Multilevel Multilingual

Multilevel Models in Stata, R and Julia

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# 1. Multilevel Multilingual

“This curious world which we inhabit is more wonderful than it is convenient…” (Thoreau, 1975)

“Mathematics is my secret. My secret weakness. I feel like a stubborn, helpless fool in the middle of a problem. Trapped and crazed. Also, thrilled.” (Schanen, 2021)

## 1.1 Introduction

Below, I describe the use of [Stata](https://www.stata.com/) (StataCorp, 2021), [R](https://www.r-project.org/) (Bates et al., 2015; R Core Team, 2023), and [Julia](https://www.julialang.org/) (Bates, 2024; Bezanson et al., 2017) to estimate multilevel models.

All of these software packages can estimate multilevel models. However, there are substantial differences between the different packages: Stata is proprietary *for cost* software, which is very well documented and very intuitive. While it costs money to purchase Stata, the price is often very reasonal for academic and educational use. R is free open source software which is less intuitive, but there are many excellent resources for learning R. There is often a cost associated with purchasing books and other materials for learning R. Julia is newer open source software, and ostensibly much faster than either Stata or R, which may be an important advantage when running multilevel models with very large data sets. At this point in time, both Stata and R feel much more *stable* than Julia which is still evolving software.

While any of these software packages can be used for learning and estimating multilevel models, I will offer my own opinion–based upon 15 years of teaching multilevel models at the doctoral level–that Stata offers the quickest pathway for learning the basic and advanced uses of multilevel models. I also believe the intuitive nature of Stata syntax contributes to accurate and replicable work in this area.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1.1: Software for Multilevel Modeling   | Software | Cost | Ease of Use | | --- | --- | --- | | Stata | some cost | learning curve, but very intuitive for both multilevel modeling and graphing. | | R | free | learning curve: intuitive for multilevel modeling; but steeper learning curve for graphing (ggplot). | | Julia | free | steep learning curve in general: steep learning curve for multilevel modeling; and very steep learning curve for graphing. Graphics libraries are very much under development and in flux. | |

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|  | **Results Will Vary Somewhat**  Estimating multilevel models is a complex endeavor. The software details of how this is accomplished are beyond the purview of this book. Suffice it to say that across different software packages there will be differences in estimation routines, resulting in some numerical differences in the results provided by different software packages. Substantively speaking, however, results should agree across software. |

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|  | **Multi-Line Commands**  Sometimes I have written commands out over multiple lines. I have done this for especially long commands, but have also sometimes done this simply for the sake of clarity. The different software packages have different approaches to multi-line commands.   1. By default, *Stata* ends a command at the end of a line. If you are going to write a multi-line command you should use the /// line continuation characters. 2. *R* is the software that most naturally can be written using multiple lines, as R commands are usually clearly encased in parentheses (()) or continued with + signs. 3. Like *Stata*, *Julia* expects commands to end at the end of a line. If you are going to write a mult-line command, all commands except for the last line should end in a character that clearly indicates continuation, like a + sign. An alternative is to encase the entire Julia command in an outer set of parentheses (()). |

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|  | **Running Statistical Packages in Quarto**  I used Quarto (<https://quarto.org/>) to create this Appendix. Quarto is a programming and publishing environment that can run multiple programming languages, including Stata, R and Julia, and that can write to multiple output formats including HTML, PDF, and MS Word. To run Stata, I used the Statamarkdown library in R to connect Stata to Quarto. Quarto has a built in connection to R, and runs R without issue. To run Julia, I used the JuliaCall library in R to connect Quarto to Julia.  Of course, each of these programs can be run by itself, if you have them installed on your computer. |

## 1.2 The Data

The examples use the simulated\_multilevel\_data.dta file from [*Multilevel Thinking*](https://agrogan1.github.io/multilevel-thinking/simulated-multi-country-data.html). Here is a [direct link](https://github.com/agrogan1/multilevel-multilingual/raw/main/simulated_multilevel_data.dta) to download the data.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1.2: Sample of Simulated Multilevel Data  Table continues below   | country | HDI | family | id | identity | intervention | physical\_punishment | | --- | --- | --- | --- | --- | --- | --- | | 1 | 69 | 1 | 1.1 | 2 | 0 | 3 | | 1 | 69 | 2 | 1.2 | 2 | 1 | 2 | | 1 | 69 | 3 | 1.3 | 1 | 1 | 3 | | 1 | 69 | 4 | 1.4 | 2 | 0 | 0 | | 1 | 69 | 5 | 1.5 | 2 | 0 | 4 | | 1 | 69 | 6 | 1.6 | 1 | 1 | 5 |      | warmth | outcome | | --- | --- | | 3 | 57.47 | | 1 | 50.1 | | 2 | 52.92 | | 5 | 60.17 | | 4 | 55.05 | | 3 | 49.81 | |

## 1.3 An Introduction To Equations and Syntax

To explain statistical syntax for each software, I consider the general case of a multilevel model with dependent variable y, independent variables x and z, clustering variable group, and a random slope for x. *i* is the index for the person, while *j* is the index for the group.

