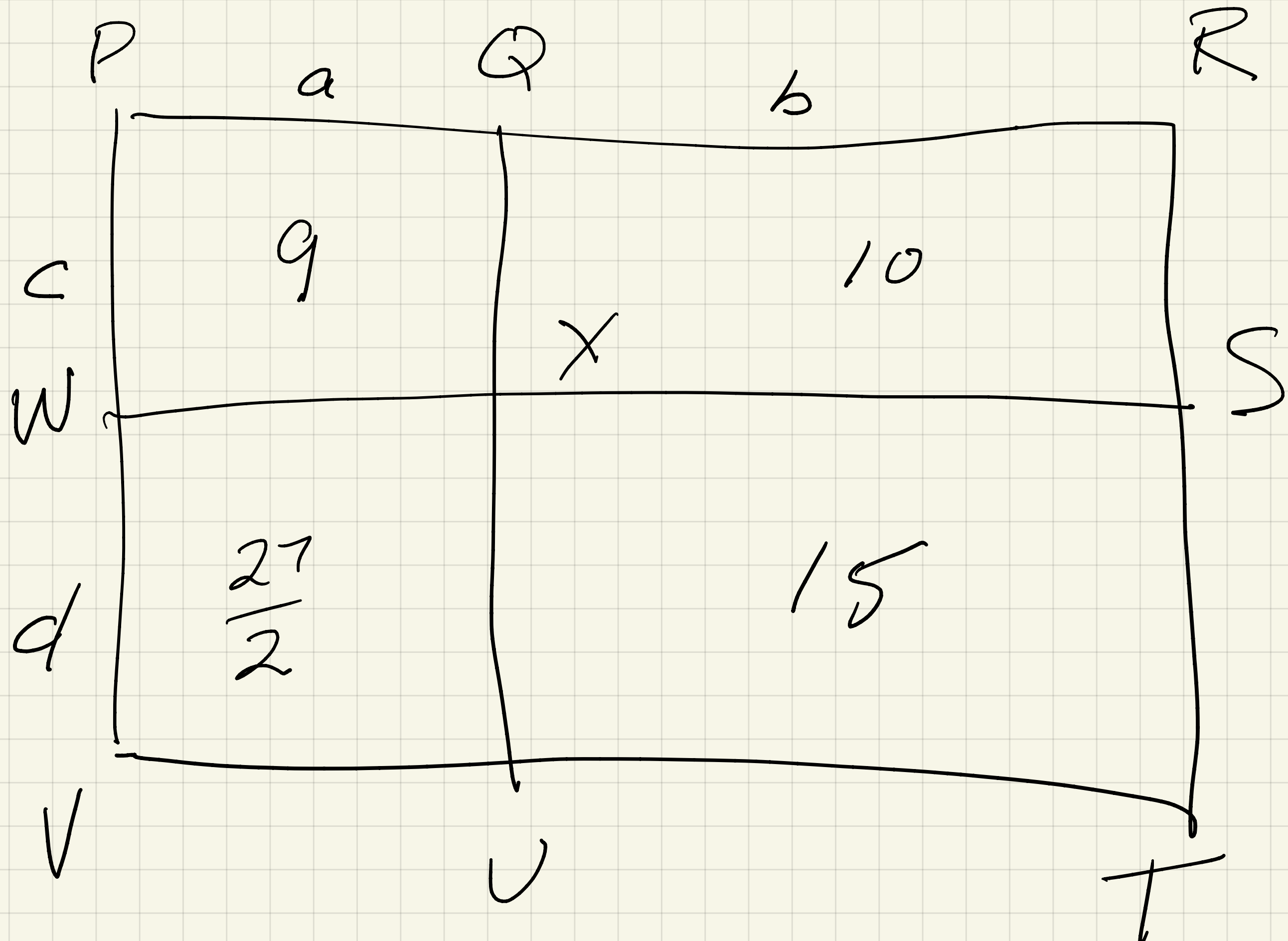
Math with Sean

Arthur Ryman

2020-06-13

2012 Pascal Contes 2020-06-13

19



$ac = 9$

$bc = 10$

$bd = 15$

$ad = ?$

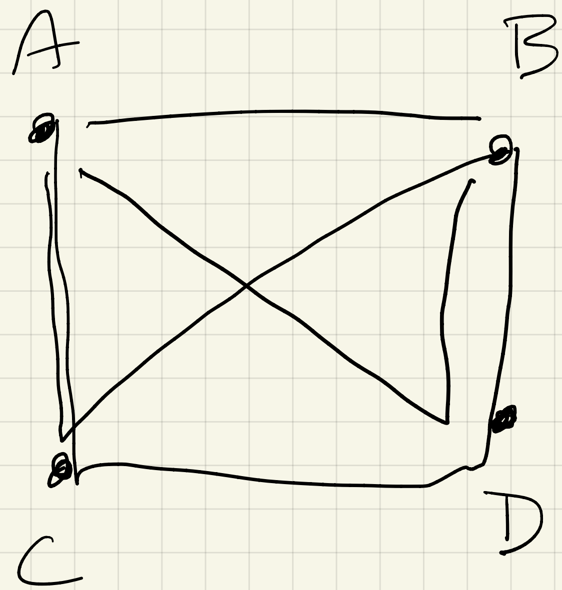
$$\frac{ac \times bd}{bc} = \frac{abcd}{bc}$$

$$= ad$$

$$\frac{9 \times 15}{10} = \frac{9 \times 3 \times \cancel{5}}{2 \times \cancel{5}}$$

$$= \frac{27}{2} = (B)$$

25



Prob A is connected to B

A+B are friends $.5 = \frac{1}{2}$

A+C, C+B are friends $.25 = \frac{1}{4}$

