Storyboard

January 30, 2025

1 Storyboard

Arthur Ryman, last updated 2025-01-23

1.1 Introduction

This notebook is the storyboard for a short video that explains the graph theory solution to the Instant Insanity puzzle. The video will combine high-quality 3D animation, mathematics, and Python code so using a Jupyter notebook to storyboard it seems appropriate.

The video is a collaboration between Will and Arthur. Will is going to produce the animation. Arthur is going to explain the math. Maybe both of us will write some Python code.

1.2 Arthur's Motivation

My motivation is to continue the tradition of sharing mathematical gems with the next generation. The gem I want to share is the graph theory solution to the Instant Insanity puzzle. It was shared with me by a mathematics professor when I was a high school student. Aside from this gem being very enjoyable, it had a significant influence on my subsequent academic and professional career.

I was a high school student during the Instant Insanity puzzle craze of 1967. The puzzle consists of four cubes whose faces are coloured red, green, blue, and white. The pattern of colours looks random. You have to line up the cubes so no colour was repeated on each row of four faces. The number of possible combinations is very high - we'll compute the exact number later.

My high school, Northview Heights Secondary School in North York, Toronto was unusual in that it had its own IBM 1130 computer. Computer Science was not even on the cirriculum back then, but one of our teachers, Mr. Wong, ran classes after school and taught us how to program in Fortran 4.

Having been unable to solve Instant Insanity by hand, I decided to write a Fortran program to find the solution by doing a brute-force search. I wrote the program, keypunched it, loaded the cards into the reader, hit the Start button, and expected to see the solution promptly emerge on the line printer. To my great dissappointment, the line printer remained silent. Instead, all I heard was the motion of the disk drive arms moving back and forth. I had never experienced a program that had spent so much time thinking. I thought maybe the number of combinations was so high that the program would never finish. But after a few minutes I heard the line printer start up and hammer out the solution. I tore the paper off the printer, pulled out my cubes, and confirmed that the program had indeed found the solution. I was elated, but then a little disappointed. Was that all there was? Just apply enough brute force and you'll solve the puzzle?

That wasn't the end of the story. One day a year or so later we were told that our math class was cancelled because we were having a special guest speaker in the auditorium. The speaker was Prof. Ross Honsberger, University of Waterloo, Faculty of Mathematics, Department of Combinatorics and Optimization. Prof. Honsberger talked to us about several topics, including graph theory. He closed by using graph theory to produce an extremely elegant solution to the Instant Insanity puzzle! I was blown away by its elegance and ingenuity. In the months following that lecture, I showed the solution to any of my friends who were interested in the puzzle.

Fast forward fifty years to the Internet and YouTube. Who would have thought that there would be an audience for math explainer videos? What a perfect medium for sharing the solution to Instant Insanity!

Fast forward to the present. I have finally retired and can spend time on my bucket list. By a stroke of pure luck I have been reconnected with Will owing to our shared past at York University centered around computer graphics. Will went on to become a CGI professional. When I asked Will if he could give me some pointers on animation, he graciously offered to collaborate. So here we are, ready to produce a video.

1.3 Prior Art

It is a general rule that if you have what seems like a good idea then the odds are high that someone has already thought of it and has acted on it. I therefore did an Internet search and found several YouTube videos. A few of these (2008, 2016) are simply videos of people explaining the solution using standard presentation aids but no animation. A couple (2018, 2019) are more recent and use simple animation such as might be implemented using PowerPoint.

The 2018 video was an episode of PBS Infinite Series, written and narrated by Tai-Danae Bradley who is a skilled content creator. This is the one to beat. If we can't significantly improve on it then there is no point in continuing this project.

I do think we can advance the state-of-the-art in several aspects. These are explained next.

