Crystals and the De Rham cohomology of Schemes[1]

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(Notes by I. Coates and O. Jussila)

Introduction

These notes are a rough summary of five talks given at I.H.E.S in November and December 1966. The purpose of these talks was to outline a possible definition of a p-adic cohomology theory, via a generalization of the De Rham cohomology which was suggested by work of Monsky-Washnitzer [?] and Manin [?].

The contents of the notes are by no means intended to be a complete theory. Rather, they outline the start of a program of work which has still not been carried out[2].

1. De Rham cohomology

1.1. Differentiable Manifolds. Let X be a differentiable manifold, and $\underline{\Omega}^{\bullet}_{X/\mathbb{C}}$ the complex of sheaves of differential forms on X

[1]This text has been transcribed by Mateo Carmona
`https://agrothendieck.github.io/`

[2]For a more detailed exposition and progress in this direction, we refer to the work of P. Berthelot, to be developed presumably in SGA 8.

2. The cohomology of Monsky and Wishnitzer

2.1. Approach via liftings.

Suppose X_0 is a scheme on a perfect field k

3. Connections on the De Rham cohomology

For the definition of a *connection* and a *stratification* on a sheaf, see Appendix I of these notes.

4. The infinitesimal topos and stratifying topos

We now turn to the definition of a more general category of coefficients for the De Rham cohomology. To this end we introduce two ringed topos, the *infinitesimal topos* and the *stratifying topos*.

We shall see later that in fact these two topos work well only in characteristic 0

5. Cěch calculations

We now consider the cohomology of the infinitesimal topos and the stratifying topos[3]

6. Comparison of the Infinitesimal and De Rham Cohomologies

6.1. The basic idea. Let X be a scheme above S, and F a quasi-coherent Module on X fortified with a stratification relative to S.

7. The crystalline topos and connecting topos

7.1. Inadequacy of infinitesimal topos. Let X_0 be a scheme above a perfect field k of characteristic $p > 0$. Then, regarding X_0 as being above $S = \operatorname{Spec} W(k)$ instead of k, the infinitesimal cohomology

$$\mathrm{H}^*((X_0/S)_{\mathrm{inf}}, \underline{O}X_0)$$

[3]For a general discussion of the cohomology of a topos, see (SGA 4 V).

is a graded module

Appendix

Let X be a scheme above the base S, and F a Module on X. For each positive integer n,

REFERENCES

[1] ARTIN, M., GROTHENDIECK, A., J.L. VERDIER — *Cohomology étale des schémas*, Sém. Géom. Alg. IHES, 1963-64 (SGA 4), à paraître dans North Holland Pub. Cie.