Math With Sean

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2020-12-05

Galois 2020

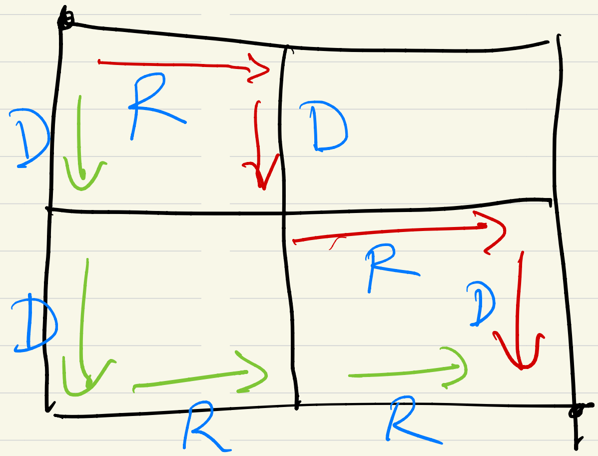
4) (c)

2 × 2

# of paths of length 4

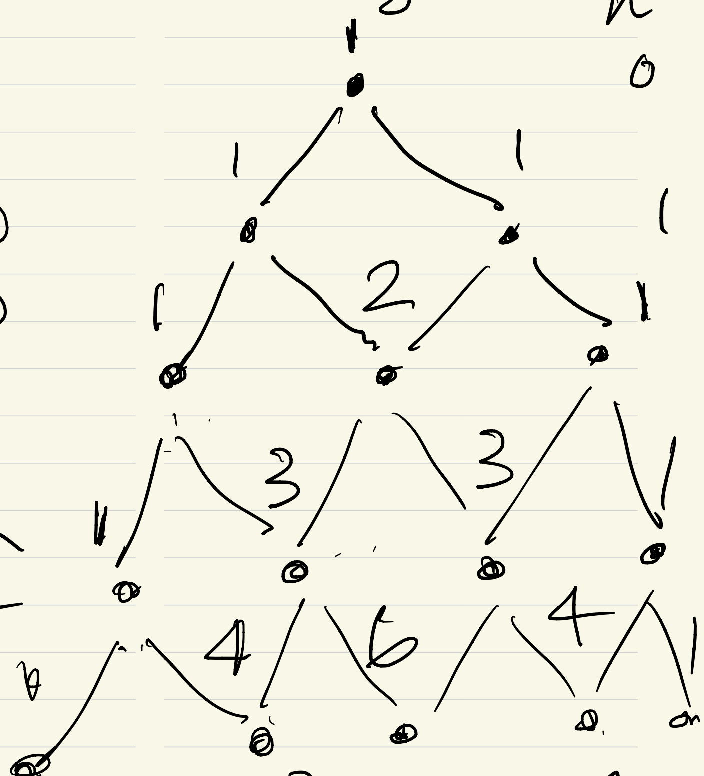
2R, 2D

$\binom{4}{2} = 6$



Moves = {L, D, U, R}

Pascal's Triangle



$\text{Path}_1 = RDRD$

$\text{Path}_2 = DDRR$

$$(R+D)^4 = RRDD + RDRD + \cdots + DDRR = \alpha R^2 D^2$$

$$(R+D)^4 = R^4 + 4DR^3 + 6D^2R^2 + 4D^3R + R^4$$

$$(R+D)^4 = R^4 + 4R^3D + 6R^2D^2 + 4RD^3 + D^4$$

$$= \binom{4}{4}R^4 + \binom{4}{3}R^3D + \binom{4}{2}R^2D^2 + \binom{4}{1}RD^3 + \binom{4}{0}D^4$$

$\binom{n}{k}$ = n choose k

$$= \frac{n!}{k!\,(n-k)!} \qquad \boxed{0! = 1}$$

$$\binom{4}{4} = \frac{4!}{4!\,0!} = 1$$

binomial coefficients

$$\binom{4}{3} = \frac{4!}{3!\,1!} = 4$$

$$\binom{4}{2} = \frac{4!}{2!\,2!} = 6$$

2 x 2 grid, paths of length 4

RR DD
RD RD
RD DR
DD RR
DR DR
DR RD

$6 = \binom{4}{2}$

3 x 3 grid, path of length 6

paths of length 6 $= \binom{6}{3}$

$$\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{3! \times \cancel{3!}}$$

$$= \frac{\cancel{6} \times 5 \times 4}{\cancel{3} \times \cancel{2} \times 1}$$

$$= 20$$

4x4 grid, paths of length 8

$$\# \text{ of paths} = \binom{8}{4} = \frac{8!}{4!\,4!}$$

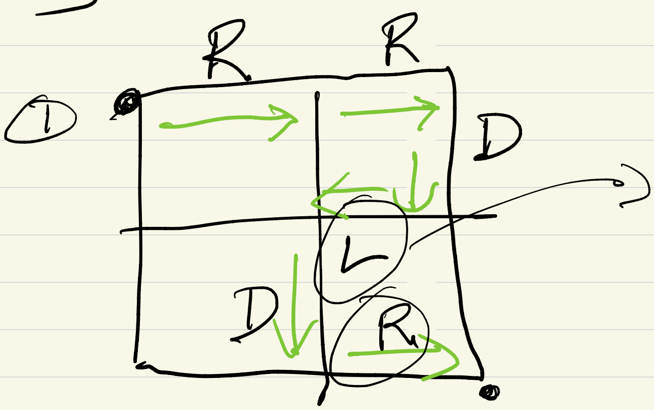
$$= \frac{\cancel{8} \times 7 \times \overset{2}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{4!}}$$

