

Logistic Regression Equation

Andy Grogan-Kaylor

2021-01-16

Contents

1	Logistic Regression	1
1.1	Equation	1
1.2	Rewriting The Equation	1
2	Graph	2

1 Logistic Regression

1.1 Equation

Logistic regression—written here with a single independent variable—models the log odds of an outcome as a function of a set of covariates:

$$\ln \left(\frac{p(outcome)}{1 - p(outcome)} \right) = \beta_0 + \beta_1 x_1$$

Here $p(outcome)$ is the probability of the outcome.

$\frac{p(outcome)}{1 - p(outcome)}$ is the *odds* of the outcome.

Hence, $\ln \left(\frac{p(outcome)}{1 - p(outcome)} \right)$ is the *log odds*.

Logistic regression returns a β coefficient for each independent variable x .

These β coefficients can then be *exponentiated* to obtain *odds ratios*: $OR = e^\beta$

1.2 Rewriting The Equation

We can take the equation:

$$\ln \left(\frac{p(outcome)}{1 - p(outcome)} \right) = \beta_0 + \beta_1 x_1$$

We exponentiate both sides of the equation:

$$\frac{p(outcome)}{1 - p(outcome)} = e^{\beta_0 + \beta_1 x_1}$$

We multiply both sides by the denominator of the fraction that is on the left hand side of the equation:

$$p(outcome) = e^{\beta_0 + \beta_1 x_1} (1 - p(outcome))$$

Then:

$$p(outcome) = e^{\beta_0 + \beta_1 x_1} - e^{\beta_0 + \beta_1 x_1} * p(outcome)$$

Then:

$$p(outcome) + e^{\beta_0 + \beta_1 x_1} * p(outcome) = e^{\beta_0 + \beta_1 x_1}$$

Then:

$$(1 + e^{\beta_0 + \beta_1 x_1}) * p(outcome) = e^{\beta_0 + \beta_1 x_1}$$

And, finally:

$$p(outcome) = \frac{e^{\beta_0 + \beta_1 x_1}}{1 + e^{\beta_0 + \beta_1 x_1}}$$

We sometimes use a shorthand, and say

$$F(z) = \frac{e^z}{1 + e^z}$$

2 Graph

We graph a logistic distribution with β_0 set to 0, and β_1 set to 1.

