

# **Multilevel Multilingual**

**Multilevel Models in Stata, R and Julia**

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# 1 Multilevel Multilingual

“This curious world which we inhabit is more wonderful than it is convenient...”  
(Thoreau, 1975)

“Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled.” (Schanen, 2021)

## 1.1 Introduction

Below, I describe the use of [Stata](#), [R](#), and [Julia](#) to estimate multilevel models.

### Results Will Vary Somewhat

Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software.

### Multi-Line Commands

Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.

1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the `///` line continuation characters.
2. *R* is the software that most naturally can be written using multiple lines, as *R* commands are usually clearly encased in parentheses `()` or continued with `+` signs.
3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a multi-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a `+` sign. An alternative is to encase the entire *Julia* command in an outer set of parentheses `()`.

## 1.2 The Data

The examples use the `simulated_multilevel_data.dta` file from [Multilevel Thinking](#). Here is a [direct link](#) to download the data.

Table 1.1: Sample of Simulated Multilevel Data

country	HDI	family	id	group	physical_punishment	warmth	outcome
1	69	1	1.1	2	2	3	59.18
1	69	2	1.2	2	4	0	61.54
1	69	3	1.3	1	4	4	51.87
1	69	4	1.4	2	0	6	51.71
1	69	5	1.5	2	3	2	55.88
1	69	6	1.6	1	5	3	60.78

## 1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable **y**, independent variables **x** and **z**, clustering variable **group**, and a random slope for **x**. *i* is the index for the person, while *j* is the index for the **group**.

$$y = \beta_0 + \beta_1 x_{ij} + \beta_2 z_{ij} + u_{0j} + u_{1j} \times x_{ij} + e_{ij} \quad (1.1)$$

### 1.3.1 Stata

In Stata `mixed`, the syntax for a multilevel model of the form described in Equation 1.1 is:

```
mixed y x || group: x
```

### 1.3.2 R

In R `lme4`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
library(lme4)

lmer(y ~ x + z + (1 + x || group), data = ...)
```

### 1.3.3 Julia

In Julia `MixedModels`, the general syntax for a multilevel model of the form described in Equation 1.1 is:

```
using MixedModels  
  
fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)
```

## 2 Descriptive Statistics

### 2.1 Descriptive Statistics

#### 2.1.1 Stata

```
use simulated_multilevel_data.dta // use data
```

We use `summarize` for *continuous* variables, and `tabulate` for *categorical* variables.

```
summarize outcome warmth physical_punishment HDI
```

```
tabulate group
```

Variable	Obs	Mean	Std. dev.	Min	Max
outcome	3,000	53.46757	6.65179	33.39014	76.75101
warmth	3,000	3.524333	1.889956	0	7
physical_p~t	3,000	2.494667	1.380075	0	5
HDI	3,000	64.76667	17.24562	33	87

arbitrary			
group			
variable	Freq.	Percent	Cum.
1	1,507	50.23	50.23
2	1,493	49.77	100.00
Total	3,000	100.00	

#### 2.1.2 R



```
library(haven) # read data in Stata format

df <- read_dta("simulated_multilevel_data.dta")
```

R's descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (`factor`) before using `summary`<sup>1</sup> to generate descriptive statistics.

```
df$country <- factor(df$country)

df$group <- factor(df$group)

summary(df)
```

	country	HDI	family	id	group
1	: 100	Min. :33.00	Min. : 1.00	Length:3000	1:1507
2	: 100	1st Qu.:53.00	1st Qu.: 25.75	Class :character	2:1493
3	: 100	Median :70.00	Median : 50.50	Mode :character	
4	: 100	Mean :64.77	Mean : 50.50		
5	: 100	3rd Qu.:81.00	3rd Qu.: 75.25		
6	: 100	Max. :87.00	Max. :100.00		
	(Other):2400				
	physical_punishment	warmth	outcome		
	Min. :0.000	Min. :0.000	Min. :33.39		
	1st Qu.:2.000	1st Qu.:2.000	1st Qu.:48.78		
	Median :3.000	Median :4.000	Median :53.64		
	Mean :2.495	Mean :3.524	Mean :53.47		
	3rd Qu.:3.250	3rd Qu.:5.000	3rd Qu.:58.06		
	Max. :5.000	Max. :7.000	Max. :76.75		

### 2.1.3 Julia

```
using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataAPI

df = DataFrame(load("simulated_multilevel_data.dta"))
```

---

<sup>1</sup>`skimr` is an excellent new alternative library for generating descriptive statistics in R.

Similarly to R, Julia relies on the idea of *variable type*. I use `transform` to convert the appropriate variables to *categorical* variables.

