# Multilevel Workshop

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# 1 Introduction

This site contains materials for a workshop on multilevel modeling.

## 1.1 Background

Multilevel models are useful when you have data that are nested or clustered inside social units such as schools, neighborhoods, states, or countries.

Multilevel models are also useful when you have longitudinal data where repeated measures are collected for study participants.

## 1.2 Simulated Multilevel Data

The data used in these workshop materials are *simulated* data on parents, children and families. The data are simulated to come from 30 hypothetical countries around the world. These are the same data used and discussed in my book *Multilevel Thinking: Discovering Variation, Universals, and Particulars in Cross-Cultural Research*.

There are two versions of the data: a *cross-sectional* data set from a single point in time; a *longitudinal* version of the data spanning several time points.

| pos | variable | label |
|-----|----------|-------|
| 1 | country | country id |
| 2 | HDI | Human Development Index |
| 3 | family | family id |
| 4 | id | unique country family id |
| 5 | identity | hypothetical identity group variable |
| 6 | intervention | recieved intervention |
| 7 | physical_punishment | physical punishment in past week |
| 8 | warmth | parental warmth in past week |
| 9 | outcome | beneficial outcome |

Figure 1.1: Simulated Multilevel Data

# 2 Two Level Cross Sectional; And Three Level Longitudinal Models

## 2.1 Cross Sectional Model

### 2.1.1 Get Data

```stata
use "simulated_multilevel_data.dta", clear
```

### 2.1.2 The Equation

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{parental warmth} + \beta_2 \text{physical punishment} + \beta_3 \text{time} +$$

$$\beta_4 \text{identity}_2 + \beta_5 \text{intervention} + \beta_6 HDI +$$

$$u_{0j} + u_{1j} \times \text{parental warmth} + e_{ij}$$

### 2.1.3 Descriptive Statistics

```stata
summarize // descriptive statistics
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     country |      3,000        15.5    8.656884         1         30
         HDI |      3,000    64.76667    17.24562        33         87
      family |      3,000        50.5    28.87088         1        100
          id |          0
    identity |      3,000    .4976667    .5000779         0          1
```

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```
-------------+----------------------------------------------------------
intervention |     3,000    .4843333    .4998378           0           1
physical_p~t |     3,000    2.478667    1.360942           0           5
      warmth |     3,000    3.521667    1.888399           0           7
     outcome |     3,000    52.43327    6.530996    29.60798    74.83553
```

### 2.1.4 Spaghetti Plot

```stata
spagplot outcome warmth, id(country) scheme(stcolor)

graph export spagplot1.png, width(1000) replace
```
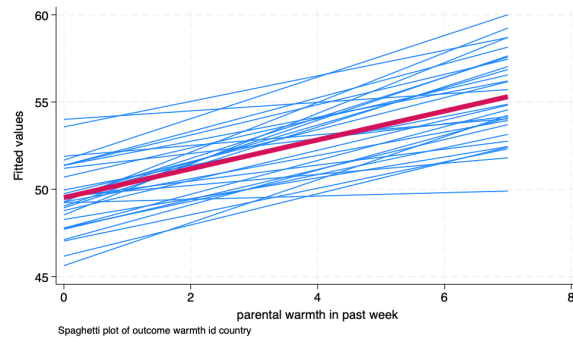


Figure 2.1: Spaghetti Plot of Outcome by Warmth by Country

### 2.1.5 Unconditional Model

#### 2.1.5.1 Model

```stata
mixed outcome || country: // unconditional model
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:   Log likelihood = -9802.8371
Iteration 1:   Log likelihood = -9802.8371

Computing standard errors ...
```

```
Mixed-effects ML regression                          Number of obs     =    3,000
Group variable: country                              Number of groups  =       30
                                                     Obs per group:
                                                                  min =      100
                                                                  avg =    100.0
                                                                  max =      100
                                                     Wald chi2(0)      =        .
Log likelihood = -9802.8371                          Prob > chi2       =        .

