

# Interactions in Logistic Regression

New Version

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## 1 Background

The purpose of this tutorial is to illustrate the idea that in *logistic regression*, the  $\beta$  parameter for an interaction term may not accurately characterize the underlying interactive relationships.

This idea may be easier to describe if we recall the formula for a logistic regression:

$$\ln \left( \frac{P(y)}{1 - P(y)} \right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \times x_2$$

**Warning**

In the above formula, the sign, and statistical significance, of  $\beta_3$  may not accurately characterize the underlying relationship.

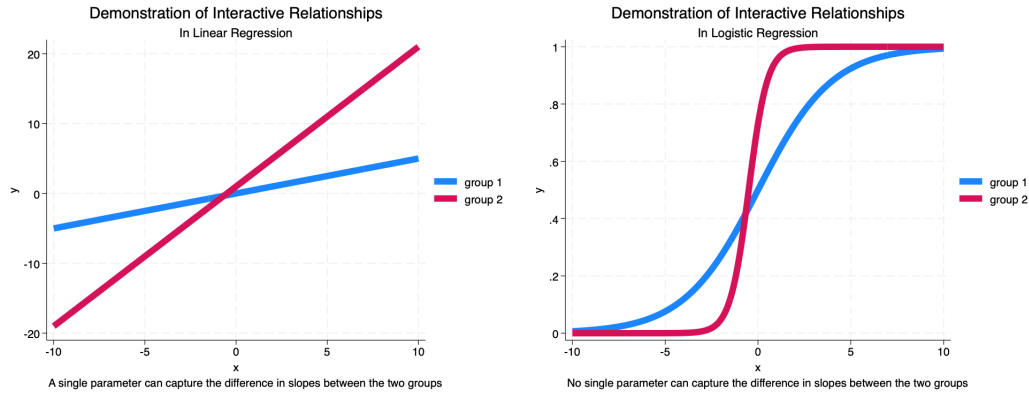


Figure 1: Demonstration of Interactive Relationships

**Key Idea**

In a *linear* model, a single parameter can capture the difference in slopes between the two groups. In a *non-linear* model, no single parameter can capture the difference in slopes between the two groups.

**Some Calculus (Not Essential To The Discussion)**

Imagine a linear model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \times x_2 + e_i$$

Here (following Ai and Norton (2003)):

$$\frac{\partial y}{\partial x_1 \partial x_2} = \beta_3$$

We use logit to describe:

$$\ln \left( \frac{P(y)}{1 - P(y)} \right)$$

In the logistic model, the quantity:

$$\frac{\partial \text{logit}(y)}{\partial x_1 \partial x_2}$$

does not have such a straightforward solution, and—importantly for this discussion—is not simply equal to  $\beta_3$ .

## 2 Get The Data

We start by obtaining *simulated data* from StataCorp.

```
clear all

graph close _all

use http://www.stata-press.com/data/r15/margex, clear
```

(Artificial data for margins)

## 3 Describe The Data

The variables are as follows:

```
describe
```

Running /Users/agrogan/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof

```
> ile.do ...
```

Contains data from <http://www.stata-press.com/data/r15/margex.dta>

Observations:	3,000	Artificial data for margins
Variables:	11	27 Nov 2016 14:27

Variable name	Storage type	Display format	Value label	Variable label
------------------	-----------------	-------------------	----------------	----------------

y	float	%6.1f	
outcome	byte	%2.0f	
sex	byte	%6.0f	sexlbl
group	byte	%2.0f	
age	float	%3.0f	
distance	float	%6.2f	
ycn	float	%6.1f	
yc	float	%6.1f	
treatment	byte	%2.0f	
agegroup	byte	%8.0g	agelab
arm	byte	%8.0g	

---

Sorted by: group

## 4 Estimate Logistic Regression

We then run a logistic regression model in which **outcome** is the dependent variable. **age** and **group** are the independent variables. We estimate an interaction of **group** and **age**.