1.4 Goals

- the target audience is math enthusiasts, especially young ones who don't have formal math training
- the video should be very self-contained - it should explain all terms
- the video should be easy to follow - it should NOT require any mental leaps
- the video should be visually appealling
- the video should take advantage of the fact that cubes a physical objects by realistically rendering them as such
- the video should use exploit 3D graphics to achieve visual continuity as one representation is morphed into another
- the video should explain the combinatorics of the puzzle, NOT simply quote the number of combinations
- the video should explain graph theory in the context of the puzzle and animate the transformation of the puzzle into a graph
- the video should explain the history of the puzzle and show the original graph theory solution published in 1947
- the video should animate the search trees for both the physical cubes and the abstract graph

- the video should explain the concept of abstraction and show how the graph abstracts the physical cubes
- the video should not be too long - no boring content

Maybe include some history of graph theory: * invented by Leonhard Euler in 1736 to solve the Seven Bridges of Königsberg puzzle * there are several YouTube videos but we could due a better animation of morphing the map into the graph * another nice example is the map of the London Underground * we could also morph that

It may also be of interest to include some background info, possibly as a separate video. * the 1947 solution was discovered by Bill Tutte and friends at Cambridge * Bill Tutte became an eminent graph theorist (the subject of his PhD thesis) * Bill Tutte moved to Canada and became a founding member of the University of Waterloo, Combinatorics and Optimization Dept. * Ross Honsberger got a math degree from University of Toronto, taught high school math, and became Head of the Math Department at Northview Heights * Ross Honsberger left Northview Heights and joined the University of Waterloo, Combinatorics and Optimization Dept. where he most certainly worked with Bill Tutte * Ross Honsberger wrote 13 books on mathematical puzzles and recreations * it seems likely that Bill Tutte showed Ross Honsberger the solution * University of Waterloo was developing a strong reputation in Computer Science * IBM provided computers to UW and hired many of its CS co-op students * it seems plausible that Ross Honsberger advocated for putting the IBM 1130 computer in NHSS * it seems plausible that Ross Honsberger was a guest speaker at NHSS because he was formerly the Head of the Math Department * therefore we most likely have a direct causal chain from Bill Tutte to Ross Honsberger/IBM to Northview Heights to Arthur/Will to you to viewer * we could present these events and relations as a graph!

1.5 Outline

The following is a tentative high-level outline. Each topic will be expanded in the remainder of this notebook.

1. Introduction
    1. Describe the 1967 puzzle
    2. Discuss the history of the puzzle and the 1947 solution
    3. Count the number of combinations
    4. Discuss symmetry - When are two combinations essentially the same?
    5. Describe the search tree
    6. Maybe show Python code to do the search
2. Graph Theory
    1. Define graphs
    2. Give some examples of graphs
    3. Discuss the solution in terms of opposite faces
    4. Morph the cubes into a graph
    5. Count the number of combinations
    6. Describe the search tree in terms of the graph
    7. Maybe show Python code to do the search
    8. Display the solution
3. Abstraction
    1. Define the power of abstraction in mathematics
    2. Describe how the graph abstracts the cube - What inessential information is discarded?

