# GEORGIA INSTITUTE OF TECHNOLOGY
## ECE 6258: Digital Image Processing

## Problem Set #04

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**Due Dates:**

Section A: Friday, Sep. 20, 2019 @5:00PM
Sections Q, Q3, QSZ: Sunday Sep. 22, 2019 @5:00PM

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**Instructions:**

- Upload ONE **PDF** file that contains the solution, codes and figures
- Late submissions will not be accepted
- Make solutions clear & neat; using a text editor (e.g., Word and LaTex) is strongly encouraged
- In the case of using a pen and paper, poor quality scans will result in a grade of 0
- Include page numbers at the bottom of every page
- You must submit the solutions in the same order as the questions
- You must start a new page for each problem AND correctly number the problem solutions
- Although collaboration is encouraged to discuss the problems, copying is not permitted
- Turn in the solutions for the **starred*** problems ONLY
- Use the following name convention for the submitted PDF: *LastName_FirstName.pdf*
- Please copy and paste the **code** for any problem right **after the problem** and **not** as an appendix at the end of the document
- **Failure to follow any of the instructions will result in a serious grade deduction**
- Remember that Piazza is a good resource to post your questions, comments, and feedback

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1.* Given the below image $A$, we would like to calculate the image resulting after interpolating $A$ when $L = 2$. Call the resulting image $\hat{A}$.

of matrix $A$ with $L = 2$ by using . The values of the interpolated pixels should be rounded to integer.

$$A = \begin{bmatrix} 90 & 93 & 156 \\ 93 & 156 & 155 \\ 156 & 155 & 155 \end{bmatrix} \Longrightarrow \hat{A} = \begin{bmatrix} 90 & ? & 93 & ? & 156 \\ ? & ? & ? & ? & ? \\ 93 & ? & 156 & ? & 155 \\ ? & ? & ? & ? & ? \\ 156 & ? & 155 & ? & 155 \end{bmatrix}$$

(a) Calculate $\hat{A}$ using **pixel replication**.

(b) Calculate $\hat{A}$ using **bilinear interpolation**.

(c) Calculate $\hat{A}$ using using the **edge-adaptive bilinear interpolation** method.

2.* Write a Python function for zooming and shrinking an image using the bicubic interpolation. The input to your function includes: original image and the transformation parameters. Your report should include:

(a) py-file with a well-commented code.

(b) Show the result when the input image is enlarged by 2.5 along rows, shrunk by 1.7 along columns, and rotated by 27.5 degrees clockwise.

(c) Show the inverse transform of the resulting image in (b).

(d) Show the difference between the original(input) image and the image obtained in (c).

(e) Calculate PSNR of the image obtain in (c) compared to the original image.

3.* The Mean Square Error (MSE) between two $M \times N$ images, $I_R$ and $I_D$ is calculated as follows:

$$\text{MSE}(I_R, I_D) = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{J=0}^{N-1} (I_R[i,j] - I_D[i,j])^2$$

Also, the Peak Signal to Noise Ratio (PSNR) can be calculated as follows:

$$\text{PSNR}(I_R, I_D) = 20 \log_{10} \frac{\max(I_R)}{\sqrt{\text{MSE}(I_R, I_D)}} \text{ dB}$$

Suppose an image $I_R$ and its degraded version, $I_D$, are given as follows:

$$I_R = \begin{bmatrix} 1 & 3 & 5 & 8 \\ 7 & 6 & 4 & 2 \\ 2 & 1 & 6 & 3 \\ 9 & 4 & 1 & 5 \end{bmatrix} \qquad I_D = \begin{bmatrix} 2 & 3 & 6 & 9 \\ 5 & 6 & 4 & 3 \\ 2 & 2 & 6 & 4 \\ 8 & 4 & 2 & 5 \end{bmatrix}$$

Calculate **MSE**, **PSNR**, and global **SSIM** by hand.

4.* The Human Visual System (HVS) is adaptive to the structural content in a scene. Therefore, objective quality assessments should focus on the structure within the image in order to correlate with HVS. Luminance content of the scene is the product of the illumination and the reflectance whereas structure is independent from the illumination. Thus, we need to separate the effect of illumination from structural components. We can observe the effect of illumination as variation of the average local luminance and contrast. Block diagram of the SSIM is provided in Fig. 1 .

