Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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# 1. Multilevel Multilingual

“This curious world which we inhabit is more wonderful than it is convenient…” (Thoreau, 1975)

“Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled.” (Schanen, 2021)

## 1.1 Introduction

Below, I describe the use of [Stata](https://www.stata.com/) (StataCorp, 2021), [R](https://www.r-project.org/) (Bates et al., 2015; R Core Team, 2023), and [Julia](https://www.julialang.org/) (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

|  |  |
| --- | --- |
|  | **Results Will Vary Somewhat**  Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software. |

|  |  |
| --- | --- |
|  | **Multi-Line Commands**  Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.   1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the /// line continuation characters. 2. *R* is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (()) or continued with + signs. 3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a + sign. An alternative is to encase the entire Julia command in an outer set of parentheses (()). |

## 1.2 The Data

The examples use the simulated\_multilevel\_data.dta file from [*Multilevel Thinking*](https://agrogan1.github.io/multilevel-thinking/simulated-multi-country-data.html). Here is a [direct link](https://github.com/agrogan1/multilevel-multilingual/raw/main/simulated_multilevel_data.dta) to download the data.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1.1: Sample of Simulated Multilevel Data   | country | HDI | family | id | group | physical\_punishment | warmth | outcome | | --- | --- | --- | --- | --- | --- | --- | --- | | 1 | 69 | 1 | 1.1 | 2 | 2 | 3 | 59.18 | | 1 | 69 | 2 | 1.2 | 2 | 4 | 0 | 61.54 | | 1 | 69 | 3 | 1.3 | 1 | 4 | 4 | 51.87 | | 1 | 69 | 4 | 1.4 | 2 | 0 | 6 | 51.71 | | 1 | 69 | 5 | 1.5 | 2 | 3 | 2 | 55.88 | | 1 | 69 | 6 | 1.6 | 1 | 5 | 3 | 60.78 | |

## 1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable y, independent variables x and z, clustering variable group, and a random slope for x. *i* is the index for the person, while *j* is the index for the group.

### Stata

In Stata mixed, the syntax for a multilevel model of the form described in [Equation 1.1](#eq-MLMsimple) is:

mixed y x || group: x

### R

In R lme4, the general syntax for a multilevel model of the form described in [Equation 1.1](#eq-MLMsimple) is:

library(lme4)  
  
lmer(y ~ x + z + (1 + x || group), data = ...)

### Julia

In Julia MixedModels, the general syntax for a multilevel model of the form described in [Equation 1.1](#eq-MLMsimple) is:

using MixedModels  
  
fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)

# 2. Descriptive Statistics

## 2.1 Descriptive Statistics

### Stata

use simulated\_multilevel\_data.dta // use data

We use summarize for *continuous* variables, and tabulate for *categorical* variables.

summarize outcome warmth physical\_punishment HDI  
  
tabulate group

Variable | Obs Mean Std. dev. Min Max  
-------------+---------------------------------------------------------  
 outcome | 3,000 53.46757 6.65179 33.39014 76.75101  
 warmth | 3,000 3.524333 1.889956 0 7  
physical\_p~t | 3,000 2.494667 1.380075 0 5  
 HDI | 3,000 64.76667 17.24562 33 87  
  
  
 arbitrary |  
 group |  
 variable | Freq. Percent Cum.  
------------+-----------------------------------  
 1 | 1,507 50.23 50.23  
 2 | 1,493 49.77 100.00  
------------+-----------------------------------  
 Total | 3,000 100.00

### R

library(haven) # read data in Stata format  
  
df <- read\_dta("simulated\_multilevel\_data.dta")

R’s descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (factor) before using summary[[1]](#footnote-36) to generate descriptive statistics.

df$country <- factor(df$country)  
  
df$group <- factor(df$group)  
  
summary(df)

country HDI family id group   
 1 : 100 Min. :33.00 Min. : 1.00 Length:3000 1:1507   
 2 : 100 1st Qu.:53.00 1st Qu.: 25.75 Class :character 2:1493   
 3 : 100 Median :70.00 Median : 50.50 Mode :character   
 4 : 100 Mean :64.77 Mean : 50.50   
 5 : 100 3rd Qu.:81.00 3rd Qu.: 75.25   
 6 : 100 Max. :87.00 Max. :100.00   
 (Other):2400   
 physical\_punishment warmth outcome   
 Min. :0.000 Min. :0.000 Min. :33.39   
 1st Qu.:2.000 1st Qu.:2.000 1st Qu.:48.78   
 Median :3.000 Median :4.000 Median :53.64   
 Mean :2.495 Mean :3.524 Mean :53.47   
 3rd Qu.:3.250 3rd Qu.:5.000 3rd Qu.:58.06   
 Max. :5.000 Max. :7.000 Max. :76.75

