Sets

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Abstract

This article contains Z Notation type declarations for concepts related to sets. It has been type checked by *f*UZZ.

1 Introduction

Typed set theory forms the mathematical foundation of Z Notation and many concepts relating to set theory are defined by its built-in mathematical tool-kit. The articles augments the tool-kit with some additional concepts.

2 Arbitrary Sets

2.1 X `\setX`, Y `\setY`, and Z `\setZ`

Let X, Y, and Z denote arbitrary sets. These will be used throughout in the statement of theorems, remarks, and examples that are parameterized by arbitrary sets.

$$[\mathsf{X}, \mathsf{Y}, \mathsf{Z}]$$

3 Families

3.1 $\mathcal{F}$ `\family`

Let X be a set. A *family* of subsets of X is a set of subsets of X. Let $\mathcal{F} X$ denote the set of all families of subsets of X.

$$\mathcal{F} X == \mathbb{P}(\mathbb{P}\, X)$$

4 Functions

4.1 const `\const`

Let X and Y be sets and let $c \in Y$ be some given point. The mapping that sends every point of X to c is called the *constant mapping* defined by c. Let $\mathrm{const}(c)$ denote the constant mapping.

$$
\begin{array}{l}
[X, Y] \\
\hline
\mathrm{const} : Y \longrightarrow (X \longrightarrow Y) \\
\hline
\forall\, c : Y \bullet \\
\qquad \mathrm{const}(c) = (\lambda\, x : X \bullet c)
\end{array}
$$

4.2 $|_{\mathsf{fun}}$ `\restrictU`

Let $f : X \longrightarrow Y$ and let $U \subseteq X$. Let $f\, |_{\mathsf{fun}}\, U$ denote the restriction of f to U.

$$
\begin{array}{l}
[X, Y] \\
\hline
\_\, |_{\mathsf{fun}}\, \_ : (X \longrightarrow Y) \times \mathbb{P}\, X \longrightarrow (X \rightarrow\!\!\!\!\!+ Y) \\
\hline
\forall\, f : X \longrightarrow Y;\ U : \mathbb{P}\, X \bullet \\
\qquad f\, |_{\mathsf{fun}}\, U = U \lhd f
\end{array}
$$