We note that the regression coefficient for the interaction terms are not statistically significant.

```
logit outcome c.age##i.group
```

Running /Users/agrogon/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof

```
> ile.do ...
```

```
Iteration 0: Log likelihood = -1366.0718
Iteration 1: Log likelihood = -1125.2149
Iteration 2: Log likelihood = -1077.539
Iteration 3: Log likelihood = -1074.8422
Iteration 4: Log likelihood = -1074.7864
Iteration 5: Log likelihood = -1074.7863
```

Logistic regression

```
Number of obs = 3,000
LR chi2(5)      = 582.57
Prob > chi2     = 0.0000
```

Log likelihood = -1074.7863

Pseudo R2 = 0.2132

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
age	.091183	.0084697	10.77	0.000	.0745827	.1077834
group						
2	-.8272665	.6648279	-1.24	0.213	-2.130305	.4757722
3	-1.085445	1.044802	-1.04	0.299	-3.133219	.9623292
group#c.age						
2	.0013522	.0139747	0.10	0.923	-.0260378	.0287421
3	-.0164596	.0259182	-0.64	0.525	-.0672584	.0343392
_cons	-5.184398	.4274964	-12.13	0.000	-6.022275	-4.34652

## 5 Margins

We use the `margins` command to estimate predicted probabilities at different values of `group` and `age`.

```
margins group, at(age = (20 30 40 50 60))
```

Running /Users/agrogan/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof

```
> ile.do ...
```

Adjusted predictions

Number of obs = 3,000

Model VCE: OIM

Expression: Pr(outcome), predict()

1.\_at: age = 20

2.\_at: age = 30

3.\_at: age = 40

4.\_at: age = 50

5.\_at: age = 60

		Delta-method					
		Margin	std. err.	z	P> z	[95% conf. interval]	
_at#group							
1 1		.0335453	.0084761	3.96	0.000	.0169325	.0501582
1 2		.0153532	.0044572	3.44	0.001	.0066173	.0240891
1 3		.0083644	.0041378	2.02	0.043	.0002545	.0164742
2 1		.0795185	.0132311	6.01	0.000	.0535861	.105451
2 2		.0378475	.0070835	5.34	0.000	.023964	.0517309
2 3		.0174959	.0054051	3.24	0.001	.0069021	.0280896
3 1		.1769607	.0155126	11.41	0.000	.1465565	.2073649
3 2		.0902771	.0095282	9.47	0.000	.0716022	.108952
3 3		.0362321	.0091647	3.95	0.000	.0182697	.0541945
4 1		.3485893	.0149941	23.25	0.000	.3192014	.3779772
4 2		.2002224	.0188064	10.65	0.000	.1633625	.2370823
4 3		.0735312	.0271556	2.71	0.007	.0203072	.1267553
5 1		.5711598	.0265203	21.54	0.000	.5191809	.6231386
5 2		.3870919	.0467552	8.28	0.000	.2954535	.4787304
5 3		.1435103	.0746567	1.92	0.055	-.0028141	.2898347

## 6 Plotting Margins

`margins` provides a lot of results, which can be difficult to understand. Therefore, we use `marginsplot` to *plot* these `margins` results.

There certainly seems to be some kind of interaction of `group` and `age`.

```
marginsplot
graph export mymarginsplot.png, width(1000) replace
```

Running `/Users/agrogan/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof`

```
> ile.do ...
```

Variables that uniquely identify margins: age group

```
file  
  /Users/agrogan/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/mymarg  
  > insplot.png saved as PNG format
```

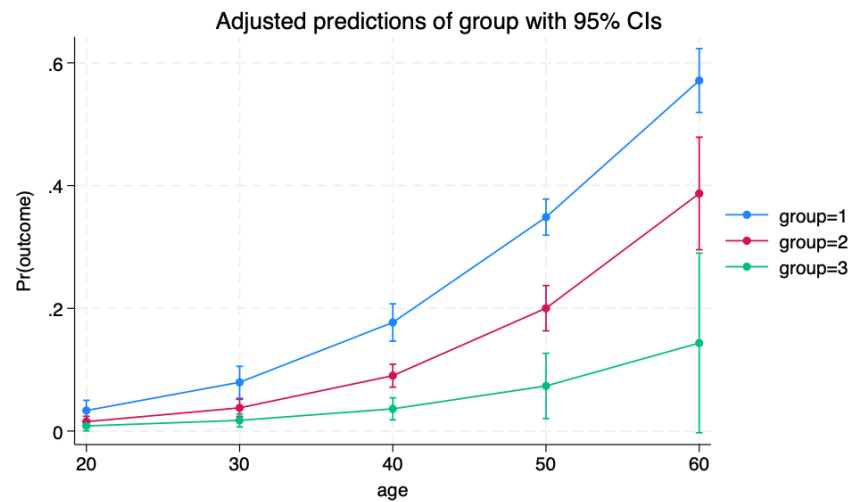


Figure 2: Margins Plot

## 7 Rerun margins, Posting Results

We again employ the `margins` command, this time using the `post` option so that the results of the margins command are *posted* as an estimation result. This will allow us to employ the `test` command to statistically test different margins against each other.

```
margins group, at(age = (20 30 40 50 60)) post
```