### Julia

using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
df = DataFrame(load("simulated\_multilevel\_data.dta"))

Similarly to R, Julia relies on the idea of *variable type*. I use transform to convert the appropriate variables to *categorical* variables.

@transform!(df, :country = categorical(:country))  
  
@transform!(df, :group = categorical(:group))

describe(df) # descriptive statistics

8×7 DataFrame  
 Row │ variable mean min median max nmissing eltyp ⋯  
 │ Symbol Union… Any Union… Any Int64 Union ⋯  
─────┼──────────────────────────────────────────────────────────────────────────  
 1 │ country 1.0 30.0 0 Union ⋯  
 2 │ HDI 64.7667 33.0 70.0 87.0 0 Union  
 3 │ family 50.5 1.0 50.5 100.0 0 Union  
 4 │ id 1.1 9.99 0 Union  
 5 │ group 1.0 2.0 0 Union ⋯  
 6 │ physical\_punishment 2.49467 0.0 3.0 5.0 0 Union  
 7 │ warmth 3.52433 0.0 4.0 7.0 0 Union  
 8 │ outcome 53.4676 33.3901 53.6426 76.751 0 Union  
 1 column omitted

# 3. Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

## 3.1 The Equation

The Intraclass Correlation Coefficient (ICC) is given by:

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

## 3.2 Run Models

### Stata

use simulated\_multilevel\_data.dta // use data

mixed outcome || country: // unconditional model

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -9856.1548   
Iteration 1: Log likelihood = -9856.1548   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 3,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 100  
 avg = 100.0  
 max = 100  
 Wald chi2(0) = .  
Log likelihood = -9856.1548 Prob > chi2 = .  
  
------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
-------------+----------------------------------------------------------------  
 \_cons | 53.46757 .3539097 151.08 0.000 52.77392 54.16122  
------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Identity |  
 var(\_cons) | 3.348734 .9702594 1.897816 5.908906  
-----------------------------+------------------------------------------------  
 var(Residual) | 40.88284 1.060908 38.8555 43.01597  
------------------------------------------------------------------------------  
LR test vs. linear model: chibar2(01) = 169.64 Prob >= chibar2 = 0.0000

estat icc // ICC

Intraclass correlation  
  
------------------------------------------------------------------------------  
 Level | ICC Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
 country | .0757091 .0203761 .0442419 .1265931  
------------------------------------------------------------------------------

### R

library(haven)  
  
df <- read\_dta("simulated\_multilevel\_data.dta")

library(lme4) # estimate multilevel models  
  
fit0 <- lmer(outcome ~ (1 | country),  
 data = df) # unconditional model  
  
summary(fit0)

Linear mixed model fit by REML ['lmerMod']  
Formula: outcome ~ (1 | country)  
 Data: df  
  
REML criterion at convergence: 19712.5  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-2.97650 -0.68006 0.00936 0.67580 3.03510   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 country (Intercept) 3.478 1.865   
 Residual 40.883 6.394   
Number of obs: 3000, groups: country, 30  
  
Fixed effects:  
 Estimate Std. Error t value  
(Intercept) 53.47 0.36 148.5

library(performance)  
  
performance::icc(fit0) # ICC

# Intraclass Correlation Coefficient  
  
 Adjusted ICC: 0.078  
 Unadjusted ICC: 0.078

### Julia

using Tables, MixedModels, MixedModelsExtras,   
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
df = DataFrame(load("simulated\_multilevel\_data.dta"))

@transform!(df, :country = categorical(:country))

m0 = fit(MixedModel,   
 @formula(outcome ~ (1 | country)), df) # unconditional model

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + (1 | country)  
 logLik -2 logLik AIC AICc BIC   
 -9856.1548 19712.3097 19718.3097 19718.3177 19736.3288  
  
