Interactions in Logistic Regression

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Background

The purpose of this tutorial is to illustrate the idea that in *logistic regression*, the β parameter for an interaction term may not accurately characterize the underlying interactive relationships.

This idea may be easier to describe if we recall the formula for a logistic regression:

$$\ln\left(\frac{P(y)}{1 - P(y)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 * x_2$$

> In the above formula, the sign, and statistical significance, of β_3 may not accurately characterize the underlying relationship.

Some Calculus (Not Essential To The Discussion)

...

Imagine a linear model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 * x_2 + e_i$$

Here (following Ai and Norton (2003)):

$$\frac{\partial y}{\partial x_1 \partial x_2} = \beta_3$$

In contrast, for the logistic model, the quantity:

$$\frac{\partial \text{logit}(y)}{\partial x_1 \partial x_2}$$

does not have such a straightforward solution, and–for the purposes of this discussion–is not simply equal to β_3.

Read more

Get The Data

We start by obtaining *simulated data* from StataCorp.

```
. clear all

. graph close _all

. use http://www.stata-press.com/data/r15/margex, clear
(Artificial data for margins)
```

Describe The Data

The variables are as follows:

```
. describe

Contains data from http://www.stata-press.com/data/r15/margex.dta
  obs:         3,000                          Artificial data for margins
 vars:            11                          27 Nov 2016 14:27
-------------------------------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
-------------------------------------------------------------------------------------------------------
y               float   %6.1f
outcome         byte    %2.0f
sex             byte    %6.0f      sexlbl
group           byte    %2.0f
age             float   %3.0f
distance        float   %6.2f
ycn             float   %6.1f
yc              float   %6.1f
treatment       byte    %2.0f
agegroup        byte    %8.0g      agelab
arm             byte    %8.0g
-------------------------------------------------------------------------------------------------------
Sorted by: group
```

Estimate Logistic Regression

We then run a logistic regression model in which `outcome` is the dependent variable. `sex`, `age` and `group` are the independent variables. We estimate an interaction of `sex` and `age`.

We note that the regression coefficient for the interaction term is not statistically significant.

```
. logit outcome sex##c.age i.group

Iteration 0:   log likelihood = -1366.0718
Iteration 1:   log likelihood =  -1118.129
Iteration 2:   log likelihood = -1070.8227
Iteration 3:   log likelihood = -1068.0102
Iteration 4:   log likelihood =   -1067.99
Iteration 5:   log likelihood =   -1067.99

Logistic regression                             Number of obs     =      3,000
                                                LR chi2(5)        =     596.16
                                                Prob > chi2       =     0.0000
Log likelihood =   -1067.99                     Pseudo R2         =     0.2182

------------------------------------------------------------------------------
     outcome |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         sex |
      female |   .5565025   .6488407     0.86   0.391    -.7152019    1.828207
         age |   .0910807   .0113215     8.04   0.000     .0688909    .1132704
             |
   sex#c.age |
      female |   -.001211   .0134012    -0.09   0.928    -.0274769     .025055
             |
       group |
          2  |  -.5854237   .1349791    -4.34   0.000    -.8499779   -.3208696
          3  |  -1.355227   .2965301    -4.57   0.000    -1.936416   -.7740391
             |
       _cons |  -5.592272   .5583131   -10.02   0.000    -6.686545   -4.497998
------------------------------------------------------------------------------
```