A+D, D+B are friends $.25 = \frac{1}{4}$

A+C, C+D, D+B are friend $.125 = \frac{1}{8}$

F - 64 outcomes

AB
AC
AD
BC
BD
CD

Total probability of a disconnected grap

$$P_D = \frac{1}{64} + \frac{6}{64} + \frac{15}{64} + \frac{4}{64}.$$

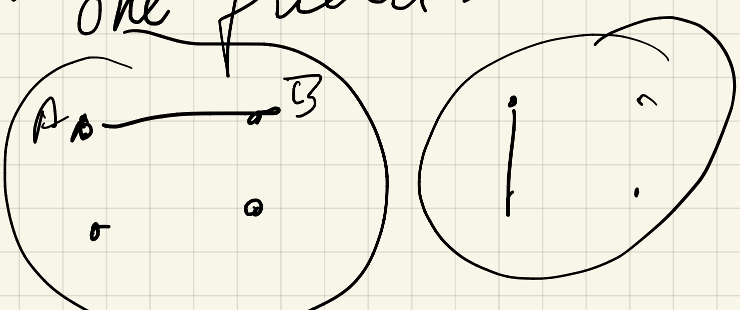
$$= \frac{26}{64} = \frac{13}{32}.$$

$$P_C = 1 - P_D = 1 - \frac{13}{32} = \boxed{\frac{19}{32}}$$ (D)

N = 0

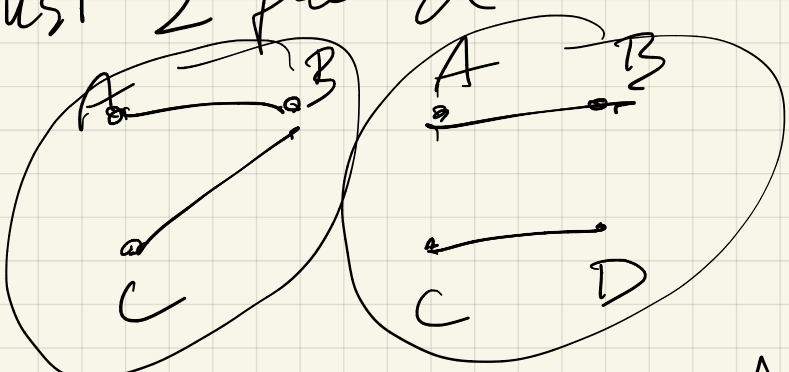
no one has any friend $\frac{1}{64}$

just one friend.