4. Wrap up
    1. Summarize the video
    2. Point to sources of more information

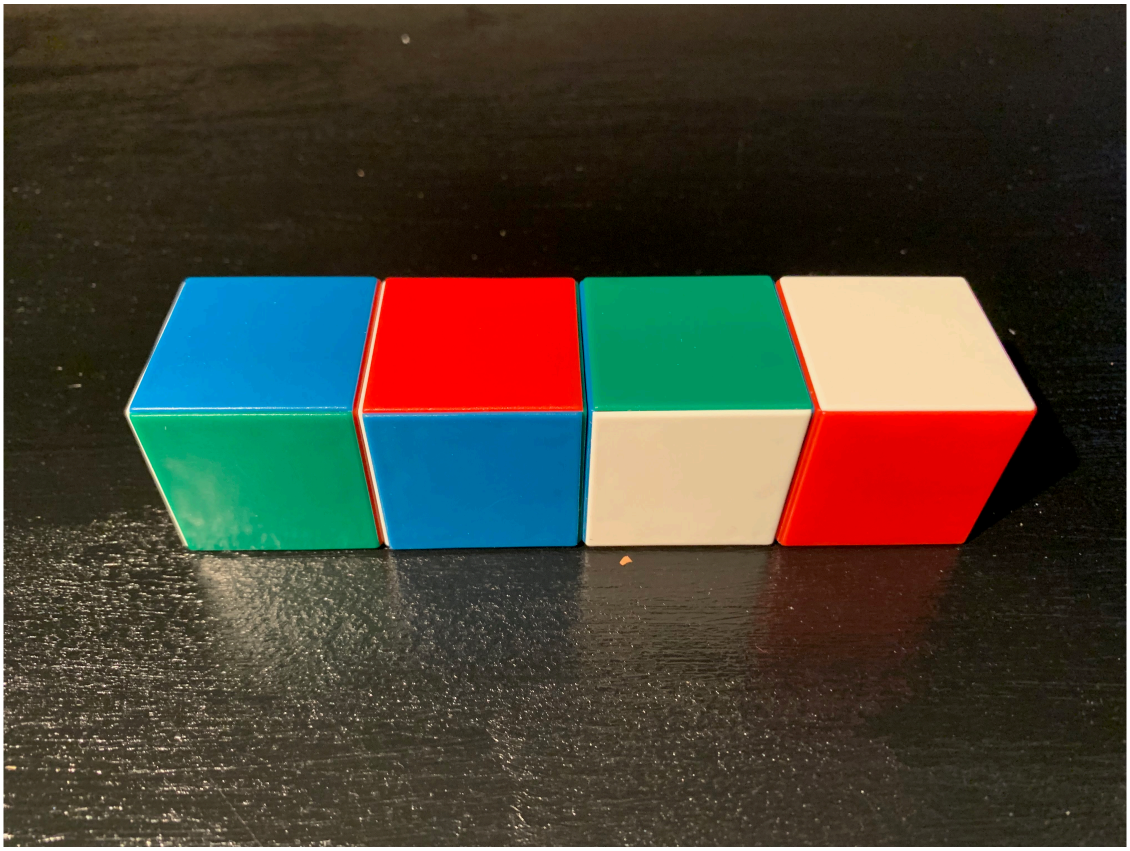
1.6 The 1967 Version of the Instant Insanity Puzzle

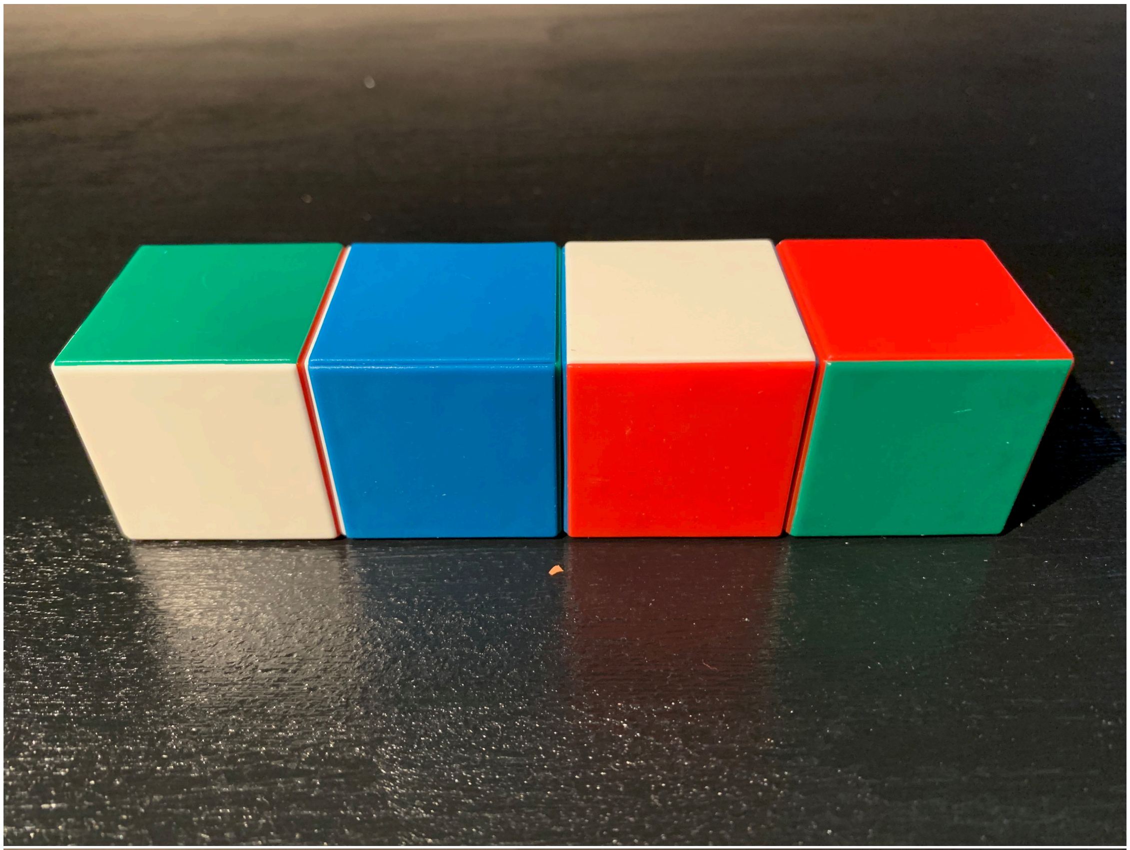
Although the current form of the puzzle was released around 1967, it is still being manufactured. I bought one manufactured in 2024 from Winning Moves Games. The packages claims that over 20 million have been sold and that there are 82,944 combinations.