Margins

We use the `margins` command to estimate predicted probabilities at different values of `sex` and `age`.

```
. margins sex, at(age = (20 30 40 50 60))
Predictive margins                              Number of obs     =      3,000
Model VCE    : OIM
Expression   : Pr(outcome), predict()
1._at        : age             =          20
2._at        : age             =          30
3._at        : age             =          40
4._at        : age             =          50
5._at        : age             =          60

             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     _at#sex |
     1#male  |   .0150645   .0047348     3.18   0.001     .0057846    .0243445
   1#female  |    .025333   .0055508     4.56   0.000     .0144536    .0362124
     2#male  |   .0364848   .0075444     4.84   0.000     .0216981    .0512714
   2#female  |   .0596255   .0086074     6.93   0.000     .0427552    .0764958
     3#male  |   .0852689   .0099016     8.61   0.000     .0658622    .1046757
   3#female  |   .1329912   .0108127    12.30   0.000     .1117987    .1541838
     4#male  |   .1849367   .0163684    11.30   0.000     .1528551    .2170182
   4#female  |    .267774   .0156218    17.14   0.000     .2371558    .2983921
     5#male  |   .3518378   .0408522     8.61   0.000      .271769    .4319066
   5#female  |   .4614446   .0314754    14.66   0.000     .3997539    .5231353
```

Plotting Margins

`margins` provides a lot of results, which can be difficult to understand. Therefore, we use `marginsplot` to *plot* these `margins` results. The key command is `marginsplot`, which could be used on its own. I have simply added the Michigan graph scheme, as well as some options to improve the graphic design of the plot.

> There certainly seems to be some kind of interaction of `sex` and `age`.

```
. marginsplot, ///
> scheme(michigan) /// michigan graph scheme
> plotopts(msize(vlarge)) /// larger plotting symbols
> plot1opts(lcolor(navy)) /// line for first group is navy
> plot2opts(lcolor(gold)) // line for second group is gold
  Variables that uniquely identify margins: age sex

. graph export mymarginsplot.png, width(500) replace
(file /Users/agrogan/Desktop/newstuff/categorical/logistic-interactions-2/mymarginsplot.png written in
>  PNG format)
```

Rerun `margins`, Posting Results

We again employ the `margins` command, this time using the `post` option so that the results of the margins command are *posted* as an estimation result. This will allow us to employ the `test` command to statistically test different margins against each other.

```
. margins sex, at(age = (20 30 40 50 60)) post
```

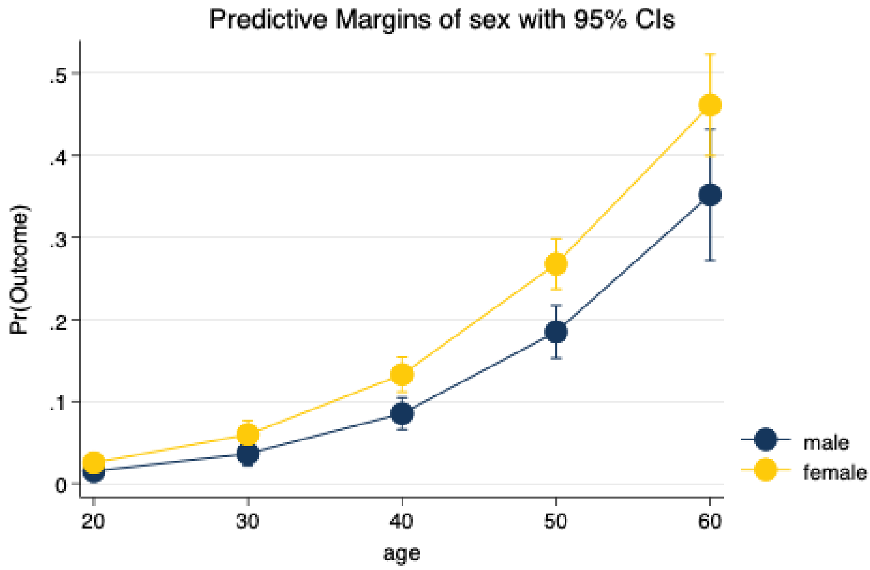

Figure 1: Margins Plot

```
Predictive margins                              Number of obs     =      3,000
Model VCE    : OIM

Expression   : Pr(outcome), predict()

1._at        : age             =          20

2._at        : age             =          30

3._at        : age             =          40

4._at        : age             =          50

5._at        : age             =          60
```