$p = \frac{6}{64}$ N=1

just 2 friend



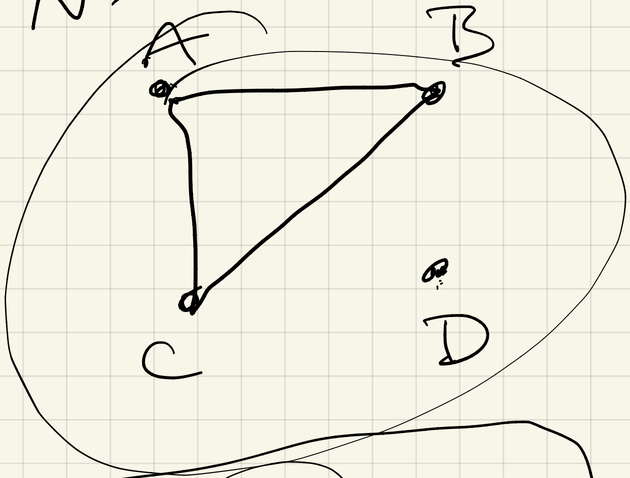
$$\frac{6 \times 5}{2} = 15$$

$p = \frac{15}{64}$

N=2

N=3 $\frac{4}{64} = p$

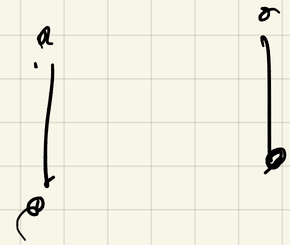
$N=3$

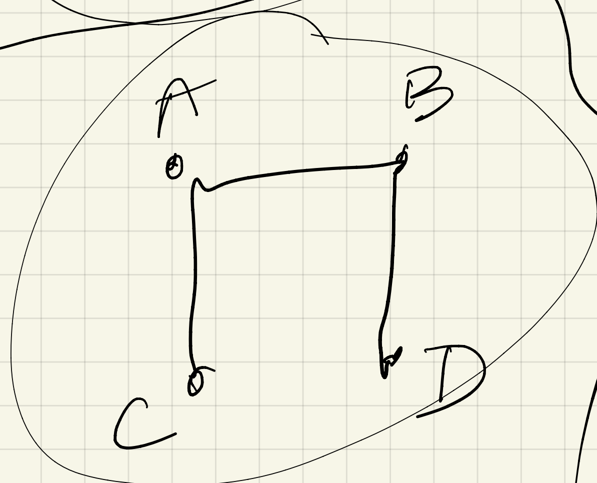


not connected

$N=4$

$p = \frac{1}{4} = \frac{4}{64}$

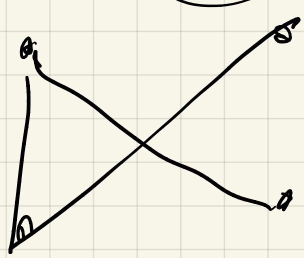


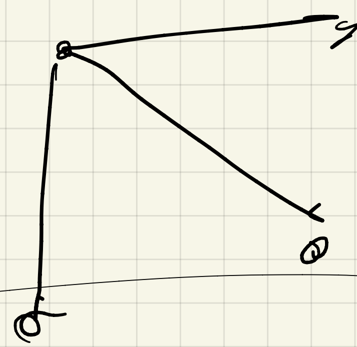


connected

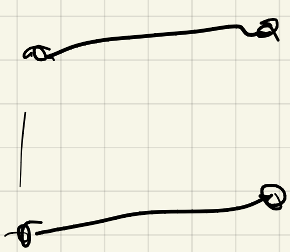
$$\boxed{\frac{6 \cdot 5 \cdot 4}{3!} = 20}$$

