Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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1 Multilevel Multilingual

"This curious world which we inhabit is more wonderful than it is convenient…" (Thoreau, 1975)

"Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled." (Schanen, 2021)

1.1 Introduction

Below, I describe the use of Stata (StataCorp, 2023), R (Bates et al., 2015; R Core Team, 2023), and Julia (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models and to visualize data.

All of these software packages can estimate multilevel models and can visualize relationships in the data. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. While it costs money to purchase Stata, the price is often very reasonable for academic and educational use. R is free open source software which is less intuitive, but there are many excellent resources for learning R. There is often a cost associated with purchasing books and other materials for learning R, which sometimes feels like it offsets the fact that R is free. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

While any of these software packages can be used for learning and estimating multilevel models, I will offer my own opinion–based upon 15 years of teaching multilevel models at the doctoral level–that Stata offers the quickest pathway for learning the basic and advanced uses of multilevel models. I also believe the intuitive nature of Stata syntax contributes to accurate and replicable work in this area.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but very intuitive for both multilevel modeling and graphing.
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (`ggplot`).
Julia	free	steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

💡 Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

💡 Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (`()`) or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire Julia command in an outer set of parentheses (`()`).

1.2 The Data

ℹ Datasets

The examples use the `simulated_multilevel_data.dta` and `simulated_multilevel_longitudinal_data.dta` files. Here is a direct link to download the cross-sectional data. Here is a direct link to download the longitudinal data.

Table 1.2: Sample of Simulated Multilevel Data

Table 1.2: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	1	0	3
1	69	2	1.2	1	1	2
1	69	3	1.3	0	1	3
1	69	4	1.4	1	0	0
1	69	5	1.5	1	0	4
1	69	6	1.6	0	1	5

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
3	57.47
1	50.1
2	52.92
5	60.17

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
4	55.05
3	49.81

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable x, independent variables x and z, clustering variable group, and a random slope for x. i is the index for the person, while j is the index for the group.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \tag{1.1}$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x z || group: x
```

1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels

fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use **summarize** for *continuous* variables, and **tabulate** for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI

tabulate identity

tabulate intervention
```

```
    Variable |        Obs        Mean    Std. dev.        Min        Max
-------------+---------------------------------------------------------
     outcome |      3,000    52.43327    6.530996   29.60798   74.83553
      warmth |      3,000    3.521667    1.888399          0          7
 physical_p~t |      3,000    2.478667    1.360942          0          5
         HDI |      3,000    64.76667    17.24562         33         87
```

```
 hypothetica |
 l identity |
       group |
    variable |       Freq.     Percent        Cum.
-------------+-----------------------------------
           0 |       1,507       50.23       50.23
           1 |       1,493       49.77      100.00
-------------+-----------------------------------
       Total |       3,000      100.00
```

```
     recieved |
  interventio |
            n |       Freq.      Percent          Cum.
  ------------+---------------------------------
            0 |       1,547        51.57         51.57
            1 |       1,453        48.43        100.00
  ------------+---------------------------------
        Total |       3,000       100.00
```

2.1.2 R

```
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`[1] to generate descriptive statistics.

```
df$country <- factor(df$country)

df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)

summary(df)
```

```
    country          HDI             family            id            identity
1      : 100   Min.   :33.00   Min.   :  1.00   Length:3000      0:1507
2      : 100   1st Qu.:53.00   1st Qu.: 25.75   Class :character  1:1493
3      : 100   Median :70.00   Median : 50.50   Mode  :character
4      : 100   Mean   :64.77   Mean   : 50.50
5      : 100   3rd Qu.:81.00   3rd Qu.: 75.25
6      : 100   Max.   :87.00   Max.   :100.00
(Other):2400
intervention physical_punishment     warmth            outcome
0:1547        Min.   :0.000       Min.   :0.000    Min.   :29.61
1:1453        1st Qu.:2.000       1st Qu.:2.000    1st Qu.:48.02
```

---

[1] **skimr** is an excellent new alternative library for generating descriptive statistics in R.

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```
Median :2.000      Median :4.000    Median :52.45
Mean   :2.479      Mean   :3.522    Mean   :52.43
3rd Qu.:3.000      3rd Qu.:5.000    3rd Qu.:56.86
Max.   :5.000      Max.   :7.000    Max.   :74.84
```

