Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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1 Multilevel Multilingual

> "This curious world which we inhabit is more wonderful than it is convenient…" (Thoreau, 1975)

> "Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled." (Schanen, 2021)

1.1 Introduction

Below, I describe the use of Stata (StataCorp, 2021), R (Bates et al., 2015; R Core Team, 2023), and Julia (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

All of these software packages can estimate multilevel models. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. R is free open source software which is less intuitive, but there are many excellent resources for learning R. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but intuitive for both multilevel modeling and graphing.
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing.
Julia	free	learning curve: steeper learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (`()`) or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire Julia command in an outer set of parentheses (`()`).

1.2 The Data

The examples use the `simulated_multilevel_data.dta` file from *Multilevel Thinking*. Here is a direct link to download the data.

Table 1.2: Sample of Simulated Multilevel Data

country	HDI	family	id	group	physical_punishment	warmth	outcome
1	69	1	1.1	2	2	3	59.18
1	69	2	1.2	2	4	0	61.54
1	69	3	1.3	1	4	4	51.87
1	69	4	1.4	2	0	6	51.71
1	69	5	1.5	2	3	2	55.88
1	69	6	1.6	1	5	3	60.78

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable `y`, independent variables `x` and `z`, clustering variable `group`, and a random slope for `x`. i is the index for the person, while j is the index for the `group`.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \tag{1.1}$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x || group: x
```

1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels

fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI

tabulate group
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     outcome |      3,000    53.46757     6.65179   33.39014   76.75101
      warmth |      3,000    3.524333    1.889956          0          7
physical_p~t |      3,000    2.494667    1.380075          0          5
         HDI |      3,000    64.76667    17.24562         33         87
```

```
  arbitrary |
      group |
   variable |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      1,507       50.23       50.23
          2 |      1,493       49.77      100.00
------------+-----------------------------------
      Total |      3,000      100.00
```

2.1.2 R

```r
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`[1] to generate descriptive statistics.

```r
df$country <- factor(df$country)

df$group <- factor(df$group)

summary(df)
```

```
    country         HDI             family              id              group   
 1      : 100   Min.   :33.00   Min.   :  1.00   Length:3000        1:1507  
 2      : 100   1st Qu.:53.00   1st Qu.: 25.75   Class :character   2:1493  
 3      : 100   Median :70.00   Median : 50.50   Mode  :character           
 4      : 100   Mean   :64.77   Mean   : 50.50                              
 5      : 100   3rd Qu.:81.00   3rd Qu.: 75.25                              
 6      : 100   Max.   :87.00   Max.   :100.00                              
 (Other):2400                                                               
 physical_punishment     warmth         outcome     
 Min.   :0.000       Min.   :0.000   Min.   :33.39  
 1st Qu.:2.000       1st Qu.:2.000   1st Qu.:48.78  
 Median :3.000       Median :4.000   Median :53.64  
 Mean   :2.495       Mean   :3.524   Mean   :53.47  
 3rd Qu.:3.250       3rd Qu.:5.000   3rd Qu.:58.06  
 Max.   :5.000       Max.   :7.000   Max.   :76.75  
```

2.1.3 Julia

```julia
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataI

df = DataFrame(load("simulated_multilevel_data.dta"))
```

[1] `skimr` is an excellent new alternative library for generating descriptive statistics in R.

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))

@transform!(df, :group = categorical(:group))
```

```
describe(df) # descriptive statistics
```

```
8×7 DataFrame
 Row   variable             mean      min      median   max      nmissing  eltyp
       Symbol               Union…    Any      Union…   Any      Int64     Union

   1   country                        1.0               30.0            0  Union
   2   HDI                  64.7667   33.0     70.0     87.0            0  Union
   3   family               50.5      1.0      50.5     100.0           0  Union
   4   id                             1.1               9.99            0  Union
   5   group                          1.0               2.0             0  Union
   6   physical_punishment  2.49467   0.0      3.0      5.0             0  Union
   7   warmth               3.52433   0.0      4.0      7.0             0  Union
   8   outcome              53.4676   33.3901  53.6426  76.751          0  Union
                                                                1 column omitted
```

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \tag{3.1}$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{var(u_{0j})}{var(u_{0j}) + var(e_{ij})} \tag{3.2}$$

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9856.1548
Iteration 1:  Log likelihood = -9856.1548
```

```
Computing standard errors ...

