

However, if one is only given the cubes, and not the solution, getting them into the correct position is none too easy. But here is a quick method of solving the problem which seems to indicate the principle on which the puzzle is based.

We make use of the diagram shown in Fig. 1, which is easily constructed from the given set of cubes.

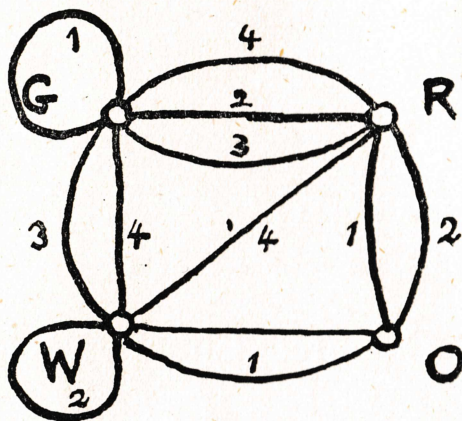


Fig. 1

Here the four points G, R, O, W represent the four colours. The line labelled 1 joining the points R and O represents the pair of opposite faces in cube 1, coloured R and O: and similarly every other pair of opposite faces has a corresponding line.

Now in the final arrangement all the faces  $x$  have different colours, and so also do all the opposite faces  $x'$ ; so in taking all the pairs of opposite faces  $x, x'$  we must take each of the colours exactly twice. In the diagram (Fig. 1) this must correspond to a set of four lines with the properties

- (i) there must be just one line with each of the labels 1, 2, 3, 4, (i.e. just one pair of faces from each cube);
- (ii) there must be just two lines ending at each of the points G, R, O, W, except that we may allow instead one line having both its ends at the point.

Conversely, if we have a set of four lines with properties (i) and (ii), then we can arrange them so that the  $x$  faces have all different colours, and the same for the  $x'$  faces. And, in particular, if the four lines form a circuit like O (1) R, R (2) G, G (4) W, W (3) O, then there is only one possible arrangement, apart from an interchange of  $x$  and  $x'$ . For if in the first cube we place the R face in position  $x$ , and therefore the O face in position  $x'$ , then as no two  $x$  faces can have the same colour, in the second cube the R face must have position  $x'$ , and so the G must be  $x$ , and therefore in the fourth cube G must be  $x'$ , and so on. Now, in order to fit in the  $y$  and  $y'$  faces