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Lecture 23
Recursive sorting I

FIT 1008&2085
Introduction to Computer Science

MONASH University
Information Technology

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For a list of size N, the worst-case time complexity for insertion sort is…

A) O(log N)

B) O(N)

C) O(N log N)

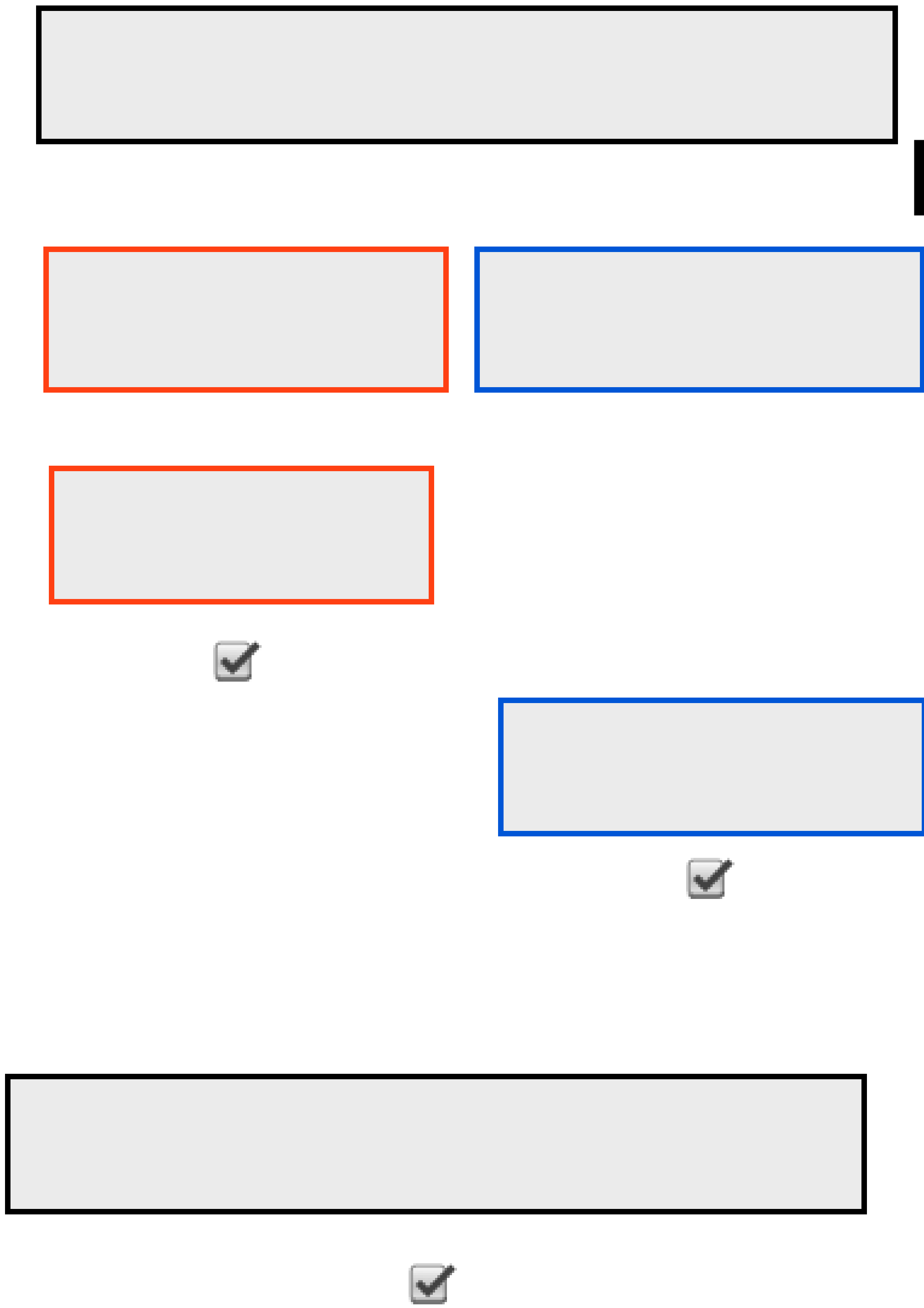
D) $O(N^2)$

Can we do better?