Mixed-effects ML regression                     Number of obs    = 3,000
Group variable: country                         Number of groups =    30
                                                Obs per group:
                                                             min =   100
                                                             avg = 100.0
                                                             max =   100
                                                Wald chi2(0)     =     .
Log likelihood = -9856.1548                     Prob > chi2      =     .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   53.46757   .3539097   151.08   0.000     52.77392    54.16122
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity            |
                  var(_cons) |   3.348734   .9702594      1.897816    5.908906
-----------------------------+------------------------------------------------
               var(Residual) |   40.88284   1.060908       38.8555    43.01597
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 169.64        Prob >= chibar2 = 0.0000
```

```
estat icc // ICC
```

```
Intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
                     country |   .0757091   .0203761      .0442419    .1265931
------------------------------------------------------------------------------
```

3.2.2 R

```r
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```r
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
   Data: df

REML criterion at convergence: 19712.5

Scaled residuals:
     Min       1Q   Median       3Q      Max 
-2.97650 -0.68006  0.00936  0.67580  3.03510 

Random effects:
 Groups   Name        Variance Std.Dev.
 country  (Intercept)  3.478   1.865   
 Residual             40.883   6.394   
Number of obs: 3000, groups:  country, 30

Fixed effects:
            Estimate Std. Error t value
(Intercept)    53.47       0.36   148.5
```

```r
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient

    Adjusted ICC: 0.078
  Unadjusted ICC: 0.078
```

3.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

```
@transform!(df, :country = categorical(:country))
```

```
m0 = fit(MixedModel,
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + (1 | country)
   logLik   -2 logLik     AIC       AICc        BIC
 -9856.1548 19712.3097 19718.3097 19718.3177 19736.3288

Variance components:
            Column   Variance Std.Dev.
country  (Intercept)   3.34871 1.82995
Residual              40.88285 6.39397
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

               Coef.  Std. Error       z  Pr(>|z|)

(Intercept)  53.4676    0.353908  151.08    <1e-99
```

```
icc(m0) # ICC
```

```
0.07570852291396266
```

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \tag{4.1}$$

$$\beta_2 \text{physical punishment}_{ij} +$$

$$\beta_3 \text{group}_{ij} + \beta_4 \text{HDI}_{ij} +$$

$$u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij}$$

4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 4.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} var(u_{0j}) & 0 \\ 0 & var(u_{1j}) \end{bmatrix} \tag{4.2}$$

As discussed in Chapter 6, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 4.3.

$$\begin{bmatrix} var(u_{0j}) & cov(u_{0j}, u_{1j}) \\ cov(u_{0j}, u_{1j}) & var(u_{1j}) \end{bmatrix} \tag{4.3}$$

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2021).

Table 4.1: Correlated and Uncorrelated Random Effects

Software	Uncorrelated Random Effects	Correlated Random Effects
Stata	default	add option: `, cov(uns)`
R	separate random effects from grouping variable with `||`	separate random effects from grouping variable with `|`
Julia	separate terms for each random effect e.g. `(1 | group) + (0 + x | group)`	separate random effects from grouping variable with `|`.

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

4.3 Run Models

4.3.1 Stata

4.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment group HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood =  -9668.198
Iteration 1:  Log likelihood = -9667.9551
Iteration 2:  Log likelihood = -9667.9534
Iteration 3:  Log likelihood = -9667.9533
Iteration 4:  Log likelihood = -9667.9532
```

```
Computing standard errors ...

Mixed-effects ML regression                     Number of obs     =  3,000
Group variable: country                         Number of groups =     30
                                                Obs per group:
                                                             min =    100
                                                             avg =  100.0
                                                             max =    100
                                                Wald chi2(4)      = 401.26
Log likelihood = -9667.9532                     Prob > chi2       = 0.0000

------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
             warmth |   .9616447   .0581825    16.53   0.000     .8476091     1.07568
physical_punishment |  -.8453802   .0798155   -10.59   0.000    -1.001816   -.6889448
              group |   1.084344   .2200539     4.93   0.000     .6530461    1.515642
                HDI |    .010557   .0204522     0.52   0.606    -.0295286    .0506426
              _cons |   49.87963   1.436612    34.72   0.000     47.06392    52.69534
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   1.83e-06   .0000173      1.76e-14    190.9774
                  var(_cons) |   3.370262   .9633726      1.924651    5.901676
-----------------------------+------------------------------------------------
               var(Residual) |   36.01906   .9346936      34.23291    37.89842
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 198.01                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

4.3.2 R

4.3.2.1 Get The Data

