Interactions in Logistic Regression

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Background

The purpose of this tutorial is to illustrate the idea that in *logistic regression*, the β parameter for an interaction term may not accurately characterize the underlying interactive relationships.

This idea may be easier to describe if we recall the formula for a logistic regression:

$$\ln\left(\frac{P(y)}{1-P(y)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \times x_2$$

> In the above formula, the sign, and statistical significance, of β_3 may not accurately characterize the underlying relationship.

Some Calculus (Not Essential To The Discussion)

...

Imagine a linear model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \times x_2 + e_i$$

Here (following Ai and Norton (2003)):

$$\frac{\partial y}{\partial x_1 \partial x_2} = \beta_3$$

We use logit to describe:

$$\ln\left(\frac{P(y)}{1-P(y)}\right)$$

In the logistic model, the quantity:

$$\frac{\partial \text{logit}(y)}{\partial x_1 \partial x_2}$$

does not have such a straightforward solution, and–importantly for this discussion–is not simply equal to β_3.

Read more

Get The Data

We start by obtaining *simulated data* from StataCorp.

```
. clear all
```

```
. graph close _all

. use http://www.stata-press.com/data/r15/margex, clear
(Artificial data for margins)
```

Describe The Data

The variables are as follows:

```
. describe
Contains data from http://www.stata-press.com/data/r15/margex.dta
 Observations:         3,000                  Artificial data for margins
    Variables:            11                  27 Nov 2016 14:27
```

Variable name	Storage type	Display format	Value label	Variable label
y	float	%6.1f		
outcome	byte	%2.0f		
sex	byte	%6.0f	sexlbl	
group	byte	%2.0f		
age	float	%3.0f		
distance	float	%6.2f		
ycn	float	%6.1f		
yc	float	%6.1f		
treatment	byte	%2.0f		
agegroup	byte	%8.0g	agelab	
arm	byte	%8.0g		

```
Sorted by: group
```

Estimate Logistic Regression

We then run a logistic regression model in which `outcome` is the dependent variable. `sex`, `age` and `group` are the independent variables. We estimate an interaction of `sex` and `age`.

We note that the regression coefficient for the interaction term is not statistically significant.

```
. logit outcome sex##c.age i.group
Iteration 0:   log likelihood = -1366.0718
Iteration 1:   log likelihood =  -1118.129
Iteration 2:   log likelihood = -1070.8227
Iteration 3:   log likelihood = -1068.0102
Iteration 4:   log likelihood =   -1067.99
Iteration 5:   log likelihood =   -1067.99
Logistic regression                                     Number of obs =  3,000
                                                        LR chi2(5)    = 596.16
                                                        Prob > chi2   = 0.0000
Log likelihood = -1067.99                               Pseudo R2     = 0.2182
```

outcome	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
sex						
female	.5565025	.6488407	0.86	0.391	-.7152019	1.828207
age	.0910807	.0113215	8.04	0.000	.0688909	.1132704
sex#c.age						
female	-.001211	.0134012	-0.09	0.928	-.0274769	.025055
group						
2	-.5854237	.1349791	-4.34	0.000	-.8499779	-.3208696

3	-1.355227	.2965301	-4.57	0.000	-1.936416	-.7740391
_cons	-5.592272	.5583131	-10.02	0.000	-6.686545	-4.497998

Margins

We use the `margins` command to estimate predicted probabilities at different values of `sex` and `age`.

```
. margins sex, at(age = (20 30 40 50 60))
Predictive margins                                      Number of obs = 3,000
Model VCE: OIM

Expression: Pr(outcome), predict()
1._at: age = 20
2._at: age = 30
3._at: age = 40
4._at: age = 50
5._at: age = 60
```