------------------------------------------------------------------------------
     outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |   52.43327   .3451217   151.93   0.000     51.75685    53.1097
------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
country: Identity            |
                 var(_cons)  |   3.178658   .9226737      1.799552   5.614658
-----------------------------+------------------------------------------------
              var(Residual)  |   39.46106   1.024013      37.50421      41.52
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 166.31         Prob >= chibar2 = 0.0000
```

## 2.1.5.2 ICC

```
estat icc
```

```
Intraclass correlation

------------------------------------------------------------------------------
                      Level |        ICC   Std. err.     [95% conf. interval]
----------------------------+-------------------------------------------------
                    country |   .0745469   .0201254      .0434963    .1248696
------------------------------------------------------------------------------
```

## 2.1.6 Conditional Model

```
mixed outcome warmth physical_punishment identity i.intervention HDI || country: warmth // mu

est store crosssectional // store estimates
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -9626.6279
Iteration 1:  Log likelihood =  -9626.607
Iteration 2:  Log likelihood =  -9626.607

Computing standard errors ...
```

```
Mixed-effects ML regression                      Number of obs    =   3,000
Group variable: country                          Number of groups =      30
                                                 Obs per group:
                                                              min =     100
                                                              avg =   100.0
                                                              max =     100
                                                 Wald chi2(5)     =  334.14
Log likelihood =  -9626.607                      Prob > chi2      =  0.0000
```

```
-------------------------------------------------------------------------------
           outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------------+-----------------------------------------------------------
            warmth |   .8345368   .0637213    13.10   0.000     .7096453    .9594282
physical_punishment|  -.9916657   .0797906   -12.43   0.000    -1.148052   -.8352791
          identity |  -.3004767   .2170295    -1.38   0.166    -.7258466    .1248933
    1.intervention |   .6396427   .2174519     2.94   0.003     .2134448    1.065841
               HDI |   -.003228   .0199257    -0.16   0.871    -.0422817    .0358256
             _cons |   51.99991   1.371257    37.92   0.000      49.3123    54.68753
-------------------------------------------------------------------------------
```

```
-------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+-------------------------------------------------
country: Independent         |
                var(warmth) |   .0227504   .0257784     .0024689    .2096436
                 var(_cons) |   2.963975   .9737647     1.556777    5.643163
```

```
--------------------------+----------------------------------------------
         var(Residual) |   34.97499    .9097109      33.23668      36.80422
--------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 205.74            Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

## 2.2 Longitudinal Model

### 2.2.1 Get Data

```
use "simulated_multilevel_longitudinal_data.dta", clear
```

### 2.2.2 The Equation

$$\text{outcome}_{ij} = \beta_0 + \beta_1 \text{parental warmth} + \beta_2 \text{physical punishment} + \beta_3 \text{time}+$$

$$\beta_4 \text{identity}_2 + \beta_5 \text{intervention} + \beta_5 HDI+$$

$$u_{0j} + u_{1j} \times \text{parental warmth}+$$

$$v_{0i} + v_{1i} \times t + e_{ij}$$

### 2.2.3 Descriptive Statistics

```
summarize // descriptive statistics
```

```
    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
     country |      9,000        15.5    8.655922          1         30
         HDI |      9,000    64.76667     17.2437         33         87
      family |      9,000        50.5    28.86767          1        100
          id |          0
```

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```
    identity |      9,000    .4976667    .5000223          0          1
-------------+--------------------------------------------------------------
intervention |      9,000    .4843333    .4997823          0          1
           t |      9,000           2    .8165419          1          3
 physical_p~t |      9,000    2.485333    1.373639          0          5
      warmth |      9,000    3.514222      1.8839          0          7
     outcome |      9,000    53.37768    6.572285   29.60798   79.02199
```

### 2.2.4 Alternate Plot

