

Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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2024-08-26

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1 Multilevel Multilingual

“This curious world which we inhabit is more wonderful than it is convenient...”
(Thoreau, 1975)

“Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled.” (Schanen, 2021)

1.1 Introduction

Below, I describe the use of [Stata](#) (StataCorp, 2023), [R](#) (Bates et al., 2015; R Core Team, 2023), and [Julia](#) (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

All of these software packages can estimate multilevel models and can visualize relationships in the data. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. While it costs money to purchase Stata, the price is often very reasonable for academic and educational use. R is *free* open source software which is less intuitive, but there are many excellent resources for learning R. There is often a cost associated with purchasing books and other materials for learning R, which sometimes feels like it offsets the fact that R is free. Julia is newer open source software, also *free*, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

While any of these software packages can be used for learning and estimating multilevel models, I will offer my own opinion—based upon 15 years of teaching multilevel models at the doctoral level—that Stata offers the quickest pathway for learning the basic and advanced uses of multilevel models. I also believe the intuitive nature of Stata syntax contributes to accurate and replicable work in this area.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but very intuitive for both multilevel modeling and graphing.

Software	Cost	Ease of Use
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (<code>ggplot</code>).
Julia	free	steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

💡 Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

💡 Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as *R* commands are usually clearly encased in parentheses `()` or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a multi-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire *Julia* command in an outer set of parentheses `()`.

💡 Running Statistical Packages in Quarto

I used Quarto (Allaire et al., 2024) (<https://quarto.org/>) to create this Appendix. Quarto is a programming and publishing environment that can run multiple programming languages, including *Stata*, *R* and *Julia*, and that can write to multiple output formats including HTML, PDF, and MS Word. To run *Stata*, I used the `Statamarkdown` library (Hemken, 2023) in *R* to connect *Stata* to Quarto. Quarto has a built in connection to *R*,

and runs R without issue. To run Julia, I used the `JuliaCall` library (Li, 2019) in R to connect Quarto to Julia.

Of course, each of these programs can be run by itself, if you have them installed on your computer.

1.2 The Data

Datasets

The examples use the `simulated_multilevel_data.dta` and `simulated_multilevel_longitudinal_data.dta` files.

Here is a [direct link](#) to download the cross-sectional data.

Here is a [direct link](#) to download the longitudinal data.

Table 1.2: Sample of Simulated Multilevel Data

Table 1.2: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	1	0	3
1	69	2	1.2	1	1	2
1	69	3	1.3	0	1	3
1	69	4	1.4	1	0	0
1	69	5	1.5	1	0	4
1	69	6	1.6	0	1	5

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
3	57.47
1	50.1
2	52.92
5	60.17
4	55.05
3	49.81