2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, Datal

df = DataFrame(load("simulated_multilevel_data.dta"))
```

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))

@transform!(df, :identity = categorical(:identity))

@transform!(df, :intervention = categorical(:intervention))
```

```
describe(df) # descriptive statistics
```

```
9×7 DataFrame
 Row   variable            mean     min     median   max      nmissing  eltype
       Symbol              Union…   Any     Union…   Any      Int64     Union

   1   country                      1.0              30.0           0   Union{
   2   HDI                 64.7667  33.0    70.0     87.0           0   Union{
   3   family              50.5     1.0     50.5     100.0          0   Union{
   4   id                           1.1              9.99           0   Union{
   5   identity                     0.0              1.0            0   Union{
   6   intervention                 0.0              1.0            0   Union{
   7   physical_punishment 2.47867  0.0     2.0      5.0            0   Union{
   8   warmth              3.52167  0.0     4.0      7.0            0   Union{
   9   outcome             52.4333  29.608  52.449   74.8355        0   Union{
                                                             1 column omitted
```

2.2 Interpretation

Examining descriptive statistics is an important first step in any analysis. It is important to examine your descriptive statistics first, before skipping ahead to more sophisticated analyses, such as multilevel models.

In examining the descriptive statistics for this data, we get a sense of the data.

- `outcome` has a mean of approximately 52 and ranges from approximately 30 to 75.
- `warmth` and `physical punishment` are both variables that represent the number of times that parents use each of these forms of discipline in a week. The average of the former is about 3.5, while the average of the latter is about 2.5.
- `HDI`, the Human Development Index has an average of about 65, and a wide range.
- `identity` is a categorical variable for a hypothetical identity group, and has values of 0 and 1.
- `intervention` is also a categorical variable, and has values of 0 and 1.

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \tag{3.1}$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{var(u_{0j})}{var(u_{0j}) + var(e_{ij})} \tag{3.2}$$

In a two level multilevel model, the ICC provides a measure of the proportion of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9802.8371
Iteration 1:  Log likelihood = -9802.8371
```

```
Computing standard errors ...

Mixed-effects ML regression                      Number of obs    = 3,000
Group variable: country                          Number of groups =    30
                                                 Obs per group:
                                                              min =    100
                                                              avg =  100.0
                                                              max =    100
                                                 Wald chi2(0)     =      .
Log likelihood = -9802.8371                      Prob > chi2      =      .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   52.43327   .3451217   151.93   0.000     51.75685    53.1097
------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity            |
                 var(_cons)  |   3.178658   .9226737      1.799552   5.614658
-----------------------------+------------------------------------------------
               var(Residual) |   39.46106   1.024013      37.50421      41.52
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 166.31       Prob >= chibar2 = 0.0000
```

```
estat icc // ICC
```

```
Intraclass correlation

------------------------------------------------------------------------------
                      Level |        ICC   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
                    country |   .0745469   .0201254      .0434963    .1248696
------------------------------------------------------------------------------
```

3.2.2 R

```r
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```r
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
   Data: df

REML criterion at convergence: 19605.9

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.3844 -0.6655 -0.0086  0.6725  3.6626

Random effects:
 Groups   Name        Variance Std.Dev.
 country  (Intercept)  3.302   1.817
 Residual             39.461   6.282
Number of obs: 3000, groups:  country, 30

Fixed effects:
            Estimate Std. Error t value
(Intercept)   52.433      0.351   149.4
```

```r
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient

    Adjusted ICC: 0.077
  Unadjusted ICC: 0.077
```

3.2.3 Julia

```julia
using Tables, MixedModels, MixedModelsExtras,
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

```julia
@transform!(df, :country = categorical(:country))
```

```julia
m0 = fit(MixedModel,
        @formula(outcome ~ (1 | country)), df) # unconditional model
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + (1 | country)
   logLik    -2 logLik     AIC        AICc       BIC
 -9802.8371 19605.6742 19611.6742 19611.6822 19629.6933

Variance components:
            Column   Variance Std.Dev.
country  (Intercept)    3.17863 1.78287
Residual               39.46106 6.28180
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

               Coef.   Std. Error      z  Pr(>|z|)

(Intercept)  52.4333    0.345121  151.93    <1e-99
```

```julia
icc(m0) # ICC
```

```
0.07454637475695493
```

3.3 Interpretation

In each case, the software finds that nearly 8% of the variation in the outcome is explainable by the clustering of the observations in each country.