```
encode id, generate(idNUMERIC) // numeric version of id

* spagplot outcome t if idNUMERIC <= 10, id(idNUMERIC) scheme(stcolor)

twoway (lfit outcome t) (scatter outcome t) if idNUMERIC <= 10, by(idNUMERIC) scheme(stcolor)

graph export spagplot2.png, width(1000) replace
```
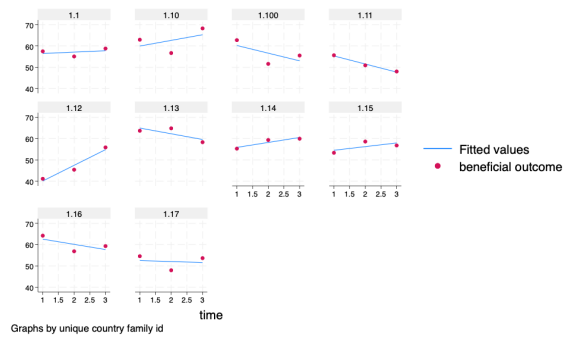


Figure 2.2: Alternate Plot of Outcome by Time by Individual; First 10 Observations

### 2.2.5 Unconditional Model

#### 2.2.5.1 Model

```
mixed outcome || country: || id: // unconditional model
```

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### 2.2.5.2 ICC

```
estat icc
```

Intraclass correlation

```
--------------------------------------------------------------------------
              Level |       ICC   Std. err.      [95% conf. interval]
--------------------+-----------------------------------------------------
            country |   .0748336   .0190847       .0450028    .1219141
         id|country |   .3462837   .0171461       .3134867    .3806097
--------------------------------------------------------------------------
```

## 2.2.6 Conditional Model

```
mixed outcome t warmth physical_punishment i.identity i.intervention HDI || country: warmth

est store longitudinal // store estimates
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood =  -28523.49
Iteration 1:  Log likelihood = -28499.987
Iteration 2:  Log likelihood = -28499.739
Iteration 3:  Log likelihood = -28499.604
Iteration 4:  Log likelihood = -28499.603

Computing standard errors ...

Mixed-effects ML regression                         Number of obs =   9,000

        Grouping information
        ----------------------------------------------------------------
                        |     No. of        Observations per group
         Group variable |     groups    Minimum     Average     Maximum
        ----------------+-----------------------------------------------
                country |         30        300       300.0         300
```

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```
              id |       3,000            3         3.0           3
            --------------------------------------------------------------
```

```
                                                 Wald chi2(6)  = 1096.15
Log likelihood = -28499.603                      Prob > chi2   =  0.0000
```

```
-------------------------------------------------------------------------------
         outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-----------------+-------------------------------------------------------------
               t |    .943864   .0658716    14.33   0.000      .814758     1.07297
          warmth |   .9134959   .0423732    21.56   0.000      .830446    .9965459
physical_punishment | -1.007897   .0497622   -20.25   0.000    -1.105429   -.9103647
      1.identity |  -.1276926   .1515835    -0.84   0.400     -.4247908    .1694057
  1.intervention |   .8589966   .1519095     5.65   0.000      .5612596    1.156734
             HDI |  -.0005657   .0196437    -0.03   0.977     -.0390666    .0379352
           _cons |   50.46724   1.338318    37.71   0.000      47.84418    53.09029
-------------------------------------------------------------------------------
```

```
--------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+--------------------------------------------------
country: Independent         |
              var(warmth) |   .0107586   .0127845      .0010478     .1104703
               var(_cons) |   3.167085   .9146761      1.798154     5.578181
-----------------------------+--------------------------------------------------
id: Independent              |
                  var(t) |   3.58e-09    7.06e-07      3.5e-177     3.7e+159
               var(_cons) |   8.387275   .4724188      7.510631     9.366242
-----------------------------+--------------------------------------------------
            var(Residual) |   26.02733   .4753701      25.11211     26.97592
--------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 1247.03          Prob > chi2 = 0.0000
```

Note: LR test is conservative and provided only for reference.