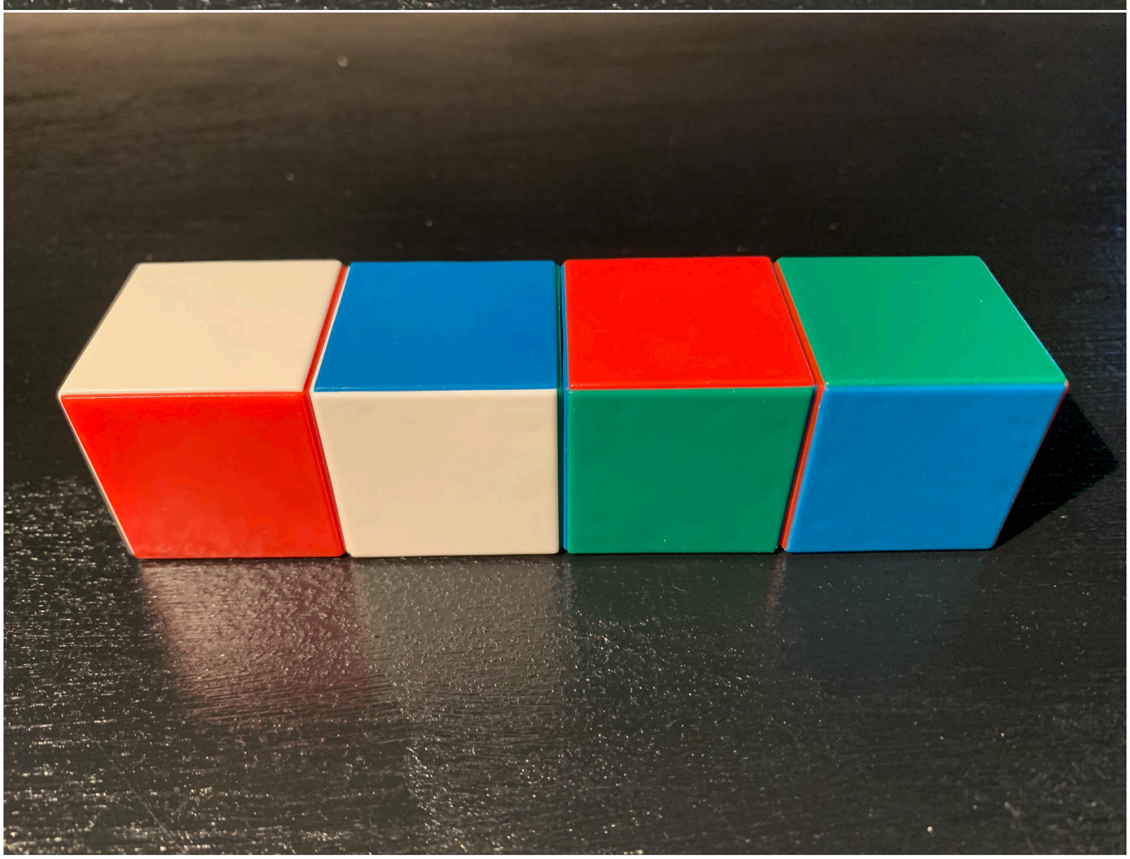
Here's the back of the package. The back describes the goal of the puzzle, namely to line up the cubes in a row of four so that no colour is repeated along the top, front, bottom, or back rows of four faces.

