Application of Fuzzy Sets to Calculate the Value of Soft Threshold for Image Denoising

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Abstract— **Fuzzy Logic is a system that nearly represents human activities of thinking and linguistics. Fuzzy logic is very useful in handling uncertainty and is used in various other applications. In this paper, fuzzy logic is used to handle wavelet transformed image statistics which is embedded with Gaussian noise spread variably across the image. Adaptive Multilevel Soft Thresholding of the noisy image was done, which removed noise from almost all types of images. We also display that Fuzzy Logic System (FLS) attains good efficiency. The remodelling of wavelet coefficient proved effective a contemporary in image denoising horizon.**

Keywords— Type-1 Fuzzy Systems, Sugeno Fuzzy Inference Engine, Adaptive Multilevel Soft Threshold, Wiener Filter, Lee Filter

I. INTRODUCTION

Various types of noise contaminate images in acquisition, storage and transmission. Noise in an image is random variation of brightness or color information. Various sources of noise in an image are: speedy transients, such as faulty switching happening during imaging, malfunctioning of camera sensor pixels, defective storage memory locations, movement of sensors during exposure, errors in analog to digital conversion, use of a noisy channel for transmission, incorrect synchronization in digital recording, erroneous bits transition during transmission of an image, etc. All these sources give rise to an unwanted signal in the image, which interferes with the original image and degrades its quality; resulting in sharp transitions in intensity levels of an image and thus it must be removed for proper retrieval of the original image.

Image Denoising is one of the most important parts of processing images in different fields. Multiple research activities are focusing on noise modeling from various sources [12]. These models give valuable insight about global knowledge of noise that exists in the image. Assumption of a model for noise doesn't pave path to good results as far as real world applications are concerned. Complications also occur when a global denoising scheme is constructed for various types' images which are corrupted with noise.

Various denoising methods have been proposed over the years till date. The applications of these methods generally depend on the type of image and noise present in the image. Primarily these methods are classified into two categories: spatial domain filtering and transform-based filtering. Spatial domain filtering operations are performed on the image directly whereas transform domain filtering is done on the resultant obtained by applying a transform on the image.

Spatial domain filtering is basically of two type's linear and non-linear filters [12]. For the purpose of image denoising we have option of various linear filters, but spatial averaging filter and wiener filter are most widely used. Averaging filter is implemented as a sliding window where the center pixel value is replaced by the average of the pixels enclosed in the neighborhood around the center pixel. Averaging filters correspond to low pass filters in the frequency domain. They remove sharp transitions in the image resulting in smoothing of the image. Applying this filter leads to blurring of edges which is not desirable and thus it turns out to be a drawback of this filter.

Besides the averaging spatial filter, Wiener filter [12, 13] is also used for denoising of an image. Wiener filter is the minimum mean square error optimal filter used for smoothing of images degraded by additive noise and blurring. Correlation Information between signal and noise is exploited to increase SNR and reduce distortion. In this method of filtering, prior knowledge about the spectra of the additive noise and the underlying image, both are required. It minimizes the noise-smoothing and mean square error. The filter coefficients are given as:

$$W(f1,f2) = \frac{H*(f1,f2)Sxx(f1,f2)}{|H(f1,f2)|2\ Sxx(f1,f2)+Snn(f1,f2)} \quad (1)$$

Where H(f1,f2) is the blurring filter and Sxx(f1,f2) and Snn(f1,f2) are the power spectra of the original image and noise respectively. In wiener filter, inverse filtering technique is used for the process of de-convolution to obtain the restored image from a degraded version of it, as well as compression operation is performed to remove the noise. Often blurry results are obtained.

Another technique for spatial removal of noise in an image is Non Local means filtering [1, 12]. This technique assumes the concept of self-similarity in an image, which states that a certain portion of an image may be similar to various globally located regions in the same image. Pixels with similar neighborhood s are used to obtain the filtered value of a noisy pixel. In a given search space around a center pixel, similar pixels are found whose neighborhood are similar to the neighborhood of the center pixel based on a certain criterion. When the matches are obtained, each matched pixel is given a weight based on its similarity to the center pixel and the center pixel is replaced by the weighted summation of all its matched pixel. It is successful in removing the low frequency noise present considerably, but the process is very slow and depends solely on the size of search space, size of neighborhood to be considered around each pixel, and matching criterion. Switched Non-Local Means is a faster variant of Non-Local Means method, designed specifically for removal of high density salt and pepper noise, but is still computationally complex.

