Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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1 Multilevel Multilingual

> "This curious world which we inhabit is more wonderful than it is convenient..." (Thoreau, 1975)

> "Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled." (Schanen, 2021)

1.1 Introduction

Below, I describe the use of Stata (StataCorp, 2021), R (Bates et al., 2015; R Core Team, 2023), and Julia (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

All of these software packages can estimate multilevel models. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. While it costs money to purchase Stata, the price is often very reasonal for academic and educational use. R is free open source software which is less intuitive, but there are many excellent resources for learning R. There is often a cost associated with purchasing books and other materials for learning R. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

While any of these software packages can be used for learning and estimating multilevel models, I will offer my own opinion–based upon 15 years of teaching multilevel models at the doctoral level–that Stata offers the quickest pathway for learning the basic and advanced uses of multilevel models. I also believe the intuitive nature of Stata syntax contributes to accurate and replicable work in this area.

Table 1.1: Software for Multilevel Modeling

Software	Cost	Ease of Use
Stata	some cost	learning curve, but very intuitive for both multilevel modeling and graphing.

Software	Cost	Ease of Use
R	free	learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (`ggplot`).
Julia	free	steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux.

Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (`()`) or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire Julia command in an outer set of parentheses (`()`).

Running Statistical Packages in Quarto

I used Quarto (https://quarto.org/) to create this Appendix. Quarto is a programming and publishing environment that can run multiple programming languages, including Stata, R and Julia, and that can write to multiple output formats including HTML, PDF, and MS Word. To run Stata, I used the `Statamarkdown` library in R to connect Stata to Quarto. Quarto has a built in connection to R, and runs R without issue. To

run Julia, I used the `JuliaCall` library in R to connect Quarto to Julia.
Of course, each of these programs can be run by itself, if you have them installed on your computer.

1.2 The Data

The examples use the `simulated_multilevel_data.dta` file from *Multilevel Thinking*. Here is a direct link to download the data.

Table 1.2: Sample of Simulated Multilevel Data

Table 1.2: Table continues below

country	HDI	family	id	identity	intervention	physical_punishment
1	69	1	1.1	2	0	3
1	69	2	1.2	2	1	2
1	69	3	1.3	1	1	3
1	69	4	1.4	2	0	0
1	69	5	1.5	2	0	4
1	69	6	1.6	1	1	5

Table 1.3: Sample of Simulated Multilevel Data

warmth	outcome
3	57.47
1	50.1
2	52.92
5	60.17
4	55.05
3	49.81

1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable `y`, independent variables `x` and `z`, clustering variable `group`, and a random slope for `x`. i is the index for the person, while j is the index for the `group`.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \tag{1.1}$$

1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x || group: x
```

1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels

fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

2 Descriptive Statistics

2.1 Descriptive Statistics

2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI

tabulate identity

tabulate intervention
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     outcome |      3,000    52.43327    6.530996   29.60798   74.83553
      warmth |      3,000    3.521667    1.888399          0          7
physical_p~t |      3,000    2.478667    1.360942          0          5
         HDI |      3,000    64.76667    17.24562         33         87


hypothetica |
 l identity |
      group |
   variable |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      1,507       50.23       50.23
          2 |      1,493       49.77      100.00
------------+-----------------------------------
      Total |      3,000      100.00
```

```
   recieved |
interventio |
          n |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      1,547       51.57       51.57
          1 |      1,453       48.43      100.00
------------+-----------------------------------
      Total |      3,000      100.00
```

2.1.2 R

```r
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`[1] to generate descriptive statistics.

```r
df$country <- factor(df$country)

df$identity <- factor(df$identity)

df$intervention <- factor(df$intervention)

summary(df)
```

```
    country          HDI            family              id              identity
 1      : 100   Min.   :33.00   Min.   :  1.00   Length:3000        1:1507
 2      : 100   1st Qu.:53.00   1st Qu.: 25.75   Class :character   2:1493
 3      : 100   Median :70.00   Median : 50.50   Mode  :character
 4      : 100   Mean   :64.77   Mean   : 50.50
 5      : 100   3rd Qu.:81.00   3rd Qu.: 75.25
 6      : 100   Max.   :87.00   Max.   :100.00
 (Other):2400
 intervention physical_punishment     warmth         outcome
 0:1547       Min.   :0.000       Min.   :0.000   Min.   :29.61
 1:1453       1st Qu.:2.000       1st Qu.:2.000   1st Qu.:48.02
```

[1] `skimr` is an excellent new alternative library for generating descriptive statistics in R.