## 2.3 Nice Table of Results

```
etable, estimates(crosssectional longitudinal) ///
showstars showstarsnote /// show stars and note
column(estimate) // column is modelname
```

```
                                    crosssectional longitudinal
-----------------------------------------------------------------
parental warmth in past week           0.835 **       0.913 **
                                      (0.064)        (0.042)
physical punishment in past week      -0.992 **      -1.008 **
                                      (0.080)        (0.050)
hypothetical identity group variable  -0.300
                                      (0.217)
recieved intervention
  1                                    0.640 **       0.859 **
                                      (0.217)        (0.152)
Human Development Index               -0.003         -0.001
                                      (0.020)        (0.020)
time                                                  0.944 **
                                                     (0.066)
hypothetical identity group variable
  1                                                  -0.128
                                                     (0.152)
Intercept                             52.000 **      50.467 **
                                      (1.371)        (1.338)
var(warmth)                            0.023          0.011
                                      (0.026)        (0.013)
var(_cons)                             2.964          3.167
                                      (0.974)        (0.915)
var(e)                                34.975         26.027
                                      (0.910)        (0.475)
var(_cons)                                            8.387
                                                     (0.472)
var(t)                                                0.000
                                                     (0.000)
Number of observations                 3000           9000
-----------------------------------------------------------------
** p<.01, * p<.05
```

## 2.4 QUESTIONS???

# 3 Cross-Classified Models

## 3.1 Introduction

A two level multilevel model imagines that *Level 1* units are nested in *Level 2* units. A three level multilevel model imagines that *Level 1* units are nested in *Level 2* units, which are in turn nested in *Level 3*.

A cross-classified model imagines that the nesting is not hierarchical, but rather that there are two sets of clusters or nestings in which individuals may be nested.

## 3.2 Get Data

```
use "simulated_multilevel_longitudinal_data.dta", clear
```

## 3.3 Cross Classified Model

We can treat these random effects as being *cross classified*.

This might be useful if we had data where individuals lived in different countries at different times.

However, because `id` is in fact nested inside `country`, in this case, estimating the random effects as cross classified will be more time consuming, but will give us equivalent results to a three level model.

### 3.3.1 Standard (Less Computationally Efficient) Syntax

The below syntax will take a very long time to run with the full sample, and thus we have commented it out.

```
* mixed outcome t warmth physical_punishment || _all: R.country || _all: R.id
```

```
* est store crossed1
```

The documentation notes that we can use a *much* more computationally efficient version of the above command, which is what we do in these notes. The user can verify that both versions of the command will produce equivalent results.

In fact, at the end of handout we verify the similarity of both sets of syntax using a random sample.

### 3.3.2 Cross Classified With Computationally Efficient Syntax

```
mixed outcome t warmth physical_punishment || _all: R.country || id:
```

```
est store crossed2 // store crossed effects result
```

```
Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28516.314
Iteration 1:  Log likelihood = -28516.277
Iteration 2:  Log likelihood = -28516.277

Computing standard errors ...

Mixed-effects ML regression                         Number of obs =    9,000

        Grouping information
        ------------------------------------------------------------
                        |    No. of      Observations per group
        Group variable |    groups    Minimum    Average    Maximum
        ----------------+-------------------------------------------
                  _all |        1      9,000    9,000.0      9,000
                    id |    3,000          3        3.0          3
        ------------------------------------------------------------


                                              Wald chi2(3)  = 1168.69
Log likelihood = -28516.277                   Prob > chi2   =  0.0000
```

```
----------------------------------------------------------------------------
          outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
------------------+---------------------------------------------------------------
                t |   .9434605    .065866     14.32   0.000     .8143654    1.072556
           warmth |   .9053924   .0380439     23.80   0.000     .8308277    .9799572
physical_punishment | -1.014385   .0499354    -20.31   0.000    -1.112257    -.916514
            _cons |    50.8301   .4123007    123.28   0.000       50.022    51.63819
----------------------------------------------------------------------------


--------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.    [95% conf. interval]
-----------------------------+--------------------------------------------
_all: Identity               |
             var(R.country)  |   3.429974    .930313     2.015668    5.836634
-----------------------------+--------------------------------------------
id: Identity                 |
                 var(_cons)  |   8.608872   .4757699     7.725107     9.59374
-----------------------------+--------------------------------------------
              var(Residual)  |   26.02862   .4752444     25.11363    26.97695
--------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 1260.84              Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

## 3.4 Three Level Model

