

Interactions in Logistic Regression

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2023-09-27

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1 Background

The purpose of this tutorial is to illustrate the idea that in *logistic regression*, the β parameter for an interaction term may not accurately characterize the underlying interactive relationships.

This idea may be easier to describe if we recall the formula for a logistic regression:

$$\ln \left(\frac{P(y)}{1 - P(y)} \right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \times x_2$$

In the above formula, the sign, and statistical significance, of β_3 may not accurately characterize the underlying relationship.

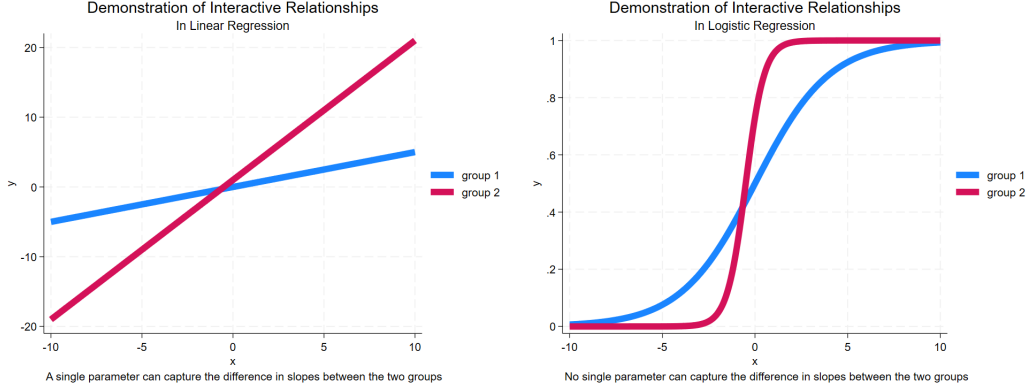


Figure 1: Demonstration of Interactive Relationships

💡 Some Calculus (Not Essential To The Discussion)

Imagine a linear model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 \times x_2 + e_i$$

Here (following Ai and Norton (2003)):

$$\frac{\partial y}{\partial x_1 \partial x_2} = \beta_3$$

We use logit to describe:

$$\ln \left(\frac{P(y)}{1 - P(y)} \right)$$

In the logistic model, the quantity:

$$\frac{\partial \text{logit}(y)}{\partial x_1 \partial x_2}$$

does not have such a straightforward solution, and—importantly for this discussion—is not simply equal to β_3 .

2 Get The Data

We start by obtaining *simulated data* from StataCorp.

```
clear all

graph close _all

use http://www.stata-press.com/data/r15/margex, clear
```

(Artificial data for margins)

3 Describe The Data

The variables are as follows:

```
describe
```

Running C:\Users\agrogan\Desktop\GitHub\newstuff\categorical\logistic-interacti
> ons-2\profile.do .

Contains data from <http://www.stata-press.com/data/r15/margex.dta>

Observations:	3,000	Artificial data for margins
Variables:	11	27 Nov 2016 14:27

Variable name	Storage type	Display format	Value label	Variable label
y	float	%6.1f		
outcome	byte	%2.0f		
sex	byte	%6.0f	sexlbl	
group	byte	%2.0f		
age	float	%3.0f		
distance	float	%6.2f		
ycn	float	%6.1f		
yc	float	%6.1f		
treatment	byte	%2.0f		
agegroup	byte	%8.0g	agelab	
arm	byte	%8.0g		

Sorted by: group

4 Estimate Logistic Regression

We then run a logistic regression model in which `outcome` is the dependent variable. `sex`, `age` and `group` are the independent variables. We estimate an interaction of `sex` and `age`.

We note that the regression coefficient for the interaction term is not statistically significant.

```
logit outcome sex##c.age i.group
```

```
Running C:\Users\agrogan\Desktop\GitHub\newstuff\categorical\logistic-interacti
> ons-2\profile.do .
```

```
Iteration 0:  Log likelihood = -1366.0718
Iteration 1:  Log likelihood = -1118.129
Iteration 2:  Log likelihood = -1070.8227
Iteration 3:  Log likelihood = -1068.0102
Iteration 4:  Log likelihood = -1067.99
Iteration 5:  Log likelihood = -1067.99
```

Logistic regression

Number of obs = 3,000

LR chi2(5) = 596.16

Prob > chi2 = 0.0000

Pseudo R2 = 0.2182

Log likelihood = -1067.99

outcome	Coefficient	Std. err.	z	P> z	[95% conf. interval]	
sex						
female	.5565025	.6488407	0.86	0.391	-.7152019	1.828207
age	.0910807	.0113215	8.04	0.000	.0688909	.1132704
sex#c.age						
female	-.001211	.0134012	-0.09	0.928	-.0274769	.025055
group						
2	-.5854237	.1349791	-4.34	0.000	-.8499779	-.3208696

3		-1.355227	.2965301	-4.57	0.000	-1.936416	-.7740391
_cons		-5.592272	.5583131	-10.02	0.000	-6.686545	-4.497998

5 Margins

We use the `margins` command to estimate predicted probabilities at different values of `sex` and `age`.

```
margins sex, at(age = (20 30 40 50 60))
```

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> ons-2\profile.do .