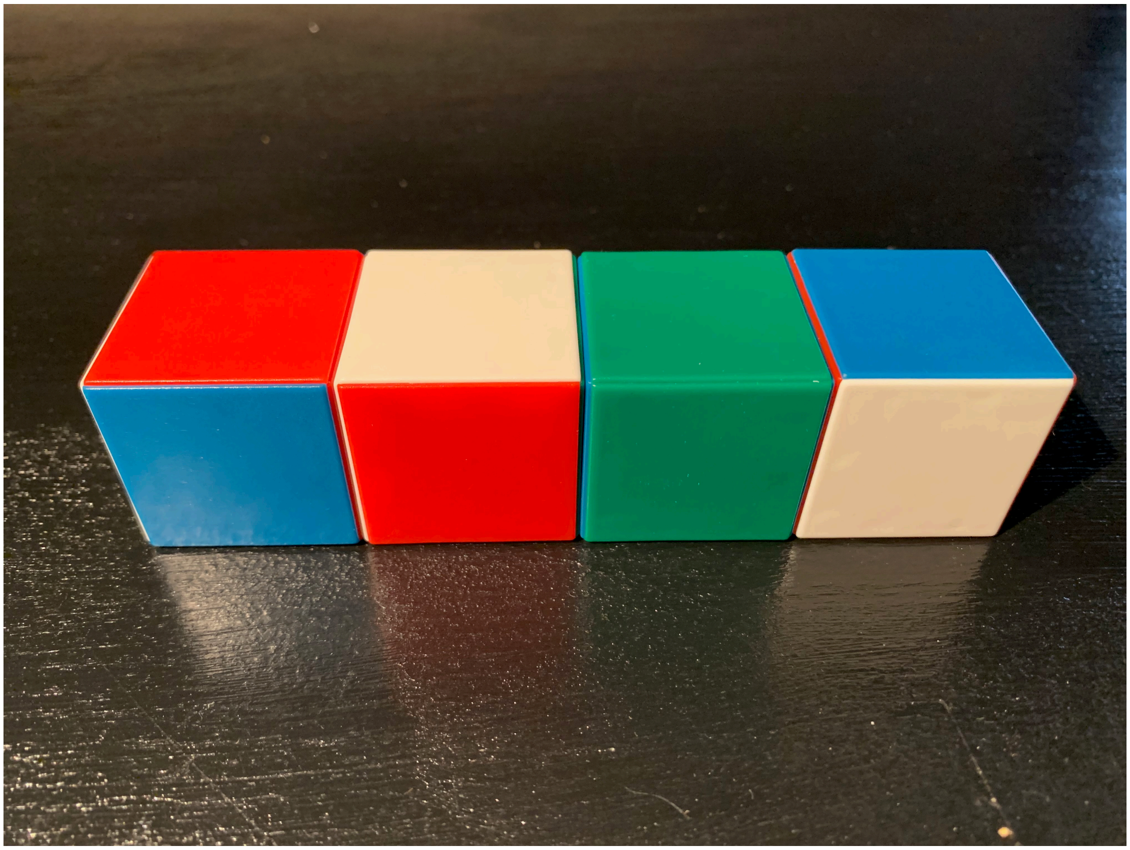
The package contains the cubes arranged in the solution. This proves that a solution exists. You are then invited to scramble the cubes and try to find the solution. The out-of-the-box solution is as follows where each image is rotated a quarter of a turn from the previous image.