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable y , independent variables \mathbf{x} and \mathbf{z} , clustering variable `group`, and a random slope for \mathbf{x} . i is the index for the person, while j is the index for the group.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \quad (1.1)$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x z || group: x
```

1.3.2 R

In R `lme4`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels

fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

2 Statistical Workflows

2.1 Statistical Software Is Best Run Using a Script

Many statistical workflows—whatever the statistical package being used—follow the same conceptual pattern.

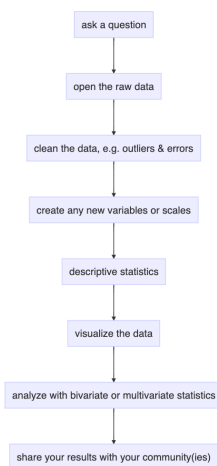


Figure 2.1: A Common Statistical Workflow

Increasingly, we want to think about workflows that are

- **documentable, transparent, and auditable:** We have a record of what we did if we want to double check our work, clarify a result, or develop a new project with a similar process. We, or others, can find the inevitable errors in our work, **and correct them**.
- **replicable:** Others can replicate our findings with the same or new data.
- **scalable:** We are developing a process that can be as easily used with *thousands* or *millions* of rows of data as it can with *ten* rows of data. We are developing a process that can be easily repeated if we are *constantly getting new or updated data*, e.g. getting new data every week, or every month.

2.2 Scripts

For most statistical workflows, we will often want to write a script or code. Data analysis scripts can be stored in a Quarto document (Allaire et al., 2024) as they are in this Appendix, or every statistical package has its own unique format for storing scripts as a text file: in Stata, scripts are stored in `.do` files; in R, scripts are stored in `.R` files, and in Julia, scripts are stored in `.jl` files.

2.3 Script Flow

A good practice when writing a script, is to have a script that begins with the raw data, moves through any necessary re-coding or cleaning of the data, generates descriptive statistics, generates the appropriate multivariate results, and then generates any necessary visualizations.

2.4 Good Statistical Workflows Allow Multiple Statistical Packages

While this Appendix focuses on the use of each individual statistical package on its own, it is certainly possible to use multiple statistical packages as part of the same workflow. For example, one might employ Stata to carry out data management tasks, and then possibly use R to run a multilevel model with a more complicated multilevel structure, such as a cross-classified model, or Julia to more quickly run a model with a large data.

2.5 Good Statistical Workflows Require Safe Workspaces

It is also *very important* to be aware that good complex workflows are *highly iterative* and *highly collaborative*. Good complex workflows require a *safe workspace* in which team members feel free to admit their own errors, and help with others' mistakes in a non-judgmental fashion. Such a *safe environment* is necessary to build an environment where the *overall error rate* is low.

2.6 Good Statistical Workflows Require Patience And Can Be Psychologically Demanding

Developing a good documented and auditable workflow that is implemented in code requires a lot of patience, and often, **many iterations**. Working through these many iterations can be psychologically demanding. It is important to remember that careful attention to getting

the details right early in the research process, while sometimes tiring and frustrating, will pay large dividends later on when the research is reviewed, presented, published and read.

2.7 Good Statistical Workflows Often Allow Multiple Principled Ways Forward

One of my most recent ideas about statistical workflows is that there are certainly *wrong* decisions that one can make with data.

For example, I would not want to write the paper that says that smoking prevents lung cancer, nor would I want to write a paper saying physical punishment is good for children.

That being said, I think there are often *multiple principled ways forward*.

Often the key is not so much to make the 100% correct decision, but to make one of *several possible principled decisions*.

Then after making a *principled decision*, one is *transparent* and *thorough* about describing the decision that one made.

For example, in implementing a multilevel analysis, I would have many choices: I could estimate only a random intercept; estimate one or more random slopes; or estimate all possible random slopes. The random effects could be correlated or uncorrelated. I could estimate only main effects, or could estimate interactions of several variables. Each of these would be a different, yet principled, approach to analyzing the data.

In science and statistics, we often want an answer that provides one clear direction. Instead, I'm increasingly convinced that the best science (and teaching!) often involves engaging in open discussion about the multiple possible alternatives, and then choosing one principled solution, and being transparent about its implementation.

3 Storing Statistical Data

3.1 Spreadsheets

Spreadsheets are sometimes used to collect and store data. Spreadsheets may sometimes be used because they are the only program that some individuals or agencies have for storing data. Spreadsheet programs may also be used because spreadsheets can be very intuitive and easy ways of managing small amounts of data.

However, spreadsheets may be problematic as a data storage solution for a number of reasons detailed below, especially as data sets grow in size. Notably, statistical programs like [Stata](#), [R](#), or [Julia](#) can all store additional information with each variable such as: a *variable label*, describing the contents of the variable, or the survey question that resulted in the variable; and a *value label*, which attaches qualitative information to each possible value of the response.

Spreadsheets do not generally contain this extra information about each variable, or column of data, which may lead to errors in working with quantitative information.

3.1.1 Data in Statistical Format

I load the data from a statistical program.

3.1.1.1 Describe The Data

Notice how a description of the data contains information that helps us to understand the variables.

pos	variable	label
1	country	country id
2	HDI	Human Development Index
3	family	family id
4	id	unique country family id
5	identity	hypothetical identity group variable
6	intervention	recieved intervention
7	physical_punishment	physical punishment in past week

pos	variable	label
8	warmth	parental warmth in past week
9	outcome	beneficial outcome

3.1.1.2 Descriptive Statistics

💡 Variable Labels and Value Labels Help Us Understand Our Data

Notice how the descriptive statistics and graph are informative in that they contain information on the *variable label* and *value label*. These help us to get an intuitive sense of the information in the data. We see this information when we list out the data as well.

Table 3.2: Table continues below

country	HDI	family	id
1 : 100	Min. :33.00	Min. : 1.00	Length:3000
2 : 100	1st Qu.:53.00	1st Qu.: 25.75	Class :character
3 : 100	Median :70.00	Median : 50.50	Mode :character
4 : 100	Mean :64.77	Mean : 50.50	NA
5 : 100	3rd Qu.:81.00	3rd Qu.: 75.25	NA
6 : 100	Max. :87.00	Max. :100.00	NA
(Other):2400	NA	NA	NA

Table 3.3: Table continues below

identity	intervention	physical_punishment	warmth
Identity B:1507	no intervention:1547	Min. :0.000	Min. :0.000
Identity A:1493	intervention :1453	1st Qu.:2.000	1st Qu.:2.000
NA	NA	Median :2.000	Median :4.000
NA	NA	Mean :2.479	Mean :3.522
NA	NA	3rd Qu.:3.000	3rd Qu.:5.000
NA	NA	Max. :5.000	Max. :7.000
NA	NA	NA	NA

outcome
Min. :29.61
1st Qu.:48.02

outcome
Median :52.45
Mean :52.43
3rd Qu.:56.86
Max. :74.84
NA

3.1.1.3 Graph

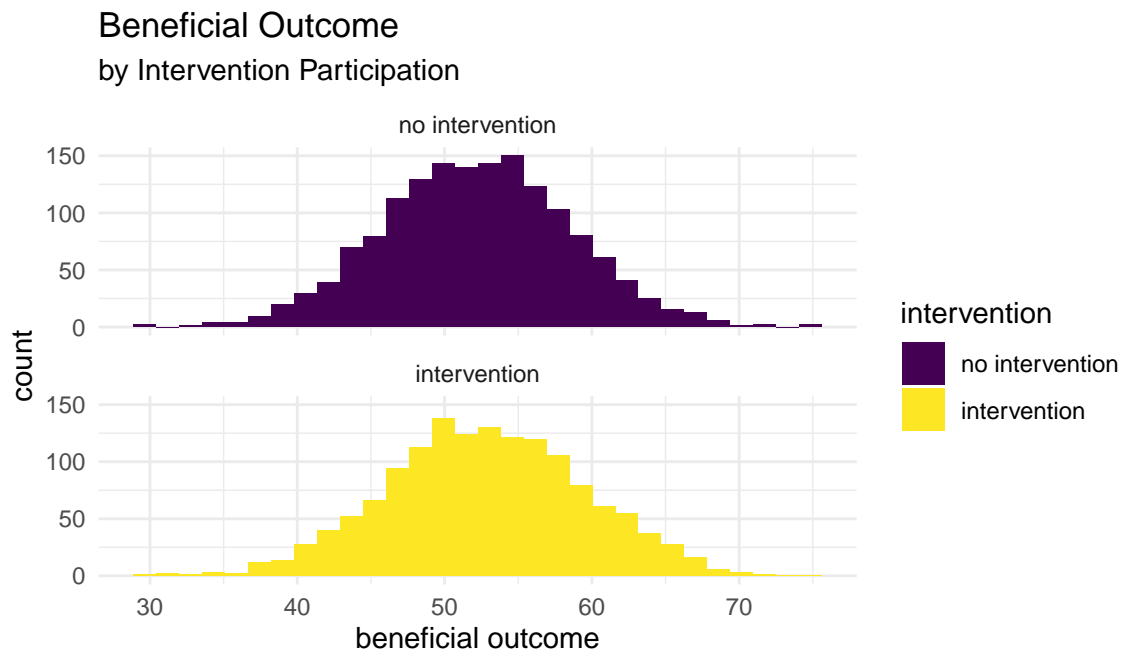


Figure 3.1: Graph from Data Stored in Statistical Software

3.1.1.4 List Out A Sample Of The Data

Table 3.5: Table continues below

country	HDI	family	id	identity	intervention
1	69	1	1.1	Identity A	no intervention
1	69	2	1.2	Identity A	intervention
1	69	3	1.3	Identity B	intervention

country	HDI	family	id	identity	intervention
1	69	4	1.4	Identity A	no intervention
1	69	5	1.5	Identity A	no intervention
1	69	6	1.6	Identity B	intervention

physical_punishment	warmth	outcome
3	3	57.47
2	1	50.1
3	2	52.92
0	5	60.17
4	4	55.05
5	3	49.81

3.1.2 Data In Spreadsheet Format

I now import the spreadsheet data file. I use the first row of data as variable names.

We see right away that the data are less informative.

3.1.2.1 Describe The Data

Notice how a description of the data no longer contains much of the information that helped us to understand the variables.

pos	variable	label
1	country	NA
2	HDI	NA
3	family	NA
4	id	NA
5	identity	NA
6	intervention	NA
7	physical_punishment	NA
8	warmth	NA
9	outcome	NA

Table 3.8: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	1	0	3
1	69	2	1.2	1	1	2
1	69	3	1.3	0	1	3
1	69	4	1.4	1	0	0
1	69	5	1.5	1	0	4
1	69	6	1.6	0	1	5

warmth	outcome
3	57.47
1	50.1
2	52.92
5	60.17
4	55.05
3	49.81

Warning

Adding this valuable information back into the data set may take a great deal of extra effort.

3.1.2.2 Descriptive Statistics

Notice here how the descriptive statistics and graph are much less informative. For example, it is now not immediately clear what the values of **identity** or **intervention** represent. The information on variable labels and value labels will have to be added back into the data when preparing a final product for dissemination.

Table 3.10: Table continues below

country	HDI	family	id
Min. : 1.0	Min. :33.00	Min. : 1.00	Length:3000
1st Qu.: 8.0	1st Qu.:53.00	1st Qu.: 25.75	Class :character
Median :15.5	Median :70.00	Median : 50.50	Mode :character
Mean :15.5	Mean :64.77	Mean : 50.50	NA
3rd Qu.:23.0	3rd Qu.:81.00	3rd Qu.: 75.25	NA

country	HDI	family	id
Max. :30.0	Max. :87.00	Max. :100.00	NA

Table 3.11: Table continues below

identity	intervention	physical_punishment	warmth
Min. :0.0000	Min. :0.0000	Min. :0.000	Min. :0.000
1st Qu.:0.0000	1st Qu.:0.0000	1st Qu.:2.000	1st Qu.:2.000
Median :0.0000	Median :0.0000	Median :2.000	Median :4.000
Mean :0.4977	Mean :0.4843	Mean :2.479	Mean :3.522
3rd Qu.:1.0000	3rd Qu.:1.0000	3rd Qu.:3.000	3rd Qu.:5.000
Max. :1.0000	Max. :1.0000	Max. :5.000	Max. :7.000

outcome
Min. :29.61
1st Qu.:48.02
Median :52.45
Mean :52.43
3rd Qu.:56.86
Max. :74.84

3.1.2.3 Graph

While the graph has an informative title, as well as informative axis labels, a crucial piece of information is missing: what each status of the intervention represents.

3.1.3 A Few Final Issues

Notice finally how spreadsheets doesn't enforce the idea of whether variables are *numeric*, or *text*, and so would allow storage of different types of information in the same column. Relatedly, *numeric* variables may be improperly stored as *text*, often necessitating recoding before graphical or statistical procedures can be employed.

Secondly, a spreadsheet would allow some of your columns to have the same name, which might make data difficult to work with in other software.

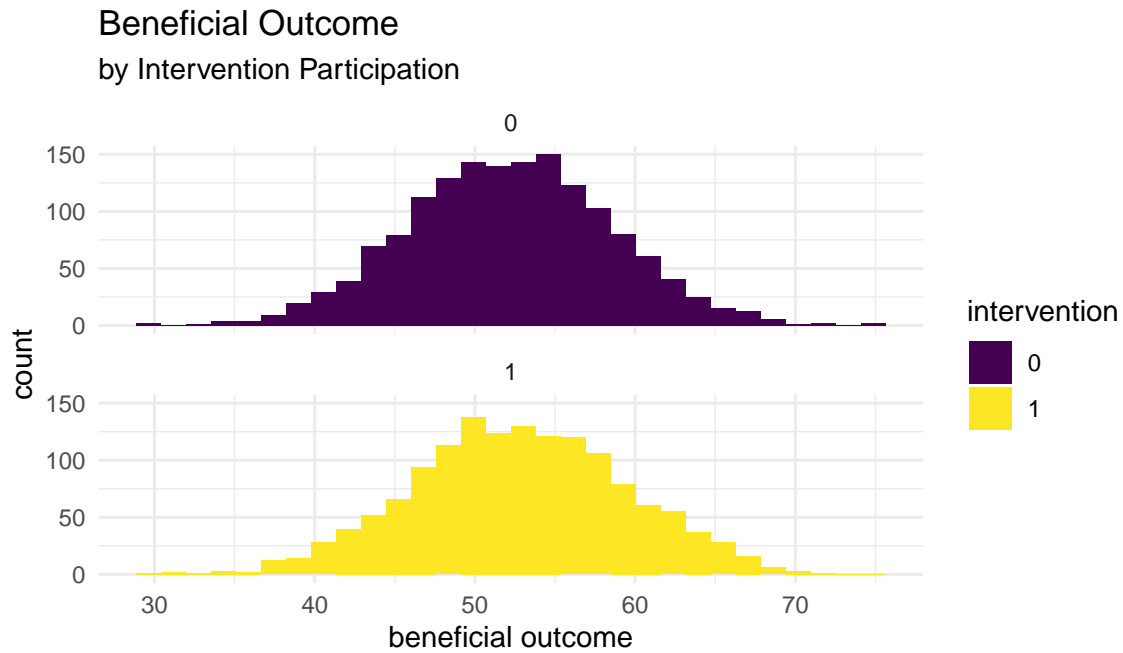


Figure 3.2: Graph from Data Stored in Spreadsheet

x	y	verylongvariablename	verylongvariablename
100	1	Smith	20
200	2	30	NA
not applicable	x	yes	60

3.2 File Organization

Files for all of your work should not be stored all together in **downloads**. Ideally, you should have a specific set of folders for your work. Each project, should be stored in its own individual folder. Ideally, each project folder would have a separate sub-folder for separate aspects of the project such as data, code or syntax, and various outputs.

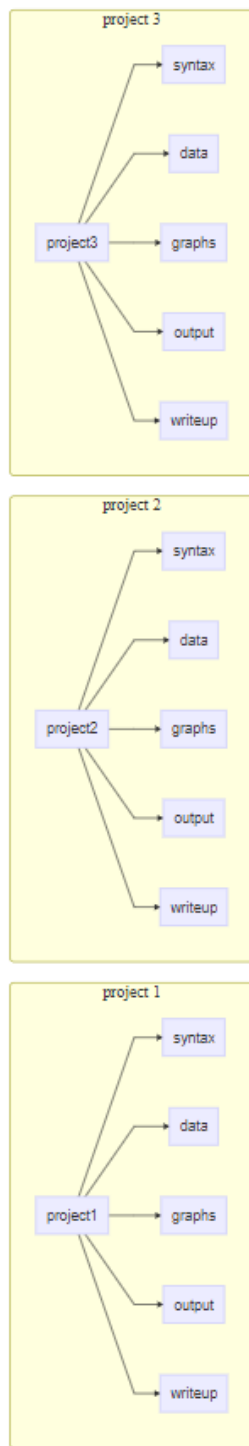


Figure 3.3: A Hypothetical Set of Folders and Subfolders

4 Descriptive Statistics

4.1 Descriptive Statistics

4.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI
```

```
tabulate identity
```

```
tabulate intervention
```

Variable	Obs	Mean	Std. dev.	Min	Max
-----+-----					
outcome	3,000	52.43327	6.530996	29.60798	74.83553
warmth	3,000	3.521667	1.888399	0	7
physical_p~t	3,000	2.478667	1.360942	0	5
HDI	3,000	64.76667	17.24562	33	87

hypothetica			
l identity			
group			
variable	Freq.	Percent	Cum.
-----+-----			
0	1,507	50.23	50.23
1	1,493	49.77	100.00
-----+-----			
Total	3,000	100.00	

recieved interventio	n	Freq.	Percent	Cum.
0	1,547	51.57	51.57	
1	1,453	48.43	100.00	
Total	3,000	100.00		

