Wavelet-Based Image Denoising Using Interval Type-2 Fuzzy Sets and Adaptive Multilevel Soft Threshold.

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***Abstract*— Successful implementation of type-1 Fuzzy Systems (FS) in diverse application areas have been accomplished till date. Nonetheless type-1 FS are not able to handle significant amount of uncertainties present in dynamic real world applications. An improved performance against these uncertainties is achieved by type-2 FS. In this paper, a type-2 FS is proposed to remove variable gaussian noise from a number of images using multilevel two dimensional wavelet transform and multilevel soft threshold. We have displayed that the proposed type-2 FS achieve a good efficiency in removing the noise from the wavelet transformed images and also has a superior performance compared to type-1 FS.**

Keywords-Type-1 Fuzzy System; Interval Type-2 Fuzzy System; Footprint of uncertainty; Adaptive Multilevel Soft Threshold

I. INTRODUCTION

Denoising of an image is a fundamental step in the processing of images in various fields. A large number of research activities have focused on modeling noise from different sources [12]. These models provide valuable information on the global knowledge of the noise present in the image, but assumption of a specific or combination of models for noise distributions doesn't lead to good results in real life applications. Difficulty also arises to form a global denoising scheme for numerous types of images corrupted with noise.

Spatial domain filtering consists of linear and non-linear filters [12]. Several linear filters have been proposed for image denoising process, but the two most frequently used are spatial averaging filter and Wiener filter [12, 13]. Averaging filters remove sharp transitions in the image resulting in smoothening of the image. The main drawback of the averaging filter is that it results in blurring of the edges. In Weiner filtering prior information about the spectra of both the underlying image and the additive noise is necessary. This statistical approach minimizes the overall mean square error in the process of inverse filtering and noise-smoothing in an image. Often the results obtained are too blurred, which is a function of the size of the window and the assumed power spectra of the original image and the additive noise. Non Linear filters [12, 1] are efficiently used for denoising of an image mostly corrupted by salt and pepper noise. However Non-Linear filters generally suffer from high computational cost and are harder to design as the powerful mathematical tools of signal analysis cannot be used to model them.

Images are also denoised in a transform domain, e.g. Fourier Transform [16], Discrete Cosine Transform, Wavelet Transform [16], etc. The transform domain coefficients are uncorrelated with respect to each other. Since noise is present in the low energy transform coefficients, these can be filtered by using a suitable filter. Proper selection of a transform depends on the original image and the type of noise also present on the image, which can be regarded as one of the drawbacks of the method. In this paper, wavelet transform is used to denoise the image.

II. PREVIOUS WORK

The proposed work in this paper deals with noise removal by employing Interval Type-2 Fuzzy Logic Systems (IT2FLS) [15,4,5] and KM algorithm [5] to calculate multilevel soft thresholding [15] on two dimensional wavelet coefficients [6]. Jamal Saedi, Mohammad Hassan and Karim Faez worked on Wavelet based multi-channel image denoising [14] which focused at calculating a shrinkage function applied to every channel of the image. U. S. Tiwary and A. Khare had worked on Multilevel Dependent Thresholding [15] where the main focus was on medical image denoising by applying soft threshold on noise sensitive wavelet coefficients. The method was tested on multiple levels of wavelet in which DB6 was found to give best result and has been incorporated in our work. Prabhakar Puvanathasan and Kosta dinka Bizhewahad worked on speckle noise reduction based on IT2FS [2] where sigmoid based fuzzy system was applied on the wavelet coefficients to extract speckle noise from tomography images.

III. THEORY

A. Soft Threshold

Wavelet coefficients are noise fragile [15]. The main purpose of thresholding is to remove the noise present in the image. There are basically two ways of selecting a threshold; hard threshold [15] and soft threshold [15, 2], and the later has been incorporated in our work as it depends on the image statistics and thus making it adaptive i.e. image dependent thresholding. Soft threshold is processing tool where the values below the threshold are attenuated to a lower value; thereby making smooth transitions between original and deleted values and thus preserving the edges in the images. In this method we calculate the soft threshold value based on mean, standard deviation and median of wavelet coefficients according to the formula given below:

$$\text{Threshold}(T) = \frac{1}{2\text{^}(\text{j}-1)}\left(\frac{\text{Standard Deviation}}{\text{Mean}}\right)(\text{Median}) \qquad (1)$$

