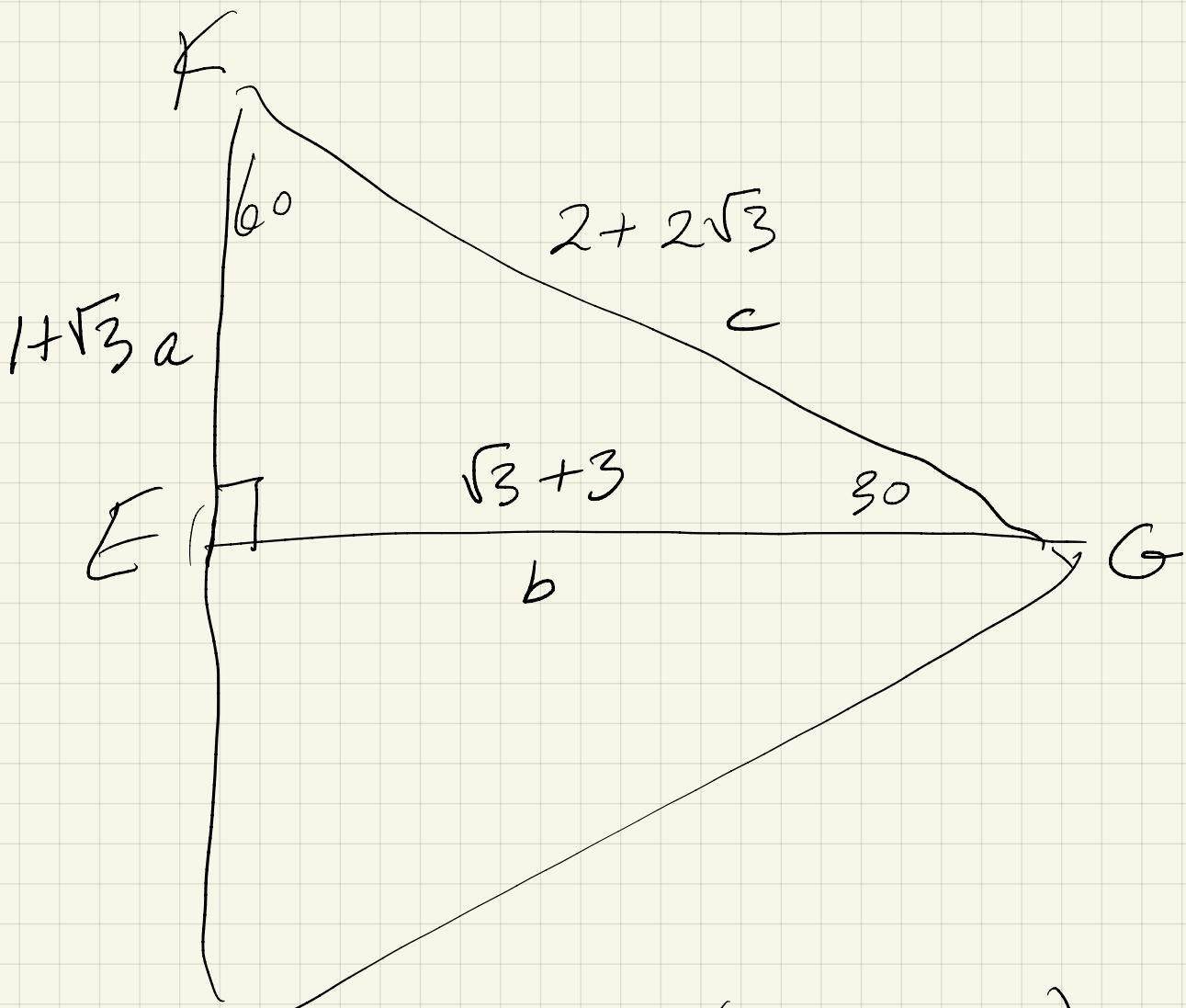


Math With Sean

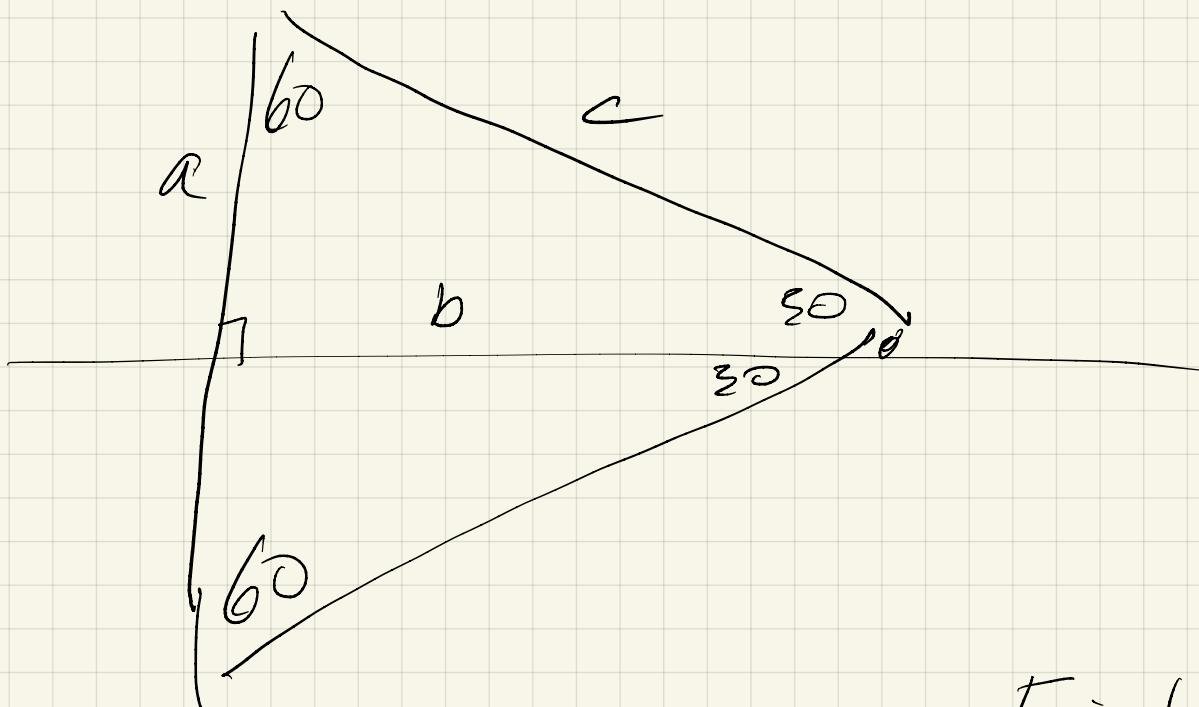
Arthur Ryman
2021-05-22





$$\begin{aligned}
 KE &= \frac{1}{2} GK = \frac{1}{2} (2 + 2\sqrt{3}) \\
 &= 1 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 c^2 &= a^2 + b^2 \\
 b^2 &= c^2 - a^2 = (2 + 2\sqrt{3})^2 - (1 + \sqrt{3})^2 \\
 &= (2(1 + \sqrt{3}))^2 - (1 + \sqrt{3})^2 = (1 + \sqrt{3})(4 - 1) \\
 b^2 &= (1 + \sqrt{3})^2 \cdot 3 \quad b = (1 + \sqrt{3})\sqrt{3} \\
 &= \sqrt{3} + 3
 \end{aligned}$$



Suppose we know a, c . Find b .

