Grothendieck, A.: Un résultat sur le dual d'une C^*-algèbre. J. Math. pur. appl., IX. Sér. **36**, 97—108 (1957).

Ist A eine C^*-Algebra und u eine lineare, hermitesche und stetige Linearform über A, so gibt es genau eine Zerlegung $u = v - w$ mit $\|u\| = \|v\| + \|w\|$, wobei v und w positive Linearformen sind (Satz 1). Während die Existenz der Zerlegung relativ leicht zu beweisen ist, erfordert der Beweis der Eindeutigkeit kompliziertere Überlegungen. Es wird gezeigt, daß der biduale Raum A'' von A als von Neumannsche Algebra aufgefaßt werden kann, und daß die positiven, normalen Linearformen bzw. die ultraschwach stetigen Linearformen über A'' die positiven Linearformen bzw. die Linearformen aus A' sind (Satz 2). Satz 1 wird dadurch zurückgeführt auf den folgenden Satz: Ist A eine von Neumannsche Algebra und u eine hermitesche, ultraschwach stetige Linearform über A, so gibt es genau eine Zerlegung $u = v - w$ mit $\|u\| = \|v\| + \|w\|$, wobei u und v positive, normale Linearformen über A sind (Satz 3). Es wird noch gezeigt, daß die Existenz der Zerlegung $u = v - w$ in positive Linearformen im wesentlichen charakteristisch dafür ist, daß eine involutive Banach-Algebra A eine C^*-Algebra ist. *E. Thoma.*

Chillingworth, H. R.: Generalised "dual" sequence spaces. Nederl. Akad. Wet., Proc., Ser. A **61**, 307—315 (1958).

If $x = \{x_k\} \in \alpha$, and $y = \{y_k\}$, the set of all y such that $\sum_{k=1}^{\infty} x_k y_k$ converges for every x is called the g-dual space of α, denoted by α^{+}. This is a generalization of the dual space α^* of α, introduced by G. Köthe and O. Toeplitz (this Zbl. 9, 257), in which absolute convergence of the above series is postulated; the possibility of removing the restriction of absolute convergence (and so obtaining the g-dual space) was considered briefly by Köthe and Toeplitz themselves [§ 16 of the above cited paper]. The g-dual space has also been employed by G. Matthews [see the following review] to extend the theory of infinite matrix rings. In the present paper, the author extends many of the theorems due to Köthe and Toeplitz, and to H. S. Allen, given in Chapter 10 of the reviewer's "Infinite matrices and Sequence spaces" (this Zbl. 40, 25), by using the g-dual space; the ideas of projective convergence and limit, and of projective bounded sets, are likewise extended, and theorems on these given in the above reference are thus generalized, and similarly for theorems on strong projective convergence and limit. Finally, some results on transformations due to Köthe and Toeplitz, and Allen, given in the reviewer's "Linear operators" (London 1953), (6. 1, II), (6. 2, I), (6. 2, VIII), and (6. 2, VII), are extended. A g-linear operation is defined as an operation which is distributive and continuous under g-p-convergence; the definition of a projective functional is extended to $f(x) = \sum_{k=1}^{\infty} u_k x_k$, where $x \in \alpha$, $u \in \alpha^{+}$. The last of the theorems cited, i. e., (6. 2. VII), states that to every linear transformation L, defined in a normal sequence space $\alpha \supseteq \Phi$ (where Φ is the space of all finite sequences), corresponds a matrix A such that $A\,x = L\,(x)$ for every x in α. This was first proved by Köthe and Toeplitz, loc. cit. 208, Satz 1, in the case when both α and $\beta = L\,(\alpha)$ are normal and contain Φ; the extension enunciated above, due to Allen, involves no restriction on β. The author here shows that the restriction that α is normal is unnecessary, on using g-linear operations in place of linear operations. *R. G. Cooke.*

Matthews, G.: Generalised rings of infinite matrices. Nederl. Akad. Wet., Proc., Ser. A **61**, 298—306 (1958).

G. Köthe and O. Toeplitz (this Zbl. 9, 257) defined a ring R of infinite matrices by the properties that, if A, B, C belong to R, then (i) $A + B$ and AB also belong to R, (ii) $(AB)C = A(BC)$, (iii) all series $\sum_{j=1}^{\infty} a_{i,j}\, b_{j,k}$ occurring in the product of two matrices of R are absolutely convergent. The author calls a set of infinite