4.1.2 R

```
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`¹ to generate descriptive statistics.

```
df$country <- factor(df$country)

df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)

summary(df)
```

```

country      HDI      family      id      identity
1      : 100  Min.   :33.00  Min.   :  1.00  Length:3000  0:1507
2      : 100  1st Qu.:53.00  1st Qu.: 25.75  Class :character 1:1493
3      : 100  Median :70.00  Median : 50.50  Mode  :character
4      : 100  Mean   :64.77  Mean   : 50.50
5      : 100  3rd Qu.:81.00  3rd Qu.: 75.25
6      : 100  Max.   :87.00  Max.   :100.00
(Other):2400
intervention physical_punishment  warmth      outcome
0:1547      Min.   :0.000      Min.   :0.000  Min.   :29.61
1:1453      1st Qu.:2.000      1st Qu.:2.000  1st Qu.:48.02
```

¹`skimr` is an excellent new alternative library for generating descriptive statistics in R.

Median	:2.000	Median	:4.000	Median	:52.45
Mean	:2.479	Mean	:3.522	Mean	:52.43
3rd Qu.	:3.000	3rd Qu.	:5.000	3rd Qu.	:56.86
Max.	:5.000	Max.	:7.000	Max.	:74.84

4.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataAPI

df = DataFrame(load("simulated_multilevel_data.dta"))
```

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))

@transform!(df, :identity = categorical(:identity))

@transform!(df, :intervention = categorical(:intervention))
```

```
describe(df) # descriptive statistics
```

9×7 DataFrame

Row	variable Symbol	mean Union...	min Any	median Union...	max Any	nmissing Int64	eltype Union
1	country		1.0		30.0	0	Union{
2	HDI	64.7667	33.0	70.0	87.0	0	Union{
3	family	50.5	1.0	50.5	100.0	0	Union{
4	id		1.1		9.99	0	Union{
5	identity		0.0		1.0	0	Union{
6	intervention		0.0		1.0	0	Union{
7	physical_punishment	2.47867	0.0	2.0	5.0	0	Union{
8	warmth	3.52167	0.0	4.0	7.0	0	Union{
9	outcome	52.4333	29.608	52.449	74.8355	0	Union{

1 column omitted

4.2 Interpretation

Examining descriptive statistics is an important first step in any analysis. It is important to examine your descriptive statistics before skipping ahead to more sophisticated analyses, such as multilevel models.

In examining the descriptive statistics for this data, we get a sense of the data.

- `outcome` has a mean of approximately 52 and ranges from approximately 30 to 75.
- `warmth` and `physical punishment` are both variables that represent the number of times that parents use each of these forms of discipline in a week. The average of the former is about 3.5, while the average of the latter is about 2.5.
- `HDI`, the Human Development Index has an average of about 65, and a wide range.
- `identity` is a categorical variable for a hypothetical identity group, and has values of 0 and 1.
- `intervention` is also a categorical variable, and has values of 0 and 1.

5 Unconditional Models

5.1 Two Level Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

5.1.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \quad (5.1)$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{\text{var}(u_{0j})}{\text{var}(u_{0j}) + \text{var}(e_{ij})} \quad (5.2)$$

In a two level multilevel model, the ICC provides a measure of the proportion of variation attributable to Level 2.

5.1.2 Run Models

5.1.2.1 Stata

```
use simulated_multilevel_data.dta // use data

mixed outcome || country: // unconditional model

estat icc // ICC
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9802.8371

Iteration 1: Log likelihood = -9802.8371

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 3,000

Number of groups = 30

Obs per group:

min = 100

avg = 100.0

max = 100

Wald chi2(0) = .

Prob > chi2 = .

Log likelihood = -9802.8371

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
+						
_cons	52.43327	.3451217	151.93	0.000	51.75685	53.1097

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
+					
country: Identity					
	var(_cons)	3.178658	.9226737	1.799552	5.614658
+					
	var(Residual)	39.46106	1.024013	37.50421	41.52

LR test vs. linear model: chibar2(01) = 166.31 Prob >= chibar2 = 0.0000

Intraclass correlation

Level	ICC	Std. err.	[95% conf. interval]	
+				
country	.0745469	.0201254	.0434963	.1248696

5.1.2.2 R

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
Data: df
```

REML criterion at convergence: 19605.9

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.3844	-0.6655	-0.0086	0.6725	3.6626

Random effects:

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.302	1.817
Residual		39.461	6.282

Number of obs: 3000, groups: country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	52.433	0.351	149.4

```
library(performance)
```

```
performance::icc(fit0) # ICC
```

Intraclass Correlation Coefficient

Adjusted ICC: 0.077
Unadjusted ICC: 0.077

5.1.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,  
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
```

```
df = DataFrame(load("simulated_multilevel_data.dta"))
```

```
@transform!(df, :country = categorical(:country))
```

```
m0 = fit(MixedModel,  
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9802.8371	19605.6742	19611.6742	19611.6822	19629.6933

Variance components:

	Column	Variance	Std.Dev.
country	(Intercept)	3.17863	1.78287
Residual		39.46106	6.28180

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	52.4333	0.345121	151.93	<1e-99

```
icc(m0) # ICC
```

```
0.07454637475695493
```

5.1.3 Interpretation

In each case, the software finds that nearly 8% of the variation in the outcome is explainable by the clustering of the observations in each country.

5.2 Three Level Model

5.2.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + v_{0i} + e_{ij} \quad (5.3)$$

As discussed in the main text, in a three level model, there are two intraclass correlation coefficients (StataCorp, 2023). The formulas for the Intraclass Correlation Coefficient (ICC) are given by (StataCorp, 2023):

$$\text{ICC} = \frac{\text{var}(u_{0j})}{\text{var}(u_{0j}) + \text{var}(v_{0i}) + \text{var}(e_{ij})} \quad (5.4)$$

Following StataCorp (2023), Equation 5.4 is the correlation of responses for person-timepoints from the same country but different persons.

$$\text{ICC} = \frac{\text{var}(u_{0j}) + \text{var}(v_{0i})}{\text{var}(u_{0j}) + \text{var}(v_{0i}) + \text{var}(e_{ij})} \quad (5.5)$$

Again, closely following StataCorp (2023), Equation 5.5 is the correlation of responses for person-timepoints from the same country and same person.

5.2.2 Run Models

5.2.2.1 Stata

```
use simulated_multilevel_longitudinal_data.dta // use data  
  
mixed outcome || country: || id: // unconditional model  
  
estat icc // ICC
```

Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0: Log likelihood = -29058.266
Iteration 1: Log likelihood = -29058.259
Iteration 2: Log likelihood = -29058.259

Computing standard errors ...

Mixed-effects ML regression

Number of obs = 9,000

Grouping information

Group variable		No. of groups	Observations per group		
			Minimum	Average	Maximum
country		30	300	300.0	300
id		3,000	3	3.0	3

Log likelihood = -29058.259

Wald chi2(0) = .
Prob > chi2 = .

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
_cons	53.37768	.3387943	157.55	0.000	52.71366	54.04171

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
country: Identity					
	var(_cons)	3.232092	.8891367	1.885043	5.54174
id: Identity					
	var(_cons)	11.72403	.5747501	10.64996	12.90641
	var(Residual)	28.23424	.5154843	27.24178	29.26287

LR test vs. linear model: chi2(2) = 1314.88 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

Intraclass correlation

Level	ICC	Std. err.	[95% conf. interval]
-------	-----	-----------	----------------------

country	.0748336	.0190847	.0450028	.1219141
id country	.3462837	.0171461	.3134867	.3806097

5.2.2.2 R

In R, the ICC for a three level model is easiest to estimate “by hand”.

```
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

```
library(lme4) # estimate multilevel models

fit0L <- lmer(outcome ~ (1 | country/id),
              data = dfL) # unconditional model

summary(fit0L)
```

Linear mixed model fit by REML ['lmerMod']

Formula: outcome ~ (1 | country/id)

Data: dfL

REML criterion at convergence: 58116.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.7858	-0.6059	-0.0062	0.6017	3.4348

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	11.724	3.424
country	(Intercept)	3.351	1.830
Residual		28.234	5.314

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	53.3777	0.3446	154.9

```
3.351 / (11.724 + 3.351 + 28.234)
```

```
[1] 0.07737422
```

```
(3.351 + 11.724) / (11.724 + 3.351 + 28.234)
```

```
[1] 0.3480801
```

5.2.2.3 Julia

In Julia, the ICC for a three level model is also easiest to estimate “by hand”.

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

```
@transform!(dfL, :country = categorical(:country))
```

```
mOL = fit(MixedModel, @formula(outcome ~  
                                (1 | country) +  
                                (1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + (1 | country) + (1 | id)

	logLik	-2 logLik	AIC	AICc	BIC
	-29058.2592	58116.5184	58124.5184	58124.5229	58152.9384

Variance components:

	Column	Variance	Std.Dev.
id	(Intercept)	11.72401	3.42403
country	(Intercept)	3.23190	1.79775
Residual		28.23426	5.31359

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	53.3777	0.338785	157.56	<1e-99

$$3.23190 / (11.72401 + 3.23190 + 28.23426)$$

0.07482952718176382

$$(3.23190 + 11.72401) / (11.72401 + 3.23190 + 28.23426)$$

0.34628041519632824

5.2.3 Interpretation

Each software suggests that almost 8% of the variation in the outcome is within time points for different individuals within the same country, while almost 35% of the variation in the outcome is within time points for the same individual within the same country.

6 Cross Sectional Multilevel Models

6.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 6.1, we consider a more substantive example.

$$\begin{aligned} \text{outcome}_{ij} = & \beta_0 + \beta_1 \text{warmth}_{ij} + \\ & \beta_2 \text{physical punishment}_{ij} + \\ & \beta_3 \text{identity}_{ij} + \beta_4 \text{intervention}_{ij} + \beta_5 \text{HDI}_j + \\ & u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij} \end{aligned} \tag{6.1}$$

6.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 6.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} \text{var}(u_{0j}) & 0 \\ 0 & \text{var}(u_{1j}) \end{bmatrix} \tag{6.2}$$

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 6.3.

$$\begin{bmatrix} \text{var}(u_{0j}) & \text{cov}(u_{0j}, u_{1j}) \\ \text{cov}(u_{0j}, u_{1j}) & \text{var}(u_{1j}) \end{bmatrix} \tag{6.3}$$

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2023).

Table 6.1: Correlated and Uncorrelated Random Effects

Software	Uncorrelated Random Effects	Correlated Random Effects
Stata	default	add option: <code>, cov(uns)</code>
R	separate random effects from grouping variable with <code> </code>	separate random effects from grouping variable with <code> </code>
Julia	separate terms for each random effect e.g. <code>(1 group) + (0 + x group)</code>	separate random effects from grouping variable with <code> </code> .

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

6.3 Run The Models

Continuous and Categorical Variables

Statistically—as noted in the main text—it is important to be clear on whether independent variables in one’s model are continuous or categorical. *Continuous* variables can be entered straightforwardly into statistical syntax. *Categorical* variables, on the other hand usually require specific attention in statistical software. In Stata, categorical variables are indicated in a statistical model by prefixing them with an `i.`. In R, categorical variables are distinguished by making them into factors e.g. `x <- factor(x)`. In Julia, categorical variables are created by using the `@transform` syntax detailed below.

6.3.1 Stata

6.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

6.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || ///  
country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9626.6279

Iteration 1: Log likelihood = -9626.607

Iteration 2: Log likelihood = -9626.607

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 3,000

Number of groups = 30

Obs per group:

min = 100

avg = 100.0

max = 100

Wald chi2(5) = 334.14

Prob > chi2 = 0.0000

Log likelihood = -9626.607

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
warmth	.8345368	.0637213	13.10	0.000	.7096453	.9594282
physical_punishment	-.9916657	.0797906	-12.43	0.000	-1.148052	-.8352791
1.identity	-.3004767	.2170295	-1.38	0.166	-.7258466	.1248933
1.intervention	.6396427	.2174519	2.94	0.003	.2134448	1.065841
HDI	-.003228	.0199257	-0.16	0.871	-.0422817	.0358256
_cons	51.99991	1.371257	37.92	0.000	49.3123	54.68753

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Independent				
var(warmth)	.0227504	.0257784	.0024689	.2096436
var(_cons)	2.963975	.9737647	1.556777	5.643163