### Stata

In Stata mixed, the syntax for a multilevel model of the form described in [Equation 1.1](#eq-MLMsimple) is:

mixed y x || group: x

### R

In R lme4, the general syntax for a multilevel model of the form described in [Equation 1.1](#eq-MLMsimple) is:

library(lme4)  
  
lmer(y ~ x + z + (1 + x || group), data = ...)

### Julia

In Julia MixedModels, the general syntax for a multilevel model of the form described in [Equation 1.1](#eq-MLMsimple) is:

using MixedModels  
  
fit(MixedModel, @formula(y ~ x + z + (1 + x | group)), data)

# 2. Descriptive Statistics

## 2.1 Descriptive Statistics

### Stata

use simulated\_multilevel\_data.dta // use data

We use summarize for *continuous* variables, and tabulate for *categorical* variables.

summarize outcome warmth physical\_punishment HDI  
  
tabulate identity  
  
tabulate intervention

Variable | Obs Mean Std. dev. Min Max  
-------------+---------------------------------------------------------  
 outcome | 3,000 52.43327 6.530996 29.60798 74.83553  
 warmth | 3,000 3.521667 1.888399 0 7  
physical\_p~t | 3,000 2.478667 1.360942 0 5  
 HDI | 3,000 64.76667 17.24562 33 87  
  
  
hypothetica |  
 l identity |  
 group |  
 variable | Freq. Percent Cum.  
------------+-----------------------------------  
 1 | 1,507 50.23 50.23  
 2 | 1,493 49.77 100.00  
------------+-----------------------------------  
 Total | 3,000 100.00  
  
  
 recieved |  
interventio |  
 n | Freq. Percent Cum.  
------------+-----------------------------------  
 0 | 1,547 51.57 51.57  
 1 | 1,453 48.43 100.00  
------------+-----------------------------------  
 Total | 3,000 100.00

### R

library(haven) # read data in Stata format  
  
df <- read\_dta("simulated\_multilevel\_data.dta")

R’s descriptive statistics functions rely heavily on whether a variable is a *numeric* variable, or a *factor* variable. Below, I convert two variables to factors (factor) before using summary[[1]](#footnote-40) to generate descriptive statistics.

df$country <- factor(df$country)  
  
df$identity <- factor(df$identity)  
  
df$intervention <- factor(df$intervention)  
  
summary(df)

country HDI family id identity  
 1 : 100 Min. :33.00 Min. : 1.00 Length:3000 1:1507   
 2 : 100 1st Qu.:53.00 1st Qu.: 25.75 Class :character 2:1493   
 3 : 100 Median :70.00 Median : 50.50 Mode :character   
 4 : 100 Mean :64.77 Mean : 50.50   
 5 : 100 3rd Qu.:81.00 3rd Qu.: 75.25   
 6 : 100 Max. :87.00 Max. :100.00   
 (Other):2400   
 intervention physical\_punishment warmth outcome   
 0:1547 Min. :0.000 Min. :0.000 Min. :29.61   
 1:1453 1st Qu.:2.000 1st Qu.:2.000 1st Qu.:48.02   
 Median :2.000 Median :4.000 Median :52.45   
 Mean :2.479 Mean :3.522 Mean :52.43   
 3rd Qu.:3.000 3rd Qu.:5.000 3rd Qu.:56.86   
 Max. :5.000 Max. :7.000 Max. :74.84

### Julia

using Tables, MixedModels, MixedModelsExtras, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
df = DataFrame(load("simulated\_multilevel\_data.dta"))

Similarly to R, Julia relies on the idea of *variable type*. I use transform to convert the appropriate variables to *categorical* variables.

@transform!(df, :country = categorical(:country))  
  
@transform!(df, :identity = categorical(:identity))  
  
@transform!(df, :intervention = categorical(:intervention))

describe(df) # descriptive statistics

9×7 DataFrame  
 Row │ variable mean min median max nmissing eltype ⋯  
 │ Symbol Union… Any Union… Any Int64 Union ⋯  
─────┼──────────────────────────────────────────────────────────────────────────  
 1 │ country 1.0 30.0 0 Union{ ⋯  
 2 │ HDI 64.7667 33.0 70.0 87.0 0 Union{  
 3 │ family 50.5 1.0 50.5 100.0 0 Union{  
 4 │ id 1.1 9.99 0 Union{  
 5 │ identity 1.0 2.0 0 Union{ ⋯  
 6 │ intervention 0.0 1.0 0 Union{  
 7 │ physical\_punishment 2.47867 0.0 2.0 5.0 0 Union{  
 8 │ warmth 3.52167 0.0 4.0 7.0 0 Union{  
 9 │ outcome 52.4333 29.608 52.449 74.8355 0 Union{ ⋯  
 1 column omitted

## 2.2 Interpretation

Examining descriptive statistics is an important first step in any analysis. It is important to examine your descriptive statistics first, before skipping ahead to more sophisticated analyses, such as multilevel models.

