Logistic Regression Equation

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1 Logistic Regression

1.1 Equation

Logistic regression–written here with a single independent variable–models the log odds of an outcome as a function of a set of covariates:

$$\ln\left(\frac{p(\text{outcome})}{1-p(\text{outcome})}\right) = \beta_0 + \beta_1 x_1$$

Here $p(\text{outcome})$ is the probability of the outcome.

$\frac{p(\text{outcome})}{1-p(\text{outcome})}$ is the *odds* of the outcome.

Hence, $\ln\left(\frac{p(\text{outcome})}{1-p(\text{outcome})}\right)$ is the *log odds*.

Logistic regression returns a β coefficient for each independent variable x.

These β coefficients can then be *exponentiated* to obtain *odds ratios*: $OR = e^{\beta}$

1.2 Rewriting The Equation

We can take the equation:

$$\ln\left(\frac{p(\text{outcome})}{1-p(\text{outcome})}\right)=\beta_0+\beta_1x_1$$

We exponentiate both sides of the equation:

$$\frac{p(\text{outcome})}{1-p(\text{outcome})}=e^{\beta_0+\beta_1x_1}$$

We multiply both sides by the denominator of the fraction that is on the left hand side of the equation:

$$p(\text{outcome})=e^{\beta_0+\beta_1x_1}(1-p(\text{outcome}))$$

Then:

$$p(\text{outcome})=e^{\beta_0+\beta_1x_1}-e^{\beta_0+\beta_1x_1}*p(\text{outcome})$$

Then:

$$p(\text{outcome})+e^{\beta_0+\beta_1x_1}*p(\text{outcome})=e^{\beta_0+\beta_1x_1}$$

Then:

$$(1+e^{\beta_0+\beta_1x_1})*p(\text{outcome})=e^{\beta_0+\beta_1x_1}$$

And, finally:

$$p(\text{outcome})=\frac{e^{\beta_0+\beta_1x_1}}{1+e^{\beta_0+\beta_1x_1}}$$

We sometimes use a shorthand, and say

$$F(z)=\frac{e^z}{1+e^z}$$

2 Graph

We graph a logistic distribution with β_0 set to 0, and β_1 set to 1.

```
N <- 100

x <- runif(N, -10, 10)

y <- exp(x)/(1 + exp(x))

library(ggplot2)


ggplot(data = NULL,
       aes(x = x,
           y = y,
           color = y)) +
  geom_point() +
  labs(title = "Logistic Function",
       y = "Probability") +
  scale_color_viridis_c(name = "probability") +
  theme_minimal()
```