Models With Three or More Levels and Cross-Classified Models

2024-08-26

Table of contents

Introduction

A two level multilevel model imagines that *Level 1* units are nested in *Level 2* units. A three level multilevel model imagines that *Level 1* units are nested in *Level 2* units, which are in turn nested in *Level 3*. As more levels are added to the model (e.g. *Level 4*), we imagine all of these levels to be hierarchically nested.

A *cross classified* model imagines that the nesting is not hierarchical, but rather that there are two sets of clusters or nestings in which individuals may be nested.

Below, I describe the use of Stata, R, and Julia to estimate these models.

Three Or More Levels

The Data

I use the *longitudinal* data from *Multilevel Thinking* to which I have added an extra level of *United Nations Region* (Arel-Bundock, Enevoldsen, and Yetman 2018). This data thus requires a four level model.

The Equation

$$\text{outcome}_{itjk} = \beta_0 + \beta_1\text{parental warmth}_{itjk} + \beta_2\text{physical punishment}_{itjk} + \beta_3\text{time}_{itjk} + \quad (1)$$

$$\beta_4\text{identity}_{itjk} + \beta_5\text{intervention}_{itjk} + \beta_6\text{HDI}_{itjk}+$$

$$w_{0k} + u_{0j} + v_{0i} + e_{itjk}$$

Here we imagine w_{0k} (region), u_{0j} (country) and v_{0i} (family) are hierarchically nested effects.

Run The Models

Stata

Get The Data

```
use "fourlevel.dta", clear
```

Unconditional Model

```
mixed outcome || UNregion: || country: || family:
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -29061.686
Iteration 1:  Log likelihood = -29061.679
Iteration 2:  Log likelihood = -29061.679

Computing standard errors ...

Mixed-effects ML regression                          Number of obs = 9,000

        Grouping information
        -------------------------------------------------------------
                        |     No. of       Observations per group
         Group variable |     groups    Minimum    Average    Maximum
        ----------------+--------------------------------------------
               UNregion |          5        600    1,800.0      3,600
                country |         30        300      300.0        300
                 family |      3,000          3        3.0          3
        -------------------------------------------------------------

                                                     Wald chi2(0)   =      .
```

```
Log likelihood = -29061.679                          Prob > chi2   =     .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   54.05906    .987367    54.75   0.000     52.12385    55.99426
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
UNregion: Identity           |
                  var(_cons) |   4.172687   3.187885      .9334852    18.65194
-----------------------------+------------------------------------------------
country: Identity            |
                  var(_cons) |   2.849348   .8710225      1.565093    5.187414
-----------------------------+------------------------------------------------
family: Identity             |
                  var(_cons) |   11.72403     .57475      10.64997    12.90641
-----------------------------+------------------------------------------------
               var(Residual) |   28.23424   .5154842      27.24177    29.26286
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 1843.44           Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Conditional Model

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI ///
|| UNregion: || country: || id:
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28503.082
Iteration 1:  Log likelihood = -28503.039
Iteration 2:  Log likelihood = -28503.039

Computing standard errors ...
```

```
Mixed-effects ML regression                         Number of obs =   9,000

        Grouping information
        -------------------------------------------------------------
                        |     No. of       Observations per group
         Group variable |     groups    Minimum    Average    Maximum
        ----------------+--------------------------------------------
               UNregion |          5        600    1,800.0      3,600
                country |         30        300      300.0        300
                     id |      3,000          3        3.0          3
        -------------------------------------------------------------

                                                    Wald chi2(6)  = 1209.42
Log likelihood = -28503.039                         Prob > chi2   =  0.0000

------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
                  t |   .9433791   .0658667    14.32   0.000     .8142827    1.072476
             warmth |   .9140704   .0379156    24.11   0.000     .8397571    .9883837
physical_punishment |  -1.008615   .0497772   -20.26   0.000    -1.106176   -.9110531
         1.identity |  -.1332133   .1516437    -0.88   0.380    -.4304294    .1640028
     1.intervention |   .8589263   .1519619     5.65   0.000     .5610865    1.156766
                HDI |   .0148561   .0196605     0.76   0.450    -.0236777    .0533899
              _cons |   50.16426   1.675219    29.94   0.000     46.88089    53.44763
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
UNregion: Identity           |
                  var(_cons) |   4.722007   3.585939      1.065898    20.91884
-----------------------------+------------------------------------------------
country: Identity            |
                  var(_cons) |   2.863495   .8656459      1.583342    5.178668
-----------------------------+------------------------------------------------
id: Identity                 |
                  var(_cons) |   8.421131   .4711947      7.546445    9.397199
-----------------------------+------------------------------------------------
               var(Residual) |   26.02919   .4752587      25.11417    26.97755
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 1844.00              Prob > chi2 = 0.0000
```

```
Note: LR test is conservative and provided only for reference.
```