```
var(Residual) | 34.97499 .9097109 33.23668 36.80422
-----
LR test vs. linear model: chi2(2) = 205.74 Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

6.3.2 R

6.3.2.1 Get The Data

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

6.3.2.2 Change Some Variables To Categorical

```
df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)
```

6.3.2.3 Run The Model

Caution

`lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call `library(lmerTest)` to provide p values for `lme4` results.

Tip

R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

```
library(lme4)

library(lmerTest)
```

```
options(scipen = 999)

fit1 <- lmer(outcome ~ warmth + physical_punishment +
             identity + intervention + HDI +
             (1 + warmth || country),
             data = df)

summary(fit1)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]
 Formula: outcome ~ warmth + physical_punishment + identity + intervention + HDI + (1 + warmth || country)
 Data: df

REML criterion at convergence: 19268.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.9774	-0.6563	0.0186	0.6645	3.6730

Random effects:

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.19120	1.786
country.1	warmth	0.02464	0.157
Residual		35.01779	5.918

Number of obs: 3000, groups: country, 30

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	52.011324	1.414976	30.293141	36.758
warmth	0.834562	0.064250	41.896457	12.989
physical_punishment	-0.991893	0.079845	2968.012381	-12.423
identity1	-0.300354	0.217179	2970.108153	-1.383
intervention1	0.639060	0.217603	2971.186718	2.937
HDI	-0.003394	0.020598	27.592814	-0.165

	Pr(> t)
(Intercept)	< 0.0000000000000002 ***
warmth	0.000000000000000277 ***
physical_punishment	< 0.0000000000000002 ***
identity1	0.16678

```

intervention1          0.00334 **
HDI                    0.87030
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

Correlation of Fixed Effects:

```

              (Intr) warmth physc_ idntt1 intrv1
warmth        -0.124
physcl_pnsh    -0.149 -0.003
identity1      -0.072 -0.012 -0.003
interventn1    -0.082  0.034  0.022 -0.018
HDI            -0.943 -0.006  0.009 -0.001  0.000

```

6.3.3 Julia

6.3.3.1 Get The Data

```

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))

```

6.3.3.2 Change Some Variables To Categorical

```

@transform!(df, :country = categorical(:country))

@transform!(df, :identity = categorical(:identity))

@transform!(df, :intervention = categorical(:intervention))

```

6.3.3.3 Run The Model

```

m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
                             identity + intervention + HDI +
                             (1 | country) +
                             (0 + warmth | country)), df)

```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9626.6070	19253.2140	19271.2140	19271.2742	19325.2713

Variance components:

	Column	Variance	Std.Dev.	Corr.
country	(Intercept)	2.963849	1.721583	
	warmth	0.022756	0.150852	.
Residual		34.974984	5.913965	

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	51.9999	1.37124	37.92	<1e-99
warmth	0.834537	0.0637228	13.10	<1e-38
physical_punishment	-0.991665	0.0797906	-12.43	<1e-34
identity: 1.0	-0.300475	0.217029	-1.38	0.1662
intervention: 1.0	0.639641	0.217452	2.94	0.0033
HDI	-0.0032286	0.0199255	-0.16	0.8713

6.4 Interpretation

Models suggest that parental warmth is associated with increases in the beneficial outcome, while physical punishment is associated with decreases in the beneficial outcome. The intervention is associated with increases in the outcome. There is insufficient evidence that either identity group or the Human Development Index are associated with the outcome.

7 Longitudinal Multilevel Models

7.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

7.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \quad (7.1)$$

$$\beta_4 \text{identity}_{itj} + \beta_5 \text{intervention}_{itj} + \beta_6 \text{HDI}_j +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0ij} + v_{1ij} \times \text{time}_{itj} + e_{itj}$$

7.3 Growth Trajectories

Remember, following the discussion in the main text, that in longitudinal multilevel models, the variable for *time* assumes an important role as we are often thinking of a *growth trajectory* over time.

As discussed in the main text, think about a model where *identity* is a (1/0) variable for membership in one of two groups:

$$\text{outcome} = \beta_0 + \beta_t \text{time} + \beta_{\text{identity}} \text{identity} + \beta_{\text{interaction}} \text{identity} \times \text{time} + u_{0i} + e_{it}$$

Then, each identity group has its own intercept and time trajectory:

Table 7.1: Slope and Intercept for Each Group

Group	Intercept	Slope (Time Trajectory)
0	β_0	β_t
1	$\beta_0 + \beta_{\text{identity}}$	$\beta_t + \beta_{\text{interaction}}$

💡 Main Effects and Interactions

Thus, again following the main text, in longitudinal multilevel models, *main effects* modify the *intercept* of the time trajectory, while *interactions with time*, modify the *slope* of the time trajectory. Below, we run models with *main effects only*, then models with *main effects, and interactions with time*.

7.4 Run The Models

⚠ Warning

Remember that we are estimating a model in which time points are nested inside families, who are in turn nested inside countries. For each software package, it is accordingly important to specify the way in which different levels of the data are nested. Pay careful attention to the syntax examples below with regard to `id` and `country`

7.4.1 Stata

7.4.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

7.4.1.2 Run The Models

7.4.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || ///
country: || id: t
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28523.888

Iteration 1: Log likelihood = -28500.378

Iteration 2: Log likelihood = -28500.105

Iteration 3: Log likelihood = -28500.086

Iteration 4: Log likelihood = -28500.086

Computing standard errors ...

Mixed-effects ML regression

Number of obs = 9,000

Grouping information

Group variable		No. of groups	Observations per group		
			Minimum	Average	Maximum
country		30	300	300.0	300
id		3,000	3	3.0	3

Log likelihood = -28500.086

Wald chi2(6) = 1208.49

Prob > chi2 = 0.0000

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
t	.9433792	.0658667	14.32	0.000	.8142828	1.072476
warmth	.9140251	.0379154	24.11	0.000	.8397122	.988338
physical_punishment	-1.00861	.0497766	-20.26	0.000	-1.106171	-.9110499
1.identity	-.1319026	.1516462	-0.87	0.384	-.4291236	.1653184
1.intervention	.8592402	.1519616	5.65	0.000	.5614009	1.15708
HDI	.0007913	.0200615	0.04	0.969	-.0385285	.040111
_cons	50.38381	1.367464	36.84	0.000	47.70363	53.06399

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Identity				
var(_cons)	3.418565	.9268849	2.009349	5.816108

```

id: Independent |
      var(t) | 1.27e-08 2.25e-06 7.3e-160 2.2e+143
      var(_cons) | 8.42116 .4720261 7.545013 9.399046
-----+-----
      var(Residual) | 26.02918 .4753157 25.11405 26.97765
-----+-----
LR test vs. linear model: chi2(3) = 1246.06 Prob > chi2 = 0.0000

```

Note: LR test is conservative and provided only for reference.

7.4.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.identity i.intervention c.HDI) || count:
```

Performing EM optimization ...

Performing gradient-based optimization:

```

Iteration 0: Log likelihood = -28522.21
Iteration 1: Log likelihood = -28498.677
Iteration 2: Log likelihood = -28498.468
Iteration 3: Log likelihood = -28498.31
Iteration 4: Log likelihood = -28498.309

```

Computing standard errors ...

Mixed-effects ML regression Number of obs = 9,000

Grouping information

```

-----+-----
Group variable | No. of      Observations per group
               | groups      Minimum   Average   Maximum
-----+-----
country | 30          300      300.0      300
id | 3,000        3         3.0         3
-----+-----

```

```

Log likelihood = -28498.309
Wald chi2(11) = 1100.25
Prob > chi2 = 0.0000

```

```

-----+-----
outcome | Coefficient Std. err.      z    P>|z|    [95% conf. interval]

```

	t	.7582075	.326177	2.32	0.020	.1189123	1.397503
	warmth	.8170757	.082662	9.88	0.000	.6550611	.9790903
	physical_punishment	-1.009031	.1112932	-9.07	0.000	-1.227162	-.7909007
	1.identity	-.2387167	.3039964	-0.79	0.432	-.8345387	.3571053
	1.intervention	.6607606	.3044503	2.17	0.030	.064049	1.257472
	HDI	.0013614	.0210842	0.06	0.949	-.0399628	.0426856
	c.t#c.warmth	.0483637	.0356074	1.36	0.174	-.0214255	.1181529
	c.t#						
	c.physical_punishment	.0005421	.0494354	0.01	0.991	-.0963496	.0974338
	identity#c.t						
	1	.0554389	.1317444	0.42	0.674	-.2027754	.3136532
	intervention#c.t						
	1	.0992811	.131925	0.75	0.452	-.1592872	.3578493
	c.t#c.HDI	-.0009551	.0038216	-0.25	0.803	-.0084453	.0065352
	_cons	50.83632	1.483548	34.27	0.000	47.92862	53.74402

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
country: Independent					
	var(warmth)	.0106014	.0127458	.0010046	.1118779
	var(_cons)	3.170088	.9153355	1.80009	5.582753
id: Independent					
	var(t)	9.47e-10	2.07e-07	1.5e-195	6.0e+176
	var(_cons)	8.39189	.4724106	7.515234	9.370809
	var(Residual)	26.01583	.4751602	25.10101	26.964

LR test vs. linear model: $\chi^2(4) = 1247.84$ Prob > $\chi^2 = 0.0000$

Note: LR test is conservative and provided only for reference.

7.4.2 R

7.4.2.1 Get The Data

