, 30
Fixed effects:
           Estimate Std. Error t value
(Intercept) 53.47 0.36 148.5
library(performance)
performance::icc(fit0) # ICC
# Intraclass Correlation Coefficient
```

Adjusted ICC: 0.078 Unadjusted ICC: 0.078

3.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
df = DataFrame(load("simulated_multilevel_data.dta"))
@transform!(df, :country = categorical(:country))
m0 = fit(MixedModel,
         @formula(outcome ~ (1 | country)), df) # unconditional model
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + (1 | country)
   logLik -2 logLik
                         AIC
                                   AICc
                                               BIC
 -9856.1548 19712.3097 19718.3097 19718.3177 19736.3288
Variance components:
           Column
                   Variance Std.Dev.
                      3.34871 1.82995
country (Intercept)
Residual
                     40.88285 6.39397
 Number of obs: 3000; levels of grouping factors: 30
 Fixed-effects parameters:
              Coef. Std. Error
                                    z Pr(>|z|)
(Intercept) 53.4676 0.353908 151.08
                                           <1e-99
icc(m0) # ICC
```

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

```
\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \beta_2 \text{physical punishment}_{ij} + \beta_3 \text{group}_{ij} + \beta_4 \text{HDI}_{ij} + u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij} \tag{4.1}
```

4.2 Run Models

4.2.1 Stata

4.2.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.2.1.2 Run The Model

```
mixed outcome warmth physical_punishment group HDI || country: warmth
```

Performing EM optimization ...

```
Performing gradient-based optimization:
```

Iteration 0: Log likelihood = -9668.198
Iteration 1: Log likelihood = -9667.9551
Iteration 2: Log likelihood = -9667.9534
Iteration 3: Log likelihood = -9667.9533

Iteration 4: Log likelihood = -9667.9532

Computing standard errors ...

Mixed-effects ML regression	Number of obs $= 3,000$
Group variable: country	Number of groups = 30
	Obs per group:
	min = 100
	avg = 100.0
	max = 100
	Wald chi2(4) = 401.26

Log likelihood = -9667.9532

outcome	Coefficient		z	P> z		interval]
warmth		.0581825	16.53	0.000	.8476091	1.07568
physical_punishment	8453802	.0798155	-10.59	0.000	-1.001816	6889448
group	1.084344	.2200539	4.93	0.000	.6530461	1.515642
HDI	.010557	.0204522	0.52	0.606	0295286	.0506426
_cons	49.87963	1.436612	34.72	0.000	47.06392	52.69534

Prob > chi2 = 0.0000

Random-effects parameters		Std. err.	[95% conf.	_
country: Independent var(warmth)		.0000173	1.76e-14	190.9774
var(warmth) var(_cons)	3.370262	.9633726	1.924651	5.901676
var(Residual)		.9346936	34.23291	37.89842
LR test vs. linear model: chi2	Prob > chi	2 = 0.0000		

Note: LR test is conservative and provided only for reference.

4.2.2 R

4.2.2.1 Get The Data

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")</pre>
```

4.2.2.2 Run The Model

```
fit1 <- lmer(outcome ~ warmth + physical_punishment +</pre>
              group + HDI +
              (1 + warmth || country),
            data = df
summary(fit1)
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ warmth + physical_punishment + group + HDI + ((1 |
    country) + (0 + warmth | country))
  Data: df
REML criterion at convergence: 19350.3
Scaled residuals:
            1Q Median
    Min
                            3Q
                                   Max
-3.4496 -0.6807 0.0016 0.6864 3.1792
Random effects:
Groups
          Name
                     Variance Std.Dev.
 country (Intercept) 3.611568 1.90041
 country.1 warmth
                      0.001876 0.04331
 Residual
                      36.049124 6.00409
Number of obs: 3000, groups: country, 30
Fixed effects:
                   Estimate Std. Error t value
(Intercept)
                   49.88754 1.48203 33.662
                    0.96155 0.05875 16.367
warmth
physical_punishment -0.84556 0.07986 -10.588
                    1.08471 0.22017 4.927
group
HDI
                    0.01044 0.02116 0.493
```

4.2.3 Julia

4.2.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
df = DataFrame(load("simulated_multilevel_data.dta"))
```

4.2.3.2 Change Country To Categorical

```
@transform!(df, :country = categorical(:country))
```

4.2.3.3 Run The Model

Variance components:

Column Variance Std.Dev. Corr.

country (Intercept) 3.2369484 1.7991521

warmth 0.0001080 0.0103903 +1.00

Residual 36.0187144 6.0015593

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(T	40, 0040	4 40405	04.70	
(Intercept)	49.9018	1.43435	34.79	<1e-99
warmth	0.961545	0.0582135	16.52	<1e-60
<pre>physical_punishment</pre>	-0.845389	0.0798149	-10.59	<1e-25
group	1.08524	0.220055	4.93	<1e-06
HDI	0.0101984	0.0204401	0.50	0.6178

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

•

Graphing Longitudinal Data

In the section on *cross-sectional* multilevel models, I employed *scatterplots* to graph the data. In longitudinal models, *time* is a variable of special interest. Often, in graphing *longitudinal* data—especially when graphing outcomes by time—it makes more sense to use *linear fit* plots, although a *scatterplot* could be employed as well.