Where j is the level upon which the threshold is calculated. It basically tries to condense coefficients of wavelets towards zero. The values which are found to be less than the threshold are made zero and the one above it are transformed by factor 'T' in the direction of zero. The soft-thresholded value $W_{soft}$is obtained using the equation below:

$$W_{soft}=\text{sign}\ (W)\ (|W|-T)_{+} \qquad (2)$$

where sign(a) is the standard signum function defined at a; and $(B)_{+}$ is the standard step function defined at B.

B. Interval Type-2 Fuzzy Sets

Uncertainty of a rule can occur due to the following stated reasons: (a) different people may have contrasting understanding of the rule antecedent and its consequent, (b) disagreement among group of experts to obtain the rule consequents, (c) noisy data for training. Type-1 FLS are not capable of handling rule uncertainties directly and type-2 FLS are introduced which handle rule uncertainties faithfully and inexact information in a better way [9]. Type-2 fuzzy sets can better approximate the noise in data and thereby assisting in superior noise removal compared to type-1 fuzzy sets and other denoising techniques.

Type-2FLS was introduced by Zadeh, which are very useful for solving linguistic uncertainties [3,4].It has membership functions which are themselves fuzzy and thus has a range between upper membership function (UMF) [3, 5] and lower membership function (LMF) [3,5]. The enclosing region between the UMF and LMF is also termed as footprint of uncertainty [5]. A type-2 fuzzy sets have fuzzy grades of membership and its membership grade can be any subset in {0, 1} - the primary membership; and there is a secondary membership grade for every primary membership which defines the various possibilities of the primary membership [9]. In Interval type-2 FLS, the secondary membership functions take on either zero or one as their value and thereby making the process of defuzzification simple.

IV. METHODOLOGY

In this subsection, the application of interval type-2fuzzy sets for denoising of multivariate gaussian noise from an image using multiple level wavelet decomposition and soft threshold will be introduced.

Any image is vulnerable to different types of signal dependent and signal independent noise which are acquired by the imaging system. All forms of noise can be either represented as a mixture of additive, multiplicative and impulse noise [15] and are modeled using additive gaussian noise, speckle noise and salt & pepper noise. In this paper, non-overlapping blocks of the image are corrupted with additive gaussian noise with variable mean and variance. The PDF of additive gaussian noise is given as follows:

$$\text{p}(z) = \frac{exp^{\frac{-(z-\mu)^2}{2*\sigma^2}}}{\sigma*\sqrt{2*pi}} \qquad (3)$$

where z represents the image values, μ is the mean and σ is the standard deviation of the noise. Variable mean and variance of the additive gaussian noise for every block is used to demonstrate the different amount of noise that can be added by an imaging device. The original input image is converted into a black & white image before the addition of the gaussian noise if needed. The block diagram of the suggested method is given below:

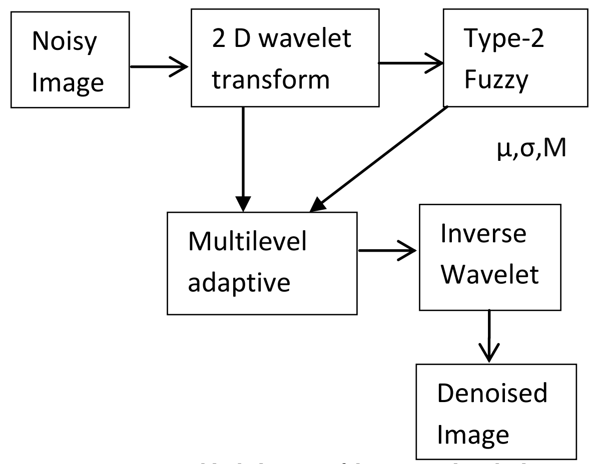


Figure 1. A block diagram of the proposed method

where µ, σ and M are the mean, standard deviation and median of the wavelet coefficient of a given level respectively.

