

Tutorial 12

Semester 1, 2019

Objectives of this tutorial

- To understand Binary Trees and Binary Search Trees.

Exercise 1 *

A binary expression tree is a binary tree used to represent algebraic expressions composed of unary and binary operators. The leaves of a binary expression tree are operands, such as constants or variable names, and the other nodes contain operations.

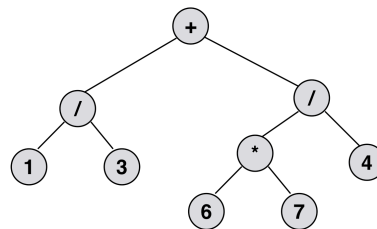


Figure 1: Expression tree

The prefix notation of an algebraic expression results from traversing the corresponding expression tree in pre-order. The infix notation results from traversing the tree in in-order; and the postfix notation (or reverse polish notation) results from traversing the tree in post-order.

Give the unambiguous mathematical expression as well as the prefix, infix and postfix notation of the expression represented by the tree above.

Exercise 2 *

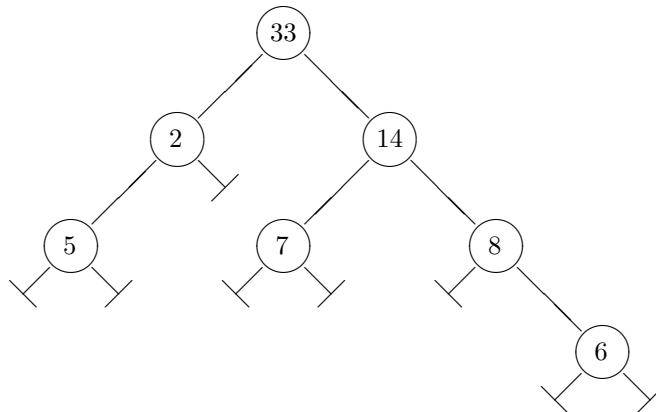
Consider a `BinaryTree` class which defines a binary tree data type implemented using linked nodes, defined as follows:

```
1 class TreeNode:
2     def __init__(self, new_item=None, left=None, right=None):
3         self.item = new_item
4         self.left = left
5         self.right = right
6
7 class BinaryTree:
8     def __init__(self):
9         self.root = None
```

Write down an attribute method for the class that returns the height of the Binary Tree.

Exercise 3 *

Add to the class above the method `sum_leaves(self)` which returns 0 if the tree is empty and, otherwise, returns the result of adding the value of every leaf in the tree. For example, for `a_tree` of the form:



the result of `a_tree.sum_leaves()` would be $5 + 7 + 6 = 18$.

Exercise 4 *

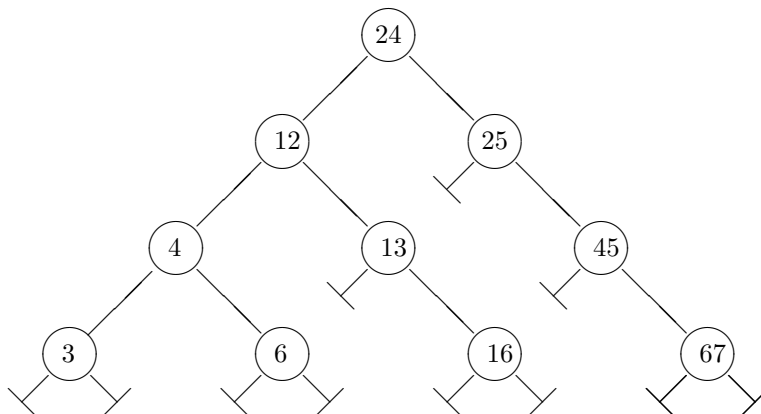
We want to extend the `BinarySearchTree` class defined in the lectures by adding a method `find_min()`. This method returns the minimum key in the tree, or `None` if the tree is empty. In doing so, it does not modify the tree. The following code shows two failed attempts at an implementation of such method:

```

1  def find_min_1(self):
2      return self.find_min_aux_1(self.root)
3
4  def find_min_aux_1(self, current):
5      if current is not None:
6          return self.find_min_aux_1(current.left)
7      else:
8          return current
9
10 def find_min_2(self):
11     if self.root is None:
12         return self.root
13     else:
14         return self.find_min_aux_2(root)
15
16 def find_min_aux_2(self, current):
17     if current.left is not None:
18         return current
19     else:
20         return self.find_min_aux_2(current.left)

```

Consider a tree `the_tree` with integer keys with the form:



1. Show the value of result after calling `result = the_tree.find_min()` for each definition above.
2. Provide a correct definition for the above method.

Exercise 5

Given a BST with numeric values and two numbers a and b , write down a function that returns a list with all items between a and b . The idea is to do this without visiting all elements, if possible.

Exercise 6

Given a BST with numeric values and a number $k > 0$, write down an algorithm that returns the k -largest element in the BST. For $k = 1$ it returns the largest element, for $k = 2$ the second largest, and so on. The idea is to do this without visiting all elements, if possible.