R

Get The Data

```r
library(haven)

df4 <- read_dta("fourlevel.dta")
```

Change Some Variables To Categorical

```r
df4$identity <- factor(df4$identity)

df4$intervention <- factor(df4$intervention)
```

Unconditional Model

> **Caution**
>
> `lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

> **Tip**
>
> R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: `options(scipen = 999)`.

```r
library(lme4)

library(lmerTest)
```

```
Attaching package: 'lmerTest'

The following object is masked from 'package:lme4':

    lmer
```

```
The following object is masked from 'package:stats':

    step
```

```r
options(scipen = 999)

fit4A <- lmer(outcome ~  (1 | UNregion/country/id),
             data = df4)

summary(fit4A)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: outcome ~ (1 | UNregion/country/id)
   Data: df4

REML criterion at convergence: 58121.4

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.7850 -0.6064 -0.0047  0.6020  3.4399

Random effects:
 Groups                 Name        Variance Std.Dev.
 id:(country:UNregion) (Intercept) 11.724   3.424
 country:UNregion      (Intercept)  2.842   1.686
 UNregion              (Intercept)  5.478   2.340
 Residual                          28.234   5.314
Number of obs: 9000, groups:
id:(country:UNregion), 3000; country:UNregion, 30; UNregion, 5

Fixed effects:
            Estimate Std. Error     df t value   Pr(>|t|)
(Intercept)   54.061      1.112  3.777    48.6 0.00000201 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conditional Model

```r
fit4B <- lmer(outcome ~ t + warmth + physical_punishment +
                identity + intervention + HDI +
```

```r
                (1 | UNregion/country/id),
              data = df4)

summary(fit4B)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula:
outcome ~ t + warmth + physical_punishment + identity + intervention +
    HDI + (1 | UNregion/country/id)
   Data: df4

REML criterion at convergence: 57026.4

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.6846 -0.6096 -0.0038  0.6138  3.6850

Random effects:
 Groups                Name        Variance Std.Dev.
 id:(country:UNregion) (Intercept)  8.438   2.905
 country:UNregion      (Intercept)  2.979   1.726
 UNregion              (Intercept)  6.178   2.486
 Residual                          26.036   5.103
Number of obs: 9000, groups:
id:(country:UNregion), 3000; country:UNregion, 30; UNregion, 5

Fixed effects:
                      Estimate Std. Error         df t value
(Intercept)           50.11857    1.78086   15.79112  28.143
t                      0.94338    0.06588 5998.37756  14.321
warmth                 0.91406    0.03793 4745.28492  24.096
physical_punishment   -1.00876    0.04980 6483.46337 -20.257
identity1             -0.13324    0.15173 2969.00938  -0.878
intervention1          0.85872    0.15205 2971.85430   5.648
HDI                    0.01560    0.02006   24.39852   0.778
                              Pr(>|t|)
(Intercept)          0.00000000000000641 ***
t                   < 0.0000000000000002 ***
warmth              < 0.0000000000000002 ***
physical_punishment < 0.0000000000000002 ***
identity1                          0.380
```

```
intervention1        0.00000001780521096 ***
HDI                                0.444
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) t      warmth physc_ idntt1 intrv1
t           -0.073
warmth      -0.071 -0.002
physcl_pnsh -0.073 -0.007 -0.012
identity1   -0.040  0.000 -0.013 -0.003
interventn1 -0.045  0.000  0.039  0.019 -0.018
HDI         -0.738  0.000 -0.005  0.005 -0.001  0.001
```

Julia

Get The Data

```julia
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df4 = DataFrame(load("fourlevel.dta"))
```

Change Some Variables To Categorical

```julia
@transform!(df4, :country = categorical(:country))

@transform!(df4, :UNregion = categorical(:UNregion))

@transform!(df4, :identity = categorical(:identity))

@transform!(df4, :intervention = categorical(:intervention))
```

Unconditional Model

```julia
m4A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                  physical_punishment +
                                  identity + intervention +
                                  HDI +
                                  (1 | UNregion) +
```

```
                                (1 | country) +
                                (1 | id)), df4)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | UNreg
    logLik   -2 logLik      AIC         AICc        BIC
 -28503.0394  57006.0787  57028.0787  57028.1081  57106.2335

Variance components:
            Column   Variance Std.Dev.
id       (Intercept)   8.42110 2.90191
country  (Intercept)   2.86347 1.69218
UNregion (Intercept)   4.72082 2.17274
Residual              26.02921 5.10188
 Number of obs: 9000; levels of grouping factors: 3000, 30, 5