In examining the descriptive statistics for this data, we get a sense of the data.

* outcome has a mean of approximately 52 and ranges from approximately 30 to 75.
* warmth and physical punishment are both variables that represent the number of times that parents use each of these forms of discipline in a week. The average of the former is about 3.5, while the average of the latter is about 2.5.
* HDI, the Human Development Index has an average of about 65, and a wide range.
* identity is a categorical variable for a hypothetical identity group, and has values of 1 and 2.
* intervention is also a categorical variable, and has values of 0 and 1.

# 3. Unconditional Model

An *unconditional* multilevel model is a model with no independent variables. One should always run an unconditional model as the first step of a multilevel model in order to get a sense of the way that variation is apportioned in the model across the different levels.

## 3.1 The Equation

The Intraclass Correlation Coefficient (ICC) is given by:

In a two level multilevel model, the ICC provides a measure of the amount of variation attributable to Level 2.

## 3.2 Run Models

### Stata

use simulated\_multilevel\_data.dta // use data

mixed outcome || country: // unconditional model

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -9802.8371   
Iteration 1: Log likelihood = -9802.8371   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 3,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 100  
 avg = 100.0  
 max = 100  
 Wald chi2(0) = .  
Log likelihood = -9802.8371 Prob > chi2 = .  
  
------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
-------------+----------------------------------------------------------------  
 \_cons | 52.43327 .3451217 151.93 0.000 51.75685 53.1097  
------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Identity |  
 var(\_cons) | 3.178658 .9226737 1.799552 5.614658  
-----------------------------+------------------------------------------------  
 var(Residual) | 39.46106 1.024013 37.50421 41.52  
------------------------------------------------------------------------------  
LR test vs. linear model: chibar2(01) = 166.31 Prob >= chibar2 = 0.0000

estat icc // ICC

Intraclass correlation  
  
------------------------------------------------------------------------------  
 Level | ICC Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
 country | .0745469 .0201254 .0434963 .1248696  
------------------------------------------------------------------------------

### R

library(haven)  
  
df <- read\_dta("simulated\_multilevel\_data.dta")

library(lme4) # estimate multilevel models  
  
fit0 <- lmer(outcome ~ (1 | country),  
 data = df) # unconditional model  
  
summary(fit0)

Linear mixed model fit by REML ['lmerMod']  
Formula: outcome ~ (1 | country)  
 Data: df  
  
REML criterion at convergence: 19605.9  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.3844 -0.6655 -0.0086 0.6725 3.6626   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 country (Intercept) 3.302 1.817   
 Residual 39.461 6.282   
Number of obs: 3000, groups: country, 30  
  
Fixed effects:  
 Estimate Std. Error t value  
(Intercept) 52.433 0.351 149.4

library(performance)  
  
performance::icc(fit0) # ICC

# Intraclass Correlation Coefficient  
  
 Adjusted ICC: 0.077  
 Unadjusted ICC: 0.077

### Julia

using Tables, MixedModels, MixedModelsExtras,   
StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
df = DataFrame(load("simulated\_multilevel\_data.dta"))

@transform!(df, :country = categorical(:country))

m0 = fit(MixedModel,   
 @formula(outcome ~ (1 | country)), df) # unconditional model

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + (1 | country)  
 logLik -2 logLik AIC AICc BIC   
 -9802.8371 19605.6742 19611.6742 19611.6822 19629.6933  
  
Variance components:  
 Column Variance Std.Dev.  
country (Intercept) 3.17863 1.78287  
Residual 39.46106 6.28180  
 Number of obs: 3000; levels of grouping factors: 30  
  
 Fixed-effects parameters:  
──────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
──────────────────────────────────────────────────  
(Intercept) 52.4333 0.345121 151.93 <1e-99  
──────────────────────────────────────────────────

icc(m0) # ICC

0.07454637475695493

## 3.3 Interpretation

In each case, the software finds that nearly 8% of the variation in the outcome is explainable by the clustering of the observations in each country.