5.2 The Equation

 $outcome_{itj} = \beta_0 + \beta_1 parental warmth_{itj} + \beta_2 physical punishment_{itj} + \beta_3 time_{itj} + (5.1)$

$$\beta_4 \text{group}_{itj} + \beta_5 \text{HDI}_{itj} +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} \, + \,$$

$$v_{0i} + v_{1i} \times \mathrm{time}_{itj} + e_{itj}$$

5.3 Run Models

5.3.1 Stata

5.3.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.3.1.2 Run The Model

5.3.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment group HDI || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28795.37
Iteration 1: Log likelihood = -28795.232
Iteration 2: Log likelihood = -28795.232

Computing standard errors ...

Log likelihood = -28795.232

Mixed-effects ML regression Number of obs = 9,000 Group variable: country Number of groups = 30 Obs per group: $\min = 300$ avg = 300.0 $\max = 300$ Wald chi2(5) = 1366.93

Prob > chi2 = 0.0000

outcome	Coefficient					interval]
	.9882371		12.98			1.137476
warmth	.9427117	.0342282	27.54	0.000	.8756256	1.009798
physical_punishment	9020727	.0452759	-19.92	0.000	9908119	8133336
group	.9861238	.1249047	7.90	0.000	.7413151	1.230933

HDI | .0073726 .020661 0.36 0.721 -.0331222 .0478674 _cons | 49.45537 1.414072 34.97 0.000 46.68384 52.2269 Random-effects parameters | Estimate Std. err. [95% conf. interval] country: Independent var(warmth) | .0024684 .0082517 3.52e-06 1.72956 var(_cons) | 3.663663 .9914845 2.155548 6.22692 _____ var(Residual) | 34.78483 .5200702 33.7803 35.81923 _____ LR test vs. linear model: chi2(2) = 805.75 Prob > chi2 = 0.0000Note: LR test is conservative and provided only for reference. 5.3.1.2.2 Interactions With Time mixed outcome c.t##(c.warmth c.physical_punishment i.group c.HDI) || country: warmth

Performing EM optimization ...

Performing gradient-based optimization: Iteration 0: Log likelihood = -28794.99 Iteration 1: Log likelihood = -28794.855 Iteration 2: Log likelihood = -28794.855

Computing standard errors ...

Number of obs = 9,000Mixed-effects ML regression Group variable: country Number of groups = 30 Obs per group: min = 300avg = 300.0 max = 300 Wald chi2(9) = 1365.73Prob > chi2 = 0.0000Log likelihood = -28794.855

outcome | Coefficient Std. err. z P>|z| [95% conf. interval]

						
t	1.047448	.3619795	2.89	0.004	.3379816	1.756915
warmth	.8869901	.0876058	10.12	0.000	.715286	1.058694
physical_punishment	893285	.1194705	-7.48	0.000	-1.127443	659127
2.group	.9648545	.3292217	2.93	0.003	.3195918	1.610117
HDI	.0120622	.022474	0.54	0.591	0319861	.0561104
c.t#c.warmth	.0277903	.0402665	0.69	0.490	0511306	.1067112
c.t#						
c.						
physical_punishment	0041479	.0553051	-0.08	0.940	1125439	.1042482
group#c.t						
2	.0105177	.1523009	0.07	0.945	2879865	.3090219
c.t#c.HDI	002342	.0044172	-0.53	0.596	0109996	.0063155
_cons	50.32233	1.572089	32.01	0.000	47.2411	53.40357

Random-effects parameters			[95% conf.	_
country: Independent var(warmth) var(_cons)	.0025661 3.66269	.0083259	4.44e-06 2.154617	1.482773 6.226305
var(Residual)			33.77713	35.8159
LR test vs. linear model: chi2(2) = 805.90			Prob > chi	2 = 0.0000

Note: LR test is conservative and provided only for reference.

5.3.2 R

5.3.2.1 Get The Data

```
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")</pre>
```

5.3.2.2 Run The Model

5.3.2.2.1 Main Effects Only

```
fit2A <- lmer(outcome ~ t + warmth + physical_punishment +</pre>
              group + HDI +
               (1 | country/id),
            data = dfL)
summary(fit2A)
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ t + warmth + physical_punishment + group + HDI + (1 |
    country/id)
  Data: dfL
REML criterion at convergence: 57088.4
Scaled residuals:
    Min
            1Q Median
                           3Q
                                   Max
-3.4471 -0.6226 0.0081 0.6153 3.1993
Random effects:
                       Variance Std.Dev.
 Groups
           Name
 id:country (Intercept) 8.864
                                2.977
         (Intercept) 3.924
                                1.981
 country
 Residual
                       26.008
                                5.100
Number of obs: 9000, groups: id:country, 3000; country, 30
Fixed effects:
                    Estimate Std. Error t value
(Intercept)
                   49.494782 1.471780 33.629
                    0.987964 0.065840 15.005
t
warmth
                    0.946259 0.038200 24.771
physical_punishment -0.926880
                              0.049970 -18.549
                              0.153550 6.420
group
                    0.985786
HDI
                    0.007543
                               0.021437 0.352
Correlation of Fixed Effects:
           (Intr) t
                         warmth physc_ group
           -0.090
          -0.085 0.008
warmth
```