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \tag{4.1}$$

$$\beta_2 \text{physical punishment}_{ij} +$$

$$\beta_3 \text{identity}_{ij} + \beta_4 \text{intervention}_{ij} + \beta_5 \text{HDI}_j +$$

$$u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij}$$

4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 4.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} var(u_{0j}) & 0 \\ 0 & var(u_{1j}) \end{bmatrix} \tag{4.2}$$

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 4.3.

$$\begin{bmatrix} var(u_{0j}) & cov(u_{0j}, u_{1j}) \\ cov(u_{0j}, u_{1j}) & var(u_{1j}) \end{bmatrix} \tag{4.3}$$

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Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2021).

Table 4.1: Correlated and Uncorrelated Random Effects

Software	Uncorrelated Random Effects	Correlated Random Effects
Stata	default	add option: `, cov(uns)`
R	separate random effects from grouping variable with `||`	separate random effects from grouping variable with `|`
Julia	separate terms for each random effect e.g. `(1 | group) + (0 + x | group)`	separate random effects from grouping variable with `|`.

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

4.3 Run Models

4.3.1 Stata

4.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:   Log likelihood = -9626.6279
Iteration 1:   Log likelihood =  -9626.607
```

```
Iteration 2:  Log likelihood =  -9626.607

Computing standard errors ...

Mixed-effects ML regression                          Number of obs     =   3,000
Group variable: country                              Number of groups  =      30

                                                     Obs per group:
                                                                   min =     100
                                                                   avg =   100.0
                                                                   max =     100

                                                     Wald chi2(5)      =  334.14
Log likelihood =  -9626.607                          Prob > chi2       =  0.0000


------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
             warmth |   .8345368   .0637213    13.10   0.000     .7096453    .9594282
physical_punishment |  -.9916657   .0797906   -12.43   0.000    -1.148052   -.8352791
         1.identity |  -.3004767   .2170295    -1.38   0.166    -.7258466    .1248933
     1.intervention |   .6396427   .2174519     2.94   0.003     .2134448    1.065841
                HDI |   -.003228   .0199257    -0.16   0.871    -.0422817    .0358256
              _cons |   51.99991   1.371257    37.92   0.000      49.3123    54.68753
------------------------------------------------------------------------------------


----------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+----------------------------------------------
country: Independent         |
                var(warmth) |   .0227504   .0257784      .0024689    .2096436
                 var(_cons) |   2.963975   .9737647      1.556777    5.643163
-----------------------------+----------------------------------------------
              var(Residual) |   34.97499   .9097109      33.23668    36.80422
----------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 205.74              Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

4.3.2 R

4.3.2.1 Get The Data

```r
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

4.3.2.2 Run The Model

> 🔥 Caution
>
> lme4 does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library lmerTest to provide p values for lme4 results.

> 💡 Tip
>
> R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: options(scipen = 999).

```r
library(lme4)

library(lmerTest)

options(scipen = 999)

fit1 <- lmer(outcome ~ warmth + physical_punishment +
               identity + intervention + HDI +
               (1 + warmth || country),
            data = df)

summary(fit1)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: outcome ~ warmth + physical_punishment + identity + intervention +
    HDI + (1 + warmth || country)
```

```
   Data: df

REML criterion at convergence: 19268.8

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9774 -0.6563  0.0186  0.6645  3.6730

Random effects:
 Groups     Name        Variance Std.Dev.
 country    (Intercept)  3.19120 1.786
 country.1  warmth       0.02464 0.157
 Residual               35.01779 5.918
Number of obs: 3000, groups:  country, 30

Fixed effects:
                      Estimate  Std. Error           df t value
(Intercept)          52.011324    1.414976    30.293141  36.758
warmth                0.834562    0.064250    41.896457  12.989
physical_punishment  -0.991893    0.079845  2968.012381 -12.423
identity             -0.300354    0.217179  2970.108153  -1.383
intervention          0.639060    0.217603  2971.186718   2.937
HDI                  -0.003394    0.020598    27.592814  -0.165
                              Pr(>|t|)
(Intercept)         < 0.0000000000000002 ***
warmth              0.000000000000000277 ***
physical_punishment < 0.0000000000000002 ***
identity                       0.16678
intervention                   0.00334 **
HDI                            0.87030
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) warmth physc_ idntty intrvn
warmth       -0.124
physcl_pnsh  -0.149 -0.003
identity     -0.072 -0.012 -0.003
interventin  -0.082  0.034  0.022 -0.018
HDI          -0.943 -0.006  0.009 -0.001  0.000
```

4.3.3 Julia

4.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

4.3.3.2 Change Country To Categorical