$$a = \frac{1}{2}c$$

$$a^2 = \frac{1}{4}c^2$$

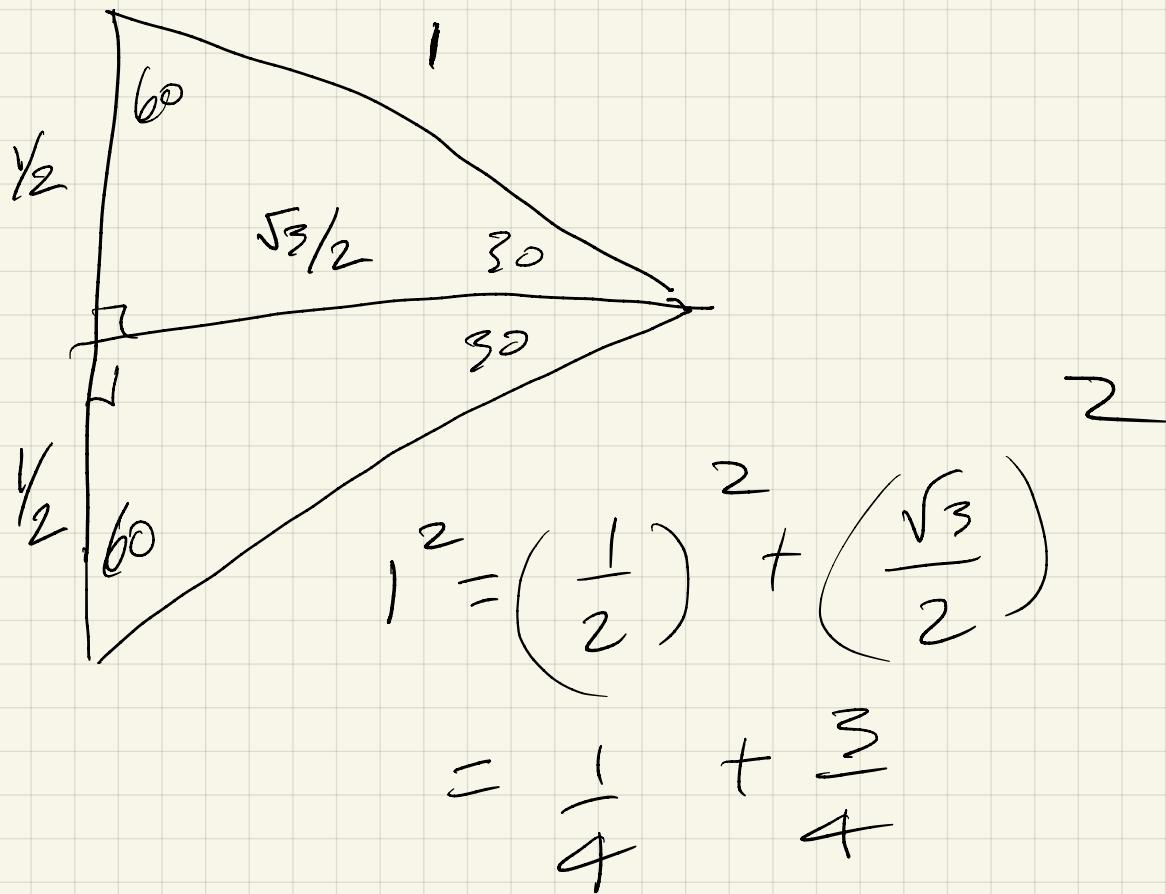
$$a^2 + b^2 = c^2$$