Overview

- To review what a "**divide and conquer**" algorithm is
- To review in more depth two different "divide and conquer" sorting algorithms:
  - **Merge Sort**
  - **Quick Sort**
- To be able to **implement** them and compare their efficiency for different classes of inputs

Divide and Conquer: **Sorting**



Divide the array into **2 parts**

Sort the first part

Sort the second part

Combine

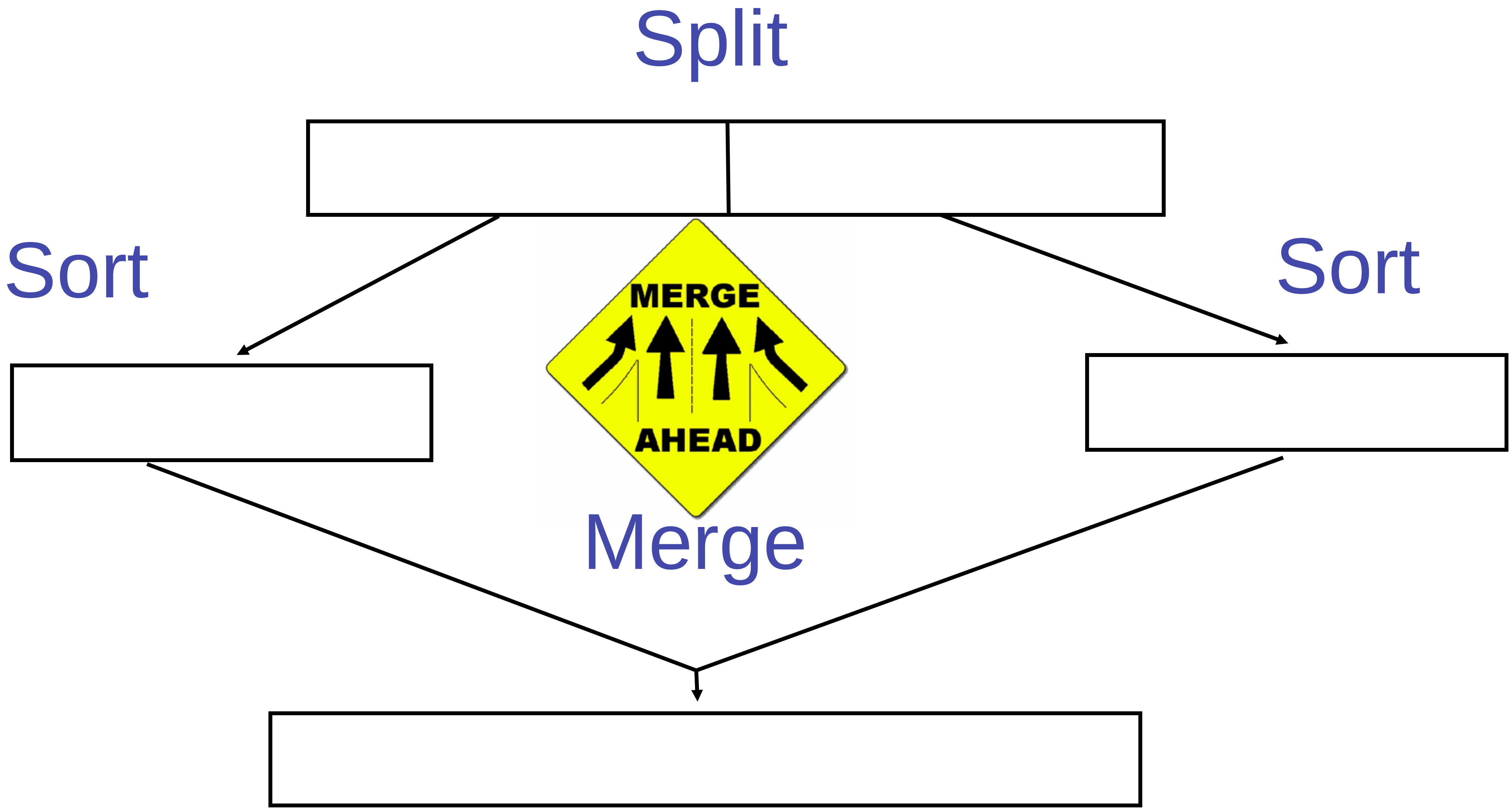
Divide and Conquer: **Sorting**

General Idea

```python
def sort(array):
    if len(array) > 1:
        split(array, first_part, second_part)
        sort(first_part)
        sort(second_part)
        combine(first_part, second_part)
```