```
@transform!(df, :country = categorical(:country))
```

4.3.3.3 Run The Model

```
m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
                identity + intervention + HDI +
                (1 | country) +
                (0 + warmth | country)), df)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country) +
   logLik   -2 logLik     AIC        AICc        BIC
 -9626.6070 19253.2140 19271.2140 19271.2742 19325.2713


Variance components:
           Column     Variance Std.Dev.   Corr.
country  (Intercept)   2.963849 1.721583
         warmth        0.022756 0.150852   .
Residual              34.974984 5.913965
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

                      Coef.   Std. Error       z  Pr(>|z|)

(Intercept)         51.9999     1.37124     37.92   <1e-99
warmth              0.834537    0.0637228   13.10   <1e-38
```

```
physical_punishment  -0.991665    0.0797906  -12.43     <1e-34
identity             -0.300475    0.217029    -1.38     0.1662
intervention          0.639641    0.217452     2.94     0.0033
HDI                  -0.0032286   0.0199255   -0.16     0.8713
```

4.4 Interpretation

Models suggest that parental warmth is associated with increases in the beneficial outcome, while physical punishment is associated with decreases in the beneficial outcome. The intervention is associated with increases in the outcome. There is insufficient evidence that either identity group or the Human Development Index are associated with the outcome.

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

5.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \quad (5.1)$$

$$\beta_4 \text{identity}_{itj} + \beta_5 \text{intervention}_{itj} + \beta_6 \text{HDI}_j +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0ij} + v_{1ij} \times \text{time}_{itj} + e_{itj}$$

5.3 Growth Trajectories

Remember, following Section 6.4, that in longitudinal multilevel models, the variable for *time* assumes an important role as we are often thinking of a *growth trajectory* over time.

As discussed in Section 6.4, think about a model where *identity* is a (1/0) variable for membership in one of two groups:

$$\text{outcome} = \beta_0 + \beta_t \text{time} + \beta_{\text{identity}} \text{identity} + \beta_{\text{interaction}} \text{identity} \times \text{time} + u_{0i} + e_{it}$$

Then, each identity group has its own intercept and time trajectory:

Table 5.1: Slope and Intercept for Each Group

Group	Intercept	Slope (Time Trajectory)
0	β_0	β_t
1	$\beta_0 + \beta_{\text{identity}}$	$\beta_t + \beta_{\text{interaction}}$

> 💡 Main Effects and Interactions
>
> Thus, again following Section 6.4, in longitudinal multilevel models, *main effects* modify the *intercept* of the time trajectory, while *interactions with time*, modify the *slope* of the time trajectory. Below, we run models with *main effects only*, then models with *main effects, and interactions with time.*

5.4 Run Models

5.4.1 Stata

5.4.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.4.1.2 Run The Model

5.4.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:   Log likelihood =   -28523.49
Iteration 1:   Log likelihood = -28499.987
Iteration 2:   Log likelihood = -28499.737
Iteration 3:   Log likelihood = -28499.604
Iteration 4:   Log likelihood = -28499.603
```

```
Computing standard errors ...

Mixed-effects ML regression                          Number of obs =    9,000

        Grouping information
        ------------------------------------------------------------
                        |      No. of        Observations per group
        Group variable  |      groups    Minimum    Average    Maximum
        ----------------+-------------------------------------------
                country |          30        300      300.0        300
                 family |       3,000          3        3.0          3
        ------------------------------------------------------------


                                                     Wald chi2(6)   = 1096.15
Log likelihood = -28499.603                          Prob > chi2    =  0.0000


------------------------------------------------------------------------------
            outcome |  Coefficient  Std. err.       z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------
                  t |    .943864    .0658716     14.33   0.000     .814758     1.07297
             warmth |    .913496    .0423731     21.56   0.000    .8304462    .9965457
physical_punishment | -1.007897    .0497622    -20.25   0.000   -1.105429   -.9103647
         1.identity |  -.1276926    .1515835     -0.84   0.400   -.4247909    .1694056
     1.intervention |   .8589966    .1519095      5.65   0.000    .5612596    1.156734
                HDI |  -.0005657    .0196437     -0.03   0.977   -.0390666    .0379352