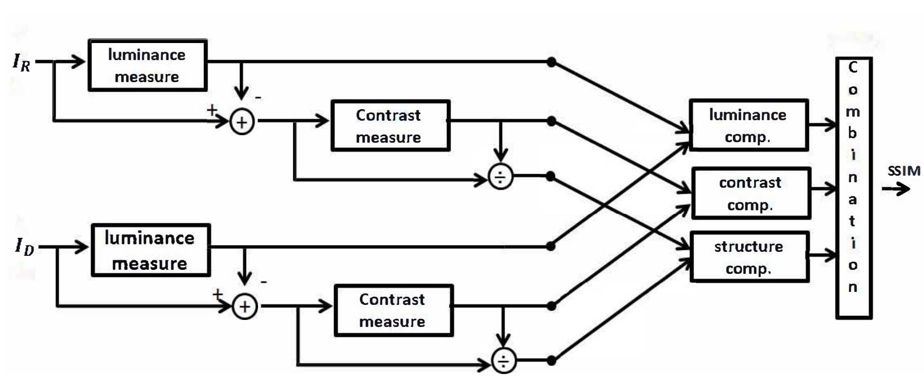



Figure 1: Block diagram of the SSIM

The purpose of this problem is to use SSIM to evaluate the quality of degraded images. You will calculate SSIM values of the same image under various degradations. Download the starter package `ch06_ssim.zip` provided with this problem set and edit the code `ch06_run_me_ssim.py` as instructed in the following step. *Hint:* You may use `dippykit.image_noise` function to create the distorted images.

(a) Unzip the file `ch06_ssim.zip` and verify its content. It should include `ch06_run_me_ssim.py` file.

(b) Read the image `barbara.png` from the `images` folder provided on the course website.

(c) Use the `image_noise` function to add Gaussian noise to the image (use default parameters).

(d) Use `image_noise` function to add Poisson noise to the image (use default parameters).

(e) Use `image_noise` function to add salt and pepper noise to the image (use default parameters).

(f) Use `image_noise` function to add speckle noise to the image (use default parameters).

(g) Use `SSIM` function to obtain MSSIM values and SSIM maps of the distorted images

(h) Report MSSIM values for each of the distorted image and show the results.

5. Suppose for a gray-scale image, you are asked to use the following fidelity metric that depicts the human visual system (HVS):

$$D = \sum_{m,n}(h[m,n] * I[m,n] - h[m,n] * f[m,n])^2$$

where $D$ is the measure of distortion between the gray-scale image $f[m,n]$ and the binary image $I[m,n]$, $*$ indicates the 2D convolution, and $h[m,n]$ is given as follows:

$$h[m,n] = (\delta[m,n] + \frac{1}{2}(\delta[m-1,n] + \delta[m+1,n])) * (\delta[m,n] + (\frac{1}{2}\delta[m,n-1] + \delta[m,n+1]))$$

(a) Calculate the Discrete Spatial Fourier Transform (DSFT) of $h[m,n]$, i.e., calculate $H(w_1, w_2)$.

(b) Is h[m,n] separable? Why? DSFT is equivalent to DTFT.

(c) Our objective here is to represent the image $f[m,n]$ by the binary image $I[m,n]$. To achieve this goal is it best for $d[m,n] = I[m,n] - f[m,n]$ to contain mostly high frequencies?OR is it best for $d[m,n] = I[m,n] - f[m,n]$ to contain mostly low frequencies? Why?

(d) If $f[m,n] = 0.5$, then determine all binary patterns $I[m,n]$ that best matches $f[m,n]$. A binary pattern is a pattern of 1's and 0's.

(e) How would you improve the distortion measure above, $D$, to better account for contrast?

6. Show that if $f[m,n]$ is real, then its 2D DFT is Hermitian Symmetric in the circular sense. That is:

$$F^*[k,l] = F[((M-k))_M, ((N-l))_N]$$

7. Suppose $F_c[k, l]$ is the DCT of an $M \times N$ image, $f[m, n]$. Determine the DCT of each of the following:

   (a) $f[M - 1 - m, n]$

   (b) $f[m, N - 1 - n]$

   (c) $(-1)^m(-1)^n f[m, n]$

8. Given an 2x2 image $f[m, n]$ and a 2-D impulse response $h[m, n]$ as follows:

$$f[m, n] = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \qquad \text{and} \qquad h[m, n] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

   Assume the origin of these arrays is at the lower left corner.

   (a) Compute the convolution between $f$ and $h$ using the direct definition of linear convolution.