```
Median :2.000       Median :4.000   Median :52.45
Mean   :2.479       Mean   :3.522   Mean   :52.43
3rd Qu.:3.000       3rd Qu.:5.000   3rd Qu.:56.86
Max.   :5.000       Max.   :7.000   Max.   :74.84
```

2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataI

df = DataFrame(load("simulated_multilevel_data.dta"))
```

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))

@transform!(df, :identity = categorical(:identity))

@transform!(df, :intervention = categorical(:intervention))
```

```
describe(df) # descriptive statistics
```

```
9×7 DataFrame
 Row   variable             mean      min     median  max      nmissing  eltype
       Symbol               Union…    Any     Union…  Any      Int64     Union

   1   country                        1.0             30.0            0  Union{
   2   HDI                  64.7667   33.0    70.0    87.0            0  Union{
   3   family               50.5      1.0     50.5    100.0           0  Union{
   4   id                             1.1             9.99            0  Union{
   5   identity                       1.0             2.0             0  Union{
   6   intervention                   0.0             1.0             0  Union{
   7   physical_punishment  2.47867   0.0     2.0     5.0             0  Union{
   8   warmth               3.52167   0.0     4.0     7.0             0  Union{
   9   outcome              52.4333   29.608  52.449  74.8355         0  Union{
                                                                   1 column omitted
```

2.2 Interpretation

Examining descriptive statistics is an important first step in any analysis. It is important to examine your descriptive statistics first, before skipping ahead to more sophisticated analyses, such as multilevel models.

In examining the descriptive statistics for this data, we get a sense of the data.

- `outcome` has a mean of approximately 52 and ranges from approximately 30 to 75.
- `warmth` and `physical punishment` are both variables that represent the number of times that parents use each of these forms of discipline in a week. The average of the former is about 3.5, while the average of the latter is about 2.5.
- `HDI`, the Human Development Index has an average of about 65, and a wide range.
- `identity` is a categorical variable for a hypothetical identity group, and has values of 1 and 2.
- `intervention` is also a categorical variable, and has values of 0 and 1.

3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \tag{3.1}$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{var(u_{0j})}{var(u_{0j}) + var(e_{ij})} \tag{3.2}$$

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

3.2 Run Models

3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9802.8371
Iteration 1:  Log likelihood = -9802.8371
```

```
Computing standard errors ...

Mixed-effects ML regression                         Number of obs    = 3,000
Group variable: country                             Number of groups =    30
                                                    Obs per group:
                                                                 min =   100
                                                                 avg = 100.0
                                                                 max =   100
                                                    Wald chi2(0)     =     .
Log likelihood = -9802.8371                         Prob > chi2      =     .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   52.43327   .3451217   151.93   0.000     51.75685     53.1097
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity            |
                  var(_cons) |   3.178658   .9226737      1.799552    5.614658
-----------------------------+------------------------------------------------
               var(Residual) |   39.46106   1.024013      37.50421       41.52
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 166.31        Prob >= chibar2 = 0.0000
```

```
estat icc // ICC
```

```
Intraclass correlation

------------------------------------------------------------------------------
                       Level |        ICC   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
                     country |   .0745469   .0201254      .0434963    .1248696
------------------------------------------------------------------------------
```

3.2.2 R

```r
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```r
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
   Data: df

REML criterion at convergence: 19605.9

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.3844 -0.6655 -0.0086  0.6725  3.6626

Random effects:
 Groups   Name        Variance Std.Dev.
 country  (Intercept)  3.302   1.817
 Residual             39.461   6.282
Number of obs: 3000, groups:  country, 30

Fixed effects:
            Estimate Std. Error t value
(Intercept)   52.433      0.351   149.4
```

```r
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient

    Adjusted ICC: 0.077
  Unadjusted ICC: 0.077
```