# 4. Cross Sectional Multilevel Models

## 4.1 The Equation

Recall the general model of [Equation 1.1](#eq-MLMsimple), and the syntax outlined in [Section 1.3](#sec-syntax). Below in [Equation 4.1](#eq-MLMsubstantive), we consider a more substantive example.

## 4.2 Correlated and Uncorrelated Random Effects

Consider the covariance matrix of random effects (e.g.  and ). In [Equation 4.2](#eq-varcovar) the covariances of the random effects are constrained to be zero.

As discussed in the Chapter on multilevel models with cross-sectional data, however, one can consider a multilevel model in which the random effects are correlated, as is the case in [Equation 4.3](#eq-varcovaruns).

Procedures for estimating models with uncorrelated and correlated random effects are detailed below (Bates et al., 2015; Bates, 2024; StataCorp, 2021).

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 4.1: Correlated and Uncorrelated Random Effects   | Software | Uncorrelated Random Effects | Correlated Random Effects | | --- | --- | --- | | Stata | default | add option: , cov(uns) | | R | separate random effects from grouping variable with || | separate random effects from grouping variable with | | | Julia | separate terms for each random effect e.g. (1 | group) + (0 + x | group) | separate random effects from grouping variable with |. | |

All models in the examples below are run with *uncorrelated* random effects, but could just as easily be run with *correlated* random effects.

## 4.3 Run Models

### Stata

#### 4.3.0.1 Get The Data

use simulated\_multilevel\_data.dta

#### 4.3.0.2 Run The Model

mixed outcome warmth physical\_punishment i.identity i.intervention HDI || country: warmth

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -9626.6279   
Iteration 1: Log likelihood = -9626.607   
Iteration 2: Log likelihood = -9626.607   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 3,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 100  
 avg = 100.0  
 max = 100  
 Wald chi2(5) = 334.14  
Log likelihood = -9626.607 Prob > chi2 = 0.0000  
  
-------------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
--------------------+----------------------------------------------------------------  
 warmth | .8345368 .0637213 13.10 0.000 .7096453 .9594282  
physical\_punishment | -.9916657 .0797906 -12.43 0.000 -1.148052 -.8352791  
 2.identity | -.3004767 .2170295 -1.38 0.166 -.7258466 .1248933  
 1.intervention | .6396427 .2174519 2.94 0.003 .2134448 1.065841  
 HDI | -.003228 .0199257 -0.16 0.871 -.0422817 .0358256  
 \_cons | 51.99991 1.371257 37.92 0.000 49.3123 54.68753  
-------------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Independent |  
 var(warmth) | .0227504 .0257784 .0024689 .2096436  
 var(\_cons) | 2.963975 .9737647 1.556777 5.643163  
-----------------------------+------------------------------------------------  
 var(Residual) | 34.97499 .9097109 33.23668 36.80422  
------------------------------------------------------------------------------  
LR test vs. linear model: chi2(2) = 205.74 Prob > chi2 = 0.0000  
  
Note: LR test is conservative and provided only for reference.

### R

#### 4.3.0.3 Get The Data

library(haven)  
  
df <- read\_dta("simulated\_multilevel\_data.dta")

#### 4.3.0.4 Run The Model

|  |  |
| --- | --- |
|  | lme4 does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library lmerTest to provide p values for lme4 results. |

|  |  |
| --- | --- |
|  | R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: options(scipen = 999). |

library(lme4)   
  
library(lmerTest)  
  
options(scipen = 999)   
  
fit1 <- lmer(outcome ~ warmth + physical\_punishment +   
 identity + intervention + HDI +  
 (1 + warmth || country),  
 data = df)  
  
summary(fit1)

Linear mixed model fit by REML. t-tests use Satterthwaite's method [  
lmerModLmerTest]  
Formula: outcome ~ warmth + physical\_punishment + identity + intervention +   
 HDI + (1 + warmth || country)  
 Data: df  
  
REML criterion at convergence: 19268.8  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.9774 -0.6563 0.0187 0.6645 3.6730   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 country (Intercept) 3.19056 1.786   
 country.1 warmth 0.02465 0.157   
 Residual 35.01782 5.918   
Number of obs: 3000, groups: country, 30  
  
Fixed effects:  
 Estimate Std. Error df t value  
(Intercept) 52.311714 1.446735 33.113738 36.158  
warmth 0.834562 0.064252 41.896966 12.989  
physical\_punishment -0.991892 0.079845 2968.010901 -12.423  
identity -0.300350 0.217179 2970.106304 -1.383  
intervention 0.639059 0.217603 2971.185215 2.937  
HDI -0.003395 0.020596 27.598517 -0.165  
 Pr(>|t|)   
(Intercept) < 0.0000000000000002 \*\*\*  
warmth 0.000000000000000277 \*\*\*  
physical\_punishment < 0.0000000000000002 \*\*\*  
identity 0.16678   
intervention 0.00334 \*\*   
HDI 0.87027   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
Correlation of Fixed Effects:  
 (Intr) warmth physc\_ idntty intrvn  
warmth -0.119   
physcl\_pnsh -0.145 -0.003   
identity -0.220 -0.012 -0.003   
interventin -0.077 0.034 0.022 -0.018   
HDI -0.922 -0.006 0.009 -0.001 0.000