              _cons |   50.46724    1.338318     37.71   0.000    47.84418    53.09029
------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                var(warmth)  |   .0107584   .0127845     .0010477    .1104715
                 var(_cons)  |   3.167089   .9146768     1.798157    5.578187
-----------------------------+------------------------------------------------
family: Independent          |
                     var(t)  |   3.74e-09   7.36e-07     1.4e-176    9.7e+158
                 var(_cons)  |   8.387276   .472419      7.510631    9.366243
-----------------------------+------------------------------------------------
               var(Residual) |   26.02733   .4753702     25.11211    26.97592
------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 1247.03           Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

5.4.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.identity i.intervention c.HDI) || count
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood =   -28522.21
Iteration 1:  Log likelihood = -28498.685
Iteration 2:  Log likelihood = -28498.469
Iteration 3:  Log likelihood =  -28498.31
Iteration 4:  Log likelihood = -28498.309

Computing standard errors ...

Mixed-effects ML regression                        Number of obs =    9,000

         Grouping information
         --------------------------------------------------------------
                         |      No. of        Observations per group
          Group variable |      groups    Minimum    Average    Maximum
         ----------------+---------------------------------------------
                 country |          30        300      300.0        300
                  family |       3,000          3        3.0          3
         --------------------------------------------------------------

                                                   Wald chi2(11) = 1100.25
Log likelihood = -28498.309                        Prob > chi2   =  0.0000
```

```
------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
                  t |   .7582075    .326177     2.32   0.020     .1189122    1.397503
             warmth |   .8170757   .082662      9.88   0.000     .6550611    .9790903
 physical_punishment|  -1.009031   .1112932    -9.07   0.000    -1.227162   -.7909007
         1.identity | -.2387167   .3039964    -0.79   0.432    -.8345387    .3571053
     1.intervention |   .6607606   .3044503     2.17   0.030      .064049    1.257472
                HDI |   .0013614   .0210842     0.06   0.949    -.0399628    .0426856
                    |
```

```
              c.t#c.warmth |    .0483637    .0356074      1.36    0.174    -.0214255     .1181529
                           |
                     c.t#|
    c.physical_punishment |    .0005421    .0494355      0.01    0.991    -.0963496     .0974338
                           |
               identity#c.t |
                         1 |    .0554389    .1317444      0.42    0.674    -.2027754     .3136532
                           |
           intervention#c.t |
                         1 |    .0992811     .131925      0.75    0.452    -.1592872     .3578493
                           |
                c.t#c.HDI |   -.0009551    .0038216     -0.25    0.803    -.0084453     .0065352
                           |
                     _cons |    50.83632    1.483548     34.27    0.000     47.92862     53.74402
------------------------------------------------------------------------------------------
```

```
------------------------------------------------------------------------------------------
  Random-effects parameters  |    Estimate    Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------------------
country: Independent         |
                var(warmth) |    .0106014    .0127458      .0010046     .1118779
                 var(_cons) |    3.170089    .9153354      1.800091     5.582753
-----------------------------+------------------------------------------------------------
family: Independent          |
                     var(t) |    1.36e-09    2.89e-07      1.7e-190     1.1e+172
                 var(_cons) |    8.391888    .4724105      7.515232     9.370806
-----------------------------+------------------------------------------------------------
            var(Residual) |    26.01583    .4751604      25.10101       26.964
------------------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 1247.84                    Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

5.4.2 R

5.4.2.1 Get The Data

```r
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

5.4.2.2 Run The Model

> 🔥 Caution
>
> lme4 does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library lmerTest to provide p values for lme4 results.

> 💡 Tip
>
> R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: options(scipen = 999).