3.2.3 Julia

```julia
using Tables, MixedModels, MixedModelsExtras,
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

```julia
@transform!(df, :country = categorical(:country))
```

```julia
m0 = fit(MixedModel,
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + (1 | country)
   logLik   -2 logLik     AIC       AICc        BIC
 -9802.8371 19605.6742 19611.6742 19611.6822 19629.6933

Variance components:
            Column   Variance Std.Dev.
country  (Intercept)   3.17863 1.78287
Residual              39.46106 6.28180
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

               Coef.  Std. Error       z  Pr(>|z|)

(Intercept)  52.4333    0.345121  151.93    <1e-99
```

```julia
icc(m0) # ICC
```

```
0.07454637475695493
```

3.3 Interpretation

In each case, the software finds that nearly 8% of the variation in the outcome is explainable by the clustering of the observations in each country.

4 Cross Sectional Multilevel Models

4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \tag{4.1}$$

$$\beta_2 \text{physical punishment}_{ij} +$$

$$\beta_3 \text{identity}_{ij} + \beta_4 \text{intervention}_{ij} + \beta_5 \text{HDI}_{ij} +$$

$$u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij}$$

4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g. u_{0j} and u_{1j}). In Equation 4.2 the covariances of the random effects are constrained to be zero.

$$\begin{bmatrix} var(u_{0j}) & 0 \\ 0 & var(u_{1j}) \end{bmatrix} \tag{4.2}$$

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in Equation 4.3.

$$\begin{bmatrix} var(u_{0j}) & cov(u_{0j}, u_{1j}) \\ cov(u_{0j}, u_{1j}) & var(u_{1j}) \end{bmatrix} \tag{4.3}$$

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2021).

Table 4.1: Correlated and Uncorrelated Random Effects

Software	Uncorrelated Random Effects	Correlated Random Effects
Stata	default	add option: `, cov(uns)`
R	separate random effects from grouping variable with `||`	separate random effects from grouping variable with `|`
Julia	separate terms for each random effect e.g. `(1 | group) + (0 + x | group)`	separate random effects from grouping variable with `|`.

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

4.3 Run Models

4.3.1 Stata

4.3.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

4.3.1.2 Run The Model

```
mixed outcome warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9626.6279
Iteration 1:  Log likelihood =  -9626.607
```

```
Iteration 2:  Log likelihood =  -9626.607

Computing standard errors ...

Mixed-effects ML regression                          Number of obs    =  3,000
Group variable: country                              Number of groups =     30
                                                     Obs per group:
                                                                  min =    100
                                                                  avg =  100.0
                                                                  max =    100
                                                     Wald chi2(5)     = 334.14
Log likelihood =  -9626.607                          Prob > chi2      = 0.0000

------------------------------------------------------------------------------------
             outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
              warmth |   .8345368   .0637213    13.10   0.000     .7096453    .9594282
physical_punishment |  -.9916657   .0797906   -12.43   0.000    -1.148052   -.8352791
          2.identity |  -.3004767   .2170295    -1.38   0.166    -.7258466    .1248933
      1.intervention |   .6396427   .2174519     2.94   0.003     .2134448    1.065841
                 HDI |   -.003228   .0199257    -0.16   0.871    -.0422817    .0358256
               _cons |   51.99991   1.371257    37.92   0.000      49.3123    54.68753
------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                var(warmth) |   .0227504   .0257784      .0024689    .2096436
                 var(_cons) |   2.963975   .9737647      1.556777    5.643163
-----------------------------+------------------------------------------------
              var(Residual) |   34.97499   .9097109      33.23668    36.80422
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 205.74                 Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

4.3.2 R

4.3.2.1 Get The Data

```r
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

4.3.2.2 Run The Model

> Caution
>
> `lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

```r
library(lme4)

library(lmerTest)

fit1 <- lmer(outcome ~ warmth + physical_punishment +
               identity + intervention + HDI +
               (1 + warmth || country),
             data = df)

summary(fit1)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: outcome ~ warmth + physical_punishment + identity + intervention +
    HDI + (1 + warmth || country)
   Data: df