### Julia

#### 4.3.0.5 Get The Data

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
df = DataFrame(load("simulated\_multilevel\_data.dta"))

#### 4.3.0.6 Change Country To Categorical

@transform!(df, :country = categorical(:country))

#### 4.3.0.7 Run The Model

m1 = fit(MixedModel, @formula(outcome ~ warmth + physical\_punishment +   
 identity + intervention + HDI +  
 (1 | country) +  
 (0 + warmth | country)), df)

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + warmth + physical\_punishment + identity + intervention + HDI + (1 | country) + (0 + warmth | country)  
 logLik -2 logLik AIC AICc BIC   
 -9626.6070 19253.2140 19271.2140 19271.2742 19325.2713  
  
Variance components:  
 Column Variance Std.Dev. Corr.  
country (Intercept) 2.963849 1.721583  
 warmth 0.022756 0.150852 .   
Residual 34.974984 5.913965  
 Number of obs: 3000; levels of grouping factors: 30  
  
 Fixed-effects parameters:  
─────────────────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
─────────────────────────────────────────────────────────────  
(Intercept) 52.3004 1.40406 37.25 <1e-99  
warmth 0.834537 0.0637228 13.10 <1e-38  
physical\_punishment -0.991665 0.0797906 -12.43 <1e-34  
identity -0.300475 0.217029 -1.38 0.1662  
intervention 0.639641 0.217452 2.94 0.0033  
HDI -0.0032286 0.0199255 -0.16 0.8713  
─────────────────────────────────────────────────────────────

## 4.4 Interpretation

Models suggest that parental warmth is associated with increases in the beneficial outcome, while physical punishment is associated with decreases in the beneficial outcome. Membership in the group represented by identity is not associated with the outcome. The intervention is associated with increases in the outcome. The Human Development Index is not associated with the outcome.

# 5. Longitudinal Multilevel Models

## 5.1 The Data

The data employed in these examples are a longitudinal extension of the data described in [Section 1.2](#sec-data).

## 5.2 The Equation

## 5.3 Run Models

### Stata

#### 5.3.0.1 Get The Data

use simulated\_multilevel\_longitudinal\_data.dta

#### 5.3.0.2 Run The Model

##### 5.3.0.2.1 Main Effects Only

mixed outcome t warmth physical\_punishment i.identity i.intervention HDI || country: warmth

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -28739.506   
Iteration 1: Log likelihood = -28739.506   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 9,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 300  
 avg = 300.0  
 max = 300  
 Wald chi2(6) = 1119.81  
Log likelihood = -28739.506 Prob > chi2 = 0.0000  
  
-------------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
--------------------+----------------------------------------------------------------  
 t | .9443446 .0756408 12.48 0.000 .7960914 1.092598  
 warmth | .9123903 .0430042 21.22 0.000 .8281035 .996677  
physical\_punishment | -.9881587 .0451732 -21.87 0.000 -1.076696 -.8996209  
 2.identity | -.1241465 .1242225 -1.00 0.318 -.367618 .1193251  
 1.intervention | .8575839 .1245179 6.89 0.000 .6135332 1.101635  
 HDI | -.0025173 .0191696 -0.13 0.896 -.0400891 .0350544  
 \_cons | 50.54528 1.304146 38.76 0.000 47.9892 53.10136  
-------------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Independent |  
 var(warmth) | .0229349 .0135353 .0072136 .0729194  
 var(\_cons) | 3.0009 .8550708 1.716768 5.245553  
-----------------------------+------------------------------------------------  
 var(Residual) | 34.31935 .5130963 33.3283 35.33988  
------------------------------------------------------------------------------  
LR test vs. linear model: chi2(2) = 767.22 Prob > chi2 = 0.0000  
  
Note: LR test is conservative and provided only for reference.

##### 5.3.0.2.2 Interactions With Time

mixed outcome c.t##(c.warmth c.physical\_punishment i.identity i.intervention c.HDI) || country: warmth

Performing EM optimization ...  
  
Performing gradient-based optimization:   
Iteration 0: Log likelihood = -28738.554   
Iteration 1: Log likelihood = -28738.554   
  
Computing standard errors ...  
  
