NUMERICAL PROJECT 3
MTH 5500 STOCHASTIC CALCULUS

- This project counts as **extra credit** (3 points on the final grade).
- The project has to be handed in by **May 14** to get the credits.
- It can be done in teams or **three people or less**.
- The codes have to be in **Python**.

A Stochastic Volatility Model: the SABR model

We consider the SABR model for the Libor Forward rates $(F_t, t \geq 0)$ given by the SDEs

$$\mathrm{d}F_t = \sigma_t F_t^\beta \mathrm{d}B_t \qquad \mathrm{d}\sigma_t = \sigma_t \mathrm{d}B'_t \ ,$$

with initial condition $\sigma_0 = 1$ and $F_0 = 0.05$. Here $(B_t, t \geq 0)$ and $(B'_t, t \geq 0)$ are two **independent** Brownian motions under the risk-neutral probability $\widetilde{\mathbf{P}}$. This model is said to be a *stochastic volatility model* since the volatility is itself a random process. The parameter β is between 0 and 1. For the project, take $\beta = 1/2$.

(1) Draw the graph of 100 paths of (σ_t) for t up to 1 at every one-hundredth using the *Euler Scheme*

$$\sigma_{t_{j+1}} - \sigma_{t_j} \approx \sigma_{t_j}(B'_{t_{j+1}} - B'_{t_j}) \ .$$

Hint: the approximation could lead to negative variance which is impossible. Add a safety feature to your code, that ensures that the variance is never negative.

(2) Using the data from the first question, draw the graph of 100 paths of (F_t) for t up to 1 at every one-hundredth using the *Euler Scheme*

$$F_{t_{j+1}} - F_{t_j} \approx \sigma_{t_j} F_{t_j}^{1/2}(B_{t_{j+1}} - B_{t_j}) \ .$$

Again F_t cannot be negative in the square root. Your code should prevent this.

(3) Draw a histogram for the distribution of F_1.

(4) Evaluate numerically the price C_0 of a European Call on the forward rate with maturity $T = 1$ and strike $K = 0.03$. Assume the risk-free rate is 0 so that

$$C_0 = \mathbf{E}[\max\{F_1 - K, 0\}] \ .$$

This can be simply done by doing the average on 100 paths generated above.

Date: Louis-Pierre Arguin, April 30 2018.