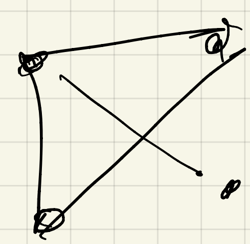
4 16





$N=4$

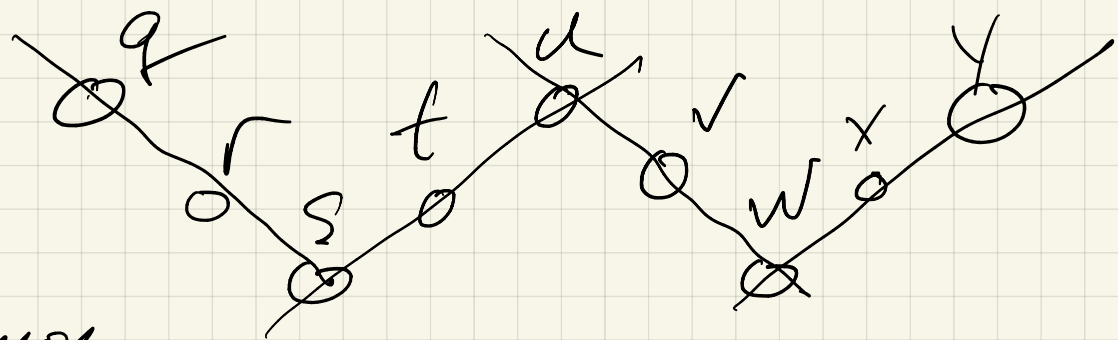




0

24.

2012, 2013, . . . , 2020 − 9 consecuti
integers



Common
Sum = C

$$q + r + s = C$$
$$s + t + u = C$$
$$u + v + w = C$$
$$w + x + y = C$$

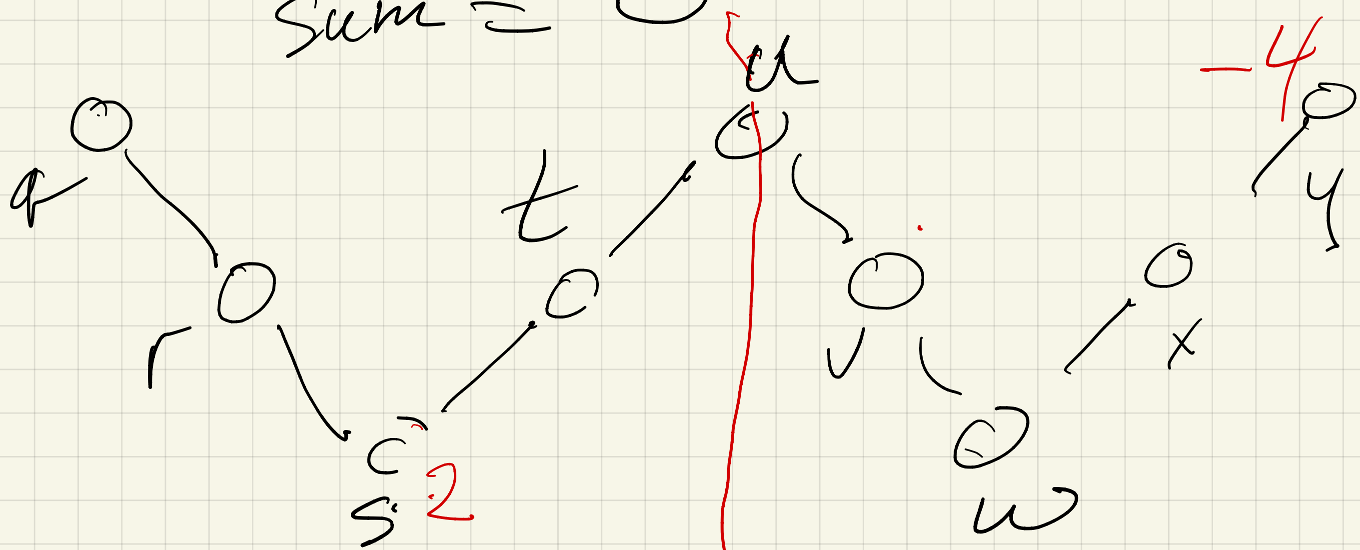
$$\overbrace{\boxed{q+r+s+t+u+v+w+x+y} + \boxed{s+u+w}} = 4C$$