Variance components:  
 Column Variance Std.Dev.  
country (Intercept) 3.34871 1.82995  
Residual 40.88285 6.39397  
 Number of obs: 3000; levels of grouping factors: 30  
  
 Fixed-effects parameters:  
──────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
──────────────────────────────────────────────────  
(Intercept) 53.4676 0.353908 151.08 <1e-99  
──────────────────────────────────────────────────

icc(m0) # ICC

0.07570852291396266

# 4. Cross Sectional Multilevel Models

## 4.1 The Equation

Recall the general model of [Equation 1.1](#eq-MLMsimple), and the syntax outlined in [Section 1.3](#sec-syntax). Below in [Equation 4.1](#eq-MLMsubstantive), we consider a more substantive example.

$$\text{outcome}\_{ij}= \beta\_0 + \beta\_1 \text{warmth}\_{ij} + \beta\_2 \text{physical punishment}\_{ij} + \\ \beta\_3 \text{group}\_{ij} + \beta\_4 \text{HDI}\_{ij} + \\ u\_{0j} + u\_{1j} \times \text{warmth}\_{ij} + e\_{ij} \qquad(4.1)$$

## 4.2 Run Models

### Stata

#### 4.2.0.1 Get The Data

use simulated\_multilevel\_data.dta

#### 4.2.0.2 Run The Model

mixed outcome warmth physical\_punishment group HDI || country: warmth

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -9668.198   
Iteration 1: Log likelihood = -9667.9551   
Iteration 2: Log likelihood = -9667.9534   
Iteration 3: Log likelihood = -9667.9533   
Iteration 4: Log likelihood = -9667.9532   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 3,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 100  
 avg = 100.0  
 max = 100  
 Wald chi2(4) = 401.26  
Log likelihood = -9667.9532 Prob > chi2 = 0.0000  
  
-------------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
--------------------+----------------------------------------------------------------  
 warmth | .9616447 .0581825 16.53 0.000 .8476091 1.07568  
physical\_punishment | -.8453802 .0798155 -10.59 0.000 -1.001816 -.6889448  
 group | 1.084344 .2200539 4.93 0.000 .6530461 1.515642  
 HDI | .010557 .0204522 0.52 0.606 -.0295286 .0506426  
 \_cons | 49.87963 1.436612 34.72 0.000 47.06392 52.69534  
-------------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Independent |  
 var(warmth) | 1.83e-06 .0000173 1.76e-14 190.9774  
 var(\_cons) | 3.370262 .9633726 1.924651 5.901676  
-----------------------------+------------------------------------------------  
 var(Residual) | 36.01906 .9346936 34.23291 37.89842  
------------------------------------------------------------------------------  
LR test vs. linear model: chi2(2) = 198.01 Prob > chi2 = 0.0000  
  
Note: LR test is conservative and provided only for reference.

### R

#### 4.2.0.3 Get The Data

library(haven)  
  
df <- read\_dta("simulated\_multilevel\_data.dta")

#### 4.2.0.4 Run The Model

fit1 <- lmer(outcome ~ warmth + physical\_punishment +   
 group + HDI +  
 (1 + warmth || country),  
 data = df)  
  
summary(fit1)

Linear mixed model fit by REML ['lmerMod']  
Formula: outcome ~ warmth + physical\_punishment + group + HDI + ((1 |   
 country) + (0 + warmth | country))  
 Data: df  
  
REML criterion at convergence: 19350.3  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.4496 -0.6807 0.0016 0.6864 3.1792   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 country (Intercept) 3.611568 1.90041   
 country.1 warmth 0.001876 0.04331   
 Residual 36.049124 6.00409   
Number of obs: 3000, groups: country, 30  
  
Fixed effects:  
 Estimate Std. Error t value  
(Intercept) 49.88754 1.48203 33.662  
warmth 0.96155 0.05875 16.367  
physical\_punishment -0.84556 0.07986 -10.588  
group 1.08471 0.22017 4.927  
HDI 0.01044 0.02116 0.493  
  
Correlation of Fixed Effects:  
 (Intr) warmth physc\_ group   
warmth -0.126   
physcl\_pnsh -0.135 -0.025   
group -0.218 -0.010 -0.019   
HDI -0.925 -0.006 0.008 -0.001