REML criterion at convergence: 19268.8

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.9774 -0.6563  0.0187  0.6645  3.6730
```

```
Random effects:
 Groups    Name        Variance Std.Dev.
 country   (Intercept)  3.19056 1.786
 country.1 warmth       0.02465 0.157
 Residual              35.01782 5.918
Number of obs: 3000, groups:  country, 30

Fixed effects:
                      Estimate Std. Error         df t value Pr(>|t|)
(Intercept)          5.231e+01  1.447e+00  3.311e+01  36.158  < 2e-16 ***
warmth               8.346e-01  6.425e-02  4.190e+01  12.989 2.77e-16 ***
physical_punishment -9.919e-01  7.984e-02  2.968e+03 -12.423  < 2e-16 ***
identity            -3.004e-01  2.172e-01  2.970e+03  -1.383  0.16678
intervention         6.391e-01  2.176e-01  2.971e+03   2.937  0.00334 **
HDI                 -3.395e-03  2.060e-02  2.760e+01  -0.165  0.87027
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) warmth physc_ idntty intrvn
warmth      -0.119
physcl_pnsh -0.145 -0.003
identity    -0.220 -0.012 -0.003
interventin -0.077  0.034  0.022 -0.018
HDI         -0.922 -0.006  0.009 -0.001  0.000
```

4.3.3 Julia

4.3.3.1 Get The Data

```julia
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

4.3.3.2 Change Country To Categorical

```julia
@transform!(df, :country = categorical(:country))
```

4.3.3.3 Run The Model

```
m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
               identity + intervention + HDI +
               (1 | country) +
               (0 + warmth | country)), df)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + warmth + physical_punishment + identity + intervention + HDI + (1 | country) +
   logLik   -2 logLik     AIC       AICc        BIC
 -9626.6070 19253.2140 19271.2140 19271.2742 19325.2713

Variance components:
            Column    Variance Std.Dev.   Corr.
country  (Intercept)   2.963849 1.721583
         warmth        0.022756 0.150852   .
Residual              34.974984 5.913965
 Number of obs: 3000; levels of grouping factors: 30

  Fixed-effects parameters:

                          Coef.  Std. Error      z  Pr(>|z|)

(Intercept)          52.3004      1.40406    37.25    <1e-99
warmth                0.834537    0.0637228  13.10    <1e-38
physical_punishment  -0.991665    0.0797906 -12.43    <1e-34
identity             -0.300475    0.217029   -1.38    0.1662
intervention          0.639641    0.217452    2.94    0.0033
HDI                  -0.0032286   0.0199255  -0.16    0.8713
```

4.4 Interpretation

Models suggest that parental warmth is associated with increases in the beneficial outcome, while physical punishment is associated with decreases in the beneficial outcome. Membership in the group represented by `identity` is not associated with the outcome. The intervention is associated with increases in the outcome. The Human Development Index is not associated with the outcome.

5 Longitudinal Multilevel Models

5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.

5.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \tag{5.1}$$

$$\beta_4 \text{identity}_{itj} + \beta_5 \text{intervention}_{itj} + \beta_6 \text{HDI}_{itj} +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0i} + v_{1i} \times \text{time}_{itj} + e_{itj}$$

5.3 Run Models

5.3.1 Stata

5.3.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

5.3.1.2 Run The Model

5.3.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28739.506
Iteration 1:  Log likelihood = -28739.506

Computing standard errors ...