$$q = 2012 + q'$$
$$r = 2012 + r'$$
$$s = 2012 + s'$$

$$q + r + s = C$$
$$= 3 * 2012 + \underbrace{q' + r' + s'}_{C'}$$

$$C = 3*2012 + c' \quad C'$$

$$\underbrace{-4, -3, -2, -1, 0, 1, 2, 3, 4}_{\text{sum} = 0}$$



$$\boxed{4c = s + u + w}$$

$-4 + -3 + -2 = -9$ ×

$c = 0$

$c = s + t + u$

$3c = w - u$

Let $A = \{2012, 2013, \ldots, 2020\}$

& Let B be a 9-tuple such that

$$\text{set}(B) = A$$

So B is a permutation of A.

or $B : 0..8 \rightarrowtail A$ is

a bijection.

Let $B = (q, r, s, t, u, v, w, y, z)$

& $q + r + s = C$ for some C

$s + t + u = C$

$u + v + w = C$

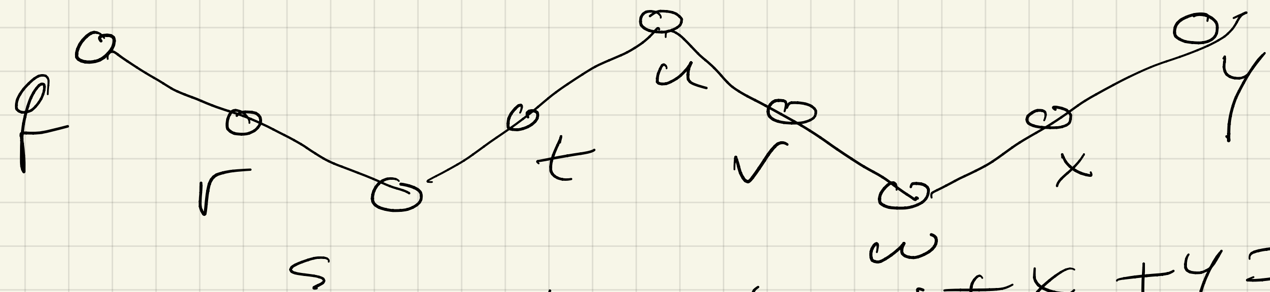
$w + y + z = C$

Let $T : A \to \{-4, \ldots, 4\} = A'$

$$T\alpha = \alpha - 2016 = \alpha'$$

Then $C \mapsto C - 3 * 2016 = C'$

$$B' = (q - 2016, \ldots, z - 2016)$$



$$q + r + s + t + u + v + w + x + y = 0$$

$$s + u + w = 4C$$

Given a solution, we can perform the following symmetries.

1. Reflect about u

ie $(q, r, s, t, u, v, w, x, y)$

$\mapsto (y, x, w, v, u, t, s, r, q)$

This reverses the sequence.

R

2. Swap q, r

$(q, r, s, t, u, v, w, x, y)$

$\mapsto (r, q, s, t, u, v, w, x, y)$

So the orbit of a solution consists of 8 points I, R, S, RS

$s+u+w = 4C$

the largest C is 2

e.g. $s=4, u=3, w=1$

$C = -2, -1, 0, 1, 2$ are the only solutions.

