From Contingency Table To Logistic Regression

With the French Skiiers Data

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1 The Data

We use the French Skiiers data that we have used in other examples.

```
use "FrenchSkiiers.dta"
```

2 Contingency Table

```
tabulate Tx Outcome [fweight = Count]
```

```
               |        Outcome
            Tx |   No Cold       Cold |     Total
---------------+----------------------+----------
       Placebo |       109         31 |       140
 Ascorbic Acid |       122         17 |       139
---------------+----------------------+----------
         Total |       231         48 |       279
```

For the sake of teaching and exposition, I re-arrange the numbers slightly.

	Develop Outcome	Do Not Develop Outcome
Exposed	a	b
Not Exposed	c	d

	Cold	No Cold
Ascorbic Acid	17 (a)	122 (b)
Placebo	31 (c)	109 (d)

2.1 Risk (R) and Risk Differences (RD)

$R = \frac{a}{a+b}$ (in Exposed)

$RD =$

risk in exposed $-$ risk in not exposed $=$

$a/(a+b) - c/(c+d) =$

$(17/139) - (31/140) =$

$-.09912641$

> How do we talk about this *risk difference*?

2.2 Odds Ratios (OR)

	Develop Outcome	Do Not Develop Outcome
Exposed	a	b
Not Exposed	c	d

$OR =$

$\frac{\text{odds that exposed person develops outcome}}{\text{odds that unexposed person develops outcome}} =$

$\frac{\frac{a}{a+b}/\frac{b}{a+b}}{\frac{c}{c+d}/\frac{d}{c+d}} =$

$\frac{a/b}{c/d} =$

$\frac{ad}{bc} =$

$(17 * 109)/(122 * 31) =$

$.4899526$

> How do we talk about this *odds ratio*?

3 Logistic Regression

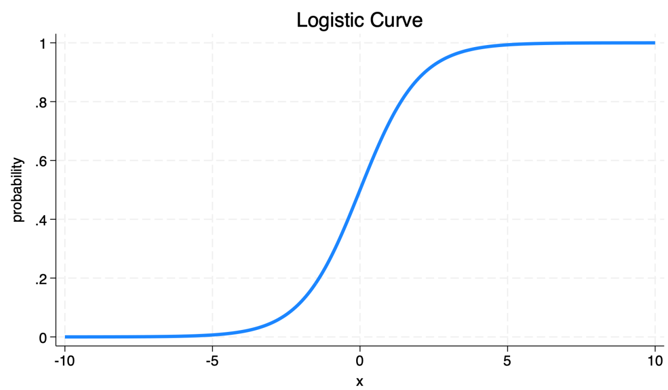
As discussed, the formula for logistic regression is:

$$\ln\left(\frac{p(\text{outcome})}{1-p(\text{outcome})}\right) = \beta_0 + \beta_1 x$$

Here $p(\text{outcome})$ is the probability of the outcome.

$\frac{p(\text{outcome})}{1-p(\text{outcome})}$ is the *odds* of the outcome.

Hence, $\ln\left(\frac{p(\text{outcome})}{1-p(\text{outcome})}\right)$[1] is the *log odds* of the outcome.

> **The logistic regression equation has the desired functional form.**
>
> The logistic regression equation is appropriate to reflect changes in the probability of an outcome that can be either 1 or 0.
>
> 
>
>
> Figure 1: Logistic Curve

Logistic regression returns a β coefficient for each independent variable x.

These β coefficients can then be *exponentiated* to obtain *odds ratios*: $OR = e^{\beta}$

> **Exponentiation "undoes" the logarithmic transformation.**
>
> If $\ln(y) = x$, then $y = e^x$
> So, if ... $\ln\left(\frac{p(\text{outcome})}{1-p(\text{outcome})}\right) = \beta_0 + \beta_1 x$ then $\frac{p(\text{outcome})}{1-p(\text{outcome})} = e^{\beta_0+\beta_1 x} = e^{\beta_0} \times e^{\beta_1 x}$

We see that the odds ratio given by logistic regression, `.4899526`, is the exact same as that given by manually calculating the odds ratio from a contingency table.

An advantage of logistic regression is that it can be extended to multiple independent variables.

```
logit Outcome Tx [fweight = Count], or
```

[1] It is sometimes useful to think of the *log odds* as a *transformed dependent variable*. We have transformed the dependent variable so that it can be expressed as a linear function of the independent variables, e.g.: $\beta_0 + \beta_1 x$

```
Iteration 0:  Log likelihood = -128.09195
Iteration 1:  Log likelihood = -125.68839
Iteration 2:  Log likelihood = -125.65611
Iteration 3:  Log likelihood =  -125.6561

Logistic regression                                     Number of obs =    279
                                                        LR chi2(1)    =   4.87
                                                        Prob > chi2   = 0.0273
Log likelihood = -125.6561                              Pseudo R2     = 0.0190

------------------------------------------------------------------------------
     Outcome | Odds ratio   Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
          Tx |   .4899526   .1613519    -2.17   0.030      .256942    .9342712
       _cons |   .2844037   .0578902    -6.18   0.000     .1908418     .423835
------------------------------------------------------------------------------
Note: _cons estimates baseline odds.
```

How do we talk about this *odds ratio*? How would we talk about it if it was > 1.0? > 2.0