Mixed-effects ML regression                     Number of obs    =   9,000
Group variable: country                         Number of groups =      30
                                                Obs per group:
                                                             min =     300
                                                             avg =   300.0
                                                             max =     300
                                                Wald chi2(6)     = 1119.81
Log likelihood = -28739.506                     Prob > chi2      =  0.0000

------------------------------------------------------------------------------------
            outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
--------------------+---------------------------------------------------------------
                  t |   .9443446   .0756408    12.48   0.000     .7960914    1.092598
             warmth |   .9123903   .0430042    21.22   0.000     .8281035     .996677
physical_punishment |  -.9881587   .0451732   -21.87   0.000    -1.076696   -.8996209
         2.identity |  -.1241465   .1242225    -1.00   0.318     -.367618    .1193251
     1.intervention |   .8575839   .1245179     6.89   0.000     .6135332    1.101635
                HDI |  -.0025173   .0191696    -0.13   0.896    -.0400891    .0350544
              _cons |   50.54528   1.304146    38.76   0.000      47.9892    53.10136
------------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                var(warmth) |   .0229349   .0135353      .0072136    .0729194
                 var(_cons) |     3.0009   .8550708      1.716768    5.245553
-----------------------------+------------------------------------------------
```

```
                var(Residual) |   34.31935   .5130963       33.3283    35.33988
-----------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 767.22               Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

5.3.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.identity i.intervention c.HDI) || count
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28738.554
Iteration 1:  Log likelihood = -28738.554

Computing standard errors ...

Mixed-effects ML regression                          Number of obs    =   9,000
Group variable: country                              Number of groups =      30
                                                     Obs per group:
                                                                  min =     300
                                                                  avg =   300.0
                                                                  max =     300
                                                     Wald chi2(11)    = 1122.75
Log likelihood = -28738.554                          Prob > chi2      =  0.0000

--------------------------------------------------------------------------------------
              outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
----------------------+---------------------------------------------------------------
                    t |   .7537359   .3719996     2.03   0.043     .0246301    1.482842
               warmth |   .8198365   .0911059     9.00   0.000     .6412723    .9984008
  physical_punishment |  -1.000348   .1198049    -8.35   0.000    -1.235162   -.7655353
           2.identity |  -.2340191   .3271243    -0.72   0.474     -.875171    .4071327
       1.intervention |   .6597456   .3275877     2.01   0.044     .0176856    1.301806
                  HDI |  -.0005531   .0210866    -0.03   0.979     -.041882    .0407757
                      |
         c.t#c.warmth |   .0463746   .0402459     1.15   0.249    -.0325059    .1252551
                      |
                  c.t#|
c.physical_punishment |   .0061255   .0551491     0.11   0.912    -.1019647    .1142157
```

```
                 |
     identity#c.t |
               2  |   .0548965   .1513015     0.36   0.717    -.241649    .3514421
                 |
 intervention#c.t |
               1  |   .0990704    .151503     0.65   0.513     -.19787    .3960108
                 |
        c.t#c.HDI |  -.0009791   .0043888    -0.22   0.823   -.0095811    .0076229
                 |
            _cons |   50.92503   1.494157    34.08   0.000    47.99654    53.85352
------------------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Independent         |
                 var(warmth) |   .0228292   .0135078      .0071588    .0728013
                  var(_cons) |   3.001849   .8552796       1.71738    5.247001
-----------------------------+------------------------------------------------
               var(Residual) |   34.31227   .5129896      33.32141    35.33258
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 767.35                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

5.3.2 R

5.3.2.1 Get The Data

```r
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

5.3.2.2 Run The Model

> Caution
>
> `lme4` does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library `lmerTest` to provide p values for `lme4` results.

5.3.2.2.1 Main Effects Only

```r
library(lme4)

library(lmerTest)

fit2A <- lmer(outcome ~ t + warmth + physical_punishment +
               identity + intervention + HDI +
               (1 | country/id),
             data = dfL)

summary(fit2A)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula:
outcome ~ t + warmth + physical_punishment + identity + intervention +
    HDI + (1 | country/id)
   Data: dfL

REML criterion at convergence: 57022.7

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.6850 -0.6094 -0.0035  0.6133  3.6792

Random effects:
 Groups     Name        Variance Std.Dev.
 id:country (Intercept)  8.438   2.905
 country    (Intercept)  3.675   1.917
 Residual               26.036   5.103
Number of obs: 9000, groups:  id:country, 3000; country, 30
```