if $B \to -B$ then $C \to -C$

Try $C=-2$ so $s+u+w = -8$

so $s, u, w \in -4, -3, -1$

$s+t+u = C = -2$

$s+u \in -7, -5, -4$

$t \in \cancel{5}, 3, 2$

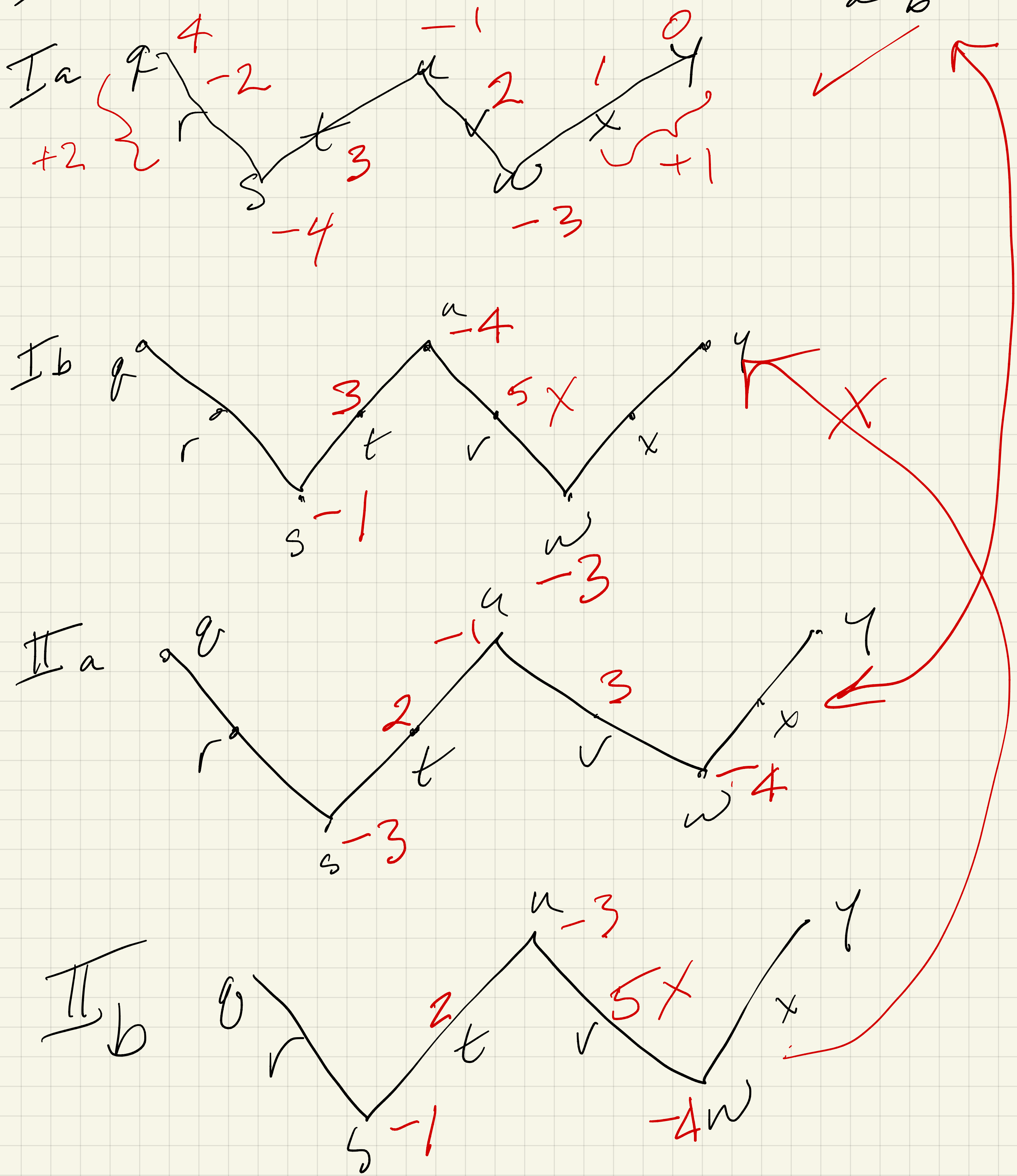
$w = \quad -3 \quad -4$

$t=3, w=-3, s+u=-5 \Rightarrow s, u \in \{-4, -1\}$

$t=2, w=-4, s+u=-4 \Rightarrow s, u \in \{-3, -1\}$

I $t=3,\ w=-3,\ s+u=-5 \Rightarrow s,u \in \{\underset{a}{-4}, \underset{b}{-1}\}$

II $t=2,\ w=-4,\ s+u=-4 \Rightarrow s,u \in \{\underset{a}{-3}, \underset{b}{-1}\}$