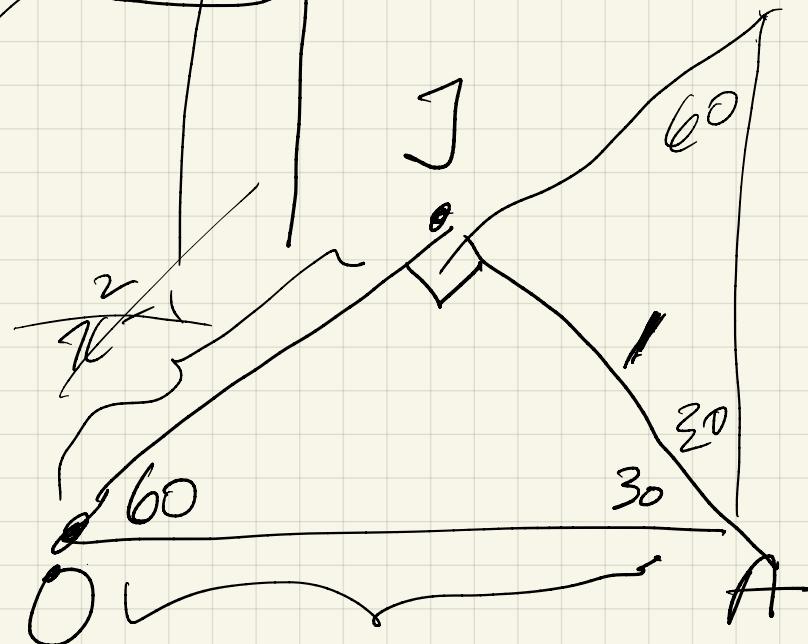
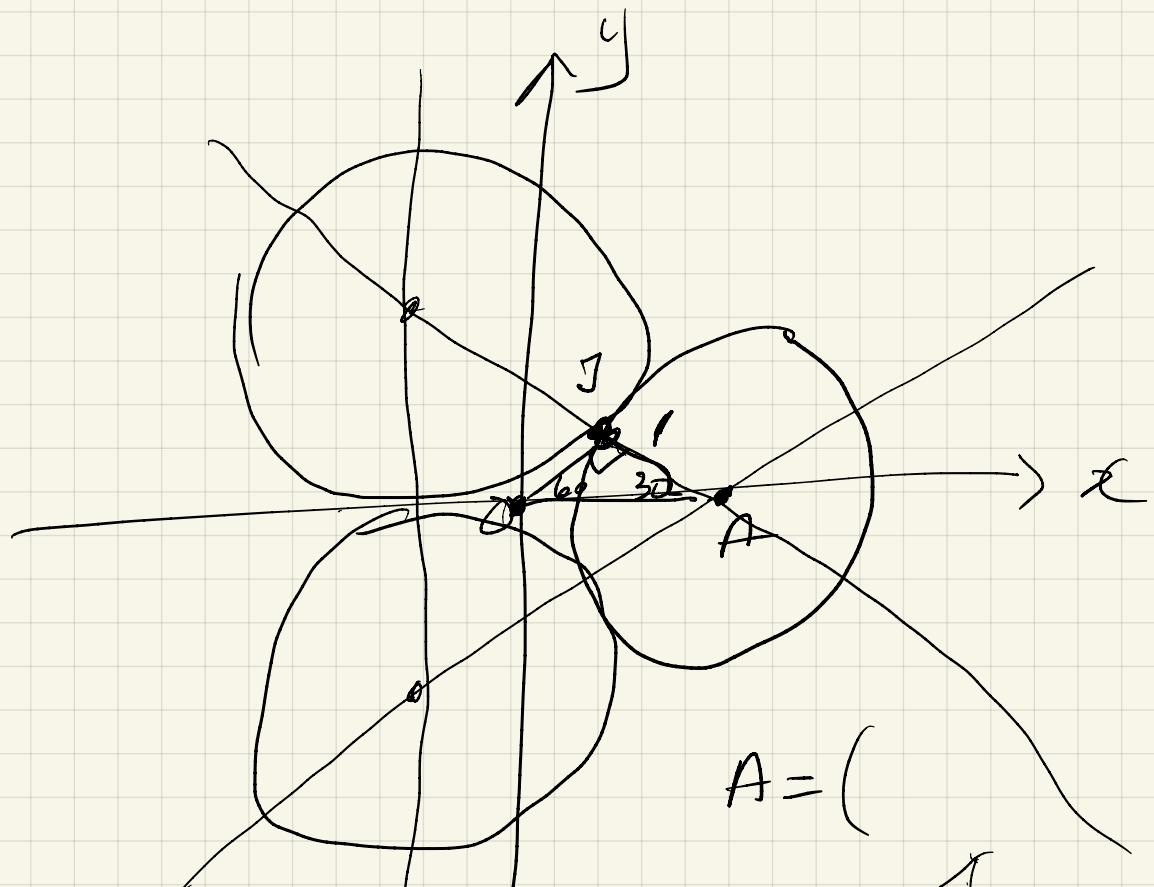
$$b^2 = c^2 - a^2 = c^2 - \frac{1}{4}c^2$$