```
@transform!(df, :country = categorical(:country))
```

```
@transform!(df, :group = categorical(:group))
```

```
describe(df) # descriptive statistics
```

8×7 DataFrame

Row	variable	mean	min	median	max	nmissing	eltyp
	Symbol	Union...	Any	Union...	Any	Int64	Union
1	country		1.0		30.0	0	Union
2	HDI	64.7667	33.0	70.0	87.0	0	Union
3	family	50.5	1.0	50.5	100.0	0	Union
4	id		1.1		9.99	0	Union
5	group		1.0		2.0	0	Union
6	physical_punishment	2.49467	0.0	3.0	5.0	0	Union
7	warmth	3.52433	0.0	4.0	7.0	0	Union
8	outcome	53.4676	33.3901	53.6426	76.751	0	Union

1 column omitted

## 3 Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

### 3.1 The Equation

$$\text{outcome}_{ij} = \beta_0 + u_{0j} + e_{ij} \quad (3.1)$$

The Intraclass Correlation Coefficient (ICC) is given by:

$$\text{ICC} = \frac{\text{var}(u_{0j})}{\text{var}(u_{0j}) + \text{var}(e_{ij})} \quad (3.2)$$

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

### 3.2 Run Models

#### 3.2.1 Stata

```
use simulated_multilevel_data.dta // use data
```

```
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...
```

```
Performing gradient-based optimization:
```

```
Iteration 0: Log likelihood = -9856.1548
```

```
Iteration 1: Log likelihood = -9856.1548
```

Computing standard errors ...

Mixed-effects ML regression  
Group variable: country

Number of obs = 3,000  
Number of groups = 30  
Obs per group:  
min = 100  
avg = 100.0  
max = 100  
Wald chi2(0) = .  
Prob > chi2 = .

Log likelihood = -9856.1548

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
_cons	53.46757	.3539097	151.08	0.000	52.77392	54.16122

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Identity				
var(_cons)	3.348734	.9702594	1.897816	5.908906
var(Residual)	40.88284	1.060908	38.8555	43.01597

LR test vs. linear model: chibar2(01) = 169.64      Prob >= chibar2 = 0.0000

```
estat icc // ICC
```

Intraclass correlation

Level	ICC	Std. err.	[95% conf. interval]	
country	.0757091	.0203761	.0442419	.1265931

### 3.2.2 R

```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

```
library(lme4) # estimate multilevel models

fit0 <- lmer(outcome ~ (1 | country),
             data = df) # unconditional model

summary(fit0)
```

```
Linear mixed model fit by REML ['lmerMod']
Formula: outcome ~ (1 | country)
Data: df
```

```
REML criterion at convergence: 19712.5
```

```
Scaled residuals:
      Min       1Q   Median       3Q      Max
-2.97650 -0.68006  0.00936  0.67580  3.03510
```

```
Random effects:
 Groups   Name      Variance Std.Dev.
country (Intercept)  3.478    1.865
Residual                40.883    6.394
Number of obs: 3000, groups: country, 30
```

```
Fixed effects:
              Estimate Std. Error t value
(Intercept)    53.47      0.36    148.5
```

```
library(performance)

performance::icc(fit0) # ICC
```

```
# Intraclass Correlation Coefficient
```

```
Adjusted ICC: 0.078
Unadjusted ICC: 0.078
```