```
mixed outcome t warmth physical_punishment || country: || id:  // 3 level w/ random intercept

est store threelevel // store random intercept model


Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -28516.314
Iteration 1:  Log likelihood = -28516.277
Iteration 2:  Log likelihood = -28516.277

Computing standard errors ...
```

```
Mixed-effects ML regression                        Number of obs  =    9,000

        Grouping information
        ----------------------------------------------------------------
                        |   No. of      Observations per group
        Group variable  |   groups   Minimum   Average   Maximum
        ----------------+-----------------------------------------------
                country |       30       300     300.0       300
                     id |    3,000         3       3.0         3
        ----------------------------------------------------------------


                                                   Wald chi2(3)   =  1168.69
Log likelihood = -28516.277                        Prob > chi2    =   0.0000


--------------------------------------------------------------------------------
           outcome | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------------+------------------------------------------------------------
                 t |    .9434605    .065866    14.32   0.000     .8143654    1.072556
            warmth |    .9053924   .0380439    23.80   0.000     .8308277    .9799572
physical_punishment|   -1.014385   .0499354   -20.31   0.000    -1.112257    -.916514
             _cons |     50.8301   .4123007   123.28   0.000       50.022    51.63819
--------------------------------------------------------------------------------


-----------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.    [95% conf. interval]
-----------------------------+-----------------------------------------------
country: Identity            |
                 var(_cons)  |   3.429974    .930313     2.015668    5.836634
-----------------------------+-----------------------------------------------
id: Identity                 |
                 var(_cons)  |   8.608872   .4757699     7.725107     9.59374
-----------------------------+-----------------------------------------------
               var(Residual) |   26.02862   .4752444     25.11363    26.97695
-----------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 1260.84           Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

## 3.5 Nice Table of Results of Three Level and Cross Classified Model

```
etable, estimates(threelevel crossed2), ///
showstars showstarsnote /// show stars and note
column(estimate) // column is modelname
```

```
invalid 'showstars'
r(198);
```

```
r(198);
```

## 3.6 Verification of Syntax Equivalence for Cross Classified Model

```
keep if family <= 5 // random sample of families

quietly mixed outcome t warmth physical_punishment || _all: R.country || _all: R.id

est store crossed1A // less efficient syntax

quietly mixed outcome t warmth physical_punishment || _all: R.country || id:

est store crossed2A // more efficient syntax

etable, estimates(crossed1A crossed2A) ///
showstars showstarsnote /// show stars and note
column(estimate) // column is modelname
```

```
(8,550 observations deleted)
```

```
-------------------------------------------------------
                                crossed1A   crossed2A
```

```
----------------------------------------------------------
time                              0.745 **    0.745 **
                                 (0.281)     (0.281)
parental warmth in past week      0.871 **    0.871 **
                                 (0.160)     (0.160)
physical punishment in past week -1.262 **   -1.262 **
                                 (0.206)     (0.206)
Intercept                        51.755 **   51.755 **
                                 (1.009)     (1.009)
var(R_country)                    2.245       2.245
                                 (1.319)     (1.319)
var(R_id)                         5.425
                                 (1.843)
var(e)                           23.638      23.638
                                 (1.933)     (1.933)
var(_cons)                                    5.425
                                             (1.843)
Number of observations            450         450
----------------------------------------------------------
** p<.01, * p<.05
```

## 3.7 QUESTIONS???