$$b^2 = c^2 \left(1 - \frac{1}{4}\right)$$

$$b^2 = c^2 \frac{3}{4}$$

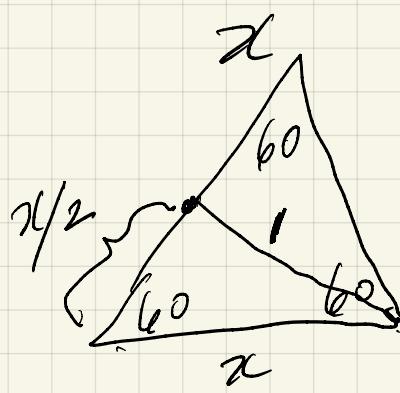
$$b = c \frac{\sqrt{3}}{2}$$





$$x^2 = \frac{4}{3}$$

$$x = \frac{2}{\sqrt{3}}$$

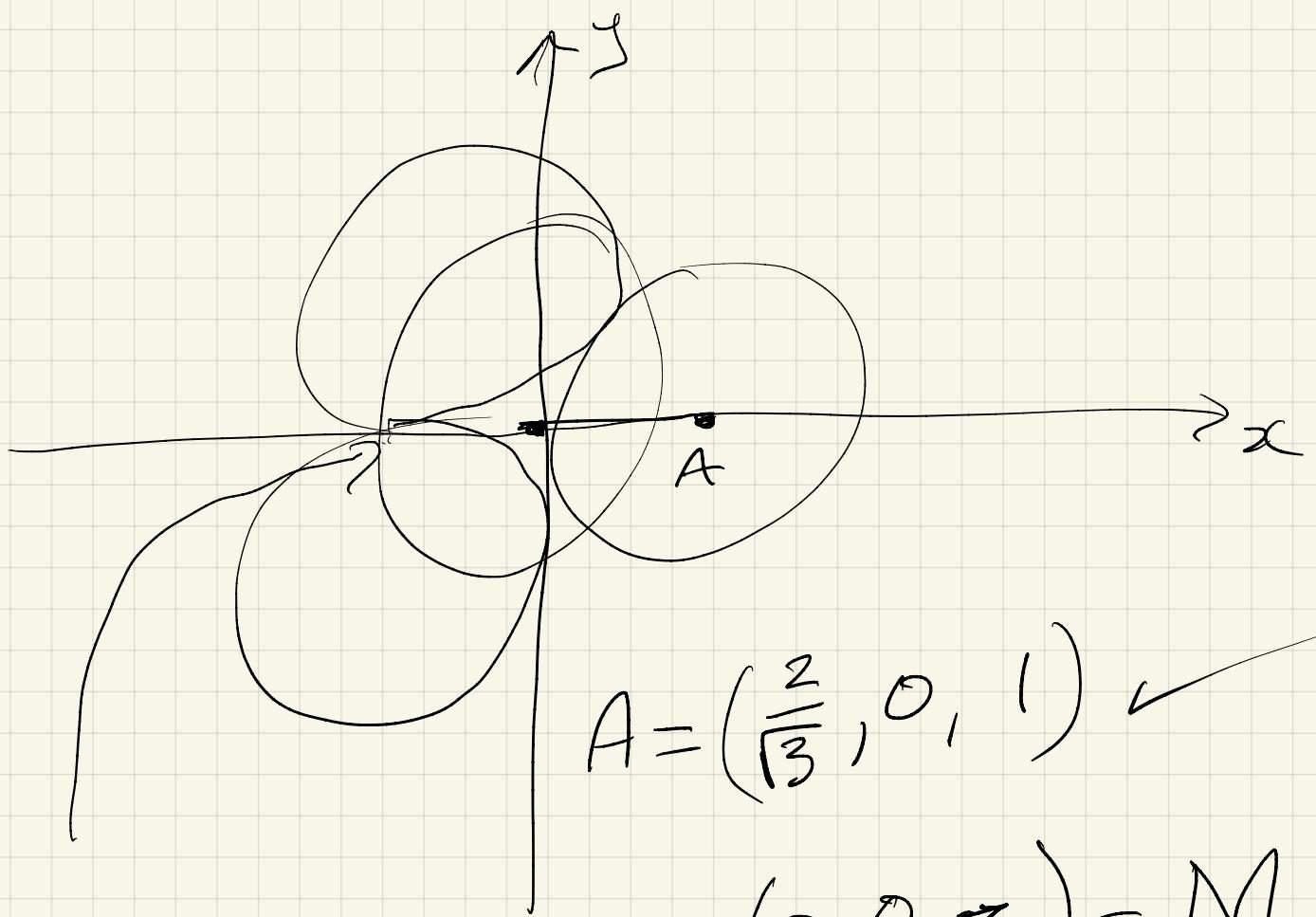


$$\left(\frac{x}{2}\right)^2 + 1^2 = x^2$$

$$\frac{x^2}{4} + 1^2 = x^2$$

$$1 = x^2 - \frac{1}{4} x^2$$

$$= \frac{3}{4} x^2$$



$$A = \left(\frac{2}{\sqrt{3}}, 0, 1\right)$$

centre of 4th sphere = $(0, 0, z) = M$

distance AZ = 2

$$4 - (AM)^2 = \left(\frac{2}{\sqrt{3}}\right)^2 + (z-1)^2 = 4$$

$$(z-1)^2 = 4 - \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 4 - \frac{4}{3}$$