This could be nicely animated by continuously rotating the row of four cubes about the horizontal axis.









1.7 The History of the Puzzle and the 1947 Graph Theory Solution

The Instant Insanity puzzle was invented many years before 1967. In fact, it was the subject of several patents. It was known under various names. (Insert some historical references here). The elegant graph theory solution was discovered in 1947 by a group of four Cambridge University students who published articles under the pseudonym Blanche Descarte. Their real names were William (Bill) Tutte, Leonard Brooks, Cedric Smith, and Harold Stone. Bill Tutte went on to become an eminent graph theorist.

The solution was published in the April, 1947 issue of Eureka, The Archimedian' Journal, No. 9. The article appears on p. 9 as *The Coloured Cubes Problem* by F. de Carteblanche who is supposedly the husband of Blanche Descartes.

EUREKA

THE ARCHIMEDEANS' JOURNAL

APRIL, 1947

PRICE 1/6

EUREKA

THE JOURNAL OF THE ARCHIMEDEANS

(The Cambridge University Mathematical Society: Junior Branch of the Mathematical Association)

Editor: *A. M. Macbeath (Clare)*

Committee: *A. R. Curtis (St. John's); A. J. Oldham (Clare); A. F. Ruston (Sidney Sussex); R. S. Scorer (Corpus)*

No. 9 APRIL, 1947

Contents

Contributions and other communications should be addressed to:

The Editor, "Eureka,"
c/o The Mathematical Faculty Library,
The Arts School,
Bene't Street, Cambridge.

$$m_1 U \int_{t_o}^{t_1} \log \frac{m}{m_1} \cdot \frac{dV}{dx} dt = m_1 \int_{t_o}^{t_1} (n + 1)\, g_o^2 (t_1 - t) dt \text{ by (x)}$$
$$= \tfrac{1}{2} m_1 (n + 1) g_o^2 (t_1 - t_o)^2$$
$$= \frac{m_1 U^2}{2(n + 1)} \left(\log \frac{m_o}{m_1} \right)^2 \text{ by (xi).}$$

Comparing this with (ix) we see that this energy deficiency is a fraction $1/(n + 1)$ of the maximum possible energy increment. If then $n = 4$, the transfer is 80% efficient.

It may be remarked in conclusion that a complete discussion of a rocket's motion in a field of force can only be undertaken after a general system of dynamical equations, such as Lagrange's, has been modified to apply to a system of variable mass.

■ ■ ■

The Coloured Cubes Problem

By F. de Carteblanche

There is a puzzle (sold commercially as the "Tantalizer") which consists of a set of 4 cubes whose faces are coloured each with one of four colours, say green, red, orange and white, or G, R, O, W for short. The problem is to stack these cubes in a vertical pile (thus forming a square prism) in such a way that each of the four vertical faces of this pile contains all four colours.

In order to make the discussion clearer we shall give names to the faces of the cubes in this way: the front, or nearest face will be named the x face; and opposite face, or back, will be named the x' face. On the right will be the y face, and on the left the y' face. The top will be the z face and the bottom the z' face. The problem then requires us to arrange the four cubes so that the x faces have all different colours, and the same for the x', y, and y' faces. If, for example, we had a set of cubes coloured and arranged in the following way, we would have a solution. (I am not sure if this is the same as the commercial version.)

Face	Cube number 1	2	3	4
x (front) ..	R	G	O	W
x' (back) ..	O	R	W	G
y (right) ..	W	O	G	R
y' (left) ..	O	R	W	G
z (top) ..	G	W	G	R
z' (bottom)	G	W	R	W

However, if one is only given the cubes, and not the solution, getting them into the correct position is none too easy. But here is a quick method of solving the problem which seems to indicate the principle on which the puzzle is based.

We make use of the diagram shown in Fig. 1, which is easily constructed from the given set of cubes.