Thenceforth, a multi-level two dimensional wavelet decomposition of the noise corrupted image is obtained. The number of levels of decomposition has been kept fixed to 4, 8, 10 and 12 respectively. The discrete wavelet transform employed in our method is Daubechies 6 wavelet [15]. Here forth, the mean, median and standard deviation of each level of wavelet coefficients is calculated. The values of mean, median and standard deviation calculated are normalized between 0.0 to 1.0, using the formula given as:

$$a_{ij}' = \frac{(aij-min(bj))*(1.0-0.0)}{(max(bj)-min(bj))} + 0.0 \qquad (4)$$

where a_{ij} is the value of mean, median and standard deviation of the j^{th} level wavelet coefficients b_j; for i=1,2 and 3 respectively, a_{ij}' is the transformed value of the same. The value of multi-level soft threshold is calculated using IT2FS with inputs a_{ij}'s.

The input dimension of every a_{ij}' is divided in three type-2 gaussian fuzzy sets as LOW, MEDIUM, and HIGH, with their mean set to 0, 0.5, and 1.0 respectively. The standard deviation of the higher and lower membership values is fixed to 0.2 and 0.15 respectively. The footprint of uncertainty [5] of every fuzzy set has the higher and lower membership function as shown in the figure below:

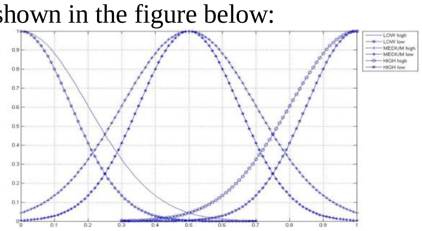


Figure 2. Interval Type-2 membership function of inputs

In the above figure, "LOW", "MEDIUM" and "HIGH" denote the interval type-2 fuzzy sets and "high" and "low" denote the higher and lower membership values of a given input a_{ij}' for each of the three fuzzy sets for a given *i*, where *i*=mean, median or standard deviation of the coefficients. Then, the process of rule inference is performed using IF-THEN Rules. In our work, we have applied fifteen or twenty seven rules to determine the value of the multilevel soft threshold. An example of a single rule used in the process of rule inference is show below:

If μ_mean_j is LOW and μ_sigma_j is LOW and μ_median_j is LOW then threshold $T=y1\_low$ (5)

where μ_mean_j, μ_sigma_j, and μ_median_j are the membership values of the mean, standard deviation and median of the j^{th}level of wavelet coefficients respectively. The firing interval of every rule antecedent is evaluated using product t-norm [5]. The consequent interval of every rule is obtained by inspecting the antecedent of the rule and the eqn. (1) used for calculation of the soft threshold. For e.g. in eqn. (5), it is inferred that the value of threshold will be low and thus *y1_low* lies in the interval {0.0 0.15}. The process of inspection is repeated to generate the output consequence interval of every rule. Thereafter Karnik-Mendel Algorithm [5] is applied to compute the crisp output *th* of the fuzzy inference engine and *th* is transformed back to the original wavelet coefficient domain; as the entire process is performed on the transformed value of mean, median and standard deviation of the wavelet coefficients eqn. (4). The inverse transformation formula used is given as follows:

$$th_j=\frac{(th-0.0)*(\max(bj)-\min(bj))}{(1.0-0.0)}+\min(bj) \qquad (6)$$

where '*th*' is the crisp output of the fuzzy inference engine, and 'th_j' is the actual calculated threshold obtained after applying inverse transformation in the j^{th} level of decomposition. Once the value of threshold is obtained, eqn. (2) is used to perform soft-thresholding on the j^{th} level wavelet coefficients. The resultant wavelet coefficients of all the levels are accumulated in the original order and the denoised image is obtained applying the inverse discrete wavelet transform.

V. RESULTS AND DISCUSSION

We observe from both the tables below, when the number of levels of wavelet decomposition was fixed at 4, the percentage of denoising has values in the range 3-8%, whereas with the increase in the number of levels to 12, the percentage of denoising has increased to 10-14%.Thereby it can be inferred that changing the number of levels affects the output image. Optimal results are obtained when the number of levels is 12; further increase showed a reduction in the percentage of denoising as the coarser details in the images are adversely affected by the soft threshold. Moreover, when fifteen numbers of rules were used, the percentage of denoising has values in the range 3-12%; but increasing the number of rules to twenty seven the percentage of denoising increases to 6-15%, resulting to better accuracy. Further increase in number of rules in the rule base might lead to a better result but it certainly increases the computational cost in the process of fuzzy inference and defuzzification.