	Delta-method Margin	std. err.	z	P>|z|	[95% conf.	interval]
_at#sex						
1#male	.0150645	.0047348	3.18	0.001	.0057846	.0243445
1#female	.025333	.0055508	4.56	0.000	.0144536	.0362124
2#male	.0364848	.0075444	4.84	0.000	.0216981	.0512714
2#female	.0596255	.0086074	6.93	0.000	.0427552	.0764958
3#male	.0852689	.0099016	8.61	0.000	.0658622	.1046757
3#female	.1329912	.0108127	12.30	0.000	.1117987	.1541838
4#male	.1849367	.0163684	11.30	0.000	.1528551	.2170182
4#female	.267774	.0156218	17.14	0.000	.2371558	.2983921
5#male	.3518378	.0408522	8.61	0.000	.271769	.4319066
5#female	.4614446	.0314754	14.66	0.000	.3997539	.5231353

Plotting Margins

`margins` provides a lot of results, which can be difficult to understand. Therefore, we use `marginsplot` to *plot* these `margins` results. The key command is `marginsplot`, which could be used on its own. I have simply added the Michigan graph scheme, as well as some options to improve the graphic design of the plot.

There certainly seems to be some kind of interaction of `sex` and `age`.

```
. marginsplot, ///
> scheme(michigan) /// michigan graph scheme
> plotopts(msize(vlarge)) /// larger plotting symbols
> plot1opts(lcolor(navy)) /// line for first group is navy
> plot2opts(lcolor(gold)) // line for second group is gold
Variables that uniquely identify margins: age sex

. graph export mymarginsplot.png, width(1000) replace
file
    /Users/agrogan/Desktop/GitHub/newstuff/categorical/logistic-interactions-2/mymargins
    > plot.png saved as PNG format
```

Rerun `margins`, Posting Results

We again employ the `margins` command, this time using the `post` option so that the results of the margins command are *posted* as an estimation result. This will allow us to employ the `test` command to statistically

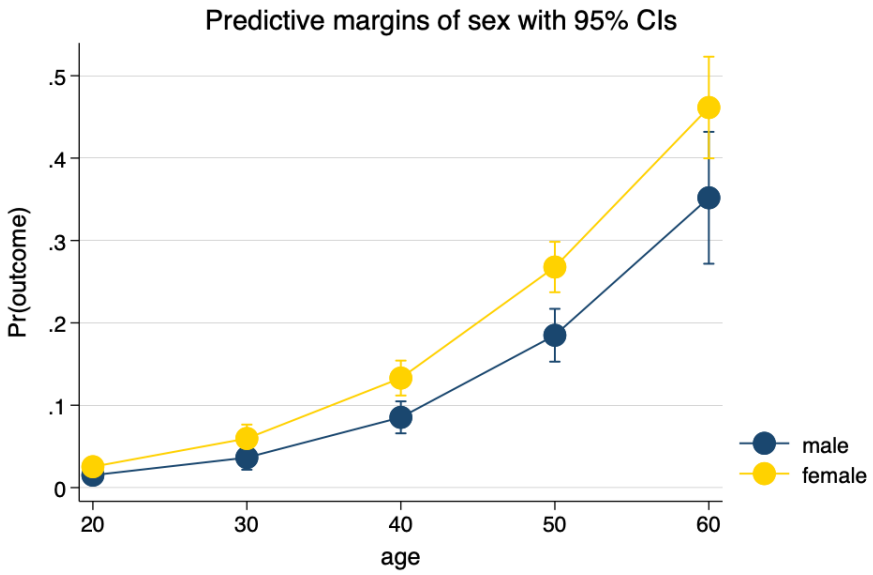

Figure 1: Margins Plot

test different margins against each other.

```
. margins sex, at(age = (20 30 40 50 60)) post
Predictive margins                                      Number of obs = 3,000
Model VCE: OIM

Expression: Pr(outcome), predict()
1._at: age = 20
2._at: age = 30
3._at: age = 40
4._at: age = 50
5._at: age = 60
```

