Math with Sean

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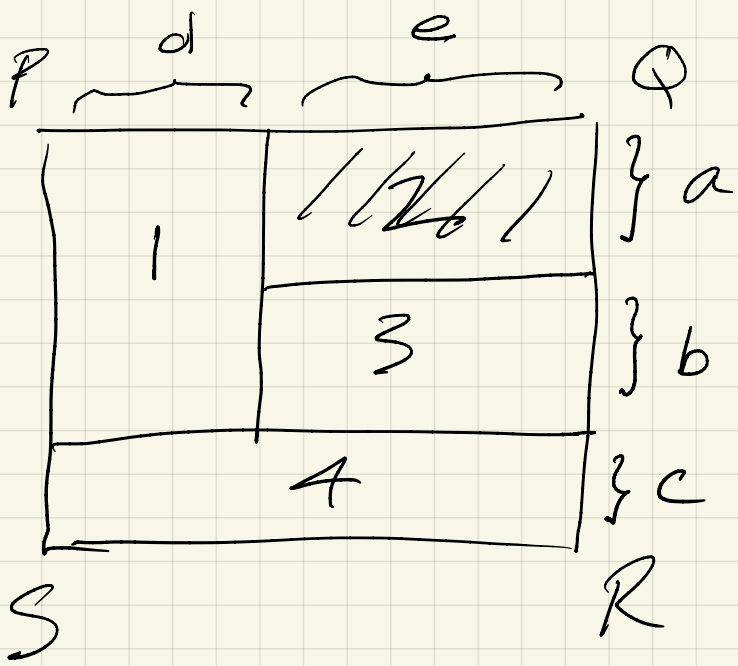
2021-04-24

Cayley 2017

22.



$a+b+c = 42$ (1)

$d+e = 42$ (2)

perimeter 1 $= 2d + 2a + 2b = P$ (3)

perimeter 2 $= 2e + 2a = P$ (4)

perimeter 3 $= 2e + 2b = P$ (5)

perimeter 4 $= 2c + 2d + 2e = P$ (6)

6 equations, 6 variables

Required to find area of (2) $= ae$

$a+b+c=42$ (1)

$d+e=42$ (2)

perimeter 1 $=2d+2a+2b=P$ (3)

perimeter 2 $=2e+2a=P$ (4)

perimeter 3 $=2e+2b=P$ (5)

perimeter 4 $=2c+2d+2e=P$ (6)

Use (6) to eliminate P.

(3') $2d+2a+2b=2c+2d+2e$

$2a+2b=2c+2e$

$a+b=c+e$

(4') $2e+2a=2c+2d+2e$

$2a=2c+2d$

$a=c+d$

(5') $2e+2b=2c+2d+2e$

$b=c+d$

$a=b$

$2a=c+e$

$a+b+c = 42$ (1)

$d+e = 42$ (2)

$a+b = c+e$ (3)

$a = c+d$ (4)

use $a=b$ to eliminate b

$2a + c = 42$ (1')

$2a = c+e$ (3')

$d + e = 42$ (2)

$a = c+d$ (4)

use (4) to eliminate a

$2(c+d) + c = 42$ (1")

$d + e = 42$ (2)

$2(c+d) = c+e$ (3)

Simplify (1") $3c + 2d = 42$ (1")

Simplify (3) $2c + 2d = c+e$ (3)

$c + 2d = e$

$d + e = 42$ (2)

$3c + 2d = 42$ (1'')

$2c + 2d = c + e$ (3)

$c + 2d = e$ (2)

$d + e = 42$

Eliminate e using (3)

$3c + 2d = 42$ (1)

$c + 3d = 42$ (2)

$c = 42 - 3d$ (2')

Eliminate c using (2)

$3(42 - 3d) + 2d = 42$ (1)