Our proposed method was tested on three different images namely a) Lena, b) MRI scan of the brain and c) Crop Images.

- In the first part, we used fifteen rules for the process of fuzzy inference. The table below shows the various results obtained for the same:

TABLE I. RESULTS OBTAINED WHEN FIFTEEN RULES ARE USEDTABLE TYPE STYLES

	Neighbourhood	***Noisy psnr***	***Denoised psnr***	***Percentage denoised***
Lena (image a)				
No of levels=4	5	18.4791	18.8183	1.835587
	7	17.2594	17.7502	2.843668
No of levels=12	5	18.4702	21.0373	13.8986
	7	17.2698	19.4802	12.79922
MRI (image b)				
No of levels=4	5	18.6401	20.0455	7.539659
	7	17.6249	19.3753	9.931404
No of levels=12	5	18.681	20.9879	12.34891
	7	17.6608	19.6104	11.03914
Crop(image c)				
No of levels=4	5	14.8914	15.5292	4.283009
	7	13.7533	14.4916	5.368166
No of levels=12	5	14.8566	16.6461	12.04515
	7	13.7384	15.4711	12.61209

- In the second part, we used twenty seven rules for the process of fuzzy inference and the results obtained are tabulated on next page:

TABLE II. RESULTS OBTAINED WHEN TWENTY SEVEN RULES ARE USED

	Neighbourhood	***Noisy psnr***	***Denoisedpsnr***	***Percentage denoised***
Lena(image a)				
No of levels=4	5	18.4791	18.7774	1.614256
	7	17.2594	17.6979	2.540645
No of levels=12	5	18.4702	21.2458	15.02745
	7	17.2698	19.6107	13.55488
MRI(image b)				
No of levels=4	5	18.6401	20.7684	11.41786
	7	17.6249	19.508	10.68432
No of levels=12	5	18.681	21.107	12.98646
	7	17.6608	19.7372	11.75711
Crop(image c)				

No of levels=4	5	14.8914	15.5814	4.633547
	7	13.7533	13.9586	6.997708
No of levels=12	5	14.8566	16.8793	13.61482
	7	13.7384	15.6381	13.82767

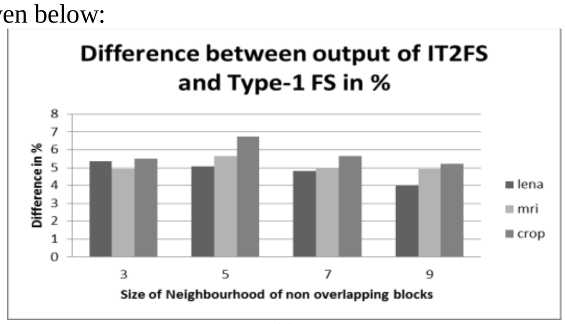
Similar experiments were performed using type-1 fuzzy sets on the images and the various parameters were kept same as that of type-2 fuzzy sets. The results obtained using interval type-2 fuzzy sets are compared with the results obtained using type-1 fuzzy sets and a significant improvement is observed in the case of the former. Moreover, when the number of rules is increased from fifteen to twenty seven, a better performance is observed for the latter for interval type-2 fuzzy sets over type-1 fuzzy sets, but further increase in the number of rules used may or may not lead to increase in better performance.

The outputs obtained are shown below:

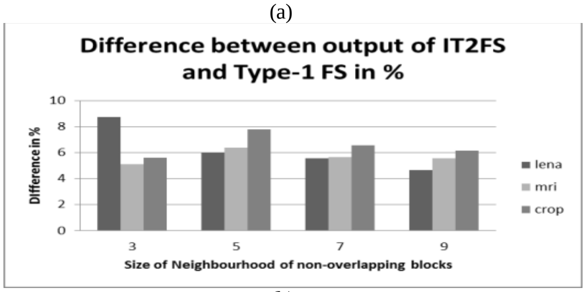
(a) (b) (c)

Figure 3. Outputs obtained when twenty seven rules were used in the fuzzy rule base (a) Lena (b) MRI scan of the brain (c) Crop

These observations areensured from the two graphs given below:



(a)



(b)

Figure 4. Difference between output of Interval Type-2 Fuzzy sets and Type-1 Fuzzy sets when (a) fifteen and (b) twenty seven numbers of rules are used and the level of wavelet decomposition is fixed at 10

It is noticeable from the above two graph plots that type-2 fuzzy sets perform better at handling rule uncertainties compared to type-1 fuzzy sets. Therefore type-2 fuzzy sets can be better used to model linguistic uncertainties as different words can have different meanings for different people.

