Lab 5. Collisions in Two Dimensions

1 Introduction

Scattering experiments are ubiquitous throughout physics as an investigative tool to study symmetries, conservation laws, nature of forces, *etc.* between interacting objects.

This week we do such an experiment, scattering cylindrical pucks to examine whether momentum, both linear and angular, and kinetic energy are conserved. The pucks move on a stream of air to reduce the effects of friction. So, except for the time of interaction, the pucks move freely in straight line motion.

For the first experiment, we use magnetic pucks that repel each other when close, never coming into contact. Our interest is in the initial and final momenta and kinetic energies, that is, before and after the magnetic forces act.

In the second experiment, we use non-magnetic pucks wrapped in velcro and launch the pucks so they collide and stick together. Now, the initial and final states are composed of different objects. This, therefore, is an inelastic collision. Once the pucks coalesce into a compound puck, the new object is freely moving. We want to see if linear momentum and kinetic energy are conserved. A little thought will convince you that as long as the collision is off-center, as we will assume from now on, the compound puck will rotate and thus, its angular momentum is of interest. We will calculate the initial and final angular momenta and compare.

As in previous experiments, we will use video acquisition to gather the relevant data.

1.1 Theory

In this lab, you are going to do your calculations using two different inertial systems. One is the lab frame, the one attached to the air table and the one you would expect to use. This will be used in the first experiment. However for the second, it is more convenient to use the center-of-mass frame, the one that moves with the center of mass of the two pucks. The center-of-mass frame is an inertial frame, moving with a constant velocity with respect to the lab frame because the sum of the external forces vanishes. Hence, an application of Newton's Laws is justified. The explanation of how to get the coordinates of each puck in this frame is explained later.[1] After the collision, the two pucks bind together to form a rotating compound system. Each puck undergoes a complicated motion relative to the lab frame but a particularly simple one in the center-of-mass frame. In this frame, the pucks just rotate about the center of mass. There is no translational motion. In general, the kinetic energy is the sum of the translational kinetic energy of the center of mass and the kinetic energy of the system relative to the center of mass. But in the center-of-mass frame, for the compound final state, there is no translational motion (why?). So, the final state relative to the center of mass is entirely rotational.

In any inertial frame we have for the kinetic energy

$$K_f = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I\omega^2 \tag{1}$$

where M is the total mass, I is the moment of inertia, and ω is the angular velocity of the combined system about its center of mass. But, for the final state in the center-of-mass frame, this simplifies to

$$K_f = \frac{1}{2}I\omega^2.$$

For the angular momentum of the final state in the center-of-mass frame, there is a similar simple expression. It is

$$|\vec{L}| = I\omega.$$

1.2 How to find the center-of-mass coordinates

Of immediate concern is how to determine the pucks' coordinates relative to their center of mass since from the videos, we find the coordinates of the pucks in the lab frame. How do we then get them in the center-of-mass frame?

We need a formula that takes the coordinates of each mass in the lab frame and converts them into coordinates in the center-of-mass frame. (This is called a coordinate transformation.) Referring to figure 1 and concentrating on particle a, its position relative to the center of mass is

$$\vec{r}_{a,cm} = \vec{r}_a - \vec{R}_{CM}. \tag{2}$$

This means $\vec{r}_{acm}$ is the location of a as seen by an observer sitting at the center-of-mass origin. We can break this equation into its coordinates along the center-of-mass axes. [2] We have

$$x_{a,cm} = x_a - \frac{m_a x_a + m_b x_b}{m_a + m_b} \tag{3}$$

The quantities on the right hand side are all measured in the lab frame via **Tracker**. Similar expressions hold for the remaining coordinates of both masses.

[1] The center-of-mass frame, more correctly, the center-of-momentum frame, is discussed in your lecture course textbook.

[2] I am assuming the orientation of the x, y, and z axes of the center-of-mass frame are the same as that