$126 - 9d + 2d = 42$

$126 - 7d = 42$

$42 + 7d = 126$

$7d = 84$

$d = 84/7 = 12$

$d = 12$

$c = 42 - 3d$

$d = 12$

$c = 42 - 3 \cdot 12$

$= 42 - 36$

$= 6$

$c + 2d = e$

$e = 6 + 2 \cdot 12$

$= 6 + 24$

$= 30$

$a = c + d$

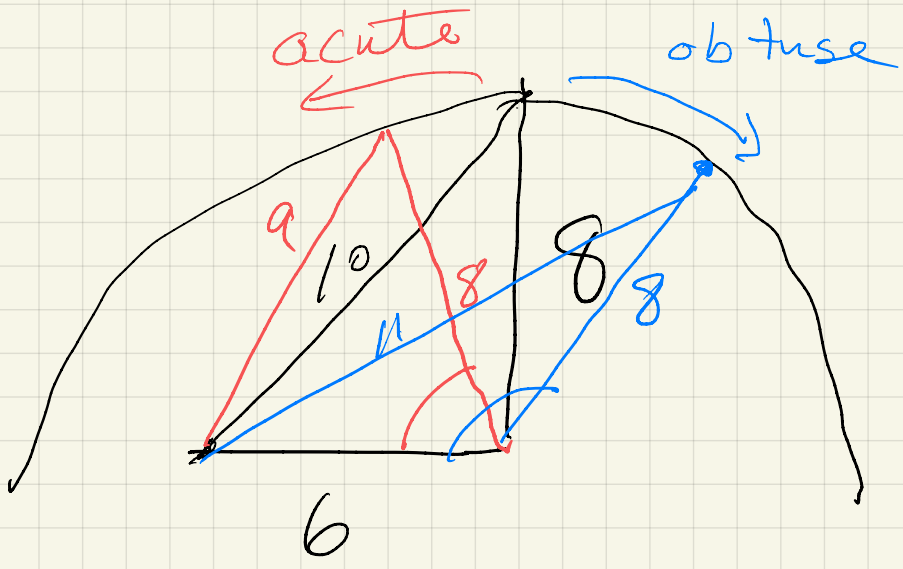
$= 6 + 12$

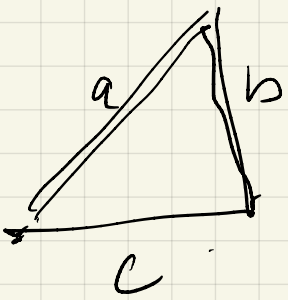
$= 18$

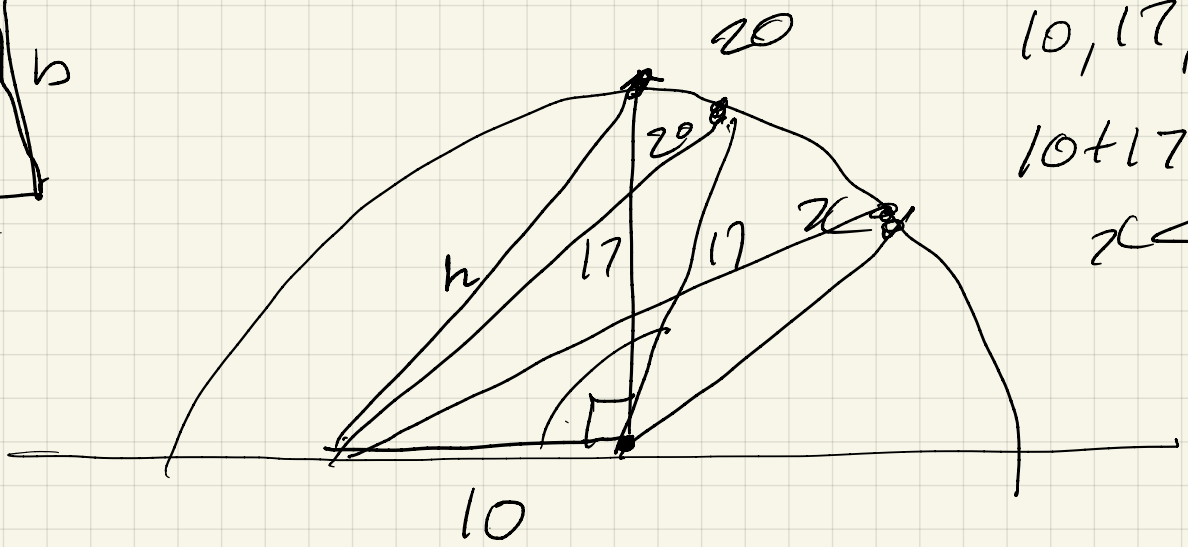
Area $= 18 \cdot 30$

$= 540$ (E)

23.







10, 17, x

$10+17 > x$

$x < 27$

$h = \sqrt{389} \approx 19.7\ldots$

acute $19^2 = 361 < 389$

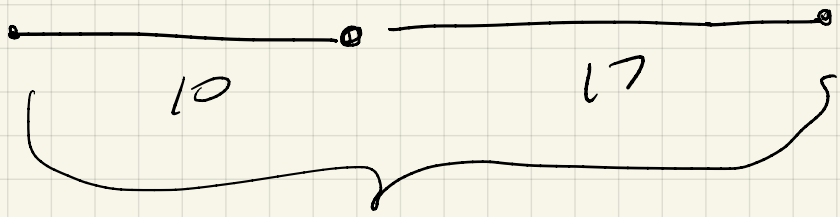
obtuse $20^2 = 400 > 389$

Does 20 17 10 form a triangle

triangle inequality

a, b, c

$\begin{cases} a+b > c \\ a+c > b \\ b+c > a \end{cases}$



27

$19 < x < 27$

$20, 21, \ldots, 26$

Sum $= 20+21+22+23+24+25+26$

$= 7 \cdot 23$

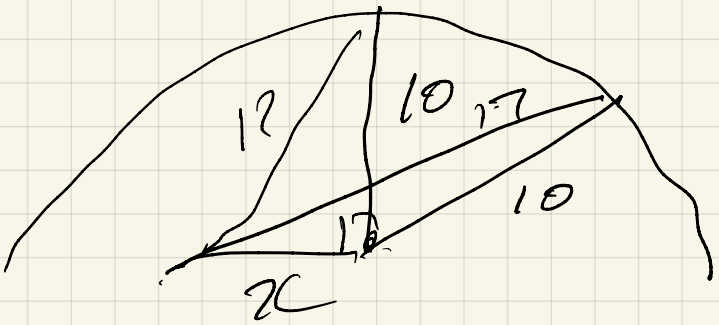
$= 161$ (A)

$x \leq 17$

$10 + x > 17$ $\quad x > 7$

$17 + x > 10$ $\quad x \geq 8$

$27 > x$



$17^2 = 10^2 + x^2$ right $<$

$17^2 > 10^2 + x^2$ obtuse

$289 - 100 > x^2$

$\sqrt{189} > x$

$13.75 > x$

$13 \geq x$

$x = 8, 9, 10, 11, 12, 13$

$$8+9+10+11+12+13 = \frac{6\times 21}{2}$$

$$= 3\times 21$$

$$= 63$$

$$161+63 = 224 \ (\text{E})$$