Image corrupted by salt and pepper noise is mostly denoised by Non Linear filters. Their functioning depends on the ordering of the pixels in a given neighborhood around a center pixel and the ranking result is used to replace the value of the center pix. Several Non-Linear filters such as max, min, median etc. are used for image denoising as they provide excellent noise removal for a certain category of noise with considerable less blurring. The main drawback of them are their High Computational cost and design complexity also no mathematical tool of signal analysis can be used for modeling.

Images are also denoised in a transform domain, where a well-known transform e.g. Fourier Transform[16], Discrete Cosine Transform, Wavelet Transform, Ridgelet Transform etc., is applied over the noisy image to represent the noisy image in the applied transform domain. The transform domain coefficients are uncorrelated with respect to each other and follow the compactness property of a transform which states that the energy of the signal is mainly concentrated in few of the transform coefficients. These coefficients carry the necessary information for the proper restoration of the image and the other coefficients can be discarded. The coefficients with the bulk of energy concentrated within them represent the coarse details of the image, and the ones having less energy associated with them, represent the rapid transitions in the intensity levels of the image. As random noise results in sharp transitions in an image, it can be filtered out be rejecting the low energy containing transform coefficients by using a suitable filter. Denoised image is then reconstructed using only the high energy coefficients left after filtering processing using the inverse of the applied. Selection of a proper transform depends on image type and the noise present in the image, which is a drawback of the method.

In this paper Wavelet Transform is used to denoise the image which is explained in the subsequent section.

II. Previous Work

Various types of noise contaminate images in acquisition, storage and transmission. The proposed work deals with noise removal by applying Fuzzy Sets on 2 Dimensional Wavelet coefficients followed by Adaptive Multilevel Soft Thresholding.

Jamal, Hassan and Karim worked on Wavelet based multi-channel image denoising [1] which focused at calculating wavelet coefficients at different levels. A shrinkage function was constructed and applied to every channel of the image and was used to remove noise from wavelet coefficients, thereby improving coefficients of wavelet. The noise free image was obtained by applying the inverse wavelet transform.

U. S. Tiwary and A. Khare had worked on Multi-level Dependent Thresholding [2] where the main focus was on medical image denoising by applying soft threshold on noise sensitive wavelet coefficients. The method was tested on multiple levels of wavelet in which DB6 was found to give best result and has been incorporated in our work.

Prabhakar and Kostadinka had worked on Speckle Noise Reduction based on IT2FS [3] where sigmoid based fuzzy system was applied on the wavelet coefficients to extract Speckle noise from Tomography images. The method showed an advantage of applying Fuzzy sets over Wiener [16] and Lee [19] filter.

III. Theory

A. Wavelet Transform

Wavelet transform is one of the most powerful tools for image denoising, Daubechies wavelets being one of the most popular. When a scaling function is combined with wavelet function in time domain, it produces a wavelet [15]. Two dimensional wavelet is composed of four frequencies LL, LH, HL and HH, where H represents high level frequency coefficients and L represents low level frequency coefficients. It is a repetitive process and is continued till the required level of decomposition is achieved is achieved. We select the wavelet based on the amount of smoothing of signal we require. This smoothing is termed as vanishing moment [15]. DBA represents orthogonal wavelets of Daubechies, with A representing the vanishing moment which is equivalent to half of the coefficients number. DB6 gives good result in the case when is used in the purpose of removing noise from still images [15]. The two dimensional wavelet decomposition structures are shown in the figure below:

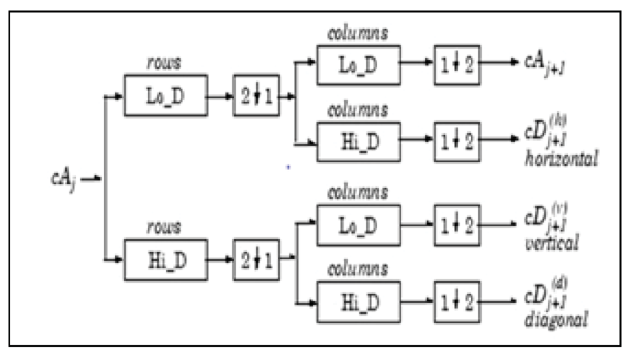

Fig. 1. Two dimensional wavelet decomposition structures

where
Lo_D is the decomposition low-pass filter
Hi_D is the decomposition high-pass filter,
cA_j and $cA_{j+1}$are the approximation coefficients at level " j" and" j+1"
$cD_{j+1}^{(h)}$ are the horizontal detail coefficients at level "j+1"
$cD_{j+1}^{(v)}$ are the vertical detail coefficients at level "j+1"
$cD_{j+1}^{(d)}$ are the diagonal detail coefficients at level "j+1"

$\boxed{2 \downarrow 1}$ Represents down sampling of columns by keeping the even indexed columns.

