Voorschoten, 30-4-69.
Palestrinalaan 11.

Dear Grothendieck,

Last fall I wrote you that I was planning to work out your notes on formal tamely ramified coverings together with a result on ''plumbing''.

In doing so it turned out to be necessary- at least by the method I followed- to write again a lot of preliminaries on tamely ramified coverings, this time for formal schemes. I am still not ready with the above mentioned ''project''! Since it took me already much time, I would appreciate it very much to have -before proceeding- your opinion about the preliminary version which I have prepared so far. Of course it is not necessary to read the notes in detail. My question is: what do you think of this set-up and do you think it worthwhile to continue in this way? To me the entire thing seems somewhat indigestible.

I have adjoined some comments on the different chapters. The notes consists of two parts. Part one (Chap. I-VII) are the preliminaries, part two (CHap.VIII-X) are your notes and the plumbing. The chapter on the analogue of the theorem of Mumford has still to be written.

I hope you don't mind too much that I bother you again with these tamely ramified coverings!

With best wishes,

Sincerely yours,

J.P. Murre

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1.

Comments.

Chap.I. Etale morphisms of formal schemes are defined as inductive systems of étale morphisms of usual schemes (1.1.2 and EGA I 10.12.3).
Crucial lemmas: 1.1.6, 1.1.13 and 1.4.9.
As to 1.1.13: étale nbhs for formal schemes are locally completions of étale nbhs of usual schemes. This is used in 7.1.5, itself used in the reduction step 9.3.3.

Chap.II. There are Kummer coverings (II-1), generalized Kummer coverings (II-2) and coverings of Kummer type (II-6)!
No attempt has been made towards completeness; for instance there is no section on the automorphisms of (generalized) kummer coverings.
It would be useful to have a charačterization of the Algebra of a generalized Kummer cov. (cf. with 2.1.8 for Kummer coverings).

Chap.III. This is a nuisance!
I have tried to write only down those things needed for Chap.VIII-X.
It was sufficient to follow the notes which I made for usual schemes and therefore I have not followed your new set-up. But one way or the other, one gets the feeling that the notion of tamely ramified covering, although natural, is very unmanageable if one has to use "the functor" instead of the geometric realization itself.
Lemma 3.2.11 is a technical lemma, used for instance in 3.5.8, 3.6.5 (and ~~many~~ at several places later on).
Very crucial are ~~are~~ 3.4.1, 3.4.2 ; also the notations introduced there are often used in the sequel.
For 3.5.5 I have given only an indication of the proof; in fact many proofs of III are standard but very unpleasant to spell out in all details.
Part 3.6 (and especially 3.6.5) are important. Here the geometric realization determines the tamely ramified covering. Usually only normality, instead of regularity, seems to be needed. The difficulty is however that the notion of normality is (perhaps) not stable by étale localization (cf. 1.7.3 and 1.7.4).

Chap.VI. 6.1.3 is used in 8.1.9, see page VIII-8 point c.
6.2.1 and 6.2.2 are technical results, used in the proof that 9.3.5 implies 9.3.2 (see ~~xxx~~ page IX-17).

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Comments (2).

Chap.VII. This is a technical chapter. The results are delicate , but they are used in an essential way in 9.3 in the reduction step 9.3.3. As pointed out before: 1.1.13 is used in 7.1.5. In 7.1.5 we have to work with Kummer coverings instead of generalized Kummer coverings; the reason (or at least one of the reasons) is that we don't have an analogous lemma for 2.1.8. As a consequence the assumption $D_i \cap D_j = \emptyset$ (see 9.0.1) has to be made.
Also 7.1.6 is a subtle lemma.

Chap.VIII. The first page of your notes ''Revêtements formels étales... Remark 8.1.6 (i.e. the isomorphism is canonical) seems to me to be correct (!) and essential for 8.1.9.iii.

Chap.IX.
This is a working-out of your notes.
The assumption $D_{i,o} \cap D_{j,o} = \emptyset$ is-as pointed out above- used in 9.3.3., in order to be able to apply 7.1.6 and 7.1.8 (page IX-13,line 4 top).
In IX-1 there are several points about which I am not certain (for instance page IX-4 below). Nevertheless I have used them (!) because it seems to me that you used them implicitely and without mentioning in your notes and probably these things are more or less standard results in étale cohomology.
In IX-2 I have not included (yet) the proof of the key-theorem 9.2.3. (this proof is explicitly given in the letter of Giraud).
As to IX-3 : the reduction step 9.3.3. seems to me to be very delicate and depends upon the results in chap.VII. (In your notes you give the motivation in the margin !). It is -in particular- at this point that one has the feeling that the notion of tamely ramified covering is cumbersome.
I hope that part IX-3 is correctly written and that I don't have overlooked difficulties; I had a lot of trouble with it.
One final remark: I needed here the results on tamely ramified coverings for usual schemes; are these included in your planned final version of SGA 1 Exp.XII (see 9.0.4,9.1,4, page IX-6 below)?

Chap.X The plumbing. X-3 is again very cumbersome. It would be nicer to work directly with the geometric realization, but there I encountered difficulties, therefore I followed an easy but long way.

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