### Julia

#### 4.2.0.5 Get The Data

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
df = DataFrame(load("simulated\_multilevel\_data.dta"))

#### 4.2.0.6 Change Country To Categorical

@transform!(df, :country = categorical(:country))

#### 4.2.0.7 Run The Model

m1 = fit(MixedModel, @formula(outcome ~ warmth + physical\_punishment +   
 group + HDI +  
 (1 + warmth | country)), df)

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + warmth + physical\_punishment + group + HDI + (1 + warmth | country)  
 logLik -2 logLik AIC AICc BIC   
 -9667.9392 19335.8783 19353.8783 19353.9385 19407.9357  
  
Variance components:  
 Column Variance Std.Dev. Corr.  
country (Intercept) 3.2369484 1.7991521  
 warmth 0.0001080 0.0103903 +1.00  
Residual 36.0187144 6.0015593  
 Number of obs: 3000; levels of grouping factors: 30  
  
 Fixed-effects parameters:  
─────────────────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
─────────────────────────────────────────────────────────────  
(Intercept) 49.9018 1.43435 34.79 <1e-99  
warmth 0.961545 0.0582135 16.52 <1e-60  
physical\_punishment -0.845389 0.0798149 -10.59 <1e-25  
group 1.08524 0.220055 4.93 <1e-06  
HDI 0.0101984 0.0204401 0.50 0.6178  
─────────────────────────────────────────────────────────────

# 5. Longitudinal Multilevel Models

## 5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in [Section 1.2](#sec-data).

|  |  |
| --- | --- |
|  | **Graphing Longitudinal Data**  In the section on *cross-sectional* multilevel models, I employed *scatterplots* to graph the data. In longitudinal models, *time* is a variable of special interest. Often, in graphing *longitudinal* data–especially when graphing outcomes by time–it makes more sense to use *linear fit* plots, although a *scatterplot* could be employed as well. |

## 5.2 The Equation

## 5.3 Run Models

### Stata

#### 5.3.0.1 Get The Data

use simulated\_multilevel\_longitudinal\_data.dta

#### 5.3.0.2 Run The Model

##### 5.3.0.2.1 Main Effects Only

mixed outcome t warmth physical\_punishment group HDI || country: warmth

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -28795.37   
Iteration 1: Log likelihood = -28795.232   
Iteration 2: Log likelihood = -28795.232   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 9,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 300  
 avg = 300.0  
 max = 300  
 Wald chi2(5) = 1366.93  
Log likelihood = -28795.232 Prob > chi2 = 0.0000  
  
-------------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
--------------------+----------------------------------------------------------------  
 t | .9882371 .0761439 12.98 0.000 .8389979 1.137476  
 warmth | .9427117 .0342282 27.54 0.000 .8756256 1.009798  
physical\_punishment | -.9020727 .0452759 -19.92 0.000 -.9908119 -.8133336  
 group | .9861238 .1249047 7.90 0.000 .7413151 1.230933  
 HDI | .0073726 .020661 0.36 0.721 -.0331222 .0478674  
 \_cons | 49.45537 1.414072 34.97 0.000 46.68384 52.2269  
-------------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Independent |  
 var(warmth) | .0024684 .0082517 3.52e-06 1.72956  
 var(\_cons) | 3.663663 .9914845 2.155548 6.22692  
-----------------------------+------------------------------------------------  
 var(Residual) | 34.78483 .5200702 33.7803 35.81923  
------------------------------------------------------------------------------  
LR test vs. linear model: chi2(2) = 805.75 Prob > chi2 = 0.0000  
  
Note: LR test is conservative and provided only for reference.

##### 5.3.0.2.2 Interactions With Time

mixed outcome c.t##(c.warmth c.physical\_punishment i.group c.HDI) || country: warmth

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -28794.99   
Iteration 1: Log likelihood = -28794.855   
Iteration 2: Log likelihood = -28794.855   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 9,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 300  
 avg = 300.0  
 max = 300  
 Wald chi2(9) = 1365.73  
Log likelihood = -28794.855 Prob > chi2 = 0.0000  
  