5.4.2.2.1 Main Effects Only

```r
library(lme4)

library(lmerTest)

options(scipen = 999)

fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
                identity + intervention + HDI +
                (1 | country/id),
              data = dfL)

summary(fit2A)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula:
outcome ~ t + warmth + physical_punishment + identity + intervention +
    HDI + (1 | country/id)
   Data: dfL

REML criterion at convergence: 57022.7

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.6850 -0.6094 -0.0035  0.6133  3.6792
```

```
Random effects:
 Groups      Name          Variance Std.Dev.
 id:country (Intercept)  8.438    2.905
 country    (Intercept)  3.675    1.917
 Residual                26.036   5.103
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                       Estimate   Std. Error          df t value
(Intercept)          50.3842343    1.4139114  29.8246912  35.635
t                     0.9433806    0.0658755 5998.3764548  14.321
warmth                0.9140307    0.0379336 4745.3497493  24.096
physical_punishment  -1.0087537    0.0497972 6483.6771808 -20.257
identity             -0.1319548    0.1517350 2968.7828107  -0.870
intervention          0.8591494    0.1520510 2971.8111995   5.650
HDI                   0.0007909    0.0207656  28.0001855   0.038
                           Pr(>|t|)
(Intercept)         < 0.0000000000000002 ***
t                   < 0.0000000000000002 ***
warmth              < 0.0000000000000002 ***
physical_punishment < 0.0000000000000002 ***
identity                           0.385
intervention              0.0000000175 ***
HDI                                0.970
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
           (Intr) t      warmth physc_ idntty intrvn
t          -0.092
warmth     -0.091 -0.002
physcl_pnsh -0.092 -0.007 -0.012
identity   -0.051  0.000 -0.013 -0.003
interventin -0.058  0.000  0.039  0.019 -0.018
HDI        -0.951  0.000 -0.004  0.005  0.000  0.002
```

5.4.2.2.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
            identity + intervention + HDI) +
            (1 | country/id),
```

```
            data = dfL)
```

```
summary(fit2B)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula:
outcome ~ t * (warmth + physical_punishment + identity + intervention +
    HDI) + (1 | country/id)
   Data: dfL

REML criterion at convergence: 57042.8

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.7118 -0.6092 -0.0024  0.6150  3.6779

Random effects:
 Groups      Name        Variance Std.Dev.
 id:country (Intercept)  8.436    2.905
 country    (Intercept)  3.675    1.917
 Residual               26.046    5.104
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                        Estimate   Std. Error          df t value
(Intercept)           50.7590272    1.5518360   43.2608620  32.709
t                      0.7552909    0.3263028 6176.7440549   2.315
warmth                 0.8170912    0.0805355 8274.9995422  10.146
physical_punishment   -1.0097729    0.1113557 8084.6084915  -9.068
identity              -0.2446453    0.3041604 8695.8966197  -0.804
intervention           0.6604671    0.3046286 8697.0843430   2.168
HDI                    0.0026692    0.0221295   36.1037733   0.121
t:warmth               0.0486211    0.0356217 6404.8723333   1.365
t:physical_punishment  0.0004964    0.0494590 6753.0158441   0.010
t:identity             0.0563140    0.1318043 5993.4518022   0.427
t:intervention         0.0995037    0.1319917 5994.1433001   0.754
t:HDI                 -0.0009379    0.0038233 5993.9090880  -0.245
                            Pr(>|t|)
(Intercept)         <0.0000000000000002 ***
t                                0.0207 *
warmth              <0.0000000000000002 ***
```

```
physical_punishment    <0.0000000000000002 ***
identity                          0.4212
intervention                      0.0302 *
HDI                               0.9047
t:warmth                          0.1723
t:physical_punishment             0.9920
t:identity                        0.6692
t:intervention                    0.4510
t:HDI                             0.8062
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
           (Intr) t       warmth physc_ idntty intrvn HDI     t:wrmt t:phy_
t           -0.421
warmth      -0.178  0.331
physcl_pnsh -0.190  0.360 -0.005
identity    -0.093  0.166 -0.013 -0.002
interventin -0.107  0.192  0.039  0.019 -0.017
HDI         -0.925  0.264 -0.007  0.012 -0.001  0.003
t:warmth     0.158 -0.377 -0.882  0.001  0.011 -0.035  0.006
t:physcl_pn  0.170 -0.402  0.004 -0.894 -0.001 -0.017 -0.010 -0.003
t:identity   0.081 -0.192  0.011  0.000 -0.867  0.014  0.001 -0.013  0.002
t:intervntn  0.093 -0.222 -0.035 -0.017  0.014 -0.867 -0.003  0.041  0.019
t:HDI        0.322 -0.765  0.015 -0.027  0.002 -0.007 -0.346 -0.016  0.029
           t:dntt t:ntrv
t
warmth
physcl_pnsh
identity
interventin
HDI
t:warmth
t:physcl_pn
t:identity
t:intervntn -0.016
t:HDI       -0.002  0.008
```

5.4.3 Julia

5.4.3.1 Get The Data