	Margin	Delta-method Std. Err.	z	P>|z|	[95% Conf.	Interval]
_at#sex						
1#male	.0150645	.0047348	3.18	0.001	.0057846	.0243445
1#female	.025333	.0055508	4.56	0.000	.0144536	.0362124
2#male	.0364848	.0075444	4.84	0.000	.0216981	.0512714
2#female	.0596255	.0086074	6.93	0.000	.0427552	.0764958
3#male	.0852689	.0099016	8.61	0.000	.0658622	.1046757
3#female	.1329912	.0108127	12.30	0.000	.1117987	.1541838
4#male	.1849367	.0163684	11.30	0.000	.1528551	.2170182
4#female	.267774	.0156218	17.14	0.000	.2371558	.2983921
5#male	.3518378	.0408522	8.61	0.000	.271769	.4319066
5#female	.4614446	.0314754	14.66	0.000	.3997539	.5231353

`margins` with `coeflegend`

We follow up by using the `margins` command with the `coeflegend` option to see the way in which Stata has labeled the different margins.

```
. margins, coeflegend

Predictive margins                              Number of obs     =      3,000
Model VCE    : OIM

Expression   : Pr(outcome), predict()
```

```
1._at        : age            =          20
2._at        : age            =          30
3._at        : age            =          40
4._at        : age            =          50
5._at        : age            =          60

------------------------------------------------------------------------------
             |     Margin  Legend
-------------+----------------------------------------------------------------
      _at#sex |
     1#male  |   .0150645  _b[1bn._at#0bn.sex]
   1#female  |    .025333  _b[1bn._at#1.sex]
     2#male  |   .0364848  _b[2._at#0bn.sex]
   2#female  |   .0596255  _b[2._at#1.sex]
     3#male  |   .0852689  _b[3._at#0bn.sex]
   3#female  |   .1329912  _b[3._at#1.sex]
     4#male  |   .1849367  _b[4._at#0bn.sex]
   4#female  |    .267774  _b[4._at#1.sex]
     5#male  |   .3518378  _b[5._at#0bn.sex]
   5#female  |   .4614446  _b[5._at#1.sex]
------------------------------------------------------------------------------
```

Testing Margins Against Each Other

Lastly, we test the margins at age 20 for men and women, and again at age 60 for men and women.

> We note that the original regression parameter for the interaction term was not statistically significant. Indeed, the margins at age 20 are not statistically significantly different by sex. However, at age 60, there is a statistically significant difference by sex.

```
. test _b[1bn._at#0bn.sex] = _b[1bn._at#1.sex] // male and female at age 20

 ( 1)  1bn._at#0bn.sex - 1bn._at#1.sex = 0

           chi2(  1) =    1.99
         Prob > chi2 =    0.1583

. test  _b[5._at#0bn.sex] =  _b[5._at#1.sex] // male and female at age 60

 ( 1)  5._at#0bn.sex - 5._at#1.sex = 0

           chi2(  1) =    5.16
         Prob > chi2 =    0.0232
```

> There is some suggestion that the difference of the differences is statistically significant, but this statistical significance is only marginal [pun intended].

```
. test _b[1bn._at#1.sex] - _b[1bn._at#0bn.sex] = _b[5._at#1.sex] - _b[5._at#0bn.sex] // test equivalen
> ce of the differences

 ( 1)  - 1bn._at#0bn.sex + 1bn._at#1.sex + 5._at#0bn.sex - 5._at#1.sex = 0

           chi2(  1) =    3.62
         Prob > chi2 =    0.0572
```

References

Ai, C., & Norton, E. C. (2003). Interaction terms in logit and probit models. *Economics Letters*. https://doi.org/10.1016/S0165-1765(03)00032-6

Karaca-Mandic, P., Norton, E. C., & Dowd, B. (2012). Interaction terms in nonlinear models. *Health Services Research*. https://doi.org/10.1111/j.1475-6773.2011.01314.x