-------------------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
--------------------------+----------------------------------------------------------------  
 t | 1.047448 .3619795 2.89 0.004 .3379816 1.756915  
 warmth | .8869901 .0876058 10.12 0.000 .715286 1.058694  
 physical\_punishment | -.893285 .1194705 -7.48 0.000 -1.127443 -.659127  
 2.group | .9648545 .3292217 2.93 0.003 .3195918 1.610117  
 HDI | .0120622 .022474 0.54 0.591 -.0319861 .0561104  
 |  
 c.t#c.warmth | .0277903 .0402665 0.69 0.490 -.0511306 .1067112  
 |  
c.t#c.physical\_punishment | -.0041479 .0553051 -0.08 0.940 -.1125439 .1042482  
 |  
 group#c.t |  
 2 | .0105177 .1523009 0.07 0.945 -.2879865 .3090219  
 |  
 c.t#c.HDI | -.002342 .0044172 -0.53 0.596 -.0109996 .0063155  
 |  
 \_cons | 50.32233 1.572089 32.01 0.000 47.2411 53.40357  
-------------------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Independent |  
 var(warmth) | .0025661 .0083259 4.44e-06 1.482773  
 var(\_cons) | 3.66269 .991533 2.154617 6.226305  
-----------------------------+------------------------------------------------  
 var(Residual) | 34.78158 .5200283 33.77713 35.8159  
------------------------------------------------------------------------------  
LR test vs. linear model: chi2(2) = 805.90 Prob > chi2 = 0.0000  
  
Note: LR test is conservative and provided only for reference.

### R

#### 5.3.0.3 Get The Data

library(haven)  
  
dfL <- read\_dta("simulated\_multilevel\_longitudinal\_data.dta")

#### 5.3.0.4 Run The Model

##### 5.3.0.4.1 Main Effects Only

fit2A <- lmer(outcome ~ t + warmth + physical\_punishment +   
 group + HDI +  
 (1 | country/id),  
 data = dfL)  
  
summary(fit2A)

Linear mixed model fit by REML ['lmerMod']  
Formula: outcome ~ t + warmth + physical\_punishment + group + HDI + (1 |   
 country/id)  
 Data: dfL  
  
REML criterion at convergence: 57088.4  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.4471 -0.6226 0.0081 0.6153 3.1993   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 id:country (Intercept) 8.864 2.977   
 country (Intercept) 3.924 1.981   
 Residual 26.008 5.100   
Number of obs: 9000, groups: id:country, 3000; country, 30  
  
Fixed effects:  
 Estimate Std. Error t value  
(Intercept) 49.494782 1.471780 33.629  
t 0.987964 0.065840 15.005  
warmth 0.946259 0.038200 24.771  
physical\_punishment -0.926880 0.049970 -18.549  
group 0.985786 0.153550 6.420  
HDI 0.007543 0.021437 0.352  
  
Correlation of Fixed Effects:  
 (Intr) t warmth physc\_ group   
t -0.090   
warmth -0.085 0.008   
physcl\_pnsh -0.085 0.003 -0.019   
group -0.154 0.000 -0.013 -0.008   
HDI -0.943 0.000 -0.003 0.003 0.000

##### 5.3.0.4.2 Interactions With Time

fit2B <- lmer(outcome ~ t \*(warmth + physical\_punishment +   
 group + HDI) +  
 (1 | country/id),  
 data = dfL)  
  
summary(fit2B)

Linear mixed model fit by REML ['lmerMod']  
Formula: outcome ~ t \* (warmth + physical\_punishment + group + HDI) +   
 (1 | country/id)  
 Data: dfL  
  
REML criterion at convergence: 57107.3  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.4431 -0.6248 0.0071 0.6183 3.1961   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 id:country (Intercept) 8.868 2.978   
 country (Intercept) 3.925 1.981   
 Residual 26.014 5.100   
Number of obs: 9000, groups: id:country, 3000; country, 30  
  
Fixed effects:  
 Estimate Std. Error t value  
(Intercept) 49.453036 1.637740 30.196  
t 1.008199 0.364915 2.763  
warmth 0.865659 0.080487 10.755  
physical\_punishment -0.908148 0.110449 -8.222  
group 0.966988 0.304936 3.171  
HDI 0.012277 0.022761 0.539  
t:warmth 0.040170 0.035364 1.136  
t:physical\_punishment -0.008932 0.049262 -0.181  
t:group 0.009180 0.131714 0.070  
t:HDI -0.002359 0.003820 -0.618  
  