Predictive margins
Model VCE: OIM

Number of obs = 3,000

Expression: Pr(outcome), predict()

1._at: age = 20
2._at: age = 30
3._at: age = 40
4._at: age = 50
5._at: age = 60

		Delta-method				[95% conf. interval]	
		Margin	std. err.	z	P> z		
<hr/>							
_at#sex							
1#male		.0150645	.0047348	3.18	0.001	.0057846	.0243445
1#female		.025333	.0055508	4.56	0.000	.0144536	.0362124
2#male		.0364848	.0075444	4.84	0.000	.0216981	.0512714
2#female		.0596255	.0086074	6.93	0.000	.0427552	.0764958
3#male		.0852689	.0099016	8.61	0.000	.0658622	.1046757
3#female		.1329912	.0108127	12.30	0.000	.1117987	.1541838
4#male		.1849367	.0163684	11.30	0.000	.1528551	.2170182
4#female		.267774	.0156218	17.14	0.000	.2371558	.2983921
5#male		.3518378	.0408522	8.61	0.000	.271769	.4319066
5#female		.4614446	.0314754	14.66	0.000	.3997539	.5231353

6 Plotting Margins

`margins` provides a lot of results, which can be difficult to understand. Therefore, we use `marginsplot` to *plot* these `margins` results.

There certainly seems to be some kind of interaction of `sex` and `age`.

```
marginsplot  
  
graph export mymarginsplot.png, width(1000) replace
```

```
Running C:\Users\agroga\Desktop\GitHub\newstuff\categorical\logistic-interacti  
> ons-2\profile.do .
```

Variables that uniquely identify margins: age sex

file `mymarginsplot.png` saved as PNG format

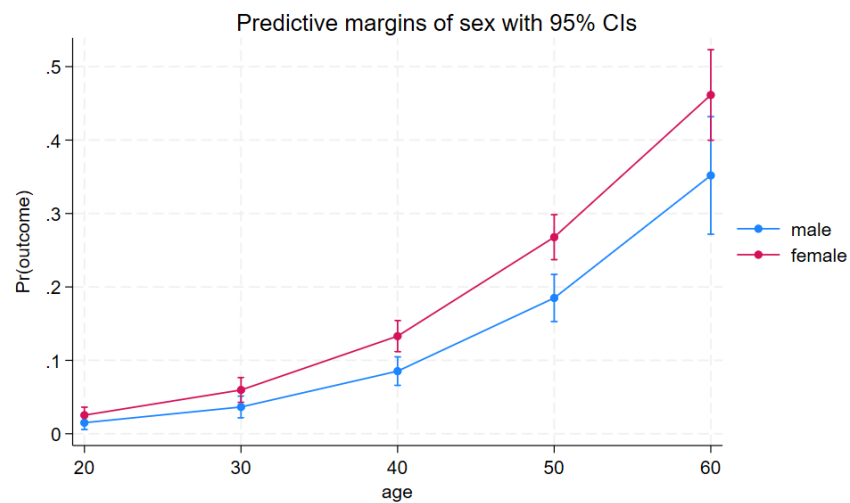


Figure 2: Margins Plot

7 Rerun margins, Posting Results

We again employ the `margins` command, this time using the `post` option so that the results of the margins command are *posted* as an estimation result. This will allow us to employ the `test` command to statistically test different margins against each other.

```
margins sex, at(age = (20 30 40 50 60)) post
```

```
Running C:\Users\agrogan\Desktop\GitHub\newstuff\categorical\logistic-interacti
> ons-2\profile.do .
```

Predictive margins

Number of obs = 3,000

Model VCE: OIM

Expression: Pr(outcome), predict()

1._at: age = 20

2._at: age = 30

3._at: age = 40

4._at: age = 50

5._at: age = 60

		Delta-method				
		Margin	std. err.	z	P> z	[95% conf. interval]

