COMMENTS

1. The topic of duality has progressed somewhat since 1975, see for example

L. Begueri Memoir SMF

M. Bester Thesis, U of Michigan

demander réf. : Breen.

(These references are due to L. Breen.)

2. The work of Mme Sinh referred to on p. 2 is as follows:

H.X. Sinh, *Gr-catégories*, Thèse Doctorat d'État Université Paris VII (1975).

See also

N. Saavedra Rivano, *Catégories tanakiennes*, Springer Lecture Notes in Mathematics 265 (1972).

Related to these notions is that of +-groupoid (=coherent symmetric monoidal groupoid with strict associativity and strict zero object) whose theory is developed also in:

M. Takeuchi, 'On Villamayor and Zelinsky's long exact sequence', Mem. Amer. Math. Soc. 33 (1981) No 249.

3. The description of groups in (1-cat) given on p. 2-3 is in effect an equivalence between G-groupoids (= group objects in the category of groupoids) and crossed modules. This is described also in:

R. Brown and C.B. Spencer, 'G-groupoids, crossed modules and the fundamental groupoid of a topological group', Proc. Kon. Akad. v. Weten. 79(1976), 296 - 302.

and is believed to have been known to Verdier in 1965.

4. The equivalence mentioned in the last comment has been generalised to

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equivalences between six categories as set out in the diagram,

$$\begin{tikzcd}
\text{poly -T-complexes} \arrow[r, leftrightarrow, "(f)"] \arrow[d, leftrightarrow, dashed, "(f)"'] & \text{cubical T-complexes} \arrow[r, leftrightarrow, "(c)"] & \omega\text{-groupoids} \arrow[d, leftrightarrow, "(d)"] \arrow[dl, leftrightarrow, "(a)"] \\
\text{simplicial T-complexes} \arrow[r, leftrightarrow, "(e)"'] & \text{crossed complexes} \arrow[r, leftrightarrow, "(d)"'] & \infty\text{-groupoids}
\end{tikzcd}$$

The categories are defined and the equivalences proved in:

a) R. Brown and P. J. Higgins, 'The algebra of cubes', J. Pure Appl. Alg. 21(1981) 233 - 260.

b) R. Brown and P. J. Higgins, 'Colimit theorems for relative homotopy groups', *ibid.* 22(1981) 11 - 41.

c) R. Brown and P. J. Higgins, 'The equivalence of ω-groupoids and cubical T-complexes', Cah. Top. Géom. Diff. (3^e Coll. sur les catégories, dédié a Charles Ehresmann) 22 (1981), 349 - 369.

d) R. Brown and P. J. Higgins, 'The equivalence of ∞-groupoids and crossed complexes' *ibid,* 370 - 386.

e) N. K. Ashley, *Simplicial T-complexes,* Ph.D. thesis, University of Wales (1978),(to be published in an issue of **Esquisses** Mathématiques, Amiens).

f) D. W. Jones, Ph.D. thesis in preparation.

Applications of the equivalence (a) are given in reference (b) above.

5. However the notion of ∞-groupoid referred to in (4) is not adequate for the purposes considered by Grothendieck since, roughly speaking, crossed complexes describe the homotopy types only of spaces X which fibre over a $K(\pi,1)$ with fibre a product of Eilenberg-MacLane spaces.

6. A notion of multiple groupoid called an *n-cat-group* has been described

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in:

J.-L. Loday, 'Spaces with finitely many homotopy groups', J. Pure Appl. Alg. 24(1982) 179 - 202.

The n-cat-groups (which may be described as n-fold categories in the category of groups) are adequate for the description of truncated homotopy types. This is Loday's main result.

7. An analysis of the structure of non-abelian chain complex on a length two normalised chain complex of a simplicial group is given in:

D. Conduché, 'Modules croisés généralisés de longeur 2' (in preparation).

8. By following through some of the equivalences in (4) above, we can associate to an ∞-groupoid G a simplicial T-complex NG, whose underlying simplicial set could be called the *nerve* of G. A direct description of NG is that $(NG)_n$ is the set of ∞-groupoid maps **to** G from $\sigma\underset{\sim}{\Delta}^n$, the homotopy ∞-groupoid of the standard n-simplex Δ^n (i.e. $\sigma\underset{\sim}{\Delta}^n$ has associated crossed complex $\pi\underset{\sim}{\Delta}^n$, the homotopy crossed complex of $\underset{\sim}{\Delta}^n$).

Now a definition of ∞-category is also given in (4)(d) above, but the corresponding functor nerve: ∞-categories $\longrightarrow$ **simplicial** sets is still unknown.

9. An indication of the use of the ideas of (4) in non-abelian situations is given in:

   a) R. Brown and P.J. Higgins, 'Crossed complexes and non-abelian extensions', Int. Conf. on Category Theory, Gummersbach (1981), Springer L.N.M. (to appear).

   b) R. Brown, 'Non-abelian cohomology and the homotopy classification of maps', Proc. Conf, *Méthodes d'algèbre homotopique en topologie*, Marseilles - Luminy (1982). Astérixes (to appear)

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The latter paper uses the equivalence between crossed complexes and simplicial T-complexes (which generalises the Dold-Kan theorem on the equivalence of chain complexes and simplicial abelian groups) to give a notion of nerve, and so classifying space BC of a crossed complex C. The classical Eilenberg-MacLane bijection

$$[X, K(A, n)] \cong H^n (X, A)$$

for a CW-complex X, is then generalised to a bijection

$$[X, BC] \cong H^0 (X, C)$$

However, as pointed out in (5), these results are still "too abelian", and much more work is needed to realise Grothendieck's programme.

10. The theory of tubular neighbourhoods has been developed in papers by D.A. Cox.

'Algebraic tubular neighbourhoods I, II', Math. Scand. 42 (1978), 211 - 228, 229 - 242.

Another of his papers seems relevant:

'Homotopy theory of simplicial schemes', Compositio Math. 39(1979) 263 - 296.

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11 A van Kampen Theorem for 2-cat-groups has now been proved by Brown and Loday.

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