5.3.2.2.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +</pre>
              group + HDI) +
               (1 | country/id),
            data = dfL)
summary(fit2B)
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ t * (warmth + physical_punishment + group + HDI) +
    (1 | country/id)
  Data: dfL
REML criterion at convergence: 57107.3
Scaled residuals:
            1Q Median
                            3Q
                                   Max
-3.4431 -0.6248 0.0071 0.6183 3.1961
Random effects:
 Groups
           Name
                       Variance Std.Dev.
 id:country (Intercept) 8.868
                                2.978
 country
            (Intercept) 3.925
                                1.981
 Residual
                       26.014
                                5.100
Number of obs: 9000, groups: id:country, 3000; country, 30
Fixed effects:
                      Estimate Std. Error t value
(Intercept)
                     49.453036 1.637740 30.196
                      1.008199
                                 0.364915
                                          2.763
warmth
                      0.865659 0.080487 10.755
physical_punishment
                     -0.908148 0.110449 -8.222
                      0.966988 0.304936 3.171
group
HDI
                      0.012277 0.022761 0.539
                                 0.035364 1.136
t:warmth
                      0.040170
t:physical_punishment -0.008932
                                 0.049262 -0.181
```

```
0.009180
                           0.131714 0.070
t:group
t:HDI
                  -0.002359 0.003820 -0.618
Correlation of Fixed Effects:
         (Intr) t
                     -0.446
warmth
         -0.159 0.278
physcl_pnsh -0.169 0.302 -0.022
        -0.274 0.459 -0.010 -0.014
group
HDI
         -0.900 0.227 -0.008 0.009 -0.001
         0.141 -0.316 -0.880 0.017 0.010 0.007
t:warmth
t:physcl_pn 0.150 -0.338 0.017 -0.892 0.010 -0.007 -0.015
```

0.237 -0.532 0.009 0.012 -0.864 0.001 -0.012 -0.008

5.3.3 Julia

t:group

t:HDI

5.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.3.3.2 Run The Model

5.3.3.2.1 Change Country To Categorical

```
@transform!(dfL, :country = categorical(:country))
```

5.3.3.2.2 Main Effects Only

```
Linear mixed model fit by maximum likelihood
outcome ~ 1 + t + warmth + physical_punishment + group + HDI + (1 + warmth | country) + (1
logLik -2 logLik AIC AICC BIC
-28533.9236 57067.8472 57089.8472 57089.8765 57168.0019
```

Variance components:

Column Variance Std.Dev. Corr.

id (Intercept) 8.851224 2.975101 country (Intercept) 3.451345 1.857780

warmth 0.000227 0.015065 +1.00

Residual 26.001212 5.099138

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	Z	Pr(> z)
(Intercept)	49.5105	1.41854	34.90	<1e-99
t	0.98814	0.0658319	15.01	<1e-50
warmth	0.946252	0.0382851	24.72	<1e-99
<pre>physical_punishment</pre>	-0.926673	0.0499547	-18.55	<1e-76
group	0.98708	0.153484	6.43	<1e-09
HDI	0.00725703	0.0206549	0.35	0.7253

5.3.3.2.3 Interactions With Time

```
Linear mixed model fit by maximum likelihood
outcome ~ 1 + t + warmth + physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + HDI + t & warmth + t & physical_punishment + group + t & warmth + t & physical_punishment + group + t & warmth + t & warmth + t & physical_punishment + group + t & warmth +
```

Variance components:

Column Variance Std.Dev. Corr. id (Intercept) 8.8593774 2.9764706

country (Intercept) 3.4464818 1.8564703

warmth 0.0002394 0.0154717 +1.00

Residual 25.9905210 5.0980899

Number of obs: 9000; levels of grouping factors: 3000, 30

${\tt Fixed-effects\ parameters:}$

	Coef.	Std. Error	z	Pr(> z)
(T	40. 4606	4 50007	04 44	44 00
(Intercept)	49.4696	1.59007	31.11	<1e-99
t	1.00761	0.36475	2.76	0.0057
warmth	0.865294	0.0804999	10.75	<1e-26
physical_punishment	-0.907816	0.110399	-8.22	<1e-15
group	0.96908	0.304797	3.18	0.0015
HDI	0.0119739	0.0220306	0.54	0.5868
t & warmth	0.0403499	0.0353477	1.14	0.2537
t & physical_punishment	-0.00900605	0.0492392	-0.18	0.8549
t & group	0.00879803	0.131655	0.07	0.9467
t & HDI	-0.00234513	0.00381848	-0.61	0.5391

Thoreau, H. D. (1975). The commercial spirit of modern times. In J. J. Moldenhauer, E. Moser, & A. C. Kern (Eds.), *Early essays and miscellanies*. Princeton University Press.