Sets

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Abstract

This article contains Z Notation type declarations for concepts related to sets. It has been type checked by fUZZ.

1 Introduction

Typed set theory forms the mathematical foundation of Z Notation and many concepts relating to set theory are defined by its built-in mathematical tool-kit. This articles augments the tool-kit with some additional concepts.

2 Arbitrary Sets

2.1 T `\setT`, U `\setU`, ..., Z `\setZ`

Let T, U, and Z denote arbitrary sets. These will be used throughout in the statement of theorems, remarks, and examples that are parameterized by arbitrary sets.

$$[\mathsf{T}, \mathsf{U}, \mathsf{V}, \mathsf{W}, \mathsf{X}, \mathsf{Y}, \mathsf{Z}]$$

3 Formal Arguments to Generic Constructions

The following typographically distinctive symbols will be used as formal arguments to generic constructions: $\mathtt{t}, \mathtt{u}, \mathtt{v}, \mathtt{w}, \mathtt{x}, \mathtt{y}, \mathtt{z}$. They denote arbitrary sets.

4 Families

4.1 $\mathcal{F}$ `\family`

Let $\mathtt{t}$ be a set. A *family* of subsets of $\mathtt{t}$ is a set of subsets of $\mathtt{t}$. Let $\mathcal{F}\mathtt{t}$ denote the set of all families of subsets of X.

$$\mathcal{F}\mathsf{t} == \mathbb{P}(\mathbb{P}\,\mathsf{t})$$

5 Functions

5.1 const \const

Let t and u be sets and let $c \in \mathsf{u}$ be some given point. The mapping that sends every point of t to c is called the *constant mapping* defined by c. Let $\mathrm{const}(c)$ denote the constant mapping.

$$
\begin{array}{l}
[\mathsf{t}, \mathsf{u}] \\
\hline
\mathrm{const} : \mathsf{u} \longrightarrow (\mathsf{t} \longrightarrow \mathsf{u}) \\
\hline
\forall\, c : \mathsf{u} \bullet \\
\qquad \mathrm{const}(c) = (\lambda\, x : \mathsf{t} \bullet c)
\end{array}
$$

5.2 $|_{\mathsf{fun}}$ \restrictU

Let t and u be sets, let $f : \mathsf{t} \longrightarrow \mathsf{u}$, and let $T \subseteq \mathsf{t}$. Let $f \,|_{\mathsf{fun}}\, T$ denote the restriction of f to T.

$$
\begin{array}{l}
[\mathsf{t}, \mathsf{u}] \\
\hline
\_ \,|_{\mathsf{fun}}\, \_ : (\mathsf{t} \longrightarrow \mathsf{u}) \times \mathbb{P}\,\mathsf{t} \longrightarrow (\mathsf{t} \nrightarrow \mathsf{u}) \\
\hline
\forall\, f : \mathsf{t} \longrightarrow \mathsf{u};\ T : \mathbb{P}\,\mathsf{t} \bullet \\
\qquad f \,|_{\mathsf{fun}}\, T = T \lhd f
\end{array}
$$

5.3 *bit*

Let *bit* denote the set of *binary digits*, namely the set $\{0, 1\} \subseteq \mathbb{Z}$.

$$bit == \{0, 1\}$$

5.4 $\mathbb{B}$ \B

We introduce the notation $\mathbb{B} = bit$.

$$\mathbb{B} == bit$$

5.5 *indicator_function*

Let t be a set, let X be a subset of t, and let $a \in \mathsf{t}$ be some element. The *indicator function* or *characteristic function* of X maps a to 1 if $a \in X$ and 0 otherwise. Let $indicator\_function(X) \in \mathsf{t} \longrightarrow \mathbb{B}$ denote this function.

$$
\begin{array}{l}
[\mathsf{t}] \\
\hline
\mathit{indicator\_function} : \mathbb{P}\,\mathsf{t} \longrightarrow \mathsf{t} \longrightarrow \mathbb{B} \\
\hline
\forall X : \mathbb{P}\,\mathsf{t} \bullet \\
\quad \mathit{indicator\_function}(X) = \\
\qquad (\lambda\, a : \mathsf{t} \bullet \mathbf{if}\ a \in X\ \mathbf{then}\ 1\ \mathbf{else}\ 0)
\end{array}
$$

5.6 $\mathbf{1}$ \indF

We introduce the notation $\mathbf{1}[\mathsf{t}](X) = \mathit{indicator\_function}[\mathsf{t}](X)$.

$$\mathbf{1}[\mathsf{t}] == \mathit{indicator\_function}[\mathsf{t}]$$