

Math with Sean

2020-04-04



2016 Pascal Contest

#25

a	b ₀	c
d	e ₀	f
g	h ₁	i

Need to count these!

a	b ₁	c
d	e ₁	f
g	h ₁	i

Assume centre = 0. Then double the count.

$$\text{Count}(e=0) \times 2 = \text{Count}()$$

$$b=0 \Rightarrow h=1 \quad \text{Wrong}^*$$

$$\text{Count}(e=0) = 2 \text{Count}(e=0, b=0)$$

$$\text{Count}() = 2 \times \text{Count}(e=0)$$

$$= 2 \times 2 \times \text{Count}(e=0, b=0)$$

$$= 4 \times \text{Count}(e=0, b=0)$$

* I forgot to consider $b=1$ and $h=1$!

Assume
 $d=0$

a	b 0	c
d 0	e 0	f 1
g	h 1	i

$$\text{Count}(e=0, b=0) = 2 \text{Count}(e=0, b=0, d=0)$$

$$\Rightarrow \text{Count}() = 4 \cdot \text{Count}(e=0, b=0) \\ = 8 \cdot \text{Count}(e=0, b=0, d=0)$$

Assume $a=0$ leads to 1 solution

a 0	b 0	c 1
d 0	e 0	f 1
g 1	h 1	i 0

Assume
 $\alpha = 1$

a 1	b 0	c 0
d 0	e 0	f 1
g 0, 1	h 1	i 0, 1

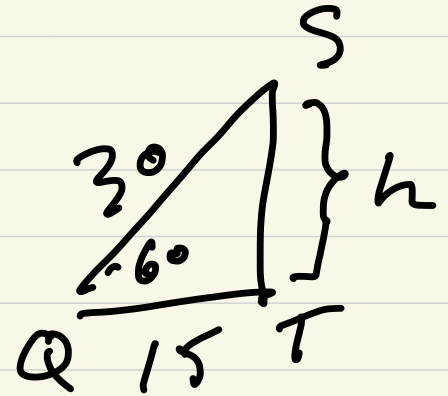
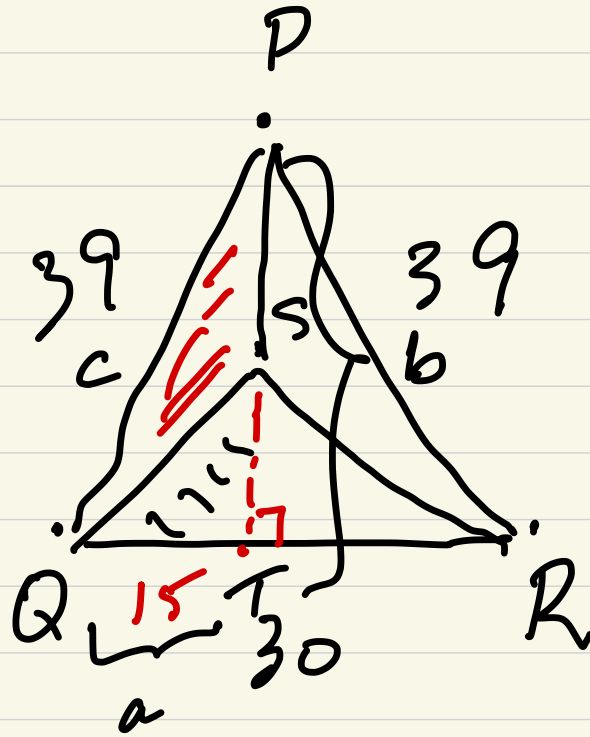
diagonal

C	g	i	
0	0	0	✓
0	0	1	✓
0	1	0	✓
0	1	1	x
1	0	0	✓
1	0	1	x
1	1	0	✓
1	1	1	x

5

WRONG!

23.



$$c = 39 \quad a = 15$$

$$b^2 = c^2 - a^2$$

$$b = \sqrt{39^2 - 15^2}$$

$$\Delta PQT = \frac{15 \times b}{2}$$

$$h^2 = 30^2 - 15^2$$

$$= 15^2(3)$$

$$h = 15\sqrt{3}$$

$$\Delta SQT = \frac{15 \cdot h}{2}$$

$$\Delta PQS = \Delta PQT - \Delta SQT$$

$$= 15 \sqrt{39^2 - 15^2} - \frac{15 \cdot 15\sqrt{3}}{2}$$

$$= \frac{15}{2} \left(\sqrt{39^2 - 15^2} - 15\sqrt{3} \right)$$

22.

$$n > 0$$

$$n \neq 4m \quad n \bmod 4 \neq 0$$

$$n = 4\underline{a} + b \quad a = \# \text{ of complete groups of } 4$$

$$0 < b < 4$$

$$b = 1, 2, 3$$

$$n = 3c + d$$

$$d = 1, 2$$

$$c = a + 3$$

$$n = 2e + f$$

$$f = 1$$

$$\begin{aligned} e &= c + 5 \\ &= a + 3 + 5 \\ &= a + 8 \end{aligned}$$

$$n = 2(a + 8) + 1$$

$$= 2a + 16 + 1$$

$$= 2a + 17 = 4a + b$$

$$17 = 2a + b$$