Correlation of Fixed Effects:  
 (Intr) t warmth physc\_ group HDI t:wrmt t:phy\_ t:grop  
t -0.446   
warmth -0.159 0.278   
physcl\_pnsh -0.169 0.302 -0.022   
group -0.274 0.459 -0.010 -0.014   
HDI -0.900 0.227 -0.008 0.009 -0.001   
t:warmth 0.141 -0.316 -0.880 0.017 0.010 0.007   
t:physcl\_pn 0.150 -0.338 0.017 -0.892 0.010 -0.007 -0.015   
t:group 0.237 -0.532 0.009 0.012 -0.864 0.001 -0.012 -0.008   
t:HDI 0.302 -0.676 0.018 -0.020 0.002 -0.336 -0.018 0.016 -0.002

### Julia

#### 5.3.0.5 Get The Data

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
dfL = DataFrame(load("simulated\_multilevel\_longitudinal\_data.dta"))

#### 5.3.0.6 Run The Model

##### 5.3.0.6.1 Change Country To Categorical

@transform!(dfL, :country = categorical(:country))

##### 5.3.0.6.2 Main Effects Only

m2A = fit(MixedModel, @formula(outcome ~ t + warmth +   
 physical\_punishment +   
 group + HDI +  
 (1 + warmth | country) +  
 (1 | id)), dfL)

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + t + warmth + physical\_punishment + group + HDI + (1 + warmth | country) + (1 | id)  
 logLik -2 logLik AIC AICc BIC   
 -28533.9236 57067.8472 57089.8472 57089.8765 57168.0019  
  
Variance components:  
 Column Variance Std.Dev. Corr.  
id (Intercept) 8.851224 2.975101  
country (Intercept) 3.451345 1.857780  
 warmth 0.000227 0.015065 +1.00  
Residual 26.001212 5.099138  
 Number of obs: 9000; levels of grouping factors: 3000, 30  
  
 Fixed-effects parameters:  
──────────────────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
──────────────────────────────────────────────────────────────  
(Intercept) 49.5105 1.41854 34.90 <1e-99  
t 0.98814 0.0658319 15.01 <1e-50  
warmth 0.946252 0.0382851 24.72 <1e-99  
physical\_punishment -0.926673 0.0499547 -18.55 <1e-76  
group 0.98708 0.153484 6.43 <1e-09  
HDI 0.00725703 0.0206549 0.35 0.7253  
──────────────────────────────────────────────────────────────

##### 5.3.0.6.3 Interactions With Time

m2B = fit(MixedModel, @formula(outcome ~ t \* (warmth +   
 physical\_punishment +   
 group + HDI) +  
 (1 + warmth | country) +  
 (1 | id)), dfL)

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + t + warmth + physical\_punishment + group + HDI + t & warmth + t & physical\_punishment + t & group + t & HDI + (1 + warmth | country) + (1 | id)  
 logLik -2 logLik AIC AICc BIC   
 -28533.0810 57066.1620 57096.1620 57096.2154 57202.7367  
  
Variance components:  
 Column Variance Std.Dev. Corr.  
id (Intercept) 8.8593774 2.9764706  
country (Intercept) 3.4464818 1.8564703  
 warmth 0.0002394 0.0154717 +1.00  
Residual 25.9905210 5.0980899  
 Number of obs: 9000; levels of grouping factors: 3000, 30  
  
 Fixed-effects parameters:  
─────────────────────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
─────────────────────────────────────────────────────────────────  
(Intercept) 49.4696 1.59007 31.11 <1e-99  
t 1.00761 0.36475 2.76 0.0057  
warmth 0.865294 0.0804999 10.75 <1e-26  
physical\_punishment -0.907816 0.110399 -8.22 <1e-15  
group 0.96908 0.304797 3.18 0.0015  
HDI 0.0119739 0.0220306 0.54 0.5868  
t & warmth 0.0403499 0.0353477 1.14 0.2537  
t & physical\_punishment -0.00900605 0.0492392 -0.18 0.8549  
t & group 0.00879803 0.131655 0.07 0.9467  
t & HDI -0.00234513 0.00381848 -0.61 0.5391  
─────────────────────────────────────────────────────────────────

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1. skimr is an excellent new alternative library for generating descriptive statistics in R. [↑](#footnote-ref-36)