spheres

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Introduction

This document illustrates some probability distributions associated with spheres of various dimensions. These probability distributions naturally arise in statistical mechanics when considering ideal gases.

I am going to use Monte Carlo simulation to approximate the distributions so I can form an intuitive understanding of them and to provide an independent check on the formulas I derive mathematically. Let *Nsamples* denote the number of samples used in each Monte Carlo run.

```
Nsample <- 1000000
```

The 1-dimensional Sphere

The 1-dimensional sphere S^1 is the circle of radius one in $\mathbb{R}^2$.

$$S^1 = \{(x, y) : \mathbb{R}^2 \mid x^2 + y^2 = 1\}$$

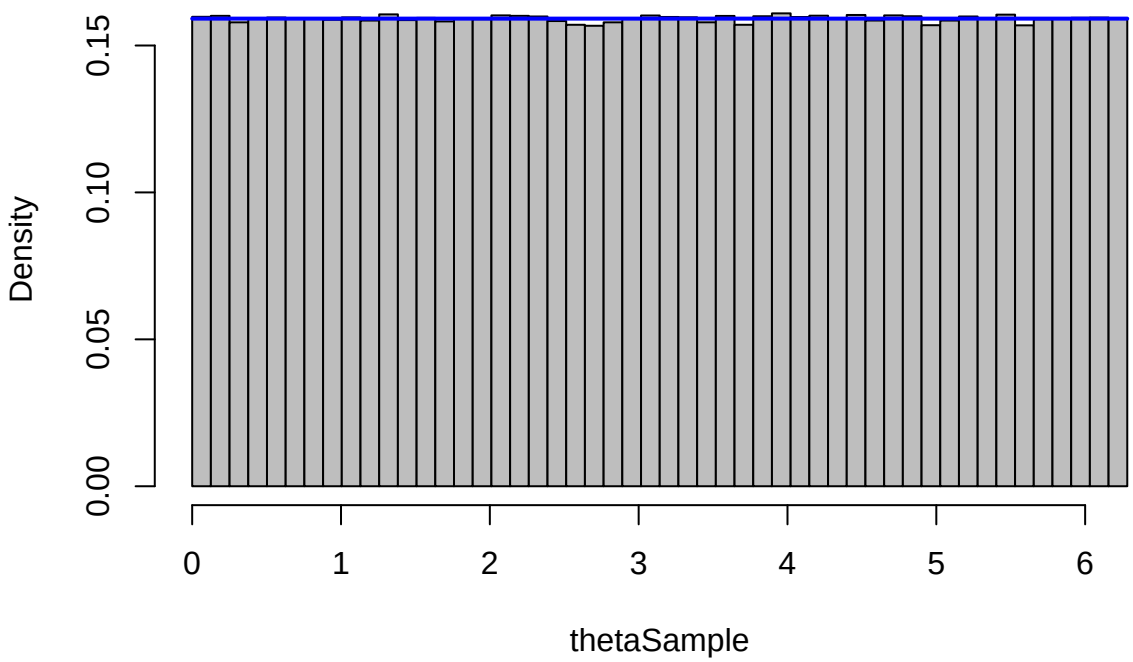
We can give the points of S^1 a real coordinate θ.

$$\theta : S^1 \longrightarrow [0, 2\pi]$$

Let $x = \cos\theta$ and $y = \sin\theta$ with $0 < \theta < 2\pi$.

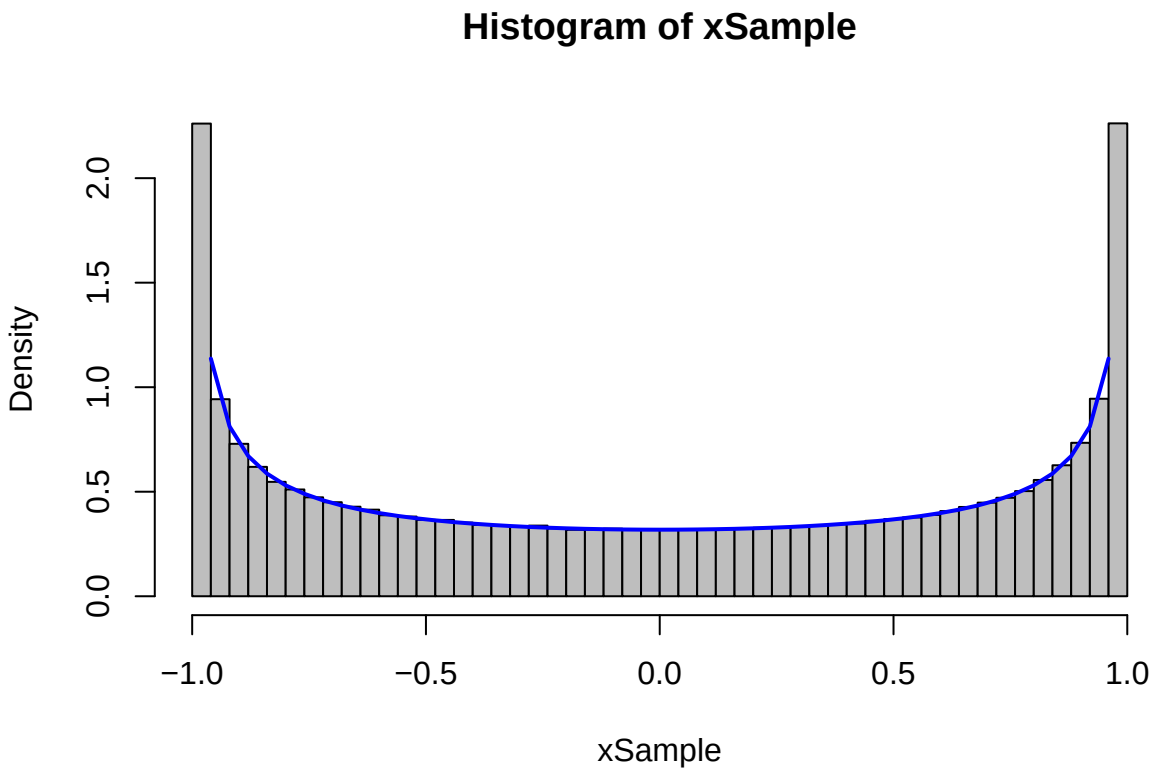
Consider the uniform probability distribution on S^1. Clearly this is a continuous probability distribution with a probability density function $\rho(\theta) = \frac{1}{2\pi}$. Let theta be a random sample from this distribution.

```
thetaSample <- runif(Nsample,0,2*pi)
Nbins <- 50
thetaBreaks <- seq(0, 2*pi, 2*pi/Nbins)
rho <- rep(1/(2*pi),times=length(thetaBreaks))
hist(thetaSample, probability = TRUE, breaks = thetaBreaks, col="grey")
lines(x=thetaBreaks,y=rho,col="blue",lwd=2)
```