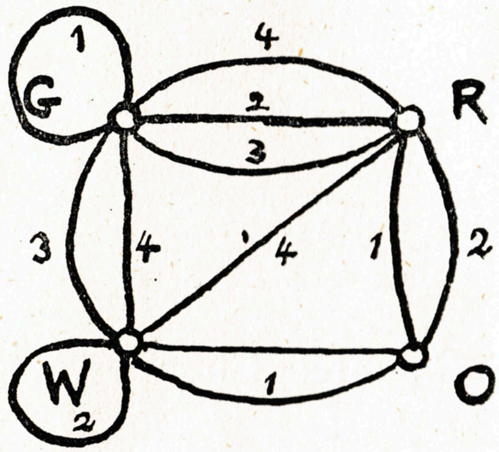


Fig. 1

Here the four points G, R, O, W represent the four colours. The line labelled 1 joining the points R and O represents the pair of opposite faces in cube 1, coloured R and O: and similarly every other pair of opposite faces has a corresponding line.

Now in the final arrangement all the faces x have different colours, and so also do all the opposite faces x'; so in taking all the pairs of opposite faces x, x' we must take each of the colours exactly twice. In the diagram (Fig. 1) this must correspond to a set of four lines with the properties

(i) there must be just one line with each of the labels 1, 2, 3, 4, (i.e. just one pair of faces from each cube);

(ii) there must be just two lines ending at each of the points G, R, O, W, except that we may allow instead one line having both its ends at the point.

Conversely, if we have a set of four lines with properties (i) and (ii), then we can arrange them so that the x faces have all different colours, and the same for the x' faces. And, in particular, if the four lines form a circuit like O (1) R, R (2) G, G (4) W, W (3) O, then there is only one possible arrangement, apart from an interchange of x and x'. For if in the first cube we place the R face in position x, and therefore the O face in position x', then as no two x faces can have the same colour, in the second cube the R face must have position x', and so the G must be x, and therefore in the fourth cube G must be x', and so on. Now, in order to fit in the y and y' faces to have all different colours, we must find a second set of four lines, distinct from the first, having properties (i) and (ii). But the only way of choosing two such sets of lines from Fig. 1 is to take the two circuits O (1) R, R (2) G, G (4) W, W (3) O and W (1) O, O (2) R, R (4) G, G (3) W, giving the solution we have already tabulated. This shows that it is an effectively unique solution, apart from a permutation of the order of the cubes, or of an interchange of x and x', or of y and y', or of x, x' and y, y'. The chance of obtaining the solution by a random arrangement of the cubes is only 1/41472.

The diagram of Fig. 1 is not the only possible one with 4 colours giving such a unique solution; in fact, there does not seem to be any reason why one should not use the same method to make a puzzle with n cubes, coloured in n different colours. But I leave that to your ingenuity.

▪ ▪ ▪

A Problem on Orchards

By G. C. Shephard

In this article* an attempt is made to generalise a problem which frequently occurs in mathematical puzzle-books,† and is, no doubt, familiar to the reader. Briefly, this is:

An eccentric farmer wishes to divide his orchard by n straight fences into enclosures, each of which must contain only one tree. Given a plan of the orchard, show how this may be done.

The following is the generalisation for the case of seven "trees":

Given six general‡ points of a plane, to find the conditions which a seventh point must satisfy so that it will be possible to divide the plane by 3 straight lines into 7 regions, each of which will contain just one of the points.

Regarding all regions as open sets of points we shall adopt the convention that if a point lies on a line we may regard it as lying in either of the two regions to whose frontiers it belongs; and similarly, if a point lies on the intersection of two lines, we may regard it as lying in any one of the four regions to whose frontiers it belongs. We shall refer to this convention by the letter Γ.

(1) Select any one of the six given points, say A.

(2) Join A to each of the remaining five points.

(3) Select any two of these five lines, say AB, AC. (This may be done in ${}_5C_2 = 10$ ways). These two lines will divide the plane

* I am indebted to E. S. Page for reading the manuscript and making several valuable suggestions.

† E.g. H. E. Dudeney: *Puzzles and Curious Problems*, No. 220.

‡ I.e. no three of which are collinear.