G E O M E T R Y

(*Published*)

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This edition is a collection of A. Grothendieck's published works on geometry reunited by Mateo Carmona. Remarks, comments, and corrections are welcome.
https://agrothendieck.github.io/

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Contents

Standard Conjectures on Algebraic Cycles

1 We state two conjectures on algebraic cycles, which arose fro an attempt at understanding the conjectures of Weil on the ζ-functions of algebraic varieties. These are not really new, and they were worked out about three years ago independently by Bombieri and myself.

The first is an existence assertion for algebraic cycles (considerably weaker than the Tate conjectures), and is inspired by and formally analogous to Lefschetz's structure theorem on the cohomology of a smooth projective variety over the complex field.

The second is a statement of positivity, generalising Weil's well-known positivity theorem in the theory of abelian varieties. It is formally analogous to the famous Hodge inequalities, and is in fact a consequence of these in characteristic zero.

Introduction

2

A weak form of conjecture 1

3 Let X be smooth and projective,

The conjecture 1 (of Lefschetz type)

4

Conjecture 2 (of Hodge type)

5 The proof of the two standard conjectures would yield results going considerably further than Weil's conjectures. They would form the basis of the so-called "theory of motives" which is a systematic theory of "arithmetic properties" of algebraic varieties, as embodied in their groups of classes of cycles for numerical equivalence. We have at present only a very small part of this theory in dimension one, as contained in the theory of abelian varieties.

Alongside the problem of resolution of singularities, the proof of the standard conjectures seems to me to be the most urgent task in algebraic geometry.

Conclusions