to have all different colours, we must find a second set of four lines, distinct from the first, having properties (i) and (ii). But the only way of choosing two such sets of lines from Fig. 1 is to take the two circuits O (1) R, R (2) G, G (4) W, W (3) O and W (1) O, O (2) R, R (4) G, G (3) W, giving the solution we have already tabulated. This shows that it is an effectively unique solution, apart from a permutation of the order of the cubes, or of an interchange of x and x', or of y and y', or of x, x' and y, y'. The chance of obtaining the solution by a random arrangement of the cubes is only 1/41472.

The diagram of Fig. 1 is not the only possible one with 4 colours giving such a unique solution; in fact, there does not seem to be any reason why one should not use the same method to make a puzzle with n cubes, coloured in n different colours. But I leave that to your ingenuity.

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A Problem on Orchards

By G. C. Shephard

In this article* an attempt is made to generalise a problem which frequently occurs in mathematical puzzle-books,† and is, no doubt, familiar to the reader. Briefly, this is:

An eccentric farmer wishes to divide his orchard by n straight fences into enclosures, each of which must contain only one tree.

Given a plan of the orchard, show how this may be done.

The following is the generalisation for the case of seven "trees":

Given six general‡ points of a plane, to find the conditions which a seventh point must satisfy so that it will be possible to divide the plane by 3 straight lines into 7 regions, each of which will contain just one of the points.

Regarding all regions as open sets of points we shall adopt the convention that if a point lies on a line we may regard it as lying in either of the two regions to whose frontiers it belongs; and similarly, if a point lies on the intersection of two lines, we may regard it as lying in any one of the four regions to whose frontiers it belongs. We shall refer to this convention by the letter Γ.

(1) Select any one of the six given points, say A.

(2) Join A to each of the remaining five points.

(3) Select any two of these five lines, say AB, AC. (This may be done in ${}_5C_2 = 10$ ways). These two lines will divide the plane

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† E.g. H. E. Dudeney: *Puzzles and Curious Problems*, No. 220.

‡ I.e. no three of which are collinear.