Now regard the x coordinate of a point on S^1 as a random variable.

```
xSample <- cos(thetaSample)
xBreaks <- seq(-1,1,2/Nbins)
xRho <- 1/(pi*sqrt(1-xBreaks^2))
hist(xSample, probability = TRUE, breaks = xBreaks, col = "grey")
lines(x=xBreaks,y=xRho,col="blue",lwd=2)
```



R Markdown

This is an R Markdown document. Markdown is a simple formatting syntax for authoring HTML, PDF, and MS Word documents. For more details on using R Markdown see http://rmarkdown.rstudio.com.

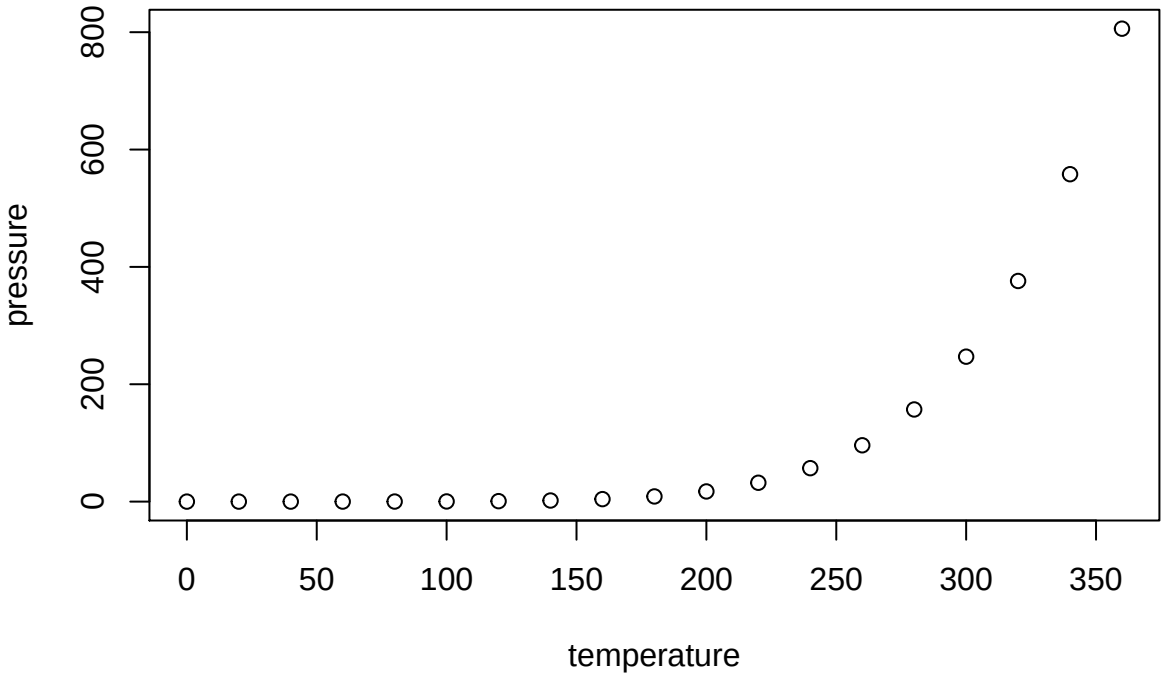
When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

```
summary(cars)
```

```
##      speed           dist
##  Min.   : 4.0   Min.   :  2.00
##  1st Qu.:12.0   1st Qu.: 26.00
##  Median :15.0   Median : 36.00
##  Mean   :15.4   Mean   : 42.98
##  3rd Qu.:19.0   3rd Qu.: 56.00
##  Max.   :25.0   Max.   :120.00
```

Including Plots

You can also embed plots, for example:



Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.