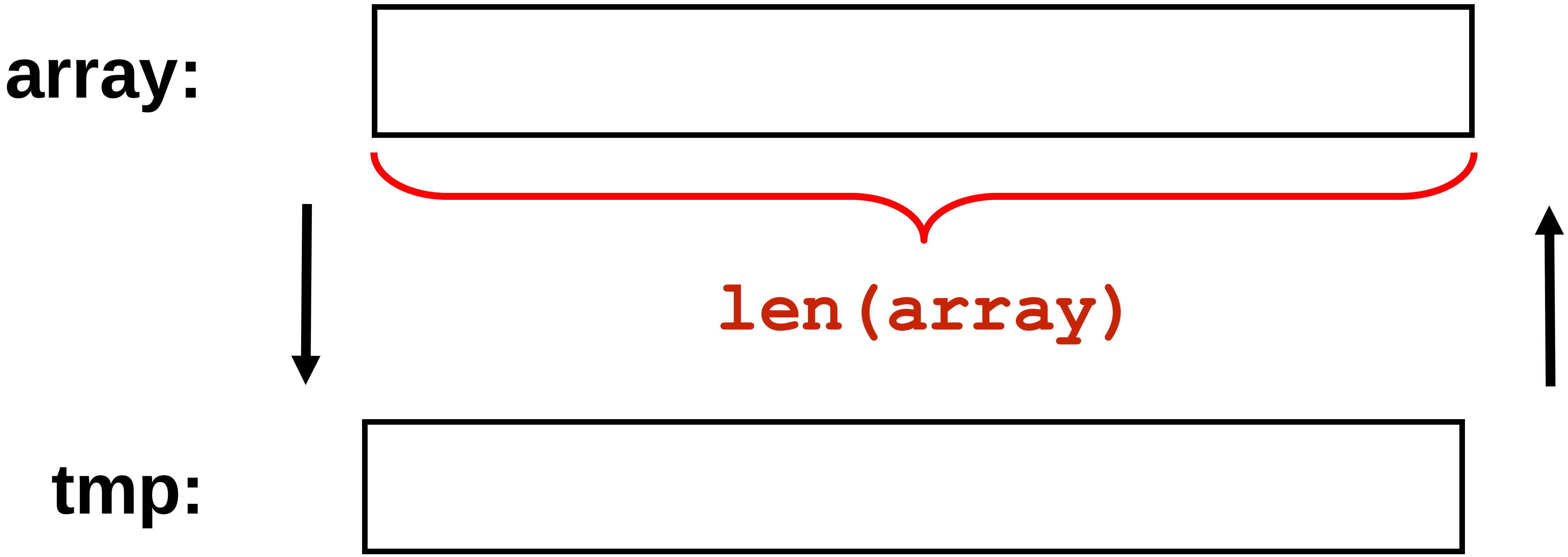
- **Merge Sort** has a simple split and a elaborate combine
- **Quick Sort** has a elaborate split and a simple combine

Merge Sort



- **Split:** In half.
- **Merge:** Take two unsorted arrays, produce a sorted one.

Merge Sort



Use a temporary array, then copy back to original array.

Merge Sort

```
def merge_sort(array):
```

Merge Sort

```python
def merge_sort(array):
    tmp = [None] * len(array)
    start = 0
    end = len(array)-1
    merge_sort_aux(array, start, end, tmp)
```

the array   start index   end index   temporary array

```python
def merge_sort_aux(array, start, end, tmp):
```

Merge Sort

```
def merge_sort_aux(array, start, end, tmp):
```

Merge Sort

```python
def merge_sort_aux(array, start, end, tmp):
    if start < end: # 2 or more still to sort
        mid = (start + end)//2

        # split into two halves
        merge_sort_aux(array, start, mid, tmp)
        merge_sort_aux(array, mid+1, end, tmp)

        # merge
        merge_arrays(array, start, mid, end, tmp)

        # copy tmp back into the original
        for i in range(start,end+1):
            array[i] = tmp[i]
```

