NUMERICAL PROJECT 2
MTH 5500 STOCHASTIC CALCULUS

- This project counts as **extra credit** (3 points on the final grade).
- The project has to be handed in by **May 7** to get the credits.
- It can be done in teams or **three people or less**.
- The codes have to be in **Python**.

(1) Consider the two processes

$$X_t = \frac{1}{\sqrt{1-t}} \exp\left(\frac{-B_t^2}{2(1-t)}\right) \qquad Y_t = 1 + \int_0^t \frac{-B_s Y_s}{1-s} \, dB_s$$

for $0 \le t < 1$.

(a) Draw the graph of 10 paths of (X_t) for t up to $99/100$ at every one-hundredth.

(b) Draw the graph of 10 paths of (Y_t) for t up to $99/100$ at every one-hundredth.
Use the appropriate martingale transform.

(c) Compute the empirical average of $|X_t - Y_t|$ over 100 paths at $t = 1/5$, $2/5$, $3/5$, $4/5$.

(d) Compute the empirical average of Y_t over 100 paths at $t = 95/100$, $t = 96/100$, $\ldots t = 99/100$.
What is your guess for the limit $\lim_{t \to 1} Y_t$?

(e) Compute the empirical average of $\max_{0 \le t < 1} Y_t$ over 100 paths.
What is your guess for the expectation $\mathbf{E}[\max_{0 \le t < 1} Y_t]$?