```julia
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.4.3.2 Run The Model

5.4.3.2.1 Change Country To Categorical

```julia
@transform!(dfL, :country = categorical(:country))
```

5.4.3.2.2 Main Effects Only

```julia
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                               physical_punishment +
                               identity + intervention +
                               HDI +
                               (1 | country) +
                               (0 + warmth | country) +
                               (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | count:
     logLik     -2 logLik        AIC          AICc          BIC
 -28499.6031   56999.2063   57021.2063   57021.2356   57099.3610

Variance components:
            Column     Variance Std.Dev.   Corr.
id       (Intercept)    8.387214 2.896069
country  (Intercept)    3.167143 1.779647
         warmth         0.010762 0.103739    .
Residual               26.027363 5.101702
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:
```

```
                            Coef.   Std. Error        z  Pr(>|z|)

(Intercept)            50.4673        1.33833     37.71    <1e-99
t                     0.943864      0.0658717     14.33    <1e-45
warmth                0.913496      0.0423744     21.56    <1e-99
physical_punishment    -1.0079      0.0497622    -20.25    <1e-90
identity             -0.127692       0.151583     -0.84    0.3996
intervention          0.858997       0.151909      5.65    <1e-07
HDI               -0.000566026      0.0196439     -0.03    0.9770
```

5.4.3.2.3 Interactions With Time

```julia
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
                          physical_punishment +
                          identity + intervention +
                            HDI) +
                          (1 | country) +
                          (0 + warmth | country) +
                          (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + t & warmth
    logLik   -2 logLik      AIC          AICc         BIC
 -28498.3091   56996.6182   57028.6182   57028.6788   57142.2979

Variance components:
           Column    Variance Std.Dev.   Corr.
id      (Intercept)   8.391746 2.896851
country (Intercept)   3.170032 1.780458
        warmth        0.010609 0.102999   .
Residual             26.015906 5.100579
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                          Coef.  Std. Error       z  Pr(>|z|)

(Intercept)            50.8364      1.48355     34.27    <1e-99
t                     0.758209     0.326177      2.32    0.0201
warmth                0.817076     0.0826636     9.88    <1e-22
```

physical_punishment	-1.00903	0.111293	-9.07	<1e-18
identity	-0.238714	0.303996	-0.79	0.4323
intervention	0.660761	0.30445	2.17	0.0300
HDI	0.00136065	0.0210842	0.06	0.9485
t & warmth	0.0483635	0.0356074	1.36	0.1744
t & physical_punishment	0.000542203	0.0494355	0.01	0.9912
t & identity	0.0554385	0.131745	0.42	0.6739
t & intervention	0.0992809	0.131925	0.75	0.4517
t & HDI	-0.000955067	0.00382162	-0.25	0.8027

5.5 Interpretation

The *main effects only model* suggests that time is associated with increases in the outcome. In the main effects model, main effects other than time, indicate whether a particular variable is associated with higher or lower intercepts of the time trajectory, at the beginning of the study time. Warmth is again associated with increases in the outcome, while physical punishment is associated with decreases in the outcome. Identity is again not associated with the outcome, while the intervention is associated with higher levels of the outcome. The Human Development Index is again not associated with the outcome.

The second model adds interactions with time to the first model. Results are largely similar to the prior model. However, here we not only examine whether main effects other than time are associated with higher or lower time trajectories, but also whether particular variables are associated with differences in the slope of the time trajectory. In this case, we find insufficient evidence that any independent variable is associated with changes in the slope of the time trajectory.

However, it may be illustrative to imagine how we would interpret the results had a particular interaction term been statistically significant. Let us consider one of the interaction terms with the largest coefficient, `intervention#time`. The interaction of the intervention with time is positive. Had this coefficient been statistically signifcant, it would have indicated that the intervention was associated with more rapid increases in the outcome over time *in addition to* the fact that the intervention is associated with higher initial levels of the outcome.

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