

Voorschoten, May 16 1967.
Palestrinalaan 11.

Dear Grothendieck,

I am trying to work out your letter with indications. However I have many difficulties and my progress is very slow. Usually I could find out from your manuscripts for what purpose you developed things in such or such a way, but this time I doubt whether I have grasped the essential points. I can understand your set-up only against the lemma of Abhyankar as background, namely instead of giving directly the covering $\underline{r}(X)$ (in your notation) of S , you give the "behaviour" of the covering over other coverings, namely the $\underline{s}(M)$, which are of a particular type; apparantly the behaviour over all these $\underline{s}(M)$ together gives better information than the $\underline{r}(X)$ alone. Is this intuitively a reasonable way of thinking over your new set-up?

My main difficulty is that I don't see that for a Galois covering the extra structure is uniquely determined. First the concept of Galois covering itself: After the abstract definition and some variations you write: (I assume that you have a copy) On peut enfin l'expliquer en revenant à la définition de $\text{Rev}^R(S)$, et en exigeant que les revêtements des $\underline{s}(M)$ correspondants à X soient principaux de groupe G . Now in all your examples there is also a group working on the $\underline{s}(M)$, for instance in example 1 a group \mathcal{G} working on Z , and this group acts also on the Galois covering X . It seems to me that one must require that the actions of G and \mathcal{G} commute; is this correct? On the same and the next page of your letter you explain (for example 1) that the functor \underline{r} (geometric realization) is fully faithful (restricted to Galois coverings) under certain conditions; this I don't understand at all (and ~~makes~~ this makes me feel very uncertain, since I expect that this is an important point ~~if~~ if one wants to see what is going on).

Corresponding
with

Some other points: in the final formulation of example 3 you introduce a scheme of groups $G(M)$; ~~with this formulation~~ do the objects in example 3a, with this formulation, consist out of couples $M = (\underline{a}, \underline{n})$ and $G(M) = \mu_{\underline{n}}$? Also you write: $\text{Hom}_S(M', M)$ est vide sauf si $\underline{n}' \geq \underline{n}$, auquel cas....etc....., it seems to me that this should besauf si $\underline{n}' = \underline{m} \cdot \underline{n}$, auquel cas..... and where $\underline{m} = (m_i)_{i \in I}$ is a set of positive integers.

2.

I realize that I cause you a lot of trouble and work by asking every time for further explanation and that my assistance in writing the exposé is not very large. Therefore I suggest that you answer this letter only in case it is clear from my questions that I have really a misconception of your new set-up (and it would not surprise me if I have...). Otherwise I shall try to work out your theory as good as I can and make a preliminary manuscript which we can discuss when I am at Bures (I plan to come June 2). In any case, I hope to complete the definitive version during my stay in Bures.

With kind regards,

Sincerely yours,

J. P. Murre

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