Sets

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Abstract

This article contains Z Notation type declarations for concepts related to sets. It has been type checked by fUZZ.

1 Introduction

Typed set theory forms the mathematical foundation of Z Notation and many concepts relating to set theory are defined by its built-in mathematical tool-kit. This articles augments the tool-kit with some additional concepts.

2 Arbitrary Sets

2.1 T \setT, U \setU, . . . , Z \setZ

Let T, U, and Z denote arbitrary sets. These will be used throughout in the statement of theorems, remarks, and examples that are parameterized by arbitrary sets.

$$[\mathsf{T}, \mathsf{U}, \mathsf{V}, \mathsf{W}, \mathsf{X}, \mathsf{Y}, \mathsf{Z}]$$

3 Formal Arguments to Generic Constructions

The following typographically distinctive symbols will be used as formal arguments to generic constructions: $\mathsf{t}, \mathsf{u}, \mathsf{v}, \mathsf{w}, \mathsf{x}, \mathsf{y}, \mathsf{z}$. They denote arbitrary sets.

4 Families

4.1 $\mathcal{F}$ \family

Let t be a set. A *family* of subsets of t is a set of subsets of t. Let $\mathcal{F}\mathsf{t}$ denote the set of all families of subsets of X.

$$\mathcal{F}\mathsf{t} == \mathbb{P}(\mathbb{P}\,\mathsf{t})$$

5 Functions

5.1 const `\const`

Let t and u be sets and let $c \in \mathsf{u}$ be some given point. The mapping that sends every point of t to c is called the *constant mapping* defined by c. Let $\mathrm{const}(c)$ denote the constant mapping.

$$
\begin{array}{|l}
\hline
[\mathsf{t}, \mathsf{u}] \\
\mathrm{const} : \mathsf{u} \longrightarrow (\mathsf{t} \longrightarrow \mathsf{u}) \\
\hline
\forall\, c : \mathsf{u} \bullet \\
\qquad \mathrm{const}(c) = (\lambda\, x : \mathsf{t} \bullet c) \\
\hline
\end{array}
$$

5.2 $|_{\mathsf{fun}}$ `\restrictU`

Let t and u be sets, let $f : \mathsf{t} \longrightarrow \mathsf{u}$, and let $T \subseteq \mathsf{t}$. Let $f\,|_{\mathsf{fun}}\,T$ denote the restriction of f to T.

$$
\begin{array}{|l}
\hline
[\mathsf{t}, \mathsf{u}] \\
\_\,|_{\mathsf{fun}}\,\_ : (\mathsf{t} \longrightarrow \mathsf{u}) \times \mathbb{P}\,\mathsf{t} \longrightarrow (\mathsf{t} \nrightarrow \mathsf{u}) \\
\hline
\forall\, f : \mathsf{t} \longrightarrow \mathsf{u};\ T : \mathbb{P}\,\mathsf{t} \bullet \\
\qquad f\,|_{\mathsf{fun}}\,T = T \lhd f \\
\hline
\end{array}
$$