```
Fixed effects:
                      Estimate Std. Error         df t value Pr(>|t|)    
(Intercept)          5.052e+01  1.430e+00  3.117e+01  35.335  < 2e-16 ***
t                    9.434e-01  6.588e-02  5.998e+03  14.321  < 2e-16 ***
warmth               9.140e-01  3.793e-02  4.745e+03  24.096  < 2e-16 ***
physical_punishment -1.009e+00  4.980e-02  6.484e+03 -20.257  < 2e-16 ***
identity            -1.320e-01  1.517e-01  2.969e+03  -0.870    0.385    
intervention         8.591e-01  1.521e-01  2.972e+03   5.650 1.75e-08 ***
HDI                  7.909e-04  2.077e-02  2.800e+01   0.038    0.970    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) t      warmth physc_ idntty intrvn
t           -0.091                                   
warmth      -0.088 -0.002                            
physcl_pnsh -0.090 -0.007 -0.012                     
identity    -0.156  0.000 -0.013 -0.003              
interventin -0.055  0.000  0.039  0.019 -0.018       
HDI         -0.941  0.000 -0.004  0.005  0.000  0.002
```

5.3.2.2.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
               identity + intervention + HDI) +
               (1 | country/id),
             data = dfL)

summary(fit2B)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: 
outcome ~ t * (warmth + physical_punishment + identity + intervention +  
    HDI) + (1 | country/id)
   Data: dfL

REML criterion at convergence: 57042.8

Scaled residuals: 
    Min      1Q  Median      3Q     Max 
```

```
-3.7118 -0.6092 -0.0024  0.6150  3.6779

Random effects:
 Groups     Name        Variance Std.Dev.
 id:country (Intercept)  8.436   2.905
 country    (Intercept)  3.675   1.917
 Residual               26.046   5.104
Number of obs: 9000, groups:  id:country, 3000; country, 30

Fixed effects:
                       Estimate Std. Error         df t value Pr(>|t|)
(Intercept)           5.100e+01  1.609e+00  4.996e+01  31.703   <2e-16 ***
t                     6.990e-01  3.747e-01  6.131e+03   1.865   0.0622 .
warmth                8.171e-01  8.054e-02  8.275e+03  10.146   <2e-16 ***
physical_punishment  -1.010e+00  1.114e-01  8.085e+03  -9.068   <2e-16 ***
identity             -2.446e-01  3.042e-01  8.696e+03  -0.804   0.4212
intervention          6.605e-01  3.046e-01  8.697e+03   2.168   0.0302 *
HDI                   2.669e-03  2.213e-02  3.610e+01   0.121   0.9047
t:warmth              4.862e-02  3.562e-02  6.405e+03   1.365   0.1723
t:physical_punishment 4.964e-04  4.946e-02  6.753e+03   0.010   0.9920
t:identity            5.631e-02  1.318e-01  5.993e+03   0.427   0.6692
t:intervention        9.950e-02  1.320e-01  5.994e+03   0.754   0.4510
t:HDI                -9.379e-04  3.823e-03  5.994e+03  -0.245   0.8062
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) t      warmth physc_ idntty intrvn HDI    t:wrmt t:phy_
t           -0.466
warmth      -0.169  0.285
physcl_pnsh -0.183  0.313 -0.005
identity    -0.278  0.450 -0.013 -0.002
interventin -0.100  0.162  0.039  0.019 -0.017
HDI         -0.892  0.230 -0.007  0.012 -0.001  0.003
t:warmth     0.150 -0.324 -0.882  0.001  0.011 -0.035  0.006
t:physcl_pn  0.164 -0.351  0.004 -0.894 -0.001 -0.017 -0.010 -0.003
t:identity   0.242 -0.519  0.011  0.000 -0.867  0.014  0.001 -0.013  0.002
t:intervntn  0.087 -0.187 -0.035 -0.017  0.014 -0.867 -0.003  0.041  0.019
t:HDI        0.310 -0.666  0.015 -0.027  0.002 -0.007 -0.346 -0.016  0.029
            t:dntt t:ntrv
t
warmth
physcl_pnsh
```

```
identity
interventin
HDI
t:warmth
t:physcl_pn
t:identity
t:intervntn -0.016
t:HDI       -0.002  0.008
```