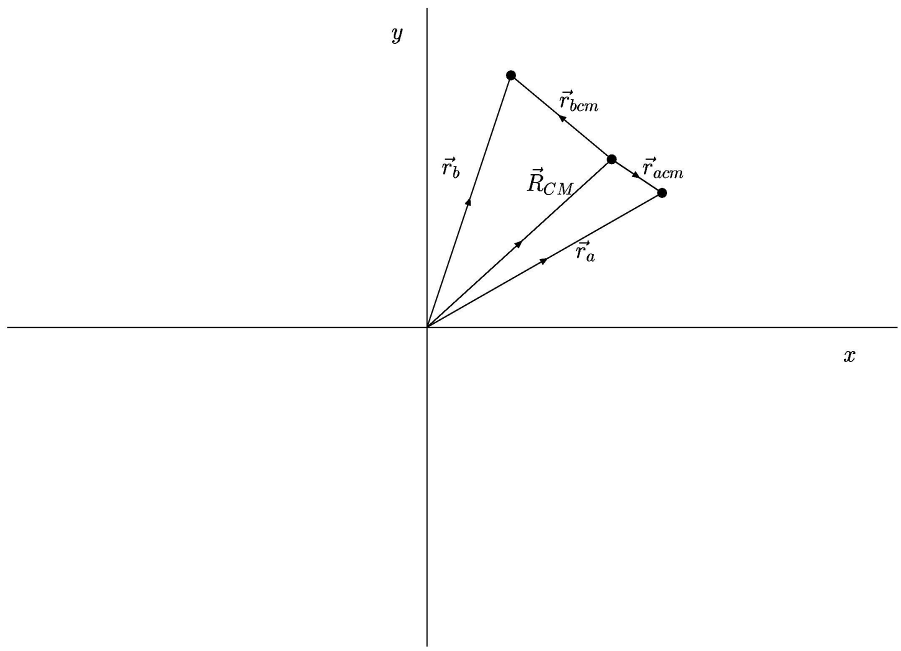

Figure 1: Diagram showing relationship between lab and center-of-mass coordinates for two masses a and b.

This experiment is an excellent example of how the relatively complicated motion of a compound system can be described much more simply in terms of the center-of-mass motion. In many physical systems, the description of the physics can take a particularly simple form if a judicious choice of coordinate system is made.

2 Experimental Procedure

Nota Bene: Make sure neither puck hits the center screw. Otherwise, an unknown external force will affect the collision and outcomes.

2.1 Experiment 1.

Before taking data, with the air on, practice launching the magnetic pucks, either by hand or elastic bands, so you get scattering and avoid the center screw at the same time. Use a moderate speed (low enough that the pucks bounce off the magnetic field and do not actually collide but high enough so that residual frictional forces due to the table not being

of the lab frame. This is not necessary but not to do so would make the calculations more complex without any gain in physical insight.

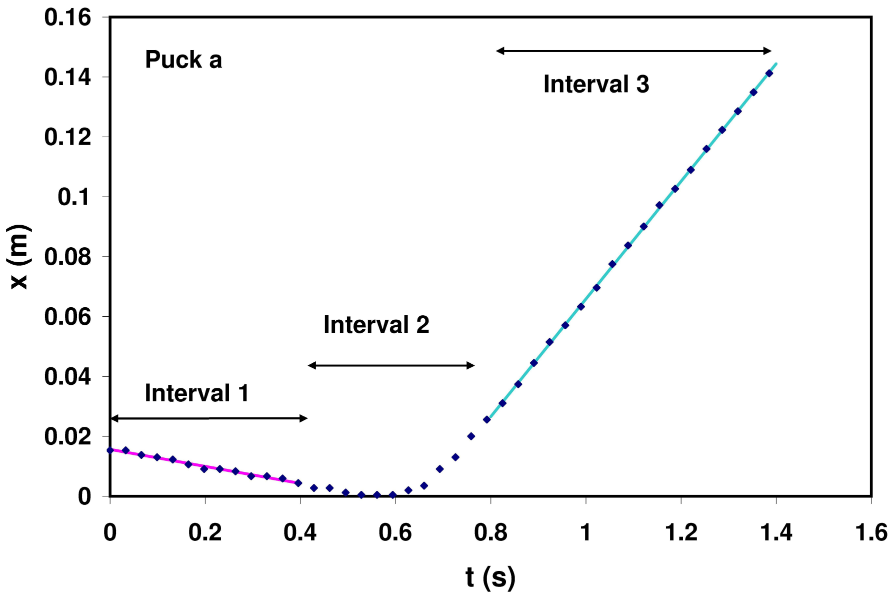

Figure 2: Example data showing the x position of puck a versus time. The times before, during, and after collision are indicated.

perfectly flat and/or level are negligible). When you can reliably achieve this, take your video and open Tracker.

Create mass points and tracks for each puck and then copy and paste the time and position coordinates into Excel for further computations. Since initial conditions are hard to reproduce in this experiment, unlike as done in previous labs, we make one determination of the coordinates.