	Margin	Delta-method std. err.	z	P>|z|	[95% conf.	interval]
_at#sex						
1#male	.0150645	.0047348	3.18	0.001	.0057846	.0243445
1#female	.025333	.0055508	4.56	0.000	.0144536	.0362124
2#male	.0364848	.0075444	4.84	0.000	.0216981	.0512714
2#female	.0596255	.0086074	6.93	0.000	.0427552	.0764958
3#male	.0852689	.0099016	8.61	0.000	.0658622	.1046757
3#female	.1329912	.0108127	12.30	0.000	.1117987	.1541838
4#male	.1849367	.0163684	11.30	0.000	.1528551	.2170182
4#female	.267774	.0156218	17.14	0.000	.2371558	.2983921
5#male	.3518378	.0408522	8.61	0.000	.271769	.4319066
5#female	.4614446	.0314754	14.66	0.000	.3997539	.5231353

margins with coeflegend

We follow up by using the `margins` command with the `coeflegend` option to see the way in which Stata has labeled the different margins.

```
. margins, coeflegend
Predictive margins                                      Number of obs = 3,000
```

```
Model VCE: OIM
Expression: Pr(outcome), predict()
1._at: age = 20
2._at: age = 30
3._at: age = 40
4._at: age = 50
5._at: age = 60
```

	Margin	Legend
_at#sex		
1#male	.0150645	_b[1bn._at#0bn.sex]
1#female	.025333	_b[1bn._at#1.sex]
2#male	.0364848	_b[2._at#0bn.sex]
2#female	.0596255	_b[2._at#1.sex]
3#male	.0852689	_b[3._at#0bn.sex]
3#female	.1329912	_b[3._at#1.sex]
4#male	.1849367	_b[4._at#0bn.sex]
4#female	.267774	_b[4._at#1.sex]
5#male	.3518378	_b[5._at#0bn.sex]
5#female	.4614446	_b[5._at#1.sex]

Testing Margins Against Each Other

Lastly, we test the margins at age 20 for men and women, and again at ages 50 and 60 for men and women.

We note that the original regression parameter for the interaction term was not statistically significant. Indeed, the margins at age 20 are not statistically significantly different by sex. However, at ages 50 & 60, there is a statistically significant difference by sex.

```
. test _b[1bn._at#0bn.sex] = _b[1bn._at#1.sex] // male and female at age 20
 ( 1)  1bn._at#0bn.sex - 1bn._at#1.sex = 0
           chi2(  1) =    1.99
         Prob > chi2 =    0.1583

. test _b[4._at#0bn.sex] = _b[4._at#1.sex] // male and female at age 50
 ( 1)  4._at#0bn.sex - 4._at#1.sex = 0
           chi2(  1) =   13.03
         Prob > chi2 =    0.0003

. test _b[5._at#0bn.sex] =  _b[5._at#1.sex] // male and female at age 60
 ( 1)  5._at#0bn.sex - 5._at#1.sex = 0
           chi2(  1) =    5.16
         Prob > chi2 =    0.0232
```

There is some suggestion that the *difference of the differences* is statistically significant. This statistical significance is only marginal [pun intended] at age 60, but truly statistically significant at age 50.

```
. test _b[1bn._at#1.sex] - _b[1bn._at#0bn.sex] = _b[5._at#1.sex] - _b[5._at#0bn.sex] // te
> st equivalence of the differences
 ( 1)  - 1bn._at#0bn.sex + 1bn._at#1.sex + 5._at#0bn.sex - 5._at#1.sex = 0
           chi2(  1) =    3.62
         Prob > chi2 =    0.0572

. test _b[1bn._at#1.sex] - _b[1bn._at#0bn.sex] = _b[4._at#1.sex] - _b[4._at#0bn.sex] // te
> st equivalence of the differences
 ( 1)  - 1bn._at#0bn.sex + 1bn._at#1.sex + 4._at#0bn.sex - 4._at#1.sex = 0
           chi2(  1) =    9.77
         Prob > chi2 =    0.0018
```

References

Ai, C., & Norton, E. C. (2003). Interaction terms in logit and probit models. *Economics Letters*. https://doi.org/10.1016/S0165-1765(03)00032-6

Karaca-Mandic, P., Norton, E. C., & Dowd, B. (2012). Interaction terms in nonlinear models. *Health Services Research*. https://doi.org/10.1111/j.1475-6773.2011.01314.x