### 3.2.3 Julia

```
using Tables, MixedModels, MixedModelsExtras,  
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta
```

```
df = DataFrame(load("simulated_multilevel_data.dta"))
```

```
@transform!(df, :country = categorical(:country))
```

```
m0 = fit(MixedModel,  
         @formula(outcome ~ (1 | country)), df) # unconditional model
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + (1 | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9856.1548	19712.3097	19718.3097	19718.3177	19736.3288

Variance components:

	Column	Variance	Std.Dev.
country	(Intercept)	3.34871	1.82995
Residual		40.88285	6.39397

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z )
(Intercept)	53.4676	0.353908	151.08	<1e-99

```
icc(m0) # ICC
```

```
0.07570852291396266
```

## 4 Cross Sectional Multilevel Models

### 4.1 The Equation

Recall the general model of Equation 1.1, and the syntax outlined in Section 1.3. Below in Equation 4.1, we consider a more substantive example.

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{warmth}_{ij} + \beta_2 \text{physical punishment}_{ij} + \beta_3 \text{group}_{ij} + \beta_4 \text{HDI}_{ij} + u_{0j} + u_{1j} \times \text{warmth}_{ij} + e_{ij} \quad (4.1)$$

### 4.2 Run Models

#### 4.2.1 Stata

##### 4.2.1.1 Get The Data

```
use simulated_multilevel_data.dta
```

##### 4.2.1.2 Run The Model

```
mixed outcome warmth physical_punishment group HDI || country: warmth
```

```
Performing EM optimization ...
```

```
Performing gradient-based optimization:
```

```
Iteration 0: Log likelihood = -9668.198
```

```
Iteration 1: Log likelihood = -9667.9551
```

```
Iteration 2: Log likelihood = -9667.9534
```

```
Iteration 3: Log likelihood = -9667.9533
```

Iteration 4: Log likelihood = -9667.9532

Computing standard errors ...

Mixed-effects ML regression  
Group variable: country

Number of obs = 3,000  
Number of groups = 30  
Obs per group:  
min = 100  
avg = 100.0  
max = 100  
Wald chi2(4) = 401.26  
Prob > chi2 = 0.0000

Log likelihood = -9667.9532

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
warmth	.9616447	.0581825	16.53	0.000	.8476091	1.07568
physical_punishment	-.8453802	.0798155	-10.59	0.000	-1.001816	-.6889448
group	1.084344	.2200539	4.93	0.000	.6530461	1.515642
HDI	.010557	.0204522	0.52	0.606	-.0295286	.0506426
_cons	49.87963	1.436612	34.72	0.000	47.06392	52.69534

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
country: Independent				
var(warmth)	1.83e-06	.0000173	1.76e-14	190.9774
var(_cons)	3.370262	.9633726	1.924651	5.901676
var(Residual)	36.01906	.9346936	34.23291	37.89842

LR test vs. linear model: chi2(2) = 198.01

Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

## 4.2.2 R

### 4.2.2.1 Get The Data



```
library(haven)

df <- read_dta("simulated_multilevel_data.dta")
```

#### 4.2.2.2 Run The Model

```
fit1 <- lmer(outcome ~ warmth + physical_punishment +
             group + HDI +
             (1 + warmth || country),
             data = df)

summary(fit1)
```

Linear mixed model fit by REML ['lmerMod']

Formula: outcome ~ warmth + physical\_punishment + group + HDI + ((1 | country) + (0 + warmth | country))

Data: df

REML criterion at convergence: 19350.3

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.4496	-0.6807	0.0016	0.6864	3.1792

Random effects:

Groups	Name	Variance	Std.Dev.
country	(Intercept)	3.611568	1.90041
country.1	warmth	0.001876	0.04331
Residual		36.049124	6.00409

Number of obs: 3000, groups: country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	49.88754	1.48203	33.662
warmth	0.96155	0.05875	16.367
physical_punishment	-0.84556	0.07986	-10.588
group	1.08471	0.22017	4.927
HDI	0.01044	0.02116	0.493

Correlation of Fixed Effects:

	(Intr)	warmth	physc_	group
warmth		-0.126		
physcl_pnsh	-0.135		-0.025	
group	-0.218	-0.010	-0.019	
HDI	-0.925	-0.006	0.008	-0.001

## 4.2.3 Julia

### 4.2.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

df = DataFrame(load("simulated_multilevel_data.dta"))
```

