

Dear Shurik,

I have been asking people about the questions you raised in your letter. Each day I postpone writing to you, thinking that I could send more complete answers the next day. But since it is getting close to the time ~~that you~~ when ~~you~~ you will be coming to visit, I'll send off these rather incomplete comments. The end of May, beginning of June seem to be fine for me, Deligne. As for me I must lecture on Mondays, but ~~on~~ other days are free. Do you want a room at Ormaille?

The answer to Question (a) seems to be no. Here is a sketch of an example that Deligne made up. Let H be the group generated by x, y with the relations $x^3 = y^3 = 1$, and $xyx^{-1}y^{-1}$ is central. Let C be the (infinite cyclic) group generated by $xyx^{-1}y^{-1}$. So: $1 \rightarrow C \rightarrow H \rightarrow \Gamma \rightarrow 1$ (where $\Gamma = \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$).

Now let G be the universal cover of $SL_2 \mathbb{R}$, and identify the infinite cyclic group C with the center of G . By a (noncommutative group analogue of) pushout, Deligne constructs an extension

$$\begin{array}{ccccccc} 1 & \rightarrow & C & \rightarrow & H & \rightarrow & \Gamma \rightarrow 1 \\ & & \downarrow & & \downarrow & & \downarrow \\ 1 & \rightarrow & G & \rightarrow & E & \rightarrow & \Gamma \rightarrow 0 \end{array}$$

which doesn't split.

The answer to (b) seems to be yes for Lie groups, but beyond that I don't know. E.g. whether a contractible group ^(or Lie group) can have elements of finite order.

The question about the cone is answered negatively. Namely, there is an involution of B^3 with a circle fixed point set in the boundary S^2 (indeed a tame circle) yet whose fixed point set in the interior is wild. (a Bing-type example)

The question about finite subgroups of the Teichmüller group seems to be related to recent work of Thurston, but I don't yet know the full story there.

9

Best, et à bientôt
Berger

10

I have been asking people about the questions you raised in your letter.
 Each day I postpone writing to you, thinking that I could send more complete
 answers the next day. But since it is getting close to the time XXXXXXXX when
 XXXXXXXX you will be coming to visit, I'll send off these rather incomplete
 comments. The end of May, beginning of June seem to be time for me, believe.
 As for me I must lecture on Mondays, but XXXXXXXX other days are free. Do
 you want a room at Urmahle?
 The answer to question (a) seems to be no. Here is a sketch of an example
 that Deligne made up. Let H be the group generated by x, y with the relations
 $x^2 = y^2 = 1$, and $xyx = yxy$. Let C be the (infinite cyclic) group
 generated by $xyz^{-1}y^{-1}$. Let $G = H \rtimes C$ (where $z = a(2\pi)$).
 Now let U be the universal cover of G/H , and identify the infinite cyclic
 group C with the center of U . By a noncommutative group analogue of
 Deligne constructs an extension

I just checked with Siebenmann and a Teichmüller
 space expert named Kravitz — The situation
 is this:

- ① It is unknown whether every finite
 subgroup $\Gamma \subseteq \text{Teich. group}$ operates
 on an algebraic curve. But
- ② It is true for $\Gamma = \text{finite solvable group}$
- ③ True for genus = 2

Concerning the hypersurface $Y \subset X$ and finite autom. u
 of X which is identity on Y — Siebenmann says
 that this is certainly solvable using Smith Theory (i.e. $u=1$).