Some Stuff About Logarithms

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1 Introduction

We consider the logarithm. One very simple way to present the logarithm is to use the constant e.

Let's first consider the exponential function with base e, $y = e^x$.[1]

```
twoway function y = exp(x), lwidth(thick) title("Exponential Function") range(-10 10)

graph export exponential.png, replace
```

[1] e is a kind of fundamental mathematical constant, like π, but without the easy geometric definition that π has. (For any $\bigcirc$, $\pi = \frac{\text{circumference}}{\text{diameter}}$.) One definition of e is

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

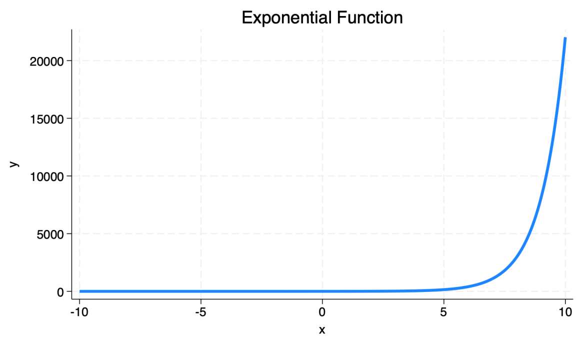

Figure 1: Graph of exponential function

2 A Definition of the Natural Logarithm

If

$$y = e^x$$

then

$$\ln(y) = x$$

.

3 Graphing Logarithmic Function

Note that in the equation above, we are taking the logarithm of y. To get some quick sense of how the logarithm behaves, we are going to graph $y = \ln(x)$.

```
twoway function y = ln(x), lwidth(thick) title("Logarithmic Function") range(-10 10)

graph export logarithmic.png, replace
```

4 Categorical Data Analysis

In categorical data analysis–in general–we are often thinking about some equation like $\ln(y) = \beta x$. This is equivalent to $y = e^{\beta x}$ so many models–particularly later in the course–will have us thinking about *exponential* relationships.

```
twoway function y = exp(x), lwidth(thick) title("Exponential Function") range(-10 10)
```

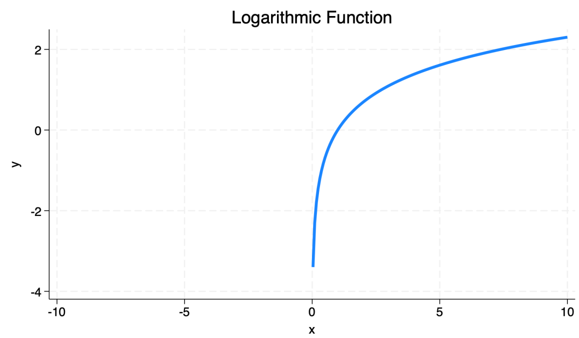

Figure 2: Graph of logarithmic function

```
graph export exponential2.png, replace
```

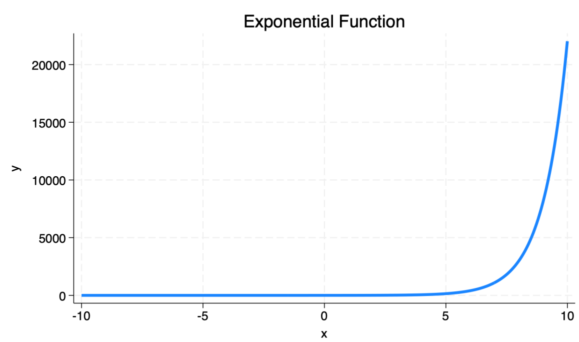

Figure 3: Graph of exponential function

5 Logistic Regression

Early on in this course, we will think about logistic regression. In logistic regression, we start by thinking about the on the *odds* of our outcome:

$$\frac{p(y)}{1-p(y)}$$

We will be working with the *log odds*:

$$\ln(\frac{p(y)}{1-p(y)}) = x$$

To *graph* these *log odds*, we need to solve for $p(y)$:

$$p(y) = \frac{e^x}{1 + e^x}$$

```
twoway function y = exp(x)/(1 + exp(x)), ///
lwidth(thick) title("Logistic Function") range(-10 10)

graph export logistic.png, replace
```

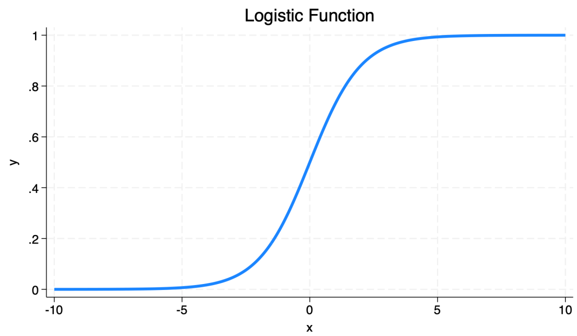

Figure 4: Logistic curve

6 Logarithmic Spiral

An interesting sidenote is that the logarithm forms the basis of the logarithmic spiral. The equation for a logarithmic spiral in polar coordinates is: $r = ae^{b\theta}$, where θ is the angle, r is the radius, and a and b are constants.

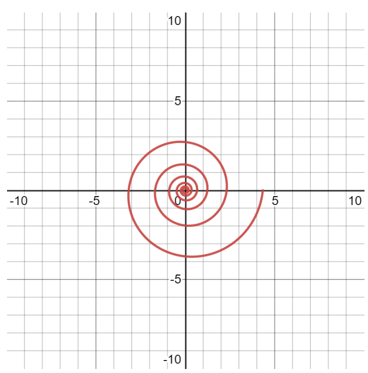

Figure 5: Desmos Graph Logarithmic Spiral

Logarithmic spirals can be found in nature in the *nautilus shell*, and in *sunflowers*.

Figure 6: Nautilus Shell, Courtesy Wikipedia

Figure 7: Sunflower, Courtesy Wikipedia