Mixed-effects ML regression Number of obs = 9,000  
Group variable: country Number of groups = 30  
 Obs per group:  
 min = 300  
 avg = 300.0  
 max = 300  
 Wald chi2(11) = 1122.75  
Log likelihood = -28738.554 Prob > chi2 = 0.0000  
  
---------------------------------------------------------------------------------------  
 outcome | Coefficient Std. err. z P>|z| [95% conf. interval]  
----------------------+----------------------------------------------------------------  
 t | .7537359 .3719996 2.03 0.043 .0246301 1.482842  
 warmth | .8198365 .0911059 9.00 0.000 .6412723 .9984008  
 physical\_punishment | -1.000348 .1198049 -8.35 0.000 -1.235162 -.7655353  
 2.identity | -.2340191 .3271243 -0.72 0.474 -.875171 .4071327  
 1.intervention | .6597456 .3275877 2.01 0.044 .0176856 1.301806  
 HDI | -.0005531 .0210866 -0.03 0.979 -.041882 .0407757  
 |  
 c.t#c.warmth | .0463746 .0402459 1.15 0.249 -.0325059 .1252551  
 |  
 c.t#|  
c.physical\_punishment | .0061255 .0551491 0.11 0.912 -.1019647 .1142157  
 |  
 identity#c.t |  
 2 | .0548965 .1513015 0.36 0.717 -.241649 .3514421  
 |  
 intervention#c.t |  
 1 | .0990704 .151503 0.65 0.513 -.19787 .3960108  
 |  
 c.t#c.HDI | -.0009791 .0043888 -0.22 0.823 -.0095811 .0076229  
 |  
 \_cons | 50.92503 1.494157 34.08 0.000 47.99654 53.85352  
---------------------------------------------------------------------------------------  
  
------------------------------------------------------------------------------  
 Random-effects parameters | Estimate Std. err. [95% conf. interval]  
-----------------------------+------------------------------------------------  
country: Independent |  
 var(warmth) | .0228292 .0135078 .0071588 .0728013  
 var(\_cons) | 3.001849 .8552796 1.71738 5.247001  
-----------------------------+------------------------------------------------  
 var(Residual) | 34.31227 .5129896 33.32141 35.33258  
------------------------------------------------------------------------------  
LR test vs. linear model: chi2(2) = 767.35 Prob > chi2 = 0.0000  
  
Note: LR test is conservative and provided only for reference.

### R

#### 5.3.0.3 Get The Data

library(haven)  
  
dfL <- read\_dta("simulated\_multilevel\_longitudinal\_data.dta")

#### 5.3.0.4 Run The Model

|  |  |
| --- | --- |
|  | lme4 does not directly provide p values in results, because of some disagreement over exactly how these p values should be calculated. Therefore, in this Appendix, I also call library lmerTest to provide p values for lme4 results. |

|  |  |
| --- | --- |
|  | R prefers to use scientific notation when possible. I find that the use of scientific notation can be confusing in reading results. I turn off scientific notation by setting a penalty for its use: options(scipen = 999). |

##### 5.3.0.4.1 Main Effects Only

library(lme4)   
  
library(lmerTest)  
  
options(scipen = 999)   
  
fit2A <- lmer(outcome ~ t + warmth + physical\_punishment +   
 identity + intervention + HDI +  
 (1 | country/id),  
 data = dfL)  
  
summary(fit2A)

Linear mixed model fit by REML. t-tests use Satterthwaite's method [  
lmerModLmerTest]  
Formula:   
outcome ~ t + warmth + physical\_punishment + identity + intervention +   
 HDI + (1 | country/id)  
 Data: dfL  
  
REML criterion at convergence: 57022.7  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.6850 -0.6094 -0.0035 0.6133 3.6792   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 id:country (Intercept) 8.438 2.905   
 country (Intercept) 3.675 1.917   
 Residual 26.036 5.103   
Number of obs: 9000, groups: id:country, 3000; country, 30  
  
Fixed effects:  
 Estimate Std. Error df t value  
(Intercept) 50.5161891 1.4296454 31.1737942 35.335  
t 0.9433806 0.0658755 5998.3764616 14.321  
warmth 0.9140307 0.0379336 4745.3496835 24.096  
physical\_punishment -1.0087537 0.0497972 6483.6770989 -20.257  
identity -0.1319548 0.1517350 2968.7829265 -0.870  
intervention 0.8591494 0.1520510 2971.8112001 5.650  
HDI 0.0007909 0.0207656 28.0001836 0.038  
 Pr(>|t|)   
(Intercept) < 0.0000000000000002 \*\*\*  
t < 0.0000000000000002 \*\*\*  
warmth < 0.0000000000000002 \*\*\*  
physical\_punishment < 0.0000000000000002 \*\*\*  
identity 0.385   
intervention 0.0000000175 \*\*\*  
HDI 0.970   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
Correlation of Fixed Effects:  
 (Intr) t warmth physc\_ idntty intrvn  
t -0.091   
warmth -0.088 -0.002   
physcl\_pnsh -0.090 -0.007 -0.012   
identity -0.156 0.000 -0.013 -0.003   
interventin -0.055 0.000 0.039 0.019 -0.018   
HDI -0.941 0.000 -0.004 0.005 0.000 0.002