$$= 4 \left(1 - \frac{1}{3}\right) = 4 \cdot \frac{2}{3}$$

$$= 4 \left(\frac{2}{3}\right)$$

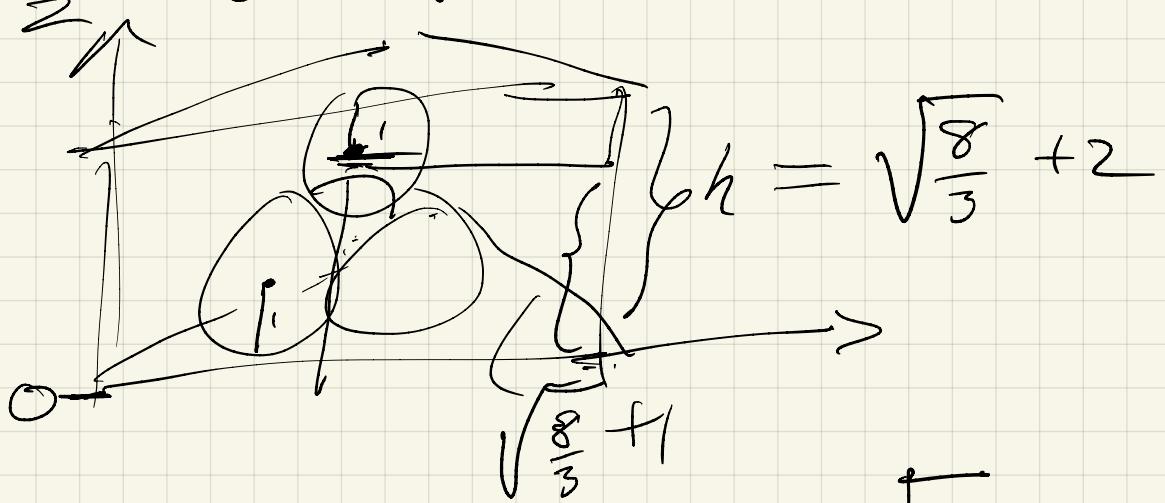
$$\begin{aligned}
 (z-1)^2 &= 4 - \left(\frac{2}{\sqrt{3}}\right)^2 \\
 &= 4 - \frac{4}{3} \\
 &= 4\left(1 - \frac{1}{3}\right) = 4 \cdot \frac{2}{3} \\
 &= 4\left(\frac{2}{3}\right)
 \end{aligned}$$

$$(z-1)^2 = \frac{8}{3}$$

$$z-1 = \sqrt{\frac{8}{3}}$$

$$z = \sqrt{\frac{8}{3}} + 1$$

What's the height of the prism?



$$h = 2 + \sqrt{\frac{8}{3}} = 2 + 2\sqrt{\frac{2}{3}}$$

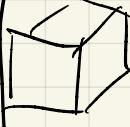
$$h = 2 + 2\sqrt{\frac{2}{3}} = 3.63$$

$$\text{area} = 6 + 4\sqrt{3} = 12.93$$

$$\text{Volume} = h \times \text{area} = \frac{12.93}{12.93} \approx 47.00 \text{ (E)}$$

Cayley 2021 #23

After 2 rolls Robbie has score 8
And Francine has score 10
probability that Robbie wins is
written $\frac{r}{400+s}$ in lowest terms.

 in normal die what is the probability of rolling any number? 1, 2, 3, 4, 5, 6

All are the same $= \frac{1}{6}$

Special die

n	P
1	$a = 1/21$
2	$2a$
3	$3a$
4	$4a$
5	$5a$
6	$6a$
Something	$1 = a + 2a + 3a + 4a + 5a + 6a$ $= 21a$ $a = 1/21$

The possible outcomes that result in a win for Robbie-

		1	2	3	4	5	6
		1	2	3	4	5	6
1							
2							
3							
4	✓						
5	✓	✓					
6	✓	✓	✓				

There are 6 outcomes that win for Robbie

$$(4, 1) \text{ Tax } a = 4a^2$$

$$(5, 1) \quad 5a^2$$

$$(5, 2) \quad 10a^2$$

$$(6, 1) \quad 6a^2$$

$$(6, 2) \quad 12a^2$$

$$(6, 3) \quad 18a^2$$

$$55a^2 \checkmark$$

probability of Robbie winning

is

$$55a^2 = 55 \times \left(\frac{1}{21}\right)^2$$

$$\frac{21}{21}$$

$$= \frac{55}{441} = \frac{5 \times 11}{3^2 \times 7^2}$$

$$\frac{42}{441}$$

$$= \frac{55}{400+41} = \frac{5}{400+5}$$

$$r = 55 \quad s = 41$$

$$r+s = 96 \quad (A)$$