The reduced $u = -1$

ie $u' = -1$

so $u' = u - 2016$

$u = u' + 2016$

$= -1 + 2016$

$= 2015$ (D)

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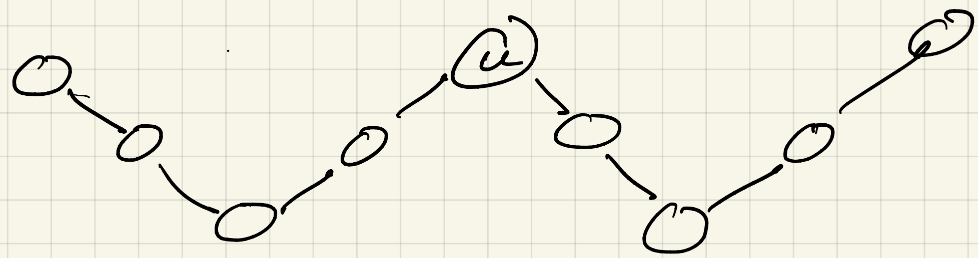
24. Assign the 9 consecutive integers

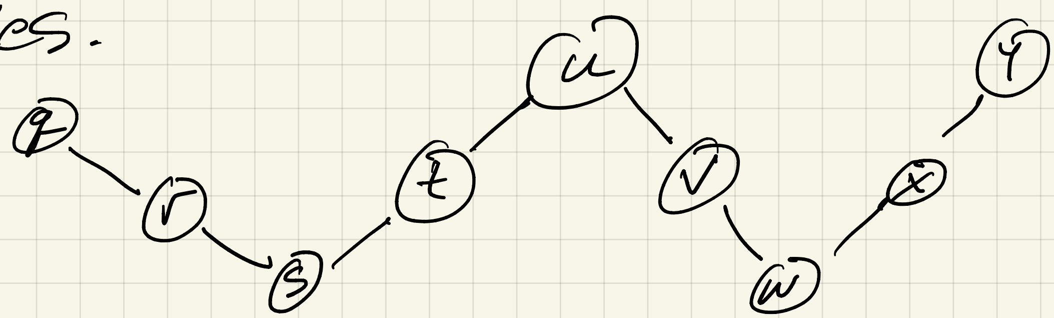
2012, 2013, ..., 2020

to the circles



Such that the sum along each of the four lines is the same.
If the sum is as small as possible, what is u?

Solution. Introduce variables for the other 8 circles.



Let C be the common sum.

$$q + r + s = C$$
$$s + t + u = C$$
$$u + v + w = C$$
$$w + x + y = C$$

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Add the four equations

$$q+r+s+s+t+u+u+v+w+w+x+y = 4c$$

$$(q+r+s+t+u+v+w+x+y) + s+u+w = 4c$$

But $q+r+s+t+u+v+w+x+y$

$$= 2012+2013+\cdots+2020$$

$$= \frac{(2012+2020)}{2}\times 9$$

$$= \frac{4032}{2}\times 9$$

$$= 2016 \times 9$$

$$= 504\times 4\times 9$$

so $504\times 4\times 9 + s+u+w = 4c$

or $s+u+w = 4c - 4\times 504\times 9$

$$= 4(c - 504\times 9)$$

Therefore $s+u+w \equiv 0 \mod 4$

Rather than work with 2012, 2013, …, 2019 let's look at there deviations from the mean 2016. Define $q' = q - 2016$, $r' = r - 2016$, etc.

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under this change, the sum along each line is still constant but is shifted. e.g.

$$C = q + r + s$$
$$= (q' + 2016) + (r' + 2016) + (s' + 2016)$$
$$= q' + r' + s' + 3 \times 2016$$

So $q' + r' + s' = C - 3 \times 2016$
$= C'$

and similarly for the other three lines.

Now we are dealing with the 9 consecutive integers

$$-4, -3, -2, -1, 0, 1, 2, 3, 4$$

and $s' + u' + w' = 4C'$.