```r
library(haven)
```

```r
df <- read_dta("simulated_multilevel_data.dta")
```

4.3.2.2 Run The Model

```r
fit1 <- lmer(outcome ~ warmth + physical_punishment +
               group + HDI +
               (1 + warmth || country),
             data = df)

summary(fit1)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ warmth + physical_punishment + group + HDI + ((1 |
    country) + (0 + warmth | country))
   Data: df

REML criterion at convergence: 19350.3

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.4496 -0.6807  0.0016  0.6864  3.1792

Random effects:
 Groups    Name        Variance  Std.Dev.
 country   (Intercept)  3.611568 1.90041
 country.1 warmth       0.001876 0.04331
 Residual              36.049124 6.00409
Number of obs: 3000, groups:  country, 30

Fixed effects:
                    Estimate Std. Error t value
(Intercept)         49.88754    1.48203  33.662
warmth               0.96155    0.05875  16.367
physical_punishment -0.84556    0.07986 -10.588
group                1.08471    0.22017   4.927
HDI                  0.01044    0.02116   0.493

Correlation of Fixed Effects:
            (Intr) warmth physc_ group
warmth      -0.126
```

```
physcl_pnsh -0.135 -0.025
group       -0.218 -0.010 -0.019
HDI         -0.925 -0.006  0.008 -0.001
```

4.3.3 Julia

4.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

4.3.3.2 Change Country To Categorical

```
@transform!(df, :country = categorical(:country))
```

4.3.3.3 Run The Model

```
m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
               group + HDI +
               (1 | country) +
               (0 + warmth | country)), df)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + warmth + physical_punishment + group + HDI + (1 | country) + (0 + warmth | co
   logLik   -2 logLik     AIC       AICc        BIC
 -9667.9532 19335.9065 19351.9065 19351.9546 19399.9574

Variance components:
            Column   Variance Std.Dev.  Corr.
country  (Intercept)   3.36982 1.83571
         warmth        0.00000 0.00000   .
Residual              36.01912 6.00159
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:
```

```
                        Coef.  Std. Error       z  Pr(>|z|)