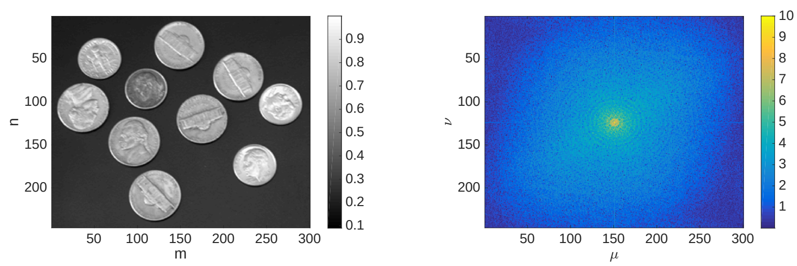
   (b) Compute the convolution between $f$ and $h$ using a 2x2 DFT.

   (c) Compute the convolution between $f$ and $h$ using a 4x4 DFT.

   (d) Discuss the results from the 2x2 DFT and the 4x4 DFT methods.

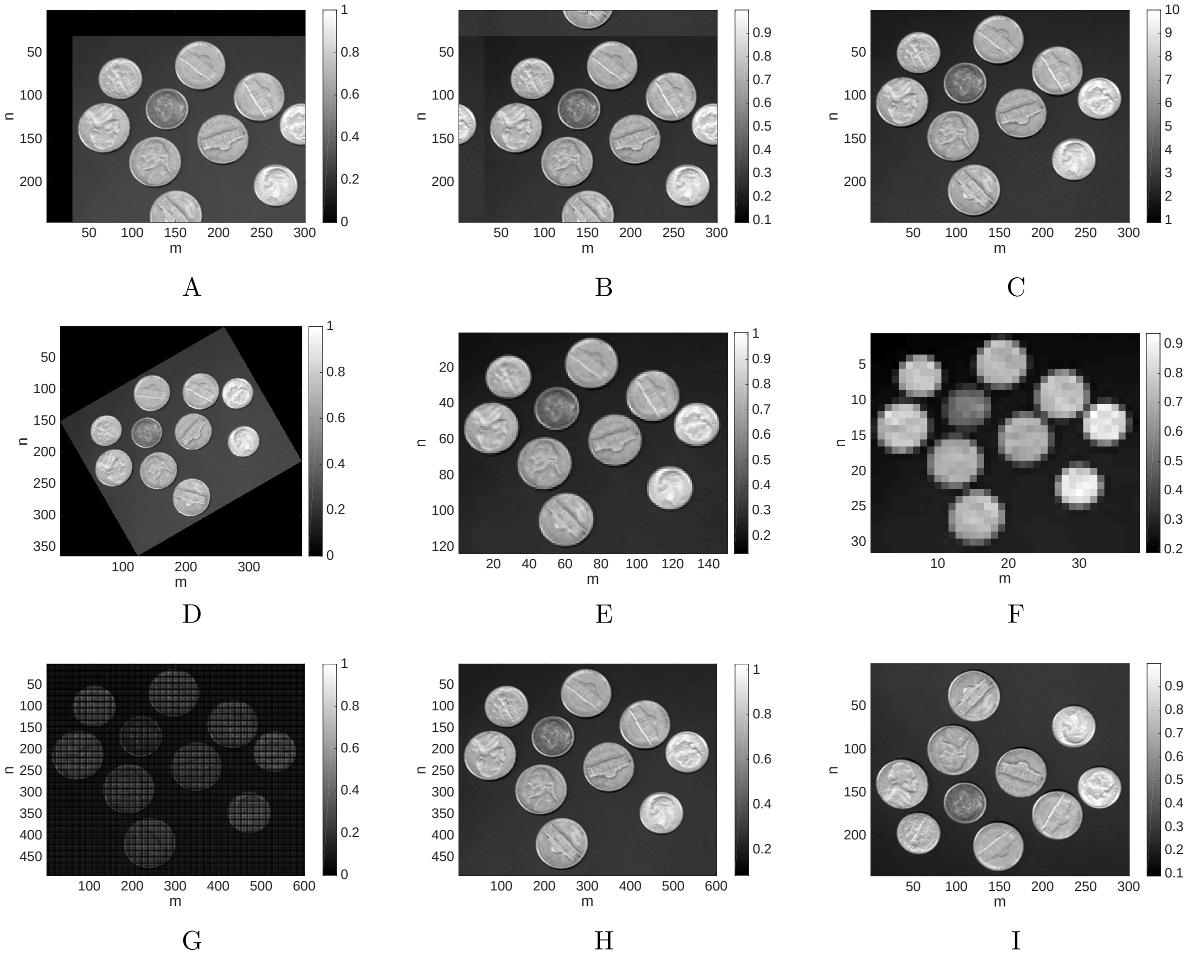
   (e) Compare the direct linear convolution method with the DFT-based method. Any pros? Any cons?