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

Sorted Lists

start: 0, mid: 5, end: 10

tmp:

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

i=0

j=0

tmp:

3										

Merge

`L:`

3	5	15	28	30	32

`R:`

10	14	22	43	50

`i=1`

`j=0`

`tmp:`

3	5									

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

`i=2`

`j=0`

tmp:

3	5	10								

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

`i=2`

`j=1`

tmp:

3	5	10	14							

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

i=2

j=2

tmp:

3	5	10	14	15						

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

`i=3`

`j=2`

tmp:

3	5	10	14	15	22					

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

i=3

j=3

tmp:

3	5	10	14	15	22	28				

Merge

L:

3	5	15	28	30	32

i=4

R:

10	14	22	43	50

j=3

tmp:

3	5	10	14	15	22	28	30			

Merge

`L:`

3	5	15	28	30	32

`i=5`

`R:`

10	14	22	43	50

`j=3`

`tmp:`

3	5	10	14	15	22	28	30	32		

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

i=6

j=3

tmp:

3	5	10	14	15	22	28	30	32	43	

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

i=6

j=4

tmp:

3	5	10	14	15	22	28	30	32	43	50

Merge

L:

3	5	15	28	30	32

R:

10	14	22	43	50

`i=6`

`j=5`

tmp:

3	5	10	14	15	22	28	30	32	43	50

```python
def merge_arrays(array, start, mid, end, tmp):
```

```python
def merge_arrays(array, start, mid, end, tmp):
    i = start
    j = mid+1
    for k in range(start, end+1):
        if i > mid: # left finished, copy right
            tmp[k] = array[j]
            j += 1
        elif j > end: # right finished, copy left
            tmp[k] = array[i]
            i += 1
        elif array[i] <= array[j]: # array[i] is the item to copy
            tmp[k] = array[i]
            i += 1
        else:
            tmp[k] = array[j] # array[j] is the item to copy
            j += 1
```

Take remaining elements

Take the smaller from either list and advance

Invariant: After the kth iteration, the first k items in tmp are in order

Invariant(2): After the kth iteration tmp holds the smallest k items from array

If N= end+1-start, what is the time complexity of merge_arrays?

```python
def merge_arrays(array, start, mid, end, tmp):
    i = start
    j = mid+1
    for k in range(start, end+1):
        if i > mid: # left finished, copy right
            tmp[k] = array[j]
            j += 1
        elif j > end: # right finished, copy left
            tmp[k] = array[i]
            i += 1
        elif array[i] <= array[j]: # array[i] is the item to copy
            tmp[k] = array[i]
            i += 1
        else:
            tmp[k] = array[j] # array[j] is the item to copy
            j += 1
```

A) O(log N)

B) O(N)

C) O(N log N)

D) O(N^2)

Merge Sort Analysis

- **Natural:** Typically the method that you would use when sorting a pile of books, CDs cards, etc.
- Most of the work is in the **merging**
- Uses more **space** than other sorts
- Close to optimal in number of comparisons. Good for languages where comparison is expensive.

If N=end+1-start, what is the (worst-case) time complexity of merge sort?

```python
def merge_sort_aux(array, start, end, tmp):
    if start < end: # 2 or more still to sort
        mid = (start + end)//2

        # split into two halves
        merge_sort_aux(array, start, mid, tmp)
        merge_sort_aux(array, mid+1, end, tmp)

        # merge
        merge_arrays(array, start, mid, end, tmp)

        # copy tmp back into the original
        for i in range(start,end+1):
            array[i] = tmp[i]
```

A) O(log N)