```
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

7.4.2.2 Change Some Variables To Categorical

```
dfL$identity <- factor(dfL$identity)

dfL$intervention <- factor(dfL$intervention)
```

7.4.2.3 Run The Models

Caution

`lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

Tip

R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

7.4.2.3.1 Main Effects Only

```
library(lme4)

library(lmerTest)

options(scipen = 999)

fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
```

```

        identity + intervention + HDI +
        (1 | country/id),
        data = dfL)

summary(fit2A)

```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]

Formula:

outcome ~ t + warmth + physical_punishment + identity + intervention + HDI + (1 | country/id)

Data: dfL

REML criterion at convergence: 57022.7

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.6850	-0.6094	-0.0035	0.6133	3.6792

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.438	2.905
country	(Intercept)	3.675	1.917
Residual		26.036	5.103

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	50.3842343	1.4139114	29.8246912	35.635
t	0.9433806	0.0658755	5998.3764548	14.321
warmth	0.9140307	0.0379336	4745.3497493	24.096
physical_punishment	-1.0087537	0.0497972	6483.6771808	-20.257
identity1	-0.1319548	0.1517350	2968.7828107	-0.870
intervention1	0.8591494	0.1520510	2971.8111995	5.650
HDI	0.0007909	0.0207656	28.0001855	0.038

Pr(>|t|)

(Intercept)	< 0.0000000000000002	***
t	< 0.0000000000000002	***
warmth	< 0.0000000000000002	***
physical_punishment	< 0.0000000000000002	***
identity1	0.385	
intervention1	0.0000000175	***

```

HDI                                0.970
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
              (Intr) t      warmth physc_ idntt1 intrv1
t              -0.092
warmth         -0.091 -0.002
physcl_pnsh    -0.092 -0.007 -0.012
identity1      -0.051  0.000 -0.013 -0.003
interventn1    -0.058  0.000  0.039  0.019 -0.018
HDI            -0.951  0.000 -0.004  0.005  0.000  0.002

```

7.4.2.3.2 Interactions With Time

```

fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
                        identity + intervention + HDI) +
              (1 | country/id),
              data = dfL)

summary(fit2B)

```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]

Formula:

```
outcome ~ t * (warmth + physical_punishment + identity + intervention +
              HDI) + (1 | country/id)
```

Data: dfL

REML criterion at convergence: 57042.8

Scaled residuals:

	Min	1Q	Median	3Q	Max
	-3.7118	-0.6092	-0.0024	0.6150	3.6779

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.436	2.905
country	(Intercept)	3.675	1.917
Residual		26.046	5.104

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	50.7590272	1.5518360	43.2608620	32.709
t	0.7552909	0.3263028	6176.7440549	2.315
warmth	0.8170912	0.0805355	8274.9995422	10.146
physical_punishment	-1.0097729	0.1113557	8084.6084915	-9.068
identity1	-0.2446453	0.3041604	8695.8966197	-0.804
intervention1	0.6604671	0.3046286	8697.0843430	2.168
HDI	0.0026692	0.0221295	36.1037733	0.121
t:warmth	0.0486211	0.0356217	6404.8723333	1.365
t:physical_punishment	0.0004964	0.0494590	6753.0158441	0.010
t:identity1	0.0563140	0.1318043	5993.4518022	0.427
t:intervention1	0.0995037	0.1319917	5994.1433001	0.754
t:HDI	-0.0009379	0.0038233	5993.9090880	-0.245

Pr(>|t|)

(Intercept)	<0.0000000000000002 ***
t	0.0207 *
warmth	<0.0000000000000002 ***
physical_punishment	<0.0000000000000002 ***
identity1	0.4212
intervention1	0.0302 *
HDI	0.9047
t:warmth	0.1723
t:physical_punishment	0.9920
t:identity1	0.6692
t:intervention1	0.4510
t:HDI	0.8062

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr)	t	warmth	physc_	idntt1	intrv1	HDI	t:wrmt	t:phy_
t	-0.421								
warmth	-0.178	0.331							
physcl_pnsh	-0.190	0.360	-0.005						
identity1	-0.093	0.166	-0.013	-0.002					
interventn1	-0.107	0.192	0.039	0.019	-0.017				
HDI	-0.925	0.264	-0.007	0.012	-0.001	0.003			
t:warmth	0.158	-0.377	-0.882	0.001	0.011	-0.035	0.006		
t:physcl_pn	0.170	-0.402	0.004	-0.894	-0.001	-0.017	-0.010	-0.003	
t:identity1	0.081	-0.192	0.011	0.000	-0.867	0.014	0.001	-0.013	0.002
t:intrvntn1	0.093	-0.222	-0.035	-0.017	0.014	-0.867	-0.003	0.041	0.019

```

t:HDI          0.322 -0.765  0.015 -0.027  0.002 -0.007 -0.346 -0.016  0.029
               t:dnt1 t:ntr1
t
warmth
physcl_pnsh
identity1
intervntn1
HDI
t:warmth
t:physcl_pn
t:identity1
t:intrvntn1 -0.016
t:HDI       -0.002  0.008

```

7.4.3 Julia

7.4.3.1 Get The Data

```

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))

```

7.4.3.2 Change Some Variables To Categorical

```

@transform!(dfL, :country = categorical(:country))

@transform!(dfL, :identity = categorical(:identity))

@transform!(dfL, :intervention = categorical(:intervention))

```

7.4.3.3 Run The Models

7.4.3.3.1 Main Effects Only

```

m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                               physical_punishment +
                               identity + intervention +
                               HDI +

```

```
(1 | country) +
(0 + warmth | country) +
(1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

```
outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | country)
logLik   -2 logLik      AIC      AICc      BIC
-28499.6031 56999.2063 57021.2063 57021.2356 57099.3610
```

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.387214	2.896069	
country	(Intercept)	3.167143	1.779647	
	warmth	0.010762	0.103739	.
Residual		26.027363	5.101702	

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	50.4673	1.33833	37.71	<1e-99
t	0.943864	0.0658717	14.33	<1e-45
warmth	0.913496	0.0423744	21.56	<1e-99
physical_punishment	-1.0079	0.0497622	-20.25	<1e-90
identity: 1.0	-0.127692	0.151583	-0.84	0.3996
intervention: 1.0	0.858997	0.151909	5.65	<1e-07
HDI	-0.000566026	0.0196439	-0.03	0.9770

7.4.3.3.2 Interactions With Time

```
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
physical_punishment +
identity + intervention +
HDI) +
(1 | country) +
(0 + warmth | country) +
(1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

```
outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + t & warmth
      logLik   -2 logLik       AIC       AICc       BIC
-28498.3091  56996.6182  57028.6182  57028.6788  57142.2979
```

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.391746	2.896851	
country	(Intercept)	3.170032	1.780458	
	warmth	0.010609	0.102999	.
Residual		26.015906	5.100579	

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	50.8364	1.48355	34.27	<1e-99
t	0.758209	0.326177	2.32	0.0201
warmth	0.817076	0.0826636	9.88	<1e-22
physical_punishment	-1.00903	0.111293	-9.07	<1e-18
identity: 1.0	-0.238714	0.303996	-0.79	0.4323
intervention: 1.0	0.660761	0.30445	2.17	0.0300
HDI	0.00136065	0.0210842	0.06	0.9485
t & warmth	0.0483635	0.0356074	1.36	0.1744
t & physical_punishment	0.000542203	0.0494355	0.01	0.9912
t & identity: 1.0	0.0554385	0.131745	0.42	0.6739
t & intervention: 1.0	0.0992809	0.131925	0.75	0.4517
t & HDI	-0.000955067	0.00382162	-0.25	0.8027

7.5 Interpretation

The *main effects only model* suggests that time is associated with increases in the outcome. In the main effects model, main effects other than time, indicate whether a particular variable is associated with higher or lower intercepts of the time trajectory, at the beginning of the study time. Warmth is again associated with increases in the outcome, while physical punishment is associated with decreases in the outcome. Identity is again not associated with the outcome, while the intervention is associated with higher levels of the outcome. The Human Development Index is again not associated with the outcome.

The second model adds interactions with time to the first model. Results are largely similar to the prior model. However, here we not only examine whether main effects other than time

are associated with higher or lower time trajectories, but also whether particular variables are associated with differences in the slope of the time trajectory. In this case, we find insufficient evidence that any independent variable is associated with changes in the slope of the time trajectory.

Which Interactions To Test?

In this example—for the sake of illustration—I test the interaction of *every* independent variable with time. In many cases, it may make sense to test only one or two interactions of time with particular variables of key interest. Also, after finding, as I did in this model, that none of the interactions of other independent variables with time are significant, I might report the model with interactions, or might report only the results of the model with only main effects.

It may be illustrative to imagine how we would interpret the results had a particular interaction term been statistically significant. Let us consider one of the interaction terms with the largest coefficient, **intervention#time**. The interaction of the intervention with time is positive. Had this coefficient been statistically significant, it would have indicated that the intervention was associated with more rapid increases in the outcome over time *in addition to* the fact that the intervention is associated with higher initial levels of the outcome.

8 Multilevel Logistic Regression

Below, I detail the procedure for multilevel logistic regression models in Stata and R.

8.1 The Data

The data employed in these examples are the cross-sectional data described in Section 1.2.