##### 5.3.0.4.2 Interactions With Time

fit2B <- lmer(outcome ~ t \*(warmth + physical\_punishment +   
 identity + intervention + HDI) +  
 (1 | country/id),  
 data = dfL)  
  
summary(fit2B)

Linear mixed model fit by REML. t-tests use Satterthwaite's method [  
lmerModLmerTest]  
Formula:   
outcome ~ t \* (warmth + physical\_punishment + identity + intervention +   
 HDI) + (1 | country/id)  
 Data: dfL  
  
REML criterion at convergence: 57042.8  
  
Scaled residuals:   
 Min 1Q Median 3Q Max   
-3.7118 -0.6092 -0.0024 0.6150 3.6779   
  
Random effects:  
 Groups Name Variance Std.Dev.  
 id:country (Intercept) 8.436 2.905   
 country (Intercept) 3.675 1.917   
 Residual 26.046 5.104   
Number of obs: 9000, groups: id:country, 3000; country, 30  
  
Fixed effects:  
 Estimate Std. Error df t value  
(Intercept) 51.0036725 1.6087742 49.9583024 31.703  
t 0.6989769 0.3746882 6131.2125222 1.865  
warmth 0.8170912 0.0805355 8274.9994610 10.146  
physical\_punishment -1.0097729 0.1113557 8084.6085126 -9.068  
identity -0.2446453 0.3041604 8695.8966126 -0.804  
intervention 0.6604671 0.3046286 8697.0843469 2.168  
HDI 0.0026692 0.0221295 36.1037721 0.121  
t:warmth 0.0486211 0.0356217 6404.8722416 1.365  
t:physical\_punishment 0.0004964 0.0494590 6753.0158846 0.010  
t:identity 0.0563140 0.1318043 5993.4518199 0.427  
t:intervention 0.0995037 0.1319917 5994.1433047 0.754  
t:HDI -0.0009379 0.0038233 5993.9091197 -0.245  
 Pr(>|t|)   
(Intercept) <0.0000000000000002 \*\*\*  
t 0.0622 .   
warmth <0.0000000000000002 \*\*\*  
physical\_punishment <0.0000000000000002 \*\*\*  
identity 0.4212   
intervention 0.0302 \*   
HDI 0.9047   
t:warmth 0.1723   
t:physical\_punishment 0.9920   
t:identity 0.6692   
t:intervention 0.4510   
t:HDI 0.8062   
---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
  
Correlation of Fixed Effects:  
 (Intr) t warmth physc\_ idntty intrvn HDI t:wrmt t:phy\_  
t -0.466   
warmth -0.169 0.285   
physcl\_pnsh -0.183 0.313 -0.005   
identity -0.278 0.450 -0.013 -0.002   
interventin -0.100 0.162 0.039 0.019 -0.017   
HDI -0.892 0.230 -0.007 0.012 -0.001 0.003   
t:warmth 0.150 -0.324 -0.882 0.001 0.011 -0.035 0.006   
t:physcl\_pn 0.164 -0.351 0.004 -0.894 -0.001 -0.017 -0.010 -0.003   
t:identity 0.242 -0.519 0.011 0.000 -0.867 0.014 0.001 -0.013 0.002  
t:intervntn 0.087 -0.187 -0.035 -0.017 0.014 -0.867 -0.003 0.041 0.019  
t:HDI 0.310 -0.666 0.015 -0.027 0.002 -0.007 -0.346 -0.016 0.029  
 t:dntt t:ntrv  
t   
warmth   
physcl\_pnsh   
identity   
interventin   
HDI   
t:warmth   
t:physcl\_pn   
t:identity   
t:intervntn -0.016   
t:HDI -0.002 0.008

### Julia

#### 5.3.0.5 Get The Data

using Tables, MixedModels, StatFiles, DataFrames, CategoricalArrays, DataFramesMeta  
  
dfL = DataFrame(load("simulated\_multilevel\_longitudinal\_data.dta"))

#### 5.3.0.6 Run The Model

##### 5.3.0.6.1 Change Country To Categorical

@transform!(dfL, :country = categorical(:country))