(Intercept)          49.8796     1.43653     34.72    <1e-99
warmth                0.961645   0.058182    16.53    <1e-60
physical_punishment  -0.84538    0.0798155  -10.59    <1e-25
group                 1.08434    0.220054     4.93    <1e-06
HDI                   0.0105571  0.0204509    0.52    0.6057
```

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

5.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1\text{parental warmth}_{itj} + \beta_2\text{physical punishment}_{itj} + \beta_3\text{time}_{itj} + \tag{5.1}$$

$$\beta_4\text{group}_{itj} + \beta_5\text{HDI}_{itj} +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0i} + v_{1i} \times \text{time}_{itj} + e_{itj}$$

5.3 Run Models

5.3.1 Stata

5.3.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.3.1.2 Run The Model

5.3.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment group HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood =  -28795.37
Iteration 1:  Log likelihood = -28795.232
Iteration 2:  Log likelihood = -28795.232

Computing standard errors ...

Mixed-effects ML regression                     Number of obs     =   9,000
Group variable: country                         Number of groups  =      30
                                                Obs per group:
                                                              min =     300
                                                              avg =   300.0
                                                              max =     300
                                                Wald chi2(5)      = 1366.93
Log likelihood = -28795.232                     Prob > chi2       =  0.0000

------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
                  t |   .9882371   .0761439    12.98   0.000     .8389979    1.137476
             warmth |   .9427117   .0342282    27.54   0.000     .8756256    1.009798
physical_punishment |  -.9020727   .0452759   -19.92   0.000    -.9908119   -.8133336
              group |   .9861238   .1249047     7.90   0.000     .7413151    1.230933
                HDI |   .0073726    .020661     0.36   0.721    -.0331222    .0478674
              _cons |   49.45537   1.414072    34.97   0.000     46.68384     52.2269
------------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0024684   .0082517      3.52e-06     1.72956
                  var(_cons) |   3.663663   .9914845      2.155548     6.22692
-----------------------------+------------------------------------------------
```

```
               var(Residual) |   34.78483   .5200702      33.7803    35.81923
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 805.75                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

5.3.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.group c.HDI) || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood =  -28794.99
Iteration 1:  Log likelihood = -28794.855
Iteration 2:  Log likelihood = -28794.855

Computing standard errors ...

Mixed-effects ML regression                          Number of obs    =   9,000
Group variable: country                              Number of groups =      30
                                                     Obs per group:
                                                                  min =     300
                                                                  avg =   300.0
                                                                  max =     300
                                                     Wald chi2(9)     = 1365.73
Log likelihood = -28794.855                          Prob > chi2      =  0.0000

---------------------------------------------------------------------------------------
              outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
----------------------+----------------------------------------------------------------
                    t |   1.047448   .3619795     2.89   0.004     .3379816    1.756915
               warmth |   .8869901   .0876058    10.12   0.000      .715286    1.058694
  physical_punishment |   -.893285   .1194705    -7.48   0.000    -1.127443   -.659127
              2.group |   .9648545   .3292217     2.93   0.003     .3195918    1.610117
                  HDI |   .0120622    .022474     0.54   0.591    -.0319861    .0561104
                      |
         c.t#c.warmth |   .0277903   .0402665     0.69   0.490    -.0511306    .1067112
                      |
                  c.t#|
c.physical_punishment |  -.0041479   .0553051    -0.08   0.940    -.1125439    .1042482
```

```
                 |
     group#c.t |
             2  |   .0105177   .1523009     0.07   0.945    -.2879865    .3090219
                 |
     c.t#c.HDI |   -.002342   .0044172    -0.53   0.596    -.0109996    .0063155
                 |
         _cons |   50.32233   1.572089    32.01   0.000      47.2411    53.40357
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0025661   .0083259      4.44e-06    1.482773
                  var(_cons) |    3.66269    .991533      2.154617    6.226305
-----------------------------+------------------------------------------------
               var(Residual) |   34.78158   .5200283      33.77713     35.8159
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 805.90                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

5.3.2 R

5.3.2.1 Get The Data

```r
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

5.3.2.2 Run The Model

5.3.2.2.1 Main Effects Only

```r
fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
               group + HDI +
               (1 | country/id),
             data = dfL)

summary(fit2A)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ t + warmth + physical_punishment + group + HDI + (1 |
    country/id)
   Data: dfL

REML criterion at convergence: 57088.4

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.4471 -0.6226  0.0081  0.6153  3.1993

Random effects:
 Groups     Name        Variance Std.Dev.
 id:country (Intercept)  8.864   2.977
 country    (Intercept)  3.924   1.981
 Residual               26.008   5.100
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                     Estimate Std. Error t value
(Intercept)         49.494782   1.471780  33.629
t                    0.987964   0.065840  15.005
warmth               0.946259   0.038200  24.771
physical_punishment -0.926880   0.049970 -18.549
group                0.985786   0.153550   6.420
HDI                  0.007543   0.021437   0.352

Correlation of Fixed Effects:
            (Intr) t      warmth physc_ group
t           -0.090
warmth      -0.085  0.008
physcl_pnsh -0.085  0.003 -0.019
group       -0.154  0.000 -0.013 -0.008
HDI         -0.943  0.000 -0.003  0.003  0.000
```