The smallest possible value of C' is -2 which occurs for

$$\{s', u', w'\} = \{-4, -3, -1\}$$

Look for any solution.

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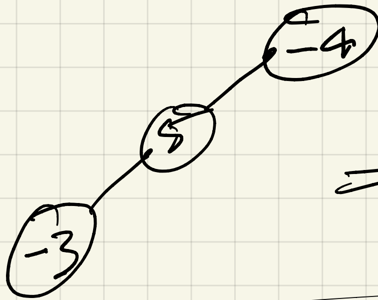
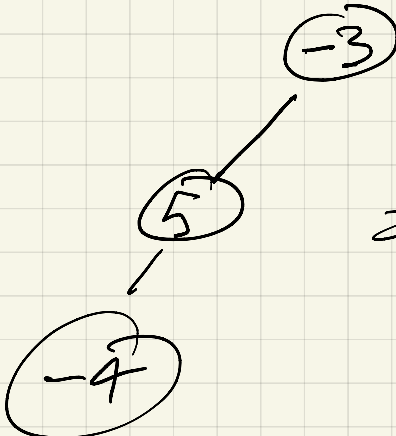
$\{s', u', w'\} = \{-4, -3, -1\}$

$s' + u' + w' = 4c' = -8$ so $c' = -2$

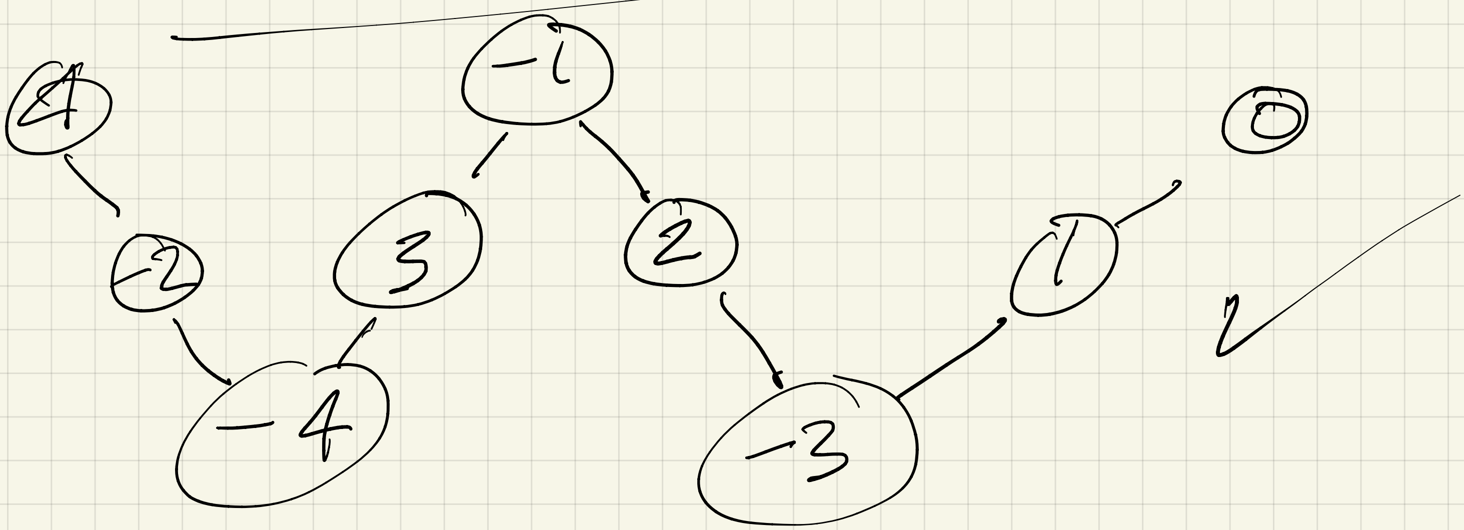


take $s' < w'$

$\Rightarrow u' \neq -4$ because we can flip it?



$\Rightarrow u' = -3$ impossible



So $u' = -1$

$$u = u' + 2016 = -1 + 2016 = 2015$$