COCAAG, WINTER 2025: WEEK 11 QUESTIONS

Problem 1: Packages in M2 Write a small package on your own (or in small groups). The package should have a function that creates an ideal in a ring (e.g. choose a set of ideals, indexed by a parameter, and have it take a ring in as parameter, and then create the ideal in that ring) Please include a test and documentation for your package and your ideal creation function.

Problem 2: Examples for primary decomposition Find the minimal primes and a primary decomposition for one of the ideals suggested in the file "week11.m2".

Problem 3: Splitting principles Which of the following hold? (prove or give a counterexample). For each that does hold, if one chooses a (irredundant) PD of each ideal on the right hand side, does combining them give a (irredundant) PD of the left hand side? We use the notation e.g.: $(I, f) := I + \langle f \rangle$

(a) $I = (I : f^\infty) \cap (I, f^\ell)$, if $I : f^\infty = f^\ell$. (We did this one already!)
(b) If $fg \in I$, and $\langle f, g \rangle = R$, then $I = (I, f) \cap (I, g)$. (We did this one too!)
(c) If $fg \in I$, then $I = (I, f) \cap (I, g)$.
(d) If $fg \in I$, then $\sqrt{I} = \sqrt{I, f} \cap \sqrt{I, g}$.
(e) if I, J are ideals in R, then

$$I = (I : J) \cap (I + J)$$

(f) if I, J are ideals in R, then

$$\sqrt{I} = \sqrt{I : J} \cap \sqrt{I + J}$$

(g) if I, J are ideals in R, then

$$I = (I : J) \cap (I : (I : J))$$

(h) if I, J are ideals in R, then

$$\sqrt{I} = \sqrt{I : J} \cap \sqrt{I : (I : J)}$$

Problem 4: Free resolutions over localizations Recall that a free resolution over a local ring is called minimal if every entry in every matrix is in the maximal ideal. In the example below, find a minimal free resolution of R_P/IR_P, where

(a) $P = \langle a, b, c \rangle$
(b) $P = \langle a, b, d \rangle$
(c) $P = \langle a, b, c, d \rangle$.

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R = QQ[a..d]
I = monomialCurveIdeal(R, {1,3,4}) -- rational quartic
F = res I
F.dd
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