$\boxed{1 \downarrow 2}$ Represents down sampling of rows by keeping the even indexed rows.

Rows $\boxed{X}$ Represents convolution of the rows of the entry with filter X.

Column $\boxed{X}$ Represents convolution of the columns of the entry with filter X.

B. Soft Threshold

Coefficients of Wavelet are fragile [15] to noise and noise is generally present at higher level of coefficients. The role of thresholding is to remove noise. Out of two ways of thresholding namely, Hard Thresholding [15] and Soft thresholding [2, 15], later is used for its various advantages over the earlier. In Soft Thresholding we attenuate values below the threshold towards 0 and above threshold are brought close to it, thus making smooth transitions deleted and original value. This also preserved edges. Statistical features like Mean, Median and Standard Deviation of image is used to calculate this value, formula is given below:

$$\text{Threshold}(T) = \frac{1}{2^{\wedge}(\text{j}-1)}\left(\frac{\text{Standard Deviation}}{\text{Mean}}\right)(\text{Median}) \quad (2)$$

Where j is the level upon which the threshold is to be calculated. It tries to condense wavelets coefficients towards zero. The equation below shows calculation of soft-threshold W_{soft} :

$$W_{soft} = \text{sign}\,(W)\,(|W|\text{-}T)_+ \quad (3)$$

$$\text{Where sign}\,(a) = \begin{cases} +1\ if\ a > 0 \\ 0\ if\ a = 0 \\ -1\ if\ a < 0 \end{cases} \quad (4)$$

$$\text{And} \qquad (B)_+ = \begin{cases} B \quad if\ B > 0 \\ 0 \quad otherwise \end{cases} \quad (5)$$

C. Fuzzy Sets

Fuzzy sets were introduced by Zadeh as an extension of sets. It is used in many areas like decision making, linguistics, etc. In set theory, membership value of an element is binary i.e. it is either part of a set or it is not. Fuzzy allows us to map this membership value between interval [0, 1]. This range is generally called "Grade of Membership". This property of fuzzy allows us to handle uncertainty in a better way.

It is represented as follows:

$$\text{A}= \bigcup_i \mu i/xi \qquad \text{i=1, 2, 3,….n} \quad (6)$$

where μ represents the membership value of member 'x'. First we fuzzify our image parameters based on the above membership value. We now apply various method to calculate rule antecedents like AND, OR and NOT. Then we apply one of various T norms available like product or minimum for consequents i.e. output. We now defuzzify this to get a crisp value. We use Sugeno method for defuzzification. This method is applied on all the three image statistics namely mean, standard deviation and median.

IV. Methodology

Different signal dependent and signal independent noise can corrupt an image that is obtained by the imaging system. Any noise form can be constituted as a combination of additive, multiplicative and impulse noise [15] and can be modeled using additive Gaussian, speckle and salt and pepper noise. In our process, we have corrupted different non-overlapping blocks of images with additive Gaussian noise of variable mean and variance. Gaussian noise distribution is equivalent to normal distribution of noise, leading to addition of noise present in real world. The PDF of additive Gaussian noise is given as follows:

$$\text{p}(z) = \frac{exp^{\frac{-(z-\mu)^2}{2*\sigma^2}}}{\sigma*\sqrt{2*pi}} \quad (7)$$

Where z, μ and σ are the image values, their mean and variance respectively. We have incorporated variable mean and variance to signify the variable amounts of noise that can be added by imaging device to generate the Noisy Image as shown in Fig. 2. Gray scale image is used for the addition of the noise. The block diagram of the process is shown below:

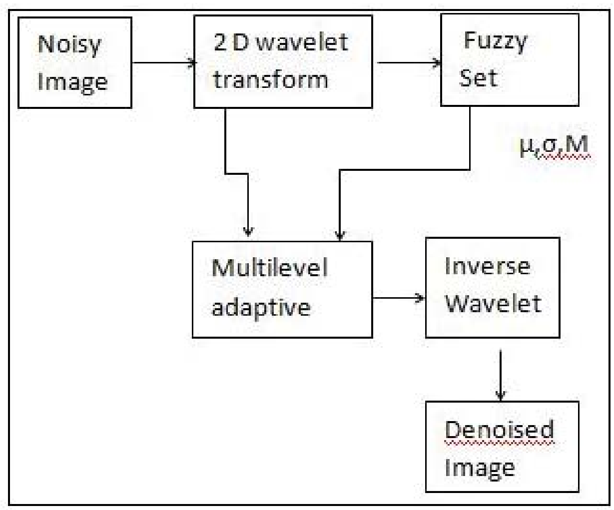

Fig. 2. A block diagram of the proposed method

Thereafter, the corrupted image was made to undergo a multilevel two dimensional wavelet decomposition where the number of decompositions levels was fixed at 4,8,10, and 12, as done in 2 D wavelet block. Daubechies Wavelet 6 was used in our method. After this, the mean, median and standard deviation of the wavelet coefficients at each level are calculated and are normalized to values 0.0 to 1.0 using the formula given below:

$$a_{ij}' = \frac{(aij - min\,(bj))*(1.0-0.0)}{(max(bj) - min(bj))} + 0.0 \quad (8)$$

In the above equation, a_{ij} is the value of mean, median and standard deviation of the j^{th} level wavelet coefficients for i=1, 2, and 3 respectively, the normalized value is represented as a_{ij}'. This normalized value is given as input to the fuzzifier which calculates the value of multi-level soft threshold using Type-1 Fuzzy sets.

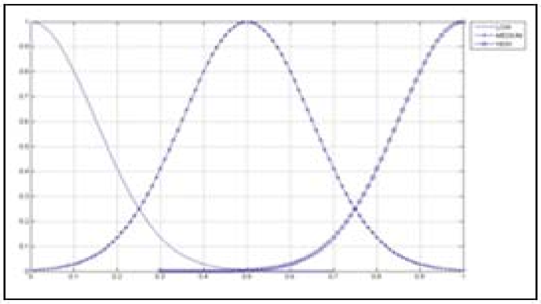

Fig. 3. Membership Functions of Type-1 Fuzzy sets

In this experiment, the input dimension of every aij' is divided into three type-1 Gaussian fuzzy sets denoted as LOW, MEDIUM and HIGH with their mean set to 0.0,0.5 and 1.0 respectively as shown above:

Gaussian membership function is incorporated as it models continuously differentiable curves generating smooth and robust output. In the above figure, “LOW”, “MEDIUM” and “HIGH” refer to the type-1 fuzzy sets for a given input aij' for each of the three fuzzy sets for a given i, where i=mean, median or standard deviation of the coefficients. Thereafter IF-THEN RULES are used for fuzzy rule inference. Twenty seven rules are used to determine for the value of multilevel soft threshold. An example of the rule used is given below:

If µ_meanj is LOW and µ_sigmaj is LOW and µ_medianj is LOW then threshold T=y1_low (9)

Where the membership values of the mean, standard deviation and median of the jth level of wavelet coefficients µ_meanj, µ_sigmaj, and µ_medianj are respectively. Since the “AND” operator is used for joining the rule antecedents, the firing interval for every rule is determined using product t-norm. In order to evaluate the rule consequent, eqn. (2) was used to calculate the value of soft threshold, after the examination of the rule antecedent. The consequent membership function of every rule is determined using this method of examination. As an example, in eqn. (9), the value of soft threshold can be deduced to be low and thus a low value will be assigned to it in the defuzzification process.

For the calculation of the crisp value of soft-threshold using type-1 fuzzy sets, defuzzification process was performed using a Sugeno defuzzifier in the form of given below:

th =0.2*mean1+0.5*sd1+0.3*med1+.3 (10)

where ‘th’ refers to the output of the defuzzification process, “mean1”,”sd1”,”med1” refer the membership value of the normalized mean, standard deviation and median of the wavelet coefficients after the rule antecedents were evaluated using product t-norm. The values of 0.2, 0.5 and 0.3 were selected based on trial and error method and it was found that these values gave the lowest error (or high PSNR).

Since the entire process was performed on the transformed value of mean, median and standard deviation of the wavelet coefficients, the value of threshold ‘th’ obtained is again transformed back to the original wavelet coefficient domain using the inverse transformation as given below:

$$th_j = \frac{(th-0.0)*(\max(bj)-\min(bj))}{(1.0-0.0)} + \min(bj) \qquad (11)$$

Where, ‘th_j’ is the value of the actual calculated threshold obtained after applying inverse transform, and ‘th’ is the crisp value of the fuzzy inference engine and ‘j’ refers to the level of wavelet decomposition. After the calculation of the soft-threshold, eqn. (3) is used to perform soft-thresholding on the j^{th} level of wavelet coefficients. The resultant wavelet coefficients of all the levels are accumulated in the original order and the denoised image is obtained by applying the inverse discrete wavelet transform.

