

Cox Proportional Hazards Model

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Cox Proportional Hazards Model

Formula

$h(t)$ the rate of occurrence.

$$h(t) = \lim_{\delta \rightarrow \infty} \frac{\text{probability of having an event before time } t + \delta}{\delta}$$

This definition per Johnson & Shih (2007).

$$h(t) = h_0(t)e^{\beta_1 x_1 + \beta_2 x_2 + \text{etc.}}$$

We don't directly estimate the hazard, but estimate the effect of covariates on the hazard.

The event (birth, death, program entry, program departure) is coded as 1, so we are estimating the association of the covariates with event occurrence.

Cox Proportional Hazards Model in Stata

Using a data set referenced frequently in Stata `help` and Stata YouTube Videos

```
. clear all

. webuse drugtr // demonstration data set from Stata
(Patient survival in drug trial)
```

Setup of Data

```
. stset // show st setup of data
-> stset studytime, failure(died)
Survival-time data settings
    Failure event: died!=0 & died<.
Observed time interval: (0, studytime]
    Exit on or before: failure
```

```
48 total observations
0 exclusions
```

```

    48 observations remaining, representing
    31 failures in single-record/single-failure data
    744 total analysis time at risk and under observation
           At risk from t =           0
           Earliest observed entry t =       0
           Last observed exit t =          39

. describe // show variables in data
Contains data from https://www.stata-press.com/data/r17/drugtr.dta
Observations:           48           Patient survival in drug trial
Variables:              8           3 Mar 2020 02:12

```

Variable name	Storage type	Display format	Value label	Variable label
studytime	byte	%8.0g		Months to death or end of exp.
died	byte	%8.0g		1 if patient died
drug	byte	%8.0g		Drug type (0=placebo)
age	byte	%8.0g		Patient's age at start of exp.
_st	byte	%8.0g		1 if record is to be used; 0 otherwise
_d	byte	%8.0g		1 if failure; 0 if censored
_t	byte	%10.0g		Analysis time when record ends
_t0	byte	%10.0g		Analysis time when record begins

Sorted by:

Kaplan-Meier Survivor Function (per Gabriela Ortiz, Stata)

$$S(t) = Pr(T > t)$$

```

. sts graph, scheme(michigan) // Kaplan-Meier Survivor Function
    Failure _d: died
    Analysis time _t: studytime

. graph export survival0.png, width(1000) replace
file /Users/agrogan/Desktop/newstuff/categorical/survival-analysis-and-event-history/survival0.png
saved as PNG format

```

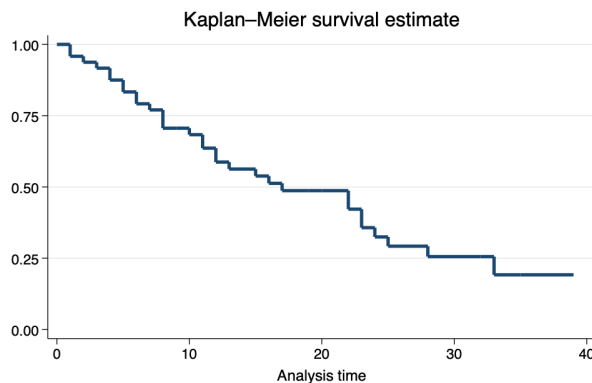


Figure 1: Kaplan-Meier Survivor Function

Cox Proportional Hazards Model

```

. stcox age drug // run Cox Proportional Hazards Model
    Failure _d: died

```

```

Analysis time _t: studytime
Iteration 0: log likelihood = -99.911448
Iteration 1: log likelihood = -83.551879
Iteration 2: log likelihood = -83.324009
Iteration 3: log likelihood = -83.323546
Refining estimates:
Iteration 0: log likelihood = -83.323546
Cox regression with Breslow method for ties
No. of subjects = 48          Number of obs = 48
No. of failures = 31
Time at risk = 744
Log likelihood = -83.323546    LR chi2(2) = 33.18
                                Prob > chi2 = 0.0000