Plot the x position of the first puck versus time (see figure 2). You should see an interval of time when the plot is linear, followed by an interval where it is not, followed by an interval where it is again linear but with a different slope. The first interval is before the pucks interact and the puck moves with constant velocity (that is, no acceleration). The second interval is when the pucks are interacting. The third interval is when the puck again moves with constant velocity. Use the Excel linest function on the position and time data for the first interval (be sure to avoid times when the pucks are interacting). This gives you the x-component of the initial velocity of first puck and its standard error ($v_{ix,a} \pm \sigma_{v_{ix,a}}$). Do the same for the third interval, giving you the x-component of the final velocity of the first puck and its uncertainty.

Repeat this for the y position of the first puck and the x and y positions of the second puck, both initial and final. After doing these calculations, you have eight velocity components ($v_{ix,a}$, $v_{iy,a}$, $v_{fx,a}$, $v_{fy,a}$, $v_{ix,b}$, $v_{iy,b}$, $v_{fx,b}$, and $v_{fy,b}$).

Calculate initial and final momentum values, initial and final kinetic energies, and uncertainties.

2.2 Experiment 2

In this experiment, use the velcro covered pucks and collide them nearly, but not exactly, head on. Again, practice launching the pucks with the air on to avoid the center screw. It is also to your advantage to launch them without rotation.

When ready, take your video, create tracks in **Tracker** (one for the center of each puck and one for the off-center white dot of each puck), and get the x and y lab coordinates of each point. Then, transform the lab coordinates to the center-of-mass coordinates at each time point and do the rest of your analysis with these coordinates.

As in the first experiment, plot the position components versus time to see when the interaction happened. Use linest to calculate the initial velocity components in center-of-mass coordinates and their uncertainties. These data will be needed to calculate the initial kinetic energy and angular momentum about the center-of-mass.

For the final state kinetic energy, you have to calculate the angular velocity about the center of mass. Perhaps the simplest way to do this is to realize that the (x, y) coordinates of the center of either puck in the center-of-mass system are the coordinates of its position vector in the that frame. The angle this vector makes is the one whose time derivative we need. So, you can form a table of angles as the puck rotates using the ATAN2 function of Excel, where the arguments are the x and y positions of one of the pucks. Plot the angle versus time for times after the collisions. You will probably notice that there are some discontinuities, which arise because ATAN2 always gives an angle between $-\pi$ and π. By appropriately adding or subtracting multiples of 2π from you angles in different intervals, you should be able to get a continuous, nearly linear plot of angle versus time. Use linest to determine the angular velocity and its uncertainty.

You will soon realize that both pucks have to rotate at the same rate. Make sure you check that this is the case within measurement errors.

If you launched your pucks with any significant rotation, you also need to find the angular velocity with which they are rotating. To do this, for each puck, calculate the angle using ATAN2 where the arguments are $x_{dot} - x_{puck}$ and $y_{dot} - y_{puck}$, where *dot* refers to the location of the dot on the puck in center-of-mass coordinates and *puck* refers to the center of the puck in center-of-mass coordinates. This rotation contributes to both the initial angular momentum and the initial kinetic energy.

The calculation of the kinetic energy is straightforward once you have transformed your lab coordinates to center-of-mass coordinates. If your pucks were rotating before the collision, you have to include the $\frac{1}{2}I\omega^2$ for each puck's rotational kinetic energy.

Likewise, the calculation of the angular momentum about the center-of-mass is not difficult if you remember the angular momentum is given by $\vec{L} = \vec{r} \times \vec{p}$. Just be careful when finding the angle between $\vec{r}$ and $\vec{p}$ or use coordinates. Either way is fine. If your pucks were rotating before collision, you have to include the $I\omega$ contribution to L from each puck. Remember, the angular momentum after the collision simplifies to $L = I\omega$, because is no translational motion (in the center of mass).

The final bit of information needed to finish this part of the experiment is to determine what the moment of inertia of the compound puck is about its center of mass. Since you have most likely not covered such a calculation yet in your lecture course, you are asked to accept that it can be shown $I = 3/2 M_T R^2$, where R is the radius of one of the pucks and M_T is the total mass of the two puck system.

3 Analysis/Report

For both experiments, calculate initial and final momenta, kinetic energies, and additionally for the second the angular momenta. Based on your knowledge of Newtonian physics and its conservation laws, explain what you expect to find based on theory, that is, give reasons why you expect either conservation or non-conservation of the measured dynamical quantities and compare to what you actually got. Account for any discrepancies between observation and expectation. If there is a non-conserved quantity, explain why.