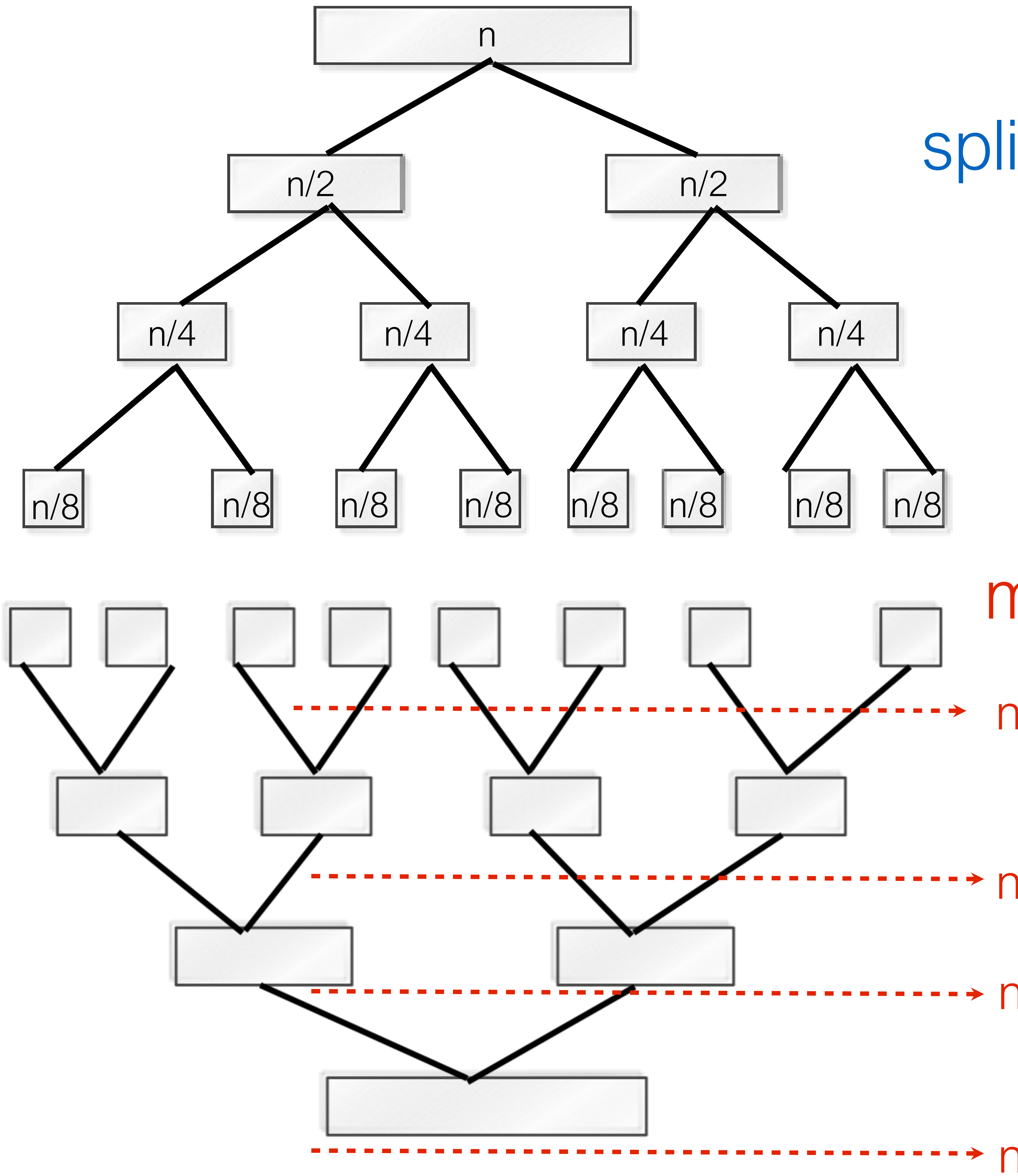
B) O(N)

C) O(N log N)

D) $O(N^2)$

Merge sort

height is O(log n)



splitting is O(1)

merging is O(n)

height is O(log n)

Total Running Time: O(n log n)

If N=end+1-start, what is the **best-case** time complexity of merge sort?

```python
def merge_sort_aux(array, start, end, tmp):
    if start < end: # 2 or more still to sort
        mid = (start + end)//2

        # split into two halves
        merge_sort_aux(array, start, mid, tmp)
        merge_sort_aux(array, mid+1, end, tmp)

        # merge
        merge_arrays(array, start, mid, end, tmp)

        # copy tmp back into the original
        for i in range(start,end+1):
            array[i] = tmp[i]
```

A) O(log N)

B) O(N)

C) O(N log N)

D) $O(N^2)$

Summary

	Best case	Worst case
Quicksort	O(?)	O(?)
Mergesort	O(n log n)	O(n log n)

Summary

Divide and Conquer and Recursive Algorithms (for sorting).

Merge Sort

- Easy: Split
- Elaborate: merge method