```

_t	Haz. ratio	Std. err.	z	P> z	[95% conf. interval]	
age	1.120325	.0417711	3.05	0.002	1.041375	1.20526
drug	.1048772	.0477017	-4.96	0.000	.0430057	.2557622

Graph Survival Curves

```

. stcurve, survival scheme(michigan) // survival curve

. graph export survival1.png, width(1000) replace
file /Users/agrogan/Desktop/newstuff/categorical/survival-analysis-and-event-history/survival1.png
saved as PNG format

```

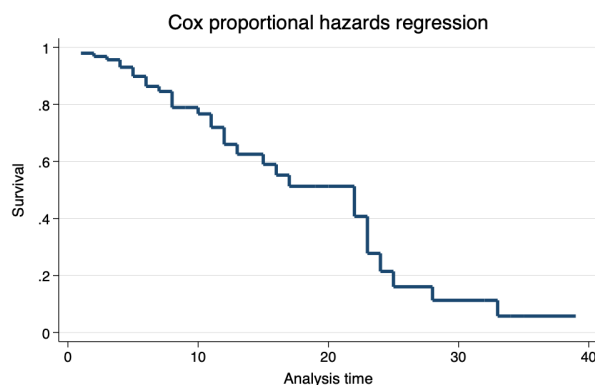


Figure 2: Survival Curve

```

. stcurve, survival at1(drug=0) at2(drug=1) scheme(michigan) // survival curve by group

. graph export survival2.png, width(1000) replace
file /Users/agrogan/Desktop/newstuff/categorical/survival-analysis-and-event-history/survival2.png
saved as PNG format

```

Proportional Hazards Assumption

```

. estat phtest // formal test of PH assumption
Test of proportional-hazards assumption
Time function: Analysis time

```

	chi2	df	Prob>chi2
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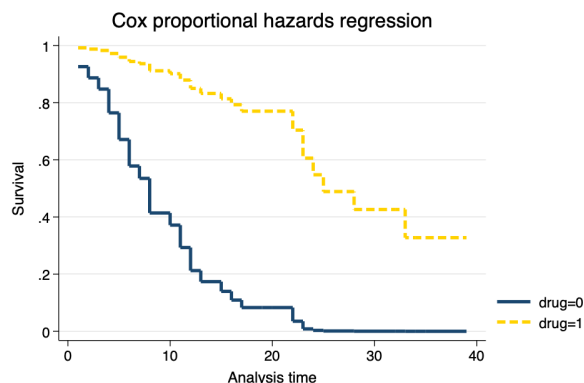


Figure 3: Survival Curve by Drug Group

Global test	0.43	2	0.8064
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```
. sthplot, by(drug) scheme(michigan) // graphical test of PH assumption
    Failure _d: died
    Analysis time _t: studytime

. graph export ph.png, width(1000) replace
file /Users/agrogan/Desktop/newstuff/categorical/survival-analysis-and-event-history/ph.png saved
as PNG format
```

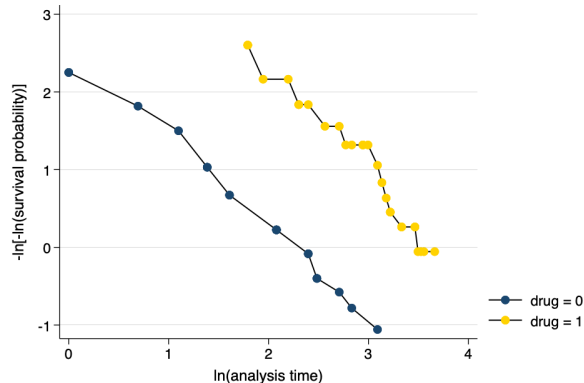


Figure 4: Graphical Assessment of Proportional Hazards Assumptions

References

Johnson, L. L., & Shih, J. H. (2007). CHAPTER 20 - An Introduction to Survival Analysis (J. I. Gallin & F. P. Ognibene, eds.). <https://doi.org/https://doi.org/10.1016/B978-012369440-9/50024-4>

Ragnar Frisch Centre for Economic Research (2020). Event History Analysis, Survival Analysis, Duration Analysis ,Transition Data Analysis, Hazard Rate Analysis. Oslo, Norway.