  Fixed-effects parameters:

                          Coef.  Std. Error       z  Pr(>|z|)

(Intercept)          50.1643      1.67514     29.95    <1e-99
t                     0.943379    0.0658668   14.32    <1e-45
warmth                0.91407     0.0379156   24.11    <1e-99
physical_punishment  -1.00861     0.0497772  -20.26    <1e-90
identity             -0.133213    0.151644    -0.88    0.3797
intervention          0.858927    0.151962     5.65    <1e-07
HDI                   0.0148553   0.0196604    0.76    0.4499
```

Conditional Model

```
m4B = fit(MixedModel, @formula(outcome ~ t + warmth +
                                physical_punishment +
                                identity + intervention +
                                HDI +
                                (1 | UNregion) +
                                (1 | country) +
                                (1 | id)), df4)
```

```
Linear mixed model fit by maximum likelihood
```

```
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | UNreg
    logLik   -2 logLik      AIC         AICc        BIC
 -28503.0394  57006.0787  57028.0787  57028.1081  57106.2335

Variance components:
            Column   Variance Std.Dev.
id       (Intercept)   8.42110 2.90191
country  (Intercept)   2.86347 1.69218
UNregion (Intercept)   4.72082 2.17274
Residual              26.02921 5.10188
 Number of obs: 9000; levels of grouping factors: 3000, 30, 5

  Fixed-effects parameters:

                          Coef.  Std. Error       z  Pr(>|z|)

(Intercept)          50.1643      1.67514     29.95    <1e-99
t                     0.943379    0.0658668   14.32    <1e-45
warmth                0.91407     0.0379156   24.11    <1e-99
physical_punishment  -1.00861     0.0497772  -20.26    <1e-90
identity             -0.133213    0.151644    -0.88    0.3797
intervention          0.858927    0.151962     5.65    <1e-07
HDI                   0.0148553   0.0196604    0.76    0.4499
```

Interpretation

Cross-Classified Models

The Data

I use the *cross-sectional* data from *Multilevel Thinking* to which I have added an extra level of a hypothetical language.

The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \tag{2}$$

$$\beta_4 \text{identity}_{itj} + \beta_5 \text{intervention}_{itj} + \beta_6 \text{HDI}_{itj} +$$

$$u_{0j} + m_{0m} + e_{ijm}$$

Here u_{0j} (country) and m_{0m} (language) are not nested hierarchically, but are *cross classified*.

Run The Models

Stata

Get The Data

```
use "crossclassified.dta", clear
```

Unconditional Model

```
mixed outcome || _all: R.country || _all: R.language
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9835.8123
Iteration 1:  Log likelihood = -9835.8111
Iteration 2:  Log likelihood = -9835.8111

Computing standard errors ...

Mixed-effects ML regression                          Number of obs    =   3,000
Group variable: _all                                 Number of groups =       1
                                                     Obs per group:
                                                                  min =   3,000
                                                                  avg = 3,000.0
                                                                  max =   3,000
                                                     Wald chi2(0)     =       .
Log likelihood = -9835.8111                          Prob > chi2      =       .

--------------------------------------------------------------------------------
        outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
----------------+---------------------------------------------------------------
          _cons |   52.43187   .3590214   146.04   0.000      51.7282    53.13554
```

```
--------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
_all: Identity               |
               var(R.country) |   3.177791   .9244633      1.796798    5.620198
-----------------------------+------------------------------------------------
_all: Identity               |
              var(R.language) |   .9566314   .3284087      .4881235     1.87482
-----------------------------+------------------------------------------------
                var(Residual) |   39.62877   1.045619      37.63148    41.73206
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 180.84                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Conditional Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || _all: R.country ||
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9663.2195
Iteration 1:  Log likelihood = -9663.2194

Computing standard errors ...

Mixed-effects ML regression                         Number of obs     =   3,000
Group variable: _all                                Number of groups  =       1
                                                    Obs per group:
                                                                  min =   3,000
                                                                  avg = 3,000.0
                                                                  max =   3,000
                                                    Wald chi2(5)      =  367.04
Log likelihood = -9663.2194                         Prob > chi2       =  0.0000