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REFERENCES

[1] H. Hwang and R. A. Haddad, Adaptive Median Filters: New Algorithms and Results, IEEE Transactions on image processing vol 4. P.no 499-502, Apr 1995.

[2] PrabakarPuvanathasan and KostadinkaBizheva, Speckle Noise Reduction Algorithm for Optical Coherence Tomography based on Interval Type II Fuzzy Set, Opt. Express 15, 15747-15758 (2007).

[3] Jerry M. Mendel, Robert I. John and Feilong Liu, Interval Type-2 Fuzzy Logic Systems Made simple, IEEE Transaction on Fuzzy Systems, Vol. 14, No. 6, December 2006.

[4] Hani Hagras and Christian Wagner, Introduction to Interval Type-2 Fuzzy Logic Controllers – Towards Better Uncertainty Handling in Real World Applications, IEEE SMC – eNewsletter, Issue #27, June 2009.

[5] Dongrui Wu, A Brief Tutorial on Interval Type-2 Fuzzy Sets and Systems.

[6] EvelyneVanraes, Maarten Jansen, AdhemarBultheel, Stabilised wavelet transforms for non-equispaced data smoothing, preprint Signal Processing stabwt - July 2000.

[7] Hani Hagras, Typ-2 FLCs: A New Generation of Fuzzy Controllers, IEEE Computational Intelligence Magazine, February 2007.

[8] CédricVonesch, Thierry Blu and Michael Unser, Generalized Daubechies Wavelet Families, IEEE Transactions on Signal Processing, Vol. 55, No.9, September 2007.

[9] Qilian Liang and Jerry M. Mendel, Interval Type-2 Fuzzy Logic Systems: Theory and Design, IEEE Transactions on Fuzzy Systems, Vol. 8, No. 5, October 2000.

[10] Jose´ V. Manjo´n , Jose´ Carbonell-Caballero a, Juan J. Lull a, Gracia´n Garcı´a-Martı´,Luı´s Martı´-Bonmatı´, Montserrat Robles, MRI denoising using Non-Local Means, *Medical Image Analysis 12* (2008) 514–523.

[11] Hari Om, MantoshBiswas, An Improved Image Denoising Method Based on Wavelet Thresholding, SciRes, Journal of Signal and Information Processing, 2012, 3, 109-116.

[12] PriyankaKamboj and Versha Rain, A Brief Study Of Various Noise Model And Filtering Techniques, Journal of Global Research in Computer Science, Vol. 4, No. 4,April 2013.

[13] Suresh Kumar, Papendra Kumar, Manoj Gupta, Ashok Kumar Nagawat, Performance Comparison of Median and Wiener Filter in Image De-noising" ,International Journal of Computer Applications (0975 – 8887) Volume 12,No.4, November 2010.

[14] Jamal Saedi, Mohammad Hassan Moradi and KarimFaez,A new wavelet-based fuzzy single and multi-channel image denoising, Elsevier -- journal, Image and Vision Computing 28(2010) 1611-1623.

[15] Ashish Khare and Uma Shanker Tiwary, Soft-thresholding for Denoising of Medical Images-A Multiresolution Approach, International Journal of Wavelets, Multiresolution and Information Processing Vol 3,N0. 4 (2005) 477-496.

[16] S.Arivazhagan, S.Deivalakshmi, K.Kannan, B.N.Gajbhiye, C.Muralidhar, SijoN.Lukose, M.P.Subramanian, Performance Analysis of Image Denoising System for Different levels of Wavelet Decomposition, International Journal Of Imaging Science And Engineering (Ijise), Vol.1,No.3, July 2007.