9. Given the image shown in Fig. 2(a) and its spectrum Fig. 2(b), match the given images [A-I] with the provided spectra [1-9]. Justify your choices.

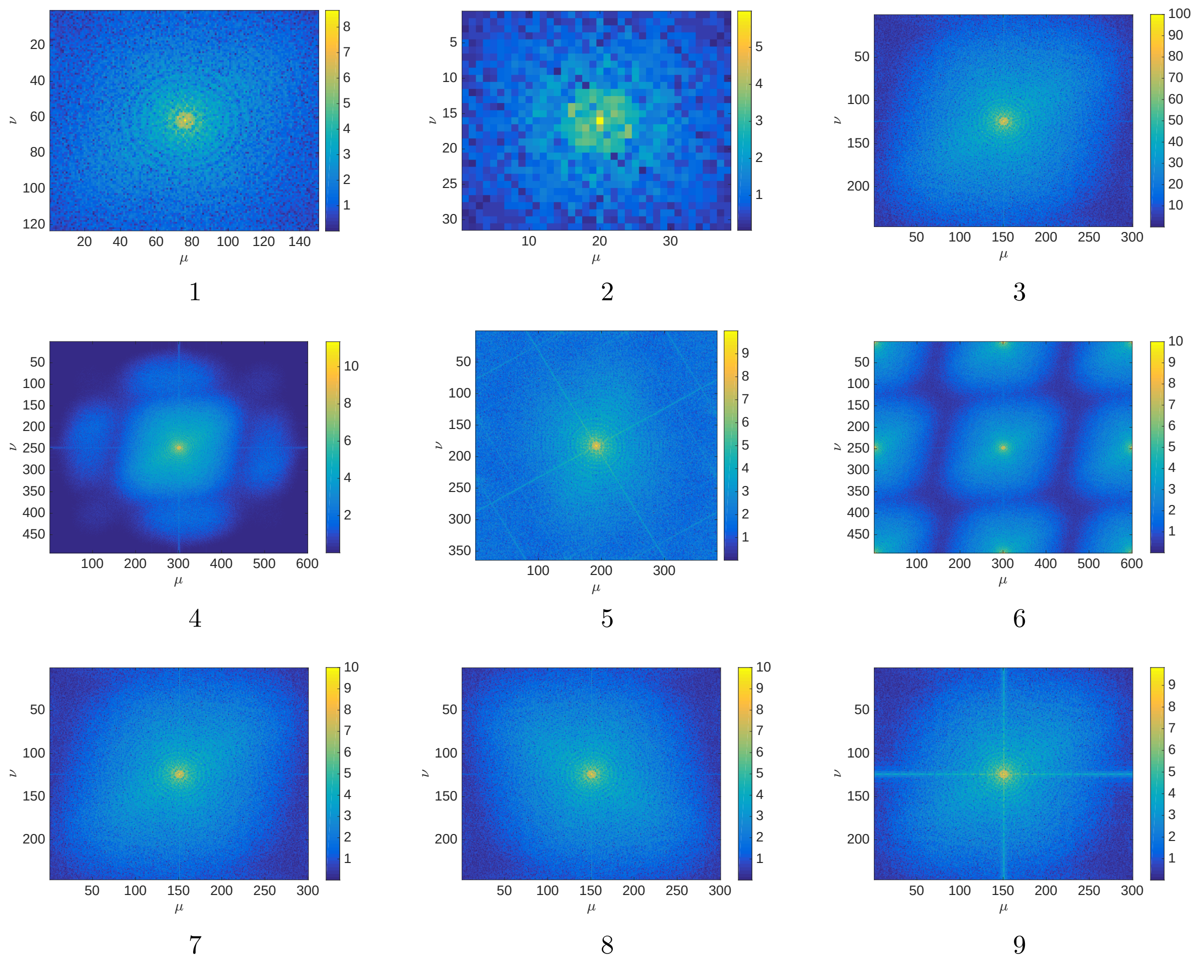



(a) (b)

Figure 2: Original image and its spectrum




Transformed Images




Spectra of Transformed Images