

From Contingency Table To Logistic Regression

With the French Skiers Data

Andy Grogan-Kaylor

2023-09-23

Table of contents

1	The Data	1
2	Contingency Table	1
2.1	Risk and Risk Differences	2
2.2	Odds Ratios	2
3	Logistic Regression	3

1 The Data

We use the French Skiers data that we have used in other examples.

```
use "FrenchSkiers.dta"
```

2 Contingency Table

```
tabulate Tx Outcome [fweight = Count]
```

Tx	Outcome		Total
	No Cold	Cold	
Placebo	109	31	140
Ascorbic Acid	122	17	139
Total	231	48	279

For the sake of teaching and exposition, I re-arrange the numbers slightly.

	Develop Outcome	Do Not Develop Outcome
Exposed	a	b
Not Exposed	c	d

	Cold	No Cold
Ascorbic Acid	17 (a)	122 (b)
Placebo	31 (c)	109 (d)

2.1 Risk and Risk Differences

$$R = \frac{a}{a+b} \text{ (in Exposed)}$$

$$RD =$$

risk in exposed – risk in not exposed =

$$a/(a+b) - c/(c+d) =$$

$$(17/139) - (31/140) =$$

$$-.09912641$$

2.2 Odds Ratios

	Develop Outcome	Do Not Develop Outcome
Exposed	a	b
Not Exposed	c	d

$$OR =$$

$$\frac{\text{odds that exposed person develops outcome}}{\text{odds that unexposed person develops outcome}} =$$

$$\frac{\frac{a}{a+b} / \frac{b}{a+b}}{\frac{c}{c+d} / \frac{d}{c+d}} =$$

$$\frac{a/b}{c/d} =$$

$$\frac{ad}{bc} =$$

$$(17 * 109) / (122 * 31) =$$

$$.4899526$$

3 Logistic Regression

As discussed, the formula for logistic regression is:

$$\ln \left(\frac{p(\text{outcome})}{1 - p(\text{outcome})} \right) = \beta_0 + \beta_1 x$$

Here $p(\text{outcome})$ is the probability of the outcome.

$\frac{p(\text{outcome})}{1 - p(\text{outcome})}$ is the *odds* of the outcome.

Hence, $\ln \left(\frac{p(\text{outcome})}{1 - p(\text{outcome})} \right)$ is the *log odds* of the outcome.

Logistic regression returns a β coefficient for each independent variable x .

These β coefficients can then be *exponentiated* to obtain *odds ratios*: $OR = e^\beta$

We see that the odds ratio given by logistic regression, .4899526, is the exact same as that given by manually calculating the odds ratio from a contingency table.

An advantage of logistic regression is that it can be extended to multiple independent variables.

```
logit Outcome Tx [fweight = Count], or
```

```
Iteration 0:  Log likelihood = -128.09195
```

```
Iteration 1:  Log likelihood = -125.68839
```

```
Iteration 2:  Log likelihood = -125.65611
```

```
Iteration 3:  Log likelihood = -125.6561
```

Logistic regression

Number of obs = 279

Log likelihood = -125.6561

LR chi2(1) = 4.87
Prob > chi2 = 0.0273
Pseudo R2 = 0.0190

Outcome	Odds ratio	Std. err.	z	P> z	[95% conf. interval]	
-----+-----						
Tx	.4899526	.1613519	-2.17	0.030	.256942	.9342712
_cons	.2844037	.0578902	-6.18	0.000	.1908418	.423835

Note: _cons estimates baseline odds.