5.3.2.2.2 Interactions With Time

```r
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
               group + HDI) +
               (1 | country/id),
             data = dfL)
```

```r
summary(fit2B)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ t * (warmth + physical_punishment + group + HDI) +
    (1 | country/id)
   Data: dfL

REML criterion at convergence: 57107.3

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.4431 -0.6248  0.0071  0.6183  3.1961

Random effects:
 Groups     Name        Variance Std.Dev.
 id:country (Intercept)  8.868   2.978
 country    (Intercept)  3.925   1.981
 Residual               26.014   5.100
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                      Estimate Std. Error t value
(Intercept)          49.453036   1.637740  30.196
t                     1.008199   0.364915   2.763
warmth                0.865659   0.080487  10.755
physical_punishment  -0.908148   0.110449  -8.222
group                 0.966988   0.304936   3.171
HDI                   0.012277   0.022761   0.539
t:warmth              0.040170   0.035364   1.136
t:physical_punishment -0.008932   0.049262  -0.181
t:group               0.009180   0.131714   0.070
t:HDI                -0.002359   0.003820  -0.618

Correlation of Fixed Effects:
            (Intr) t      warmth physc_ group  HDI    t:wrmt t:phy_ t:grop
t           -0.446
warmth      -0.159  0.278
physcl_pnsh -0.169  0.302 -0.022
group       -0.274  0.459 -0.010 -0.014
HDI         -0.900  0.227 -0.008  0.009 -0.001
t:warmth     0.141 -0.316 -0.880  0.017  0.010  0.007
```

```
t:physcl_pn  0.150 -0.338  0.017 -0.892  0.010 -0.007 -0.015
t:group      0.237 -0.532  0.009  0.012 -0.864  0.001 -0.012 -0.008
t:HDI        0.302 -0.676  0.018 -0.020  0.002 -0.336 -0.018  0.016 -0.002
```

5.3.3 Julia

5.3.3.1 Get The Data

```julia
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.3.3.2 Run The Model

5.3.3.2.1 Change Country To Categorical

```julia
@transform!(dfL, :country = categorical(:country))
```

5.3.3.2.2 Main Effects Only

```julia
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                 physical_punishment +
                                 group + HDI +
                                 (1 | country) +
                                 (0 + warmth | country) +
                                 (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + group + HDI + (1 | country) + (0 + warmth
    logLik   -2 logLik       AIC          AICc         BIC
 -28533.9968  57067.9935  57087.9935  57088.0180  57159.0433

Variance components:
            Column    Variance Std.Dev.  Corr.
id       (Intercept)   8.85263 2.97534
country  (Intercept)   3.65031 1.91058
         warmth        0.00000 0.00000   .
Residual              26.00093 5.09911
```

```
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                           Coef.  Std. Error       z  Pr(>|z|)

(Intercept)          49.4945       1.42422     34.75    <1e-99
t                     0.987965     0.0658315   15.01    <1e-50
warmth                0.946255     0.0381869   24.78    <1e-99
physical_punishment  -0.926774     0.0499549  -18.55    <1e-76
group                 0.985819     0.153487     6.42    <1e-09
HDI                   0.00754357   0.0207101    0.36    0.7157
```

5.3.3.2.3 Interactions With Time

```
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
                                 physical_punishment +
                                 group + HDI) +
                                 (1 | country) +
                                 (0 + warmth | country) +
                                 (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + group + HDI + t & warmth + t & physical_pun
    logLik   -2 logLik      AIC         AICc         BIC
 -28533.1579  57066.3158  57094.3158  57094.3626  57193.7855

Variance components:
            Column    Variance Std.Dev.  Corr.
id       (Intercept)   8.86088 2.97672
country  (Intercept)   3.65109 1.91078
         warmth        0.00000 0.00000    .
Residual              25.99020 5.09806
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                               Coef.  Std. Error      z  Pr(>|z|)

(Intercept)              49.4526       1.59495     31.01    <1e-99
```

t	1.00828	0.364747	2.76	0.0057
warmth	0.865674	0.0804504	10.76	<1e-26
physical_punishment	-0.908024	0.110399	-8.22	<1e-15
group	0.967016	0.304798	3.17	0.0015
HDI	0.0122774	0.0220761	0.56	0.5781
t & warmth	0.0401613	0.0353475	1.14	0.2559
t & physical_punishment	-0.00895284	0.0492392	-0.18	0.8557
t & group	0.00918249	0.131654	0.07	0.9444
t & HDI	-0.00235908	0.00381845	-0.62	0.5367

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