Integers

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Abstract

This article contains Z Notation type declarations for the integers, $\mathbb{Z}$, and some related objects. It has been type checked by *f*UZZ.

1 Introduction

The integers, $\mathbb{Z}$, are built-in to Z Notation. This article provides type declarations for some related objects so that they can be used and type checked in formal Z specifications.

2 Integers

2.1 *AddIntegerSequences*

Let l be a natural number and let x and y be two integer sequences of length l. Their sum $z = x + y$ is the integer sequence of length l defined by point-wise addition of of the terms in x and y. Let the schema *AddIntegerSequences* denote this situation.

AddIntegerSequences

$$l : \mathbb{N}$$
$$x, y, z : \operatorname{seq} \mathbb{Z}$$

$$l = \# x = \# y$$
$$z = (\lambda\, i : 1 \,..\, l \bullet x\, i + y\, i)$$

- The sequence z is defined by pointwise addition of the sequences x and y.

2.2 *add_int_seq*

Let the function $add\_int\_seq(x, y) = z$ be the sum of two equal-length integer sequences.

$$add\_int\_seq == \{\, AddIntegerSequences \bullet (x, y) \mapsto z \,\}$$

2.3 $+$ \addSeqZ

We introduce the notation $x + y = add\_int\_seq(x, y)$.

$$(\_ + \_) == add\_int\_seq$$