8.2 The Equation

To explain statistical syntax for Stata and R, I consider the general case of a multilevel model with *categorical* dependent variable y , independent variables \mathbf{x} and \mathbf{z} , clustering variable **group**, and a random slope for \mathbf{x} . i is the index for the person, while j is the index for the group.

$$\ln\left(\frac{p(y)}{1-p(y)}\right) = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} \quad (8.1)$$

Correlated and Uncorrelated Random Effects in Logistic Regression

The reader is referred to the discussion of correlated and uncorrelated random effects in Section 6.2

8.2.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 8.1 is:

```
melogit y x z || group:
```

8.2.2 R

In R `lme4`, the syntax for a multilevel model of the form described in Equation 8.1 is:

```
library(lme4)

glmer(y ~ x + z + (1 | group), data = ...)
```

8.3 Run Models

Less Variation In Logistic Than Linear Models

Note that in *logistic* regression models, there is less variation to work with—due to the fact that the outcome is 1/0, than there is in *linear* models. Therefore, in the models below, I do not attempt to estimate a random slope in addition to a random intercept, as I do in Section 6.

8.3.1 Stata

8.3.1.1 Get The Data

```
use simulated_multilevel_data.dta

generate outcome_category = outcome > 52 // dichotomous outcome
```

8.3.1.2 Run The Model

As suggested in Equation 8.1, odds ratios are obtained by exponentiating the β coefficients: e^β . Stata provides the odds ratios automatically with option `, or`.

```
melogit outcome_category warmth physical_punishment i.identity i.intervention HDI || ///
country:, or
```

Fitting fixed-effects model:

```
Iteration 0:  Log likelihood = -1965.6466
Iteration 1:  Log likelihood = -1963.7805
```

Iteration 2: Log likelihood = -1963.7791
 Iteration 3: Log likelihood = -1963.7791

Refining starting values:

Grid node 0: Log likelihood = -1908.9697

Fitting full model:

Iteration 0: Log likelihood = -1908.9697 (not concave)
 Iteration 1: Log likelihood = -1903.703
 Iteration 2: Log likelihood = -1902.2851
 Iteration 3: Log likelihood = -1901.3176
 Iteration 4: Log likelihood = -1901.2662
 Iteration 5: Log likelihood = -1901.2661

Mixed-effects logistic regression Number of obs = 3,000
 Group variable: country Number of groups = 30

Obs per group:
 min = 100
 avg = 100.0
 max = 100

Integration method: mvaghermite Integration pts. = 7

Log likelihood = -1901.2661 Wald chi2(5) = 219.75
 Prob > chi2 = 0.0000

outcome_category		Odds ratio	Std. err.	z	P> z	[95% conf. interval]	
warmth		1.292603	.0278565	11.91	0.000	1.239142	1.34837
physical_punishment		.7524276	.0222773	-9.61	0.000	.7100077	.797382
1.identity		.9517262	.0748541	-0.63	0.529	.8157636	1.11035
1.intervention		1.191581	.0940459	2.22	0.026	1.020803	1.390929
HDI		.9990491	.0061371	-0.15	0.877	.9870928	1.01115
_cons		.9115548	.3901774	-0.22	0.829	.3939478	2.109244
-----+-----							
country							
var(_cons)		.2897697	.0880892			.1596945	.5257944
-----+-----							

Note: Estimates are transformed only in the first equation to odds ratios.
 Note: _cons estimates baseline odds (conditional on zero random effects).

LR test vs. logistic model: $\text{chibar2}(01) = 125.03$ Prob $\geq \text{chibar2} = 0.0000$

8.3.2 R

8.3.2.1 Get The Data

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")

df$outcome_category <- 0 # initialize to 0

df$outcome_category[df$outcome > 52] <- 1 # dichotomous outcome
```

8.3.2.2 Change Some Variables To Categorical

```
df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)
```

8.3.2.3 Run The Model

Caution

`lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call `library lmerTest` to provide p values for `lme4` results.

Tip

R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

```
library(lme4)

library(lmerTest)
```

```
options(scipen = 999)

fit3 <- glmer(outcome_category ~ warmth + physical_punishment +
              identity + intervention + HDI +
              (1 | country),
              family = binomial(link = "logit"),
              data = df)

summary(fit3)
```

```
Generalized linear mixed model fit by maximum likelihood (Laplace
Approximation) [glmerMod]
Family: binomial ( logit )
Formula: outcome_category ~ warmth + physical_punishment + identity +
intervention + HDI + (1 | country)
Data: df
```

AIC	BIC	logLik	deviance	df.resid
3816.6	3858.7	-1901.3	3802.6	2993

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.0109	-0.8798	0.4369	0.8428	2.8223

Random effects:

Groups	Name	Variance	Std.Dev.
country	(Intercept)	0.2894	0.5379

Number of obs: 3000, groups: country, 30

Fixed effects:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-0.0926371	0.4277643	-0.217	0.8286
warmth	0.2566693	0.0215443	11.914	<0.0000000000000002 ***
physical_punishment	-0.2844595	0.0295990	-9.610	<0.0000000000000002 ***
identity1	-0.0494765	0.0786286	-0.629	0.5292
intervention1	0.1752879	0.0789030	2.222	0.0263 *
HDI	-0.0009513	0.0061388	-0.155	0.8769

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

```

              (Intr) warmth physc_ idntt1 intrv1
warmth        -0.158
physcl_pnsh   -0.170 -0.082
identity1     -0.086 -0.014  0.002
interventn1   -0.102  0.055  0.006 -0.020
HDI           -0.930 -0.007  0.012 -0.001  0.004

```

8.3.2.4 Calculate Odds Ratios

R requires one to use a bit of extra syntax to extract the odds ratios. As suggested in Equation 8.1, odds ratios are obtained by exponentiating the β coefficients: e^β .

```
exp(fixef(fit3))
```

(Intercept)	warmth	physical_punishment	identity1
0.9115242	1.2926176	0.7524208	0.9517275
intervention1	HDI		
1.1915893	0.9990492		

9 # Models With Three or More Levels and Cross-Classified Models

9.1 Introduction

A two level multilevel model imagines that *Level 1* units are nested in *Level 2* units. A three level multilevel model imagines that *Level 1* units are nested in *Level 2* units, which are in turn nested in *Level 3*. As more levels are added to the model (e.g. *Level 4*), we imagine all of these levels to be hierarchically nested.

A *cross classified* model imagines that the nesting is not hierarchical, but rather that there are two sets of clusters or nestings in which individuals may be nested.

Below, I describe the use of Stata, R, and Julia to estimate these models.

9.2 Three Or More Levels

9.2.1 The Data

I use the *longitudinal* data from *Multilevel Thinking* (Section 1.2) to which I have added an extra level of *United Nations Region* (Arel-Bundock et al., 2018). This data thus requires a four level model.

9.2.2 The Equation

$$\text{outcome}_{itjk} = \beta_0 + \beta_1 \text{parental warmth}_{itjk} + \beta_2 \text{physical punishment}_{itjk} + \beta_3 \text{time}_{itjk} + \quad (9.1)$$

$$\beta_4 \text{identity}_{itjk} + \beta_5 \text{intervention}_{itjk} + \beta_6 \text{HDI}_{itjk} +$$

$$w_{0k} + u_{0j} + v_{0i} + e_{itjk}$$

Here we imagine w_{0k} (region), u_{0j} (country) and v_{0i} (family) are hierarchically nested effects.

9.2.3 Run The Models

9.2.3.1 Stata

9.2.3.1.1 Get The Data

```
use "fourlevel.dta", clear
```

9.2.3.1.2 Unconditional Model

```
mixed outcome || UNregion: || country: || family:
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -29061.686

Iteration 1: Log likelihood = -29061.679

Iteration 2: Log likelihood = -29061.679

Computing standard errors ...

Mixed-effects ML regression

Number of obs = 9,000

Grouping information

Group variable		No. of groups	Observations per group		
			Minimum	Average	Maximum
UNregion		5	600	1,800.0	3,600
country		30	300	300.0	300
family		3,000	3	3.0	3

Log likelihood = -29061.679

Wald chi2(0) = .

Prob > chi2 = .

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
_cons	54.05906	.987367	54.75	0.000	52.12385	55.99426

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
UNregion: Identity					
	var(_cons)	4.172687	3.187885	.9334852	18.65194
country: Identity					
	var(_cons)	2.849348	.8710225	1.565093	5.187414
family: Identity					
	var(_cons)	11.72403	.57475	10.64997	12.90641
	var(Residual)	28.23424	.5154842	27.24177	29.26286
LR test vs. linear model: chi2(3) = 1843.44			Prob > chi2 = 0.0000		

Note: LR test is conservative and provided only for reference.

9.2.3.1.3 Conditional Model

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI ///
|| UNregion: || country: || id:
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28503.082

Iteration 1: Log likelihood = -28503.039

Iteration 2: Log likelihood = -28503.039

Computing standard errors ...

Mixed-effects ML regression Number of obs = 9,000

Grouping information

Group variable		No. of groups	Observations per group		
			Minimum	Average	Maximum

UNregion	5	600	1,800.0	3,600
country	30	300	300.0	300
id	3,000	3	3.0	3

Log likelihood = -28503.039

Wald chi2(6) = 1209.42
Prob > chi2 = 0.0000

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
t	.9433791	.0658667	14.32	0.000	.8142827	1.072476
warmth	.9140704	.0379156	24.11	0.000	.8397571	.9883837
physical_punishment	-1.008615	.0497772	-20.26	0.000	-1.106176	-.9110531
1.identity	-.1332133	.1516437	-0.88	0.380	-.4304294	.1640028
1.intervention	.8589263	.1519619	5.65	0.000	.5610865	1.156766
HDI	.0148561	.0196605	0.76	0.450	-.0236777	.0533899
_cons	50.16426	1.675219	29.94	0.000	46.88089	53.44763

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
UNregion: Identity				
var(_cons)	4.722007	3.585939	1.065898	20.91884
country: Identity				
var(_cons)	2.863495	.8656459	1.583342	5.178668
id: Identity				
var(_cons)	8.421131	.4711947	7.546445	9.397199
var(Residual)	26.02919	.4752587	25.11417	26.97755

LR test vs. linear model: chi2(3) = 1844.00 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

9.2.3.2 R

9.2.3.2.1 Get The Data

```
library(haven)

df4 <- read_dta("fourlevel.dta")
```

9.2.3.2.2 Change Some Variables To Categorical

```
df4$identity <- factor(df4$identity)

df4$intervention <- factor(df4$intervention)
```