##### 5.3.0.6.2 Main Effects Only

m2A = fit(MixedModel, @formula(outcome ~ t + warmth +   
 physical\_punishment +   
 identity + intervention +   
 HDI +  
 (1 | country) +   
 (0 + warmth | country) +  
 (1 | id)), dfL)

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + t + warmth + physical\_punishment + identity + intervention + HDI + (1 | country) + (0 + warmth | country) + (1 | id)  
 logLik -2 logLik AIC AICc BIC   
 -28499.6031 56999.2063 57021.2063 57021.2356 57099.3610  
  
Variance components:  
 Column Variance Std.Dev. Corr.  
id (Intercept) 8.387351 2.896092  
country (Intercept) 3.166939 1.779590  
 warmth 0.010760 0.103732 .   
Residual 26.027290 5.101695  
 Number of obs: 9000; levels of grouping factors: 3000, 30  
  
 Fixed-effects parameters:  
───────────────────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
───────────────────────────────────────────────────────────────  
(Intercept) 50.5949 1.35491 37.34 <1e-99  
t 0.943864 0.0658716 14.33 <1e-45  
warmth 0.913496 0.0423739 21.56 <1e-99  
physical\_punishment -1.0079 0.0497622 -20.25 <1e-90  
identity -0.127692 0.151584 -0.84 0.3996  
intervention 0.858997 0.15191 5.65 <1e-07  
HDI -0.000565882 0.0196433 -0.03 0.9770  
───────────────────────────────────────────────────────────────

##### 5.3.0.6.3 Interactions With Time

m2B = fit(MixedModel, @formula(outcome ~ t \* (warmth +   
 physical\_punishment +   
 identity + intervention +   
 HDI) +  
 (1 | country) +  
 (0 + warmth | country) +  
 (1 | id)), dfL)

Linear mixed model fit by maximum likelihood  
 outcome ~ 1 + t + warmth + physical\_punishment + identity + intervention + HDI + t & warmth + t & physical\_punishment + t & identity + t & intervention + t & HDI + (1 | country) + (0 + warmth | country) + (1 | id)  
 logLik -2 logLik AIC AICc BIC   
 -28498.3091 56996.6182 57028.6182 57028.6788 57142.2979  
  
Variance components:  
 Column Variance Std.Dev. Corr.  
id (Intercept) 8.391746 2.896851  
country (Intercept) 3.170031 1.780458  
 warmth 0.010609 0.102999 .   
Residual 26.015906 5.100579  
 Number of obs: 9000; levels of grouping factors: 3000, 30  
  
 Fixed-effects parameters:  
──────────────────────────────────────────────────────────────────  
 Coef. Std. Error z Pr(>|z|)  
──────────────────────────────────────────────────────────────────  
(Intercept) 51.0751 1.54284 33.10 <1e-99  
t 0.702771 0.374539 1.88 0.0606  
warmth 0.817076 0.0826636 9.88 <1e-22  
physical\_punishment -1.00903 0.111293 -9.07 <1e-18  
identity -0.238714 0.303996 -0.79 0.4323  
intervention 0.660761 0.30445 2.17 0.0300  
HDI 0.00136065 0.0210842 0.06 0.9485  
t & warmth 0.0483635 0.0356074 1.36 0.1744  
t & physical\_punishment 0.000542203 0.0494355 0.01 0.9912  
t & identity 0.0554385 0.131745 0.42 0.6739  
t & intervention 0.0992809 0.131925 0.75 0.4517  
t & HDI -0.000955067 0.00382162 -0.25 0.8027  
──────────────────────────────────────────────────────────────────

## 5.4 Interpretation

The *main effects only model* suggests that time is associated with increases in the outcome. In the main effects model, main effects other than time, indicate whether a particular variable is associated with higher or lower intercepts of the time trajectory, at the beginning of the study time. Warmth is again associated with increases in the outcome, while physical punishment is associated with decreases in the outcome. Identity is again not associated with the outcome, while the intervention is associated with higher levels of the outcome. The Human Development Index is again not associated with the outcome.

The second model adds interactions with time to the first model. Results are largly similar to the prior model. However, here we not only examine whether main effects other than time are associated with higher or lower time trajectories, but also whether particular variables are associated with differences in the slope of the time trajectory. In this case, we find that no independent variable is associated with changes in the slope of the time trajectory.

However, it may be illustrative to imagine how we would interpret the results had a particular interaction term been statistically significant. Let us consider one of the interaction terms with the largest coefficient, intervention#time. The interaction of the intervention with time is positive. Had this coefficient been statistically signifcant, it would have indicated that the intervention was associated with more rapid increases in the outcome over time *in addition to* the fact that the intervention is associated with higher initial levels of the outcome.

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1. skimr is an excellent new alternative library for generating descriptive statistics in R. [↑](#footnote-ref-40)