------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
```

```
               warmth |   .8331461   .0579811    14.37   0.000     .7195052     .946787
physical_punishment |  -.9979749    .080268   -12.43   0.000    -1.155297   -.8406525
         1.identity |  -.2922428   .2191421    -1.33   0.182    -.7217534    .1372678
     1.intervention |   .6097458   .2195139     2.78   0.005     .1795064    1.039985
                HDI |  -.0015879   .0204157    -0.08   0.938    -.0416021    .0384262
              _cons |   51.92255   1.411069    36.80   0.000     49.15691     54.6882
--------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
_all: Identity               |
              var(R.country) |   3.361218   .9603072      1.920024    5.884192
-----------------------------+------------------------------------------------
_all: Identity               |
             var(R.language) |   1.121946   .3269535      .6337502    1.986214
-----------------------------+------------------------------------------------
               var(Residual) |   35.11959   .9263999      33.35002    36.98306
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 227.02                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

R

Get The Data

```r
library(haven)

dfCC <- read_dta("crossclassified.dta")
```

Change Some Variables To Categorical

```r
dfCC$identity <- factor(dfCC$identity)

dfCC$intervention <- factor(dfCC$intervention)
```

Unconditional Model

```r
library(lme4)

library(lmerTest)

options(scipen = 999)

fitCC_A <- lmer(outcome ~
                  (1 | country) +
                  (1 | language),
                data = dfCC)

summary(fitCC_A)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: outcome ~ (1 | country) + (1 | language)
   Data: dfCC

REML criterion at convergence: 19671.8

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.3899 -0.6602 -0.0104  0.6798  3.6924

Random effects:
 Groups   Name        Variance Std.Dev.
 language (Intercept)  0.9604  0.980
 country  (Intercept)  3.2919  1.814
 Residual             39.6276  6.295
Number of obs: 3000, groups:  language, 100; country, 30

Fixed effects:
            Estimate Std. Error      df t value            Pr(>|t|)
(Intercept)  52.4319     0.3643 33.4284   143.9 <0.0000000000000002 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conditional Model

```r
fitCC_B <- lmer(outcome ~ t + warmth + physical_punishment +
                identity + intervention + HDI +
```

```
                (1 | country) +
                (1 | language),
              data = dfCC)
```

```
Error in model.frame.default(data = dfCC, drop.unused.levels = TRUE, formula = outcome ~ : i
```

```
summary(fitCC_B)
```

```
Error in h(simpleError(msg, call)): error in evaluating the argument 'object' in selecting a
```

Julia

Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfCC = DataFrame(load("crossclassified.dta"))
```

Change Some Variables To Categorical

```
@transform!(dfCC, :country = categorical(:country))

@transform!(dfCC, :language = categorical(:language))

@transform!(dfCC, :identity = categorical(:identity))

@transform!(dfCC, :intervention = categorical(:intervention))
```

Unconditional Model

```
mCCA = fit(MixedModel, @formula(outcome ~
                                 (1 | country) +
                                 (1 | language)), dfCC)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + (1 | country) + (1 | language)
   logLik   -2 logLik     AIC       AICc        BIC
 -9835.8111 19671.6222 19679.6222 19679.6356 19703.6477

Variance components:
            Column    Variance Std.Dev.
language (Intercept)   0.956631 0.978075
country  (Intercept)   3.177768 1.782629
Residual              39.628773 6.295139
 Number of obs: 3000; levels of grouping factors: 100, 30

  Fixed-effects parameters:

               Coef.  Std. Error       z  Pr(>|z|)

(Intercept)  52.4319     0.35902  146.04    <1e-99
```

Conditional Model

```
mCCA = fit(MixedModel, @formula(outcome ~ warmth +
                                 physical_punishment +
                                 identity + intervention +
                                 HDI +
                                 (1 | country) +
                                 (1 | language)), dfCC)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country) +
   logLik   -2 logLik     AIC       AICc        BIC
 -9663.2194 19326.4388 19344.4388 19344.4990 19398.4962

Variance components:
            Column   Variance Std.Dev.
language (Intercept)   1.12193 1.05921
country  (Intercept)   3.36119 1.83335
Residual              35.11960 5.92618
 Number of obs: 3000; levels of grouping factors: 100, 30

  Fixed-effects parameters:
```

	Coef.	Std. Error	z	Pr(>|z|)
(Intercept)	51.9226	1.41106	36.80	<1e-99
warmth	0.833146	0.0579811	14.37	<1e-46
physical_punishment	-0.997975	0.080268	-12.43	<1e-34
identity	-0.292243	0.219142	-1.33	0.1823
intervention	0.609746	0.219514	2.78	0.0055
HDI	-0.00158794	0.0204156	-0.08	0.9380

Interpretation

Arel-Bundock, Vincent, Nils Enevoldsen, and CJ Yetman. 2018. "Countrycode: An r Package to Convert Country Names and Country Codes." *Journal of Open Source Software* 3 (28): 848. https://doi.org/10.21105/joss.00848.