9.2.3.2.3 Unconditional Model

Caution

`lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

Tip

R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

```
library(lme4)

library(lmerTest)

options(scipen = 999)

fit4A <- lmer(outcome ~ (1 | UNregion/country/id),
              data = df4)

summary(fit4A)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: outcome ~ (1 | UNregion/country/id)
Data: df4
```


REML criterion at convergence: 58121.4

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.7850	-0.6064	-0.0047	0.6020	3.4399

Random effects:

Groups	Name	Variance	Std.Dev.
id:(country:UNregion)	(Intercept)	11.724	3.424
country:UNregion	(Intercept)	2.842	1.686
UNregion	(Intercept)	5.478	2.340
Residual		28.234	5.314

Number of obs: 9000, groups:

id:(country:UNregion), 3000; country:UNregion, 30; UNregion, 5

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	54.061	1.112	3.777	48.6	0.00000201 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

9.2.3.2.4 Conditional Model

```
fit4B <- lmer(outcome ~ t + warmth + physical_punishment +  
              identity + intervention + HDI +  
              (1 | UNregion/country/id),  
              data = df4)  
  
summary(fit4B)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]

Formula:

outcome ~ t + warmth + physical_punishment + identity + intervention +
HDI + (1 | UNregion/country/id)

Data: df4

REML criterion at convergence: 57026.4

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.6846	-0.6096	-0.0038	0.6138	3.6850

Random effects:

Groups	Name	Variance	Std.Dev.
id:(country:UNregion)	(Intercept)	8.438	2.905
country:UNregion	(Intercept)	2.979	1.726
UNregion	(Intercept)	6.178	2.486
Residual		26.036	5.103

Number of obs: 9000, groups:

id:(country:UNregion), 3000; country:UNregion, 30; UNregion, 5

Fixed effects:

	Estimate	Std. Error	df	t value
(Intercept)	50.11857	1.78086	15.79112	28.143
t	0.94338	0.06588	5998.37756	14.321
warmth	0.91406	0.03793	4745.28492	24.096
physical_punishment	-1.00876	0.04980	6483.46337	-20.257
identity1	-0.13324	0.15173	2969.00938	-0.878
intervention1	0.85872	0.15205	2971.85430	5.648
HDI	0.01560	0.02006	24.39852	0.778

Pr(>|t|)

(Intercept)	0.00000000000000641	***
t	< 0.0000000000000002	***
warmth	< 0.0000000000000002	***
physical_punishment	< 0.0000000000000002	***
identity1	0.380	
intervention1	0.00000001780521096	***
HDI	0.444	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr) t	warmth	physc_ idntt1	intrv1
t	-0.073			
warmth	-0.071	-0.002		
physcl_pnsh	-0.073	-0.007	-0.012	
identity1	-0.040	0.000	-0.013	-0.003
interventn1	-0.045	0.000	0.039	0.019
HDI	-0.738	0.000	-0.005	0.005

9.2.3.3 Julia

9.2.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df4 = DataFrame(load("fourlevel.dta"))
```

9.2.3.3.2 Change Some Variables To Categorical

```
@transform!(df4, :country = categorical(:country))

@transform!(df4, :UNregion = categorical(:UNregion))

@transform!(df4, :identity = categorical(:identity))

@transform!(df4, :intervention = categorical(:intervention))
```

9.2.3.3.3 Unconditional Model

```
m4A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                physical_punishment +
                                identity + intervention +
                                HDI +
                                (1 | UNregion) +
                                (1 | country) +
                                (1 | id)), df4)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 UNreg				
logLik	-2 logLik	AIC	AICc	BIC
-28503.0394	57006.0787	57028.0787	57028.1081	57106.2335

Variance components:

	Column	Variance	Std.Dev.
id	(Intercept)	8.42110	2.90191
country	(Intercept)	2.86347	1.69218
UNregion	(Intercept)	4.72082	2.17274
Residual		26.02921	5.10188

Number of obs: 9000; levels of grouping factors: 3000, 30, 5

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	50.1643	1.67514	29.95	<1e-99
t	0.943379	0.0658668	14.32	<1e-45
warmth	0.91407	0.0379156	24.11	<1e-99
physical_punishment	-1.00861	0.0497772	-20.26	<1e-90
identity	-0.133213	0.151644	-0.88	0.3797
intervention	0.858927	0.151962	5.65	<1e-07
HDI	0.0148553	0.0196604	0.76	0.4499

9.2.3.3.4 Conditional Model

```
m4B = fit(MixedModel, @formula(outcome ~ t + warmth +
                                physical_punishment +
                                identity + intervention +
                                HDI +
                                (1 | UNregion) +
                                (1 | country) +
                                (1 | id)), df4)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | UNregion)

logLik	-2 logLik	AIC	AICc	BIC
-28503.0394	57006.0787	57028.0787	57028.1081	57106.2335

Variance components:

	Column	Variance	Std.Dev.
id	(Intercept)	8.42110	2.90191
country	(Intercept)	2.86347	1.69218
UNregion	(Intercept)	4.72082	2.17274
Residual		26.02921	5.10188

Number of obs: 9000; levels of grouping factors: 3000, 30, 5

Fixed-effects parameters:

Coef.	Std. Error	z	Pr(> z)
-------	------------	---	----------

(Intercept)	50.1643	1.67514	29.95	<1e-99
t	0.943379	0.0658668	14.32	<1e-45
warmth	0.91407	0.0379156	24.11	<1e-99
physical_punishment	-1.00861	0.0497772	-20.26	<1e-90
identity	-0.133213	0.151644	-0.88	0.3797
intervention	0.858927	0.151962	5.65	<1e-07
HDI	0.0148553	0.0196604	0.76	0.4499

9.2.4 Interpretation

There is group level variation attributable to individual, country, and region.

As in other models, parental warmth, and participation in the intervention are associated with increases in the outcome. Parental use of physical punishment is associated with decreases in the outcome.

9.3 Cross-Classified Models

9.3.1 The Data

I use the *cross-sectional* data from *Multilevel Thinking* (Section 1.2) to which I have added an extra level of a hypothetical language.

9.3.2 The Equation

$$\text{outcome}_{ijm} = \beta_0 + \beta_1 \text{parental warmth}_{ijm} + \beta_2 \text{physical punishment}_{ijm} + \beta_3 \text{time}_{ijm} + (9.2)$$

$$\beta_4 \text{identity}_{ijm} + \beta_5 \text{intervention}_{ijm} + \beta_6 \text{HDI}_{ijm} +$$

$$u_{0j} + m_{0m} + e_{ijm}$$

Here u_{0j} (country) and m_{0m} (language) are not nested hierarchically, but are *cross classified*.

9.3.3 Run The Models

9.3.3.1 Stata

9.3.3.1.1 Get The Data

```
use "crossclassified.dta", clear
```

9.3.3.1.2 Unconditional Model

```
mixed outcome || _all: R.country || _all: R.language
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9835.8123

Iteration 1: Log likelihood = -9835.8111

Iteration 2: Log likelihood = -9835.8111

Computing standard errors ...

Mixed-effects ML regression

Group variable: _all

Number of obs = 3,000

Number of groups = 1

Obs per group:

min = 3,000

avg = 3,000.0

max = 3,000

Wald chi2(0) = .

Prob > chi2 = .

Log likelihood = -9835.8111

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
_cons	52.43187	.3590214	146.04	0.000	51.7282	53.13554

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]
_all: Identity			

	var(R.country)		3.177791	.9244633	1.796798	5.620198
-----+-----						
_all: Identity						
	var(R.language)		.9566314	.3284087	.4881235	1.87482
-----+-----						
	var(Residual)		39.62877	1.045619	37.63148	41.73206

LR test vs. linear model: chi2(2) = 180.84					Prob > chi2 = 0.0000	

Note: LR test is conservative and provided only for reference.

9.3.3.1.3 Conditional Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || _all: R.country ||
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -9663.2195

Iteration 1: Log likelihood = -9663.2194

Computing standard errors ...

Mixed-effects ML regression

Group variable: _all

Number of obs = 3,000

Number of groups = 1

Obs per group:

min = 3,000

avg = 3,000.0

max = 3,000

Wald chi2(5) = 367.04

Log likelihood = -9663.2194

Prob > chi2 = 0.0000

	outcome		Coefficient	Std. err.	z	P> z	[95% conf. interval]
-----+-----							
	warmth		.8331461	.0579811	14.37	0.000	.7195052 .946787
physical_punishment			-.9979749	.080268	-12.43	0.000	-1.155297 -.8406525
1.identity			-.2922428	.2191421	-1.33	0.182	-.7217534 .1372678
1.intervention			.6097458	.2195139	2.78	0.005	.1795064 1.039985
HDI			-.0015879	.0204157	-0.08	0.938	-.0416021 .0384262
_cons			51.92255	1.411069	36.80	0.000	49.15691 54.6882

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
<hr/>				
_all: Identity				
var(R.country)	3.361218	.9603072	1.920024	5.884192
<hr/>				
_all: Identity				
var(R.language)	1.121946	.3269535	.6337502	1.986214
<hr/>				
var(Residual)	35.11959	.9263999	33.35002	36.98306
<hr/>				
LR test vs. linear model: chi2(2) = 227.02			Prob > chi2 = 0.0000	

Note: LR test is conservative and provided only for reference.

9.3.3.2 R

9.3.3.2.1 Get The Data

```
library(haven)

dfCC <- read_dta("crossclassified.dta")
```

9.3.3.2.2 Change Some Variables To Categorical

```
dfCC$identity <- factor(dfCC$identity)

dfCC$intervention <- factor(dfCC$intervention)
```

9.3.3.2.3 Unconditional Model

```
library(lme4)

library(lmerTest)

options(scipen = 999)

fitCC_A <- lmer(outcome ~
```



```

      (1 | country) +
      (1 | language),
  data = dfCC)

summary(fitCC_A)

```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [lmerModLmerTest]

Formula: outcome ~ (1 | country) + (1 | language)
Data: dfCC

REML criterion at convergence: 19671.8

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.3899	-0.6602	-0.0104	0.6798	3.6924

Random effects:

Groups	Name	Variance	Std.Dev.
language	(Intercept)	0.9604	0.980
country	(Intercept)	3.2919	1.814
Residual		39.6276	6.295

Number of obs: 3000, groups: language, 100; country, 30

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	52.4319	0.3643	33.4284	143.9	<0.0000000000000002 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

9.3.3.2.4 Conditional Model