Running /Users/agrogan/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof

```
> ile.do ...
```

Adjusted predictions  
Model VCE: OIM

Number of obs = 3,000

Expression: Pr(outcome), predict()

1.\_at: age = 20

2.\_at: age = 30

3.\_at: age = 40

4.\_at: age = 50

5.\_at: age = 60

		Delta-method					
		Margin	std. err.	z	P> z	[95% conf. interval]	
_at#group							
1 1		.0335453	.0084761	3.96	0.000	.0169325	.0501582
1 2		.0153532	.0044572	3.44	0.001	.0066173	.0240891
1 3		.0083644	.0041378	2.02	0.043	.0002545	.0164742
2 1		.0795185	.0132311	6.01	0.000	.0535861	.105451
2 2		.0378475	.0070835	5.34	0.000	.023964	.0517309
2 3		.0174959	.0054051	3.24	0.001	.0069021	.0280896
3 1		.1769607	.0155126	11.41	0.000	.1465565	.2073649
3 2		.0902771	.0095282	9.47	0.000	.0716022	.108952
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5 3		.1435103	.0746567	1.92	0.055	-.0028141	.2898347

## 8 margins with coeflegend

We follow up by using the `margins` command with the `coeflegend` option to see the way in which Stata has labeled the different margins.

```
margins, coeflegend
```

Running /Users/agrogon/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof

```
> ile.do ...
```



Adjusted predictions  
Model VCE: OIM

Number of obs = 3,000

Expression: Pr(outcome), predict()

1.\_at: age = 20  
2.\_at: age = 30  
3.\_at: age = 40  
4.\_at: age = 50  
5.\_at: age = 60

		Margin	Legend
_at#group			
1 1		.0335453	_b[1bn._at#1bn.group]
1 2		.0153532	_b[1bn._at#2.group]
1 3		.0083644	_b[1bn._at#3.group]
2 1		.0795185	_b[2._at#1bn.group]
2 2		.0378475	_b[2._at#2.group]
2 3		.0174959	_b[2._at#3.group]
3 1		.1769607	_b[3._at#1bn.group]
3 2		.0902771	_b[3._at#2.group]
3 3		.0362321	_b[3._at#3.group]
4 1		.3485893	_b[4._at#1bn.group]
4 2		.2002224	_b[4._at#2.group]
4 3		.0735312	_b[4._at#3.group]
5 1		.5711598	_b[5._at#1bn.group]
5 2		.3870919	_b[5._at#2.group]
5 3		.1435103	_b[5._at#3.group]

## 9 Testing Margins Against Each Other

Lastly, we test the margins at age 20 across some of the groups, and again at ages 50 and 60 for some of the groups.

We note that the original regression parameter for the interaction term was not statistically significant. Indeed, the margins at age 20 are not statistically significantly different by group (1 vs. 2), though they are close to being significantly

different. However, at ages 50 & 60, there is a statistically significant difference by group (1 vs. 2).

```
test _b[1bn._at#1bn.group] = _b[1bn._at#2.group] // groups 1 & 2 at age 20
test _b[4._at#1bn.group] = _b[4._at#2.group] // groups 1 & 2 at age 50
test _b[5._at#1bn.group] = _b[5._at#2.group] // groups 1 & 2 at age 60
```

Running /Users/agrogon/Desktop/GitHub/newstuff/categorical/logistic-interactions-3/prof

```
> ile.do ...
```

```
( 1) 1bn._at#1bn.group - 1bn._at#2.group = 0
```

```
      chi2( 1) =      3.61
Prob > chi2 =      0.0575
```

```
( 1) 4._at#1bn.group - 4._at#2.group = 0
```

```
      chi2( 1) =     38.05
Prob > chi2 =      0.0000
```

```
( 1) 5._at#1bn.group - 5._at#2.group = 0
```

```
      chi2( 1) =     11.73
Prob > chi2 =      0.0006
```

Ai, Chunrong, and Edward C. Norton. 2003. "Interaction Terms in Logit and Probit Models." *Economics Letters*. [https://doi.org/10.1016/S0165-1765\(03\)00032-6](https://doi.org/10.1016/S0165-1765(03)00032-6).

Karaca-Mandic, Pinar, Edward C. Norton, and Bryan Dowd. 2012. "Interaction Terms in Nonlinear Models." *Health Services Research*. <https://doi.org/10.1111/j.1475-6773.2011.01314.x>.