### 4.2.3.2 Change Country To Categorical

```
@transform!(df, :country = categorical(:country))
```

### 4.2.3.3 Run The Model

```
m1 = fit(MixedModel, @formula(outcome ~ warmth + physical_punishment +
                               group + HDI +
                               (1 + warmth | country)), df)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + warmth + physical\_punishment + group + HDI + (1 + warmth | country)

	logLik	-2 logLik	AIC	AICc	BIC
	-9667.9392	19335.8783	19353.8783	19353.9385	19407.9357

Variance components:

	Column	Variance	Std.Dev.	Corr.
country	(Intercept)	3.2369484	1.7991521	
	warmth	0.0001080	0.0103903	+1.00
Residual		36.0187144	6.0015593	

Number of obs: 3000; levels of grouping factors: 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z )
(Intercept)	49.9018	1.43435	34.79	<1e-99
warmth	0.961545	0.0582135	16.52	<1e-60
physical_punishment	-0.845389	0.0798149	-10.59	<1e-25
group	1.08524	0.220055	4.93	<1e-06
HDI	0.0101984	0.0204401	0.50	0.6178

## 5 Longitudinal Multilevel Models

### 5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in Section 1.2.



#### Graphing Longitudinal Data

In the section on *cross-sectional* multilevel models, I employed *scatterplots* to graph the data. In longitudinal models, *time* is a variable of special interest. Often, in graphing *longitudinal* data—especially when graphing outcomes by time—it makes more sense to use *linear fit* plots, although a *scatterplot* could be employed as well.

### 5.2 The Equation

$$\text{outcome}_{itj} = \beta_0 + \beta_1 \text{parental warmth}_{itj} + \beta_2 \text{physical punishment}_{itj} + \beta_3 \text{time}_{itj} + \quad (5.1)$$

$$\beta_4 \text{group}_{itj} + \beta_5 \text{HDI}_{itj} +$$

$$u_{0j} + u_{1j} \times \text{parental warmth}_{itj} +$$

$$v_{0i} + v_{1i} \times \text{time}_{itj} + e_{itj}$$

## 5.3 Run Models

### 5.3.1 Stata

#### 5.3.1.1 Get The Data

```
use simulated_multilevel_longitudinal_data.dta
```

#### 5.3.1.2 Run The Model

##### 5.3.1.2.1 Main Effects Only

```
mixed outcome t warmth physical_punishment group HDI || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28795.37

Iteration 1: Log likelihood = -28795.232

Iteration 2: Log likelihood = -28795.232

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 9,000

Number of groups = 30

Obs per group:

min = 300

avg = 300.0

max = 300

Wald chi2(5) = 1366.93

Prob > chi2 = 0.0000

Log likelihood = -28795.232

-----							
	outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
-----+-----							
	t	.9882371	.0761439	12.98	0.000	.8389979	1.137476
	warmth	.9427117	.0342282	27.54	0.000	.8756256	1.009798
physical_punishment		-.9020727	.0452759	-19.92	0.000	-.9908119	-.8133336
group		.9861238	.1249047	7.90	0.000	.7413151	1.230933

HDI		.0073726	.020661	0.36	0.721	-.0331222	.0478674
_cons		49.45537	1.414072	34.97	0.000	46.68384	52.2269

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
-----+-----					
country: Independent					
var(warmth)		.0024684	.0082517	3.52e-06	1.72956
var(_cons)		3.663663	.9914845	2.155548	6.22692
-----+-----					
var(Residual)		34.78483	.5200702	33.7803	35.81923

LR test vs. linear model: chi2(2) = 805.75                      Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.

### 5.3.1.2.2 Interactions With Time

```
mixed outcome c.t##(c.warmth c.physical_punishment i.group c.HDI) || country: warmth
```

Performing EM optimization ...

Performing gradient-based optimization:

Iteration 0: Log likelihood = -28794.99

Iteration 1: Log likelihood = -28794.855

Iteration 2: Log likelihood = -28794.855

Computing standard errors ...

