Logistic Regression Equation

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1 Logistic Regression

1.1 Equation

Logistic regression–written here with a single independent variable–models the log odds of an outcome as a function of a set of covariates:

$$\ln\left(\frac{p(outcome)}{1 - p(outcome)}\right) = \beta_0 + \beta_1 x_1$$

Here $p(outcome)$ is the probability of the outcome.

$\frac{p(outcome)}{1-p(outcome)}$ is the *odds* of the outcome.

Hence, $\ln\left(\frac{p(outcome)}{1-p(outcome)}\right)$ is the *log odds*.

Logistic regression returns a β coefficient for each independent variable x.

These β coefficients can then be *exponentiated* to obtain *odds ratios*: $OR = e^\beta$

1.2 Rewriting The Equation

We can take the equation:

$$\ln\left(\frac{p(outcome)}{1 - p(outcome)}\right) = \beta_0 + \beta_1 x_1$$

We exponentiate both sides of the equation:

$$\frac{p(outcome)}{1-p(outcome)} = e^{\beta_0+\beta_1 x_1}$$

We multiply both sides by the denominator of the fraction that is on the left hand side of the equation:

$$p(outcome) = e^{\beta_0+\beta_1 x_1}(1-p(outcome))$$

Then:

$$p(outcome) = e^{\beta_0+\beta_1 x_1} - e^{\beta_0+\beta_1 x_1} * p(outcome)$$

Then:

$$p(outcome) + e^{\beta_0+\beta_1 x_1} * p(outcome) = e^{\beta_0+\beta_1 x_1}$$

Then:

$$(1+e^{\beta_0+\beta_1 x_1}) * p(outcome) = e^{\beta_0+\beta_1 x_1}$$

And, finally:

$$p(outcome) = \frac{e^{\beta_0+\beta_1 x_1}}{1+e^{\beta_0+\beta_1 x_1}}$$

We sometimes use a shorthand, and say

$$F(z) = \frac{e^z}{1+e^z}$$

2 Graph

We graph a logistic distribution with β_0 set to 0, and β_1 set to 1.