5.3.3 Julia

5.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

5.3.3.2 Run The Model

5.3.3.2.1 Change Country To Categorical

```
@transform!(dfL, :country = categorical(:country))
```

5.3.3.2.2 Main Effects Only

```
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                 physical_punishment +
                                 identity + intervention +
                                 HDI +
                                 (1 | country) +
                                 (0 + warmth | country) +
                                 (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + (1 | count
    logLik   -2 logLik       AIC          AICc         BIC
 -28499.6031  56999.2063  57021.2063  57021.2356  57099.3610
```

```
Variance components:
            Column    Variance Std.Dev.   Corr.
id       (Intercept)   8.387351 2.896092
country  (Intercept)   3.166939 1.779590
         warmth        0.010760 0.103732   .
Residual              26.027290 5.101695
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                            Coef.  Std. Error       z  Pr(>|z|)

(Intercept)           50.5949        1.35491     37.34    <1e-99
t                      0.943864      0.0658716   14.33    <1e-45
warmth                 0.913496      0.0423739   21.56    <1e-99
physical_punishment   -1.0079        0.0497622  -20.25    <1e-90
identity              -0.127692      0.151584    -0.84    0.3996
intervention           0.858997      0.15191      5.65    <1e-07
HDI                   -0.000565882   0.0196433   -0.03    0.9770
```

5.3.3.2.3 Interactions With Time

```
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +
                                 physical_punishment +
                                 identity + intervention +
                                   HDI) +
                                 (1 | country) +
                                 (0 + warmth | country) +
                                 (1 | id)), dfL)
```

```
Linear mixed model fit by maximum likelihood
 outcome ~ 1 + t + warmth + physical_punishment + identity + intervention + HDI + t & warmth
    logLik   -2 logLik      AIC         AICc        BIC
 -28498.3091  56996.6182  57028.6182  57028.6788  57142.2979

Variance components:
            Column    Variance Std.Dev.   Corr.
id       (Intercept)   8.391746 2.896851
country  (Intercept)   3.170031 1.780458
         warmth        0.010609 0.102999   .
```

```
Residual              26.015906 5.100579
 Number of obs: 9000; levels of grouping factors: 3000, 30

  Fixed-effects parameters:

                                Coef.  Std. Error      z  Pr(>|z|)

(Intercept)               51.0751      1.54284     33.10    <1e-99
t                          0.702771    0.374539    1.88    0.0606
warmth                     0.817076    0.0826636   9.88    <1e-22
physical_punishment       -1.00903     0.111293   -9.07    <1e-18
identity                  -0.238714    0.303996   -0.79    0.4323
intervention               0.660761    0.30445     2.17    0.0300
HDI                        0.00136065  0.0210842   0.06    0.9485
t & warmth                 0.0483635   0.0356074   1.36    0.1744
t & physical_punishment    0.000542203 0.0494355   0.01    0.9912
t & identity               0.0554385   0.131745    0.42    0.6739
t & intervention           0.0992809   0.131925    0.75    0.4517
t & HDI                   -0.000955067 0.00382162 -0.25    0.8027
```

5.4 Interpretation

The *main effects only model* suggests that time is associated with increases in the outcome. In the main effects model, main effects other than time, indicate whether a particular variable is associated with higher or lower intercepts of the time trajectory, at the beginning of the study time. Warmth is again associated with increases in the outcome, while physical punishment is associated with decreases in the outcome. Identity is again not associated with the outcome, while the intervention is associated with higher levels of the outcome. The Human Development Index is again not associated with the outcome.

The second model adds interactions with time to the first model. Results are largely similar to the prior model. However, here we not only examine whether main effects other than time are associated with higher or lower time trajectories, but also whether particular variables are associated with differences in the slope of the time trajectory. In this case, we find that no independent variable is associated with changes in the slope of the time trajectory.

However, it may be illustrative to imagine how we would interpret the results had a particular interaction term been statistically significant. Let us consider one of the interaction terms with the largest coefficient, `intervention#time`. The interaction of the intervention with time is positive. Had this coefficient been statistically signifcant, it would have indicated that the intervention was associated with more rapid increases in the outcome over time *in addition to* the fact that the intervention is associated with higher initial levels of the outcome.

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