Mixed-effects ML regression

Group variable: country

Number of obs = 9,000

Number of groups = 30

Obs per group:

min = 300

avg = 300.0

max = 300

Wald chi2(9) = 1365.73

Prob > chi2 = 0.0000

Log likelihood = -28794.855

outcome		Coefficient	Std. err.	z	P> z	[95% conf. interval]
---------	--	-------------	-----------	---	------	----------------------

t		1.047448	.3619795	2.89	0.004	.3379816	1.756915
warmth		.8869901	.0876058	10.12	0.000	.715286	1.058694
physical_punishment		-.893285	.1194705	-7.48	0.000	-1.127443	-.659127
2.group		.9648545	.3292217	2.93	0.003	.3195918	1.610117
HDI		.0120622	.022474	0.54	0.591	-.0319861	.0561104
c.t#c.warmth		.0277903	.0402665	0.69	0.490	-.0511306	.1067112
c.t#							
c.							
physical_punishment		-.0041479	.0553051	-0.08	0.940	-.1125439	.1042482
group#c.t							
2		.0105177	.1523009	0.07	0.945	-.2879865	.3090219
c.t#c.HDI		-.002342	.0044172	-0.53	0.596	-.0109996	.0063155
_cons		50.32233	1.572089	32.01	0.000	47.2411	53.40357

Random-effects parameters		Estimate	Std. err.	[95% conf. interval]	
country: Independent					
var(warmth)		.0025661	.0083259	4.44e-06	1.482773
var(_cons)		3.66269	.991533	2.154617	6.226305
var(Residual)		34.78158	.5200283	33.77713	35.8159

LR test vs. linear model:  $\chi^2(2) = 805.90$  Prob >  $\chi^2 = 0.0000$

Note: LR test is conservative and provided only for reference.

## 5.3.2 R

### 5.3.2.1 Get The Data

```
library(haven)

dfL <- read_dta("simulated_multilevel_longitudinal_data.dta")
```

### 5.3.2.2 Run The Model

#### 5.3.2.2.1 Main Effects Only

```
fit2A <- lmer(outcome ~ t + warmth + physical_punishment +  
              group + HDI +  
              (1 | country/id),  
              data = dfL)  
  
summary(fit2A)
```

Linear mixed model fit by REML ['lmerMod']

Formula: outcome ~ t + warmth + physical\_punishment + group + HDI + (1 |  
country/id)

Data: dfL

REML criterion at convergence: 57088.4

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.4471	-0.6226	0.0081	0.6153	3.1993

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.864	2.977
country	(Intercept)	3.924	1.981
Residual		26.008	5.100

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	49.494782	1.471780	33.629
t	0.987964	0.065840	15.005
warmth	0.946259	0.038200	24.771
physical_punishment	-0.926880	0.049970	-18.549
group	0.985786	0.153550	6.420
HDI	0.007543	0.021437	0.352

Correlation of Fixed Effects:

	(Intr) t	warmth	physc_ group
t	-0.090		
warmth	-0.085	0.008	



physcl_pnsh	-0.085	0.003	-0.019		
group	-0.154	0.000	-0.013	-0.008	
HDI	-0.943	0.000	-0.003	0.003	0.000

### 5.3.2.2.2 Interactions With Time

```
fit2B <- lmer(outcome ~ t *(warmth + physical_punishment +
  group + HDI) +
  (1 | country/id),
  data = dfL)

summary(fit2B)
```

Linear mixed model fit by REML ['lmerMod']  
 Formula: outcome ~ t \* (warmth + physical\_punishment + group + HDI) +  
 (1 | country/id)  
 Data: dfL

REML criterion at convergence: 57107.3

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.4431	-0.6248	0.0071	0.6183	3.1961

Random effects:

Groups	Name	Variance	Std.Dev.
id:country	(Intercept)	8.868	2.978
country	(Intercept)	3.925	1.981
Residual		26.014	5.100

Number of obs: 9000, groups: id:country, 3000; country, 30

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	49.453036	1.637740	30.196
t	1.008199	0.364915	2.763
warmth	0.865659	0.080487	10.755
physical_punishment	-0.908148	0.110449	-8.222
group	0.966988	0.304936	3.171
HDI	0.012277	0.022761	0.539
t:warmth	0.040170	0.035364	1.136
t:physical_punishment	-0.008932	0.049262	-0.181

t:group	0.009180	0.131714	0.070
t:HDI	-0.002359	0.003820	-0.618

Correlation of Fixed Effects:

	(Intr)	t	warmth	physc_	group	HDI	t:wrmt	t:phy_	t:grop
t	-0.446								
warmth	-0.159	0.278							
physcl_pnsh	-0.169	0.302	-0.022						
group	-0.274	0.459	-0.010	-0.014					
HDI	-0.900	0.227	-0.008	0.009	-0.001				
t:warmth	0.141	-0.316	-0.880	0.017	0.010	0.007			
t:physcl_pn	0.150	-0.338	0.017	-0.892	0.010	-0.007	-0.015		
t:group	0.237	-0.532	0.009	0.012	-0.864	0.001	-0.012	-0.008	
t:HDI	0.302	-0.676	0.018	-0.020	0.002	-0.336	-0.018	0.016	-0.002

### 5.3.3 Julia

#### 5.3.3.1 Get The Data

```
using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta

dfL = DataFrame(load("simulated_multilevel_longitudinal_data.dta"))
```

#### 5.3.3.2 Run The Model

##### 5.3.3.2.1 Change Country To Categorical

```
@transform!(dfL, :country = categorical(:country))
```

##### 5.3.3.2.2 Main Effects Only

```
m2A = fit(MixedModel, @formula(outcome ~ t + warmth +
                                physical_punishment +
                                group + HDI +
                                (1 + warmth | country) +
                                (1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + t + warmth + physical\_punishment + group + HDI + (1 + warmth | country) + (1

logLik	-2 logLik	AIC	AICc	BIC
-28533.9236	57067.8472	57089.8472	57089.8765	57168.0019

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.851224	2.975101	
country	(Intercept)	3.451345	1.857780	
	warmth	0.000227	0.015065	+1.00
Residual		26.001212	5.099138	

Number of obs: 9000; levels of grouping factors: 3000, 30

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z )
(Intercept)	49.5105	1.41854	34.90	<1e-99
t	0.98814	0.0658319	15.01	<1e-50
warmth	0.946252	0.0382851	24.72	<1e-99
physical_punishment	-0.926673	0.0499547	-18.55	<1e-76
group	0.98708	0.153484	6.43	<1e-09
HDI	0.00725703	0.0206549	0.35	0.7253

### 5.3.3.2.3 Interactions With Time

```
m2B = fit(MixedModel, @formula(outcome ~ t * (warmth +  
    physical_punishment +  
    group + HDI) +  
    (1 + warmth | country) +  
    (1 | id)), dfL)
```

Linear mixed model fit by maximum likelihood

outcome ~ 1 + t + warmth + physical\_punishment + group + HDI + t & warmth + t & physical\_punishment

logLik	-2 logLik	AIC	AICc	BIC
-28533.0810	57066.1620	57096.1620	57096.2154	57202.7367

Variance components:

	Column	Variance	Std.Dev.	Corr.
id	(Intercept)	8.8593774	2.9764706	

```
country (Intercept) 3.4464818 1.8564703
          warmth    0.0002394 0.0154717 +1.00
Residual          25.9905210 5.0980899
Number of obs: 9000; levels of grouping factors: 3000, 30
```

Fixed-effects parameters:

	Coef.	Std. Error	z	Pr(> z )
(Intercept)	49.4696	1.59007	31.11	<1e-99
t	1.00761	0.36475	2.76	0.0057
warmth	0.865294	0.0804999	10.75	<1e-26
physical_punishment	-0.907816	0.110399	-8.22	<1e-15
group	0.96908	0.304797	3.18	0.0015
HDI	0.0119739	0.0220306	0.54	0.5868
t & warmth	0.0403499	0.0353477	1.14	0.2537
t & physical_punishment	-0.00900605	0.0492392	-0.18	0.8549
t & group	0.00879803	0.131655	0.07	0.9467
t & HDI	-0.00234513	0.00381848	-0.61	0.5391

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