_at#sex						
1#male		.0150645	.0047348	3.18	0.001	.0057846 .0243445
1#female		.025333	.0055508	4.56	0.000	.0144536 .0362124
2#male		.0364848	.0075444	4.84	0.000	.0216981 .0512714
2#female		.0596255	.0086074	6.93	0.000	.0427552 .0764958
3#male		.0852689	.0099016	8.61	0.000	.0658622 .1046757
3#female		.1329912	.0108127	12.30	0.000	.1117987 .1541838
4#male		.1849367	.0163684	11.30	0.000	.1528551 .2170182
4#female		.267774	.0156218	17.14	0.000	.2371558 .2983921
5#male		.3518378	.0408522	8.61	0.000	.271769 .4319066
5#female		.4614446	.0314754	14.66	0.000	.3997539 .5231353

8 margins with coeflegend

We follow up by using the `margins` command with the `coeflegend` option to see the way in which Stata has labeled the different margins.

```
margins, coeflegend
```

```
Running C:\Users\agrogan\Desktop\GitHub\newstuff\categorical\logistic-interacti
> ons-2\profile.do .
```

Predictive margins

Number of obs = 3,000

Model VCE: OIM

Expression: Pr(outcome), predict()

1._at: age = 20

2._at: age = 30

3._at: age = 40

4._at: age = 50

5._at: age = 60

	Margin	Legend
_at#sex		
1#male	.0150645	_b[1bn._at#0bn.sex]
1#female	.025333	_b[1bn._at#1.sex]
2#male	.0364848	_b[2._at#0bn.sex]
2#female	.0596255	_b[2._at#1.sex]
3#male	.0852689	_b[3._at#0bn.sex]
3#female	.1329912	_b[3._at#1.sex]
4#male	.1849367	_b[4._at#0bn.sex]
4#female	.267774	_b[4._at#1.sex]
5#male	.3518378	_b[5._at#0bn.sex]
5#female	.4614446	_b[5._at#1.sex]

9 Testing Margins Against Each Other

Lastly, we test the margins at age 20 for men and women, and again at ages 50 and 60 for men and women.

We note that the original regression parameter for the interaction term was not statistically significant. Indeed, the margins at age 20 are not statistically significantly different by sex. However, at ages 50 & 60, there is a statistically significant difference by sex.

```
test _b[1bn._at#0bn.sex] = _b[1bn._at#1.sex] // male and female at age 20

test _b[4._at#0bn.sex] = _b[4._at#1.sex] // male and female at age 50

test _b[5._at#0bn.sex] = _b[5._at#1.sex] // male and female at age 60
```

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> ons-2\profile.do .

```
( 1) 1bn._at#0bn.sex - 1bn._at#1.sex = 0
```

```
      chi2( 1) =      1.99
Prob > chi2 =      0.1583
```

```
( 1) 4._at#0bn.sex - 4._at#1.sex = 0
```

```
      chi2( 1) =     13.03
Prob > chi2 =      0.0003
```

```
( 1) 5._at#0bn.sex - 5._at#1.sex = 0
```

```
      chi2( 1) =      5.16
Prob > chi2 =      0.0232
```

There is some suggestion that the *difference of the differences* is statistically significant. This statistical significance is only marginal [pun intended] at age 60, but truly statistically significant at age 50.

```
test _b[1bn._at#1.sex] - _b[1bn._at#0bn.sex] = _b[5._at#1.sex] - _b[5._at#0bn.sex] // test

test _b[1bn._at#1.sex] - _b[1bn._at#0bn.sex] = _b[4._at#1.sex] - _b[4._at#0bn.sex] // test
```

Running C:\Users\agrogon\Desktop\GitHub\newstuff\categorical\logistic-interacti

```
> ons-2\profile.do .
```

```
( 1) - 1bn._at#0bn.sex + 1bn._at#1.sex + 5._at#0bn.sex - 5._at#1.sex = 0
```

```
      chi2( 1) =      3.62  
Prob > chi2 =      0.0572
```

```
( 1) - 1bn._at#0bn.sex + 1bn._at#1.sex + 4._at#0bn.sex - 4._at#1.sex = 0
```

```
      chi2( 1) =      9.77  
Prob > chi2 =      0.0018
```

10 References

Ai, C., & Norton, E. C. (2003). Interaction terms in logit and probit models. *Economics Letters*. [https://doi.org/10.1016/S0165-1765\(03\)00032-6](https://doi.org/10.1016/S0165-1765(03)00032-6)

Karaca-Mandic, P., Norton, E. C., & Dowd, B. (2012). Interaction terms in nonlinear models. *Health Services Research*. <https://doi.org/10.1111/j.1475-6773.2011.01314.x>