```

fitCC_B <- lmer(outcome ~ t + warmth + physical_punishment +
  identity + intervention + HDI +
  (1 | country) +
  (1 | language),
  data = dfCC)

```

Error in model.frame.default(data = dfCC, drop.unused.levels = TRUE, formula = outcome ~ : i

```
summary(fitCC_B)
```

Error in h(simpleError(msg, call)): error in evaluating the argument 'object' in selecting a

9.3.3.3 Julia

9.3.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
dfCC = DataFrame(load("crossclassified.dta"))
```

9.3.3.3.2 Change Some Variables To Categorical

```
@transform!(dfCC, :country = categorical(:country))  
  
@transform!(dfCC, :language = categorical(:language))  
  
@transform!(dfCC, :identity = categorical(:identity))  
  
@transform!(dfCC, :intervention = categorical(:intervention))
```

9.3.3.3.3 Unconditional Model

```
mCCA = fit(MixedModel, @formula(outcome ~  
                                (1 | country) +  
                                (1 | language)), dfCC)
```

Linear mixed model fit by maximum likelihood
outcome ~ 1 + (1 | country) + (1 | language)
logLik -2 logLik AIC AICc BIC
-9835.8111 19671.6222 19679.6222 19679.6356 19703.6477

Variance components:

	Column	Variance	Std.Dev.
language	(Intercept)	0.956631	0.978075
country	(Intercept)	3.177768	1.782629
Residual		39.628773	6.295139

Number of obs: 3000; levels of grouping factors: 100, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	52.4319	0.35902	146.04	<1e-99

9.3.3.3.4 Conditional Model

```
mCCA = fit(MixedModel, @formula(outcome ~ warmth +  
                                physical_punishment +  
                                identity + intervention +  
                                HDI +  
                                (1 | country) +  
                                (1 | language)), dfCC)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country) -

	logLik	-2 logLik	AIC	AICc	BIC
	-9663.2194	19326.4388	19344.4388	19344.4990	19398.4962

Variance components:

	Column	Variance	Std.Dev.
language (Intercept)		1.12193	1.05921
country (Intercept)		3.36119	1.83335
Residual		35.11960	5.92618

Number of obs: 3000; levels of grouping factors: 100, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z)
(Intercept)	51.9226	1.41106	36.80	<1e-99
warmth	0.833146	0.0579811	14.37	<1e-46
physical_punishment	-0.997975	0.080268	-12.43	<1e-34
identity	-0.292243	0.219142	-1.33	0.1823
intervention	0.609746	0.219514	2.78	0.0055
HDI	-0.00158794	0.0204156	-0.08	0.9380

9.3.4 Interpretation

There is group level variation attributable to both language and country.

As in other models, parental warmth, and participation in the intervention are associated with increases in the outcome. Parental use of physical punishment is associated with decreases in the outcome.

10 Reshaping Data

10.1 Introduction

Cross-sectional analyses (Section 6) make use of data in *wide* format: every row is a person, or family, and every person, or family, has a single row of data.

Longitudinal analyses (Section 7) make use of *long* data: every row is a person-timepoint, or family-timepoint, and every person, or family, has multiple rows of data.

Data therefore sometimes need to be *reshaped*, most often from *wide* format to *long* format. Almost any software that is capable of estimating multilevel models is capable of reshaping data.

Below, I detail the procedure for reshaping data in Stata and R.

10.2 Data in Wide Format

Note

The data below are in *wide* format.

Every individual in the data set has a *single row of data*. Every row in the data set is an *individual* or *family*.

Table 10.1: Data in Wide Format

Table 10.1: Table continues below

id	physical_punishment1	warmth1	outcome1	physical_punishment2
1.1	3	3	57.47	3
1.10	2	0	62.9	3
1.100	2	5	62.71	1
1.11	4	4	55.61	2
1.12	5	4	41.15	5
1.13	4	5	63.66	3

Table 10.2: Data in Wide Format

Table 10.2: Table continues below

warmth2	outcome2	physical_punishment3	warmth3	outcome3	country	HDI
4	55.06	1	2	58.77	1	69
0	56.67	2	0	68.22	1	69
4	51.58	2	5	55.51	1	69
5	50.9	3	3	48.02	1	69
5	45.4	3	4	55.86	1	69
3	64.81	3	3	58.3	1	69

Table 10.3: Data in Wide Format

family	identity	intervention
1	1	0
10	1	0
100	1	1
11	1	1
12	0	0
13	0	1

10.3 Data Management

Because reshaping your data dramatically changes the structure of your data...

1. It is a good idea to have your raw data saved in a location where it will not be changed, and can be retrieved again if the reshape command does not work correctly, or if you simply want to modify your reshaping data workflow. (CF Section 2.3)
2. Usually we want to work with only a *subset* of your data, to keep only the data in which we are interested.
 - In Stata, the command to keep only variables of interest would be: `keep y x z t`.
 - In R, one option would be to use the subset function: `mysubset <- subset(mydata, select = c(y, x, z, t))`

10.4 Reshaping Data From Wide To Long

Usually, we are most interested in reshaping data from *wide* to *long*.

10.4.1 Stata

In Stata, I only list variables that vary over time, or are *time varying*. Stata assumes that variables that are *not listed* do *not vary over time*, or are *time invariant*.

Notice also that our *time varying* data are in the *stub-time* format, e.g. `warmth1`, `warmth2`, `physical_punishment1` `physical_punishment2`, etc. Because the variables are named in this way, Stata knows to use the *stub* (e.g. `warmth`) as the variable name, and the numeric value, (e.g. 1, 2, 3) as the timepoint.

The `id` variable, whatever it is named, has to uniquely identify the observations. A useful Stata command here is `isid`, e.g. `isid id`. If your `id` variable is not unique, it is often due to missing values. `drop if id == .` usually solves the problem (assuming that your `id` variable is indeed named `id`, and not something else).

```
use simulated_multilevel_longitudinal_data_WIDE.dta, clear

describe

reshape long outcome physical_punishment warmth, i(id) j(wave)
```

Contains data from `simulated_multilevel_longitudinal_data_WIDE.dta`

Observations: 3,000
Variables: 15 3 Jul 2024 14:29

Variable name	Storage type	Display format	Value label	Variable label
id	str7	%9s		unique country family id
physical_puni~1	float	%9.0g		1 physical_punishment
warmth1	float	%9.0g		1 warmth
outcome1	float	%9.0g		1 outcome
physical_puni~2	float	%9.0g		2 physical_punishment
warmth2	float	%9.0g		2 warmth
outcome2	float	%9.0g		2 outcome
physical_puni~3	float	%9.0g		3 physical_punishment
warmth3	float	%9.0g		3 warmth
outcome3	float	%9.0g		3 outcome
country	float	%9.0g		country id
HDI	float	%9.0g		Human Development Index
family	float	%9.0g		family id
identity	float	%9.0g		hypothetical identity group variable

intervention	float	%9.0g	recieved intervention
--------------	-------	-------	-----------------------

Sorted by: id

(j = 1 2 3)

Data	Wide	->	Long
Number of observations	3,000	->	9,000
Number of variables	15	->	10
j variable (3 values)		->	wave
xij variables:			
	outcome1 outcome2 outcome3	->	outcome
physical_punishment1	physical_punishment2	physical_punishment3	->physical_punishment
	warmth1 warmth2 warmth3	->	warmth

10.4.2 R

In R, I only list variables that vary over time, or are *time varying*.

Notice also that our *time varying* data are in the *stub-time* format, e.g. `warmth1`, `warmth2`, `physical_punishment1` `physical_punishment2`, etc. In order to facilitate reshaping the data, it is helpful in R to insert an underscore (`_`) to separate the *stub* from the *time* variable.

```
library(dplyr) # data wrangling
```

```
library(tidyr) # tidy (reshape data)
```

```
# rename variables with "_" separator
```

```
df <- simulated_multilevel_longitudinal_data_WIDE %>%
  rename(outcome_1 = outcome1,
         outcome_2 = outcome2,
         outcome_3 = outcome3,
         physical_punishment_1 = physical_punishment1,
         physical_punishment_2 = physical_punishment2,
         physical_punishment_3 = physical_punishment3,
         warmth_1 = warmth1,
         warmth_2 = warmth2,
         warmth_3 = warmth3)
```



```
# pivot_longer() to long data

dfL <- df %>%
  pivot_longer(cols = c(outcome_1,
                        outcome_2,
                        outcome_3,
                        physical_punishment_1,
                        physical_punishment_2,
                        physical_punishment_3,
                        warmth_1,
                        warmth_2,
                        warmth_3),
              names_pattern = "(.+)_(.+)",
              names_to = c(".value", "t"))
```

10.5 Data in Long Format

i Note

The data below are in *long* format.

Every individual/family in the data set has a *multiple rows of data*. Every row in the data set is an *individual-timepoint* or *family-timepoint*.

Table 10.4: Data in Long Format

Table 10.4: Table continues below

country	HDI	family	id	identity	intervention	t
1	69	1	1.1	1	0	1
1	69	1	1.1	1	0	2
1	69	1	1.1	1	0	3
1	69	2	1.2	1	1	1
1	69	2	1.2	1	1	2
1	69	2	1.2	1	1	3

Table 10.5: Data in Long Format

physical_punishment	warmth	outcome
3	3	57.47
3	4	55.06
1	2	58.77
2	1	50.1
3	0	53.31
3	1	49.79

11 Aggregating Data

In many instances, we may wish to aggregate data. For example, we may wish to create *contextual variables* representing the average level of an indicator across a group. In the examples I am using in this book, the group under consideration is the country. Aggregating data is also an important part of discussions of *within* and *between* variation, and is an important part of the correlated random effects model.

In the examples below, I create a group level variable for `warmth`, representing the average level of parental warmth in each country. If warmth is denoted by $warmth_{ij}$ then the country level variable is denoted by $\overline{warmth}_{.j}$.

Below, I detail the procedure for aggregating data in Stata and R.

11.0.1 Stata

11.0.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

11.0.1.2 Create A Group Level Variable

```
bysort country: egen mean_warmth = mean(warmth)
```

11.0.2 R

11.0.2.1 Get The Data

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

11.0.2.2 Create A Group Level Variable

```
library(dplyr)

df <- df %>%
  group_by(country) %>%
  mutate(mean_warmth = mean(warmth))
```

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