V. RESULTS

The three input images taken for our proposed method are shown below:

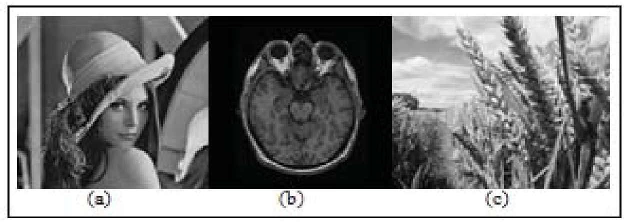

Fig. 4. The original images used in our work. (a) Lena (b) MRI scan of the brain(c)Crop

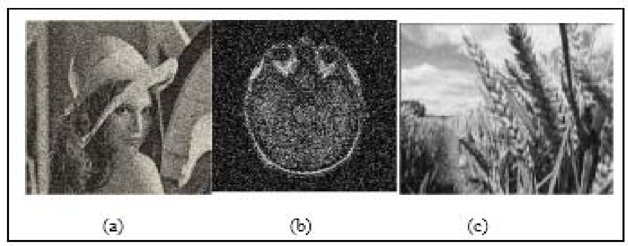

Fig. 5. Images corrupted with variable Gaussian noise with 5x5 non-overlapping blocks (a) Lena (b) MRI scan of the brain (c) Crop

The outputs obtained after the process of thresholding using the calculated soft threshold and type-1 fuzzy sets using twenty seven rules are given below:

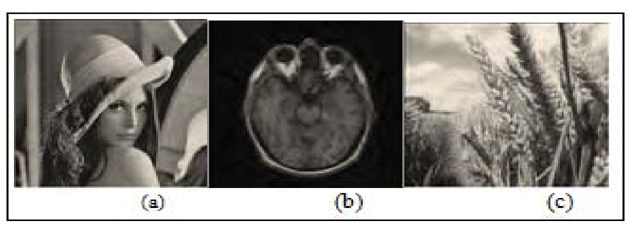

Fig. 6. Outputs obtained using fuzzy rule base (a) Lena (b) MRI scan of the brain (c) Crop

The various values of the Signal to Noise Ratio are shown in the table below:

TABLE I. RESULTS OBTAINED WHEN THE LEVEL OF WAVELET DECOMPOSITION WAS FIXED TO 8

	Size of Neighborhood	*Noisy PSNR*	*Denoised PSNR*	*Percentage Denoised*
Lena (image a)	3	20.49	21.65	5.67
	5	18.46	20.21	9.47
	7	17.24	18.45	7.01
	9	16.43	17.65	7.42
MRI (image b)	3	20.62	21.68	5.14
	5	18.60	19.77	6.29
	7	17.62	18.61	5.59
	9	16.89	17.83	5.53
Crop (image c)	3	17.26	18.42	6.72
	5	14.86	15.78	6.15
	7	13.74	14.69	6.89
	9	13.05	13.93	6.71

TABLE II. RESULTS OBTAINED WHEN THE LEVEL OF WAVELET DECOMPOSITION WAS FIXED TO 12

	Size of Neighborhood	*Noisy PSNR*	*Denoised PSNR*	*Percentage Denoised*
Lena (image a)	3	20.47	22.84	10.37
	5	18.47	20.72	12.17
	7	17.26	18.92	9.96
	9	16.42	17.97	9.47
MRI (image b)	3	20.65	23.12	11.98
	5	18.58	20.42	9.89
	7	17.66	19.21	8.74
	9	16.95	18.36	8.32
Crop (image c)	3	17.27	19.45	12.66
	5	14.88	16.51	10.96
	7	13.74	15.37	11.85
	9	13.06	14.52	11.19

It is observed from both the tables above that the percentage of denoising has values in the range 5-9% when the number of levels of wavelet decomposition was fixed at 8, which significantly increased to 8-14% when the number of levels was fixed at 12. Thus we can infer that changing the number of levels affects the output image. Optimal results are obtained when the number of levels is 12; but a further increase in the number of levels of wavelet decomposition showed a reduction in the percentage of denoising as the coarser details in the images are adversely affected by the soft threshold.

VI. CONCLUSION

It is observed that type-1 fuzzy sets can be used significantly to denoise an image corrupted with a variable Gaussian noise. We domains such as image classification, image clustering, image de-blurring etc.

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