$$= 7 \times 2 \times 5$$

$$= 70$$

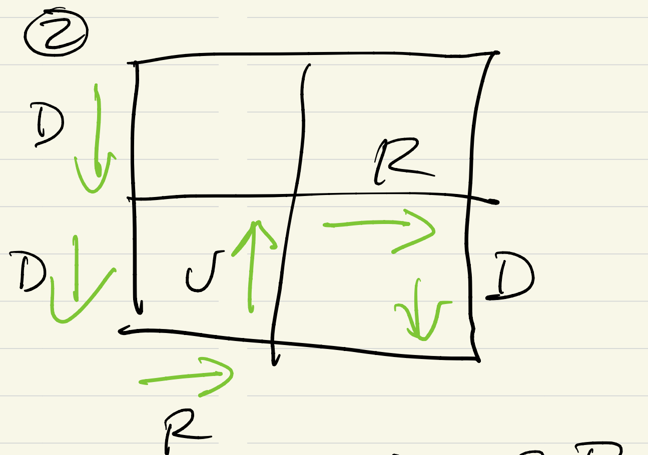
2 × 2 grid − 6 paths of length 4

Try to count paths of length 6



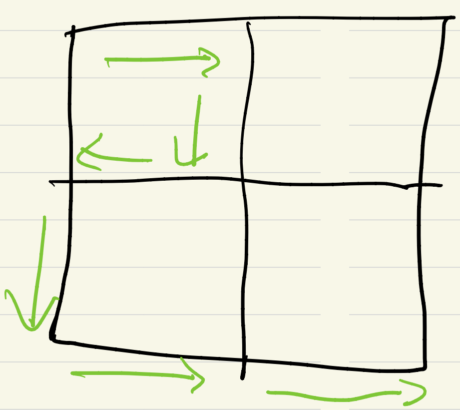
3R, 1L, 2D

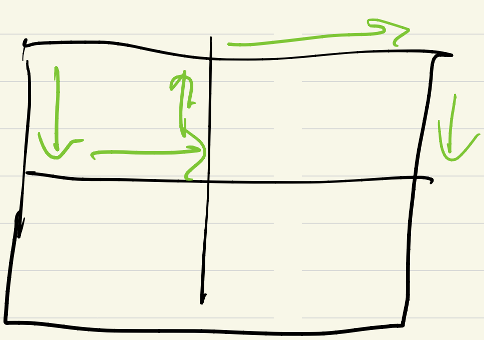
RRDLDR



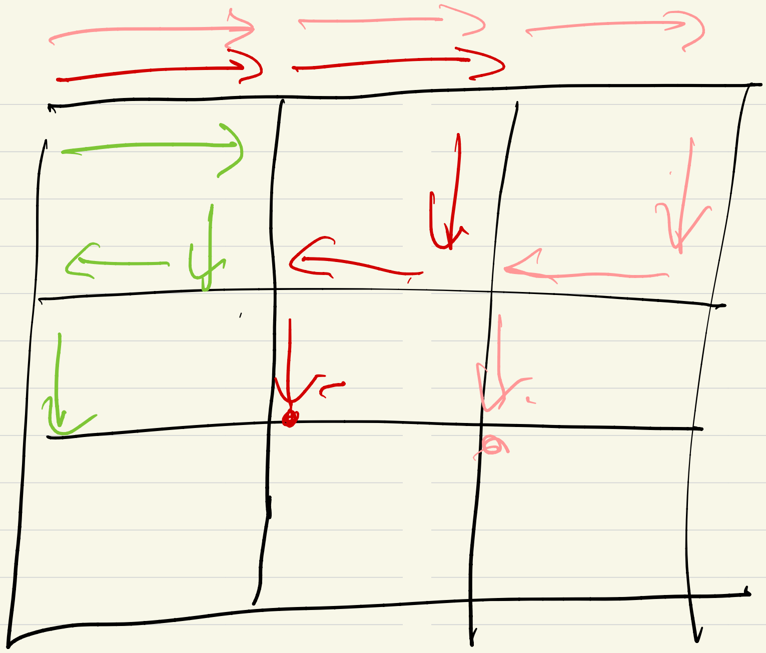
3D, 1U, 2R

DD RURD





The number of paths of length 6 = 4.



$6 + 2 = 8$

R(DLD) D ~~R~~ R R   4R, 1L, 3D

R R (D L D) D R R

R R R (D L D) D R

RL is illegal

LR is illegal

Can't end on DLD

Can't start on DLD

3x3 grid

$$\overbrace{A \quad \underline{DLD} \quad B}^{8}$$

$\#A > 0$ $\#B > 0$

#A	DLD	#B	Total
1	3	4	8
2	3	3	8
3	3	2	8
4	3	1	8

$$\{4R, 1L, 3D\} - \{DLD\}$$

$$= \{\underbrace{4R, 1D}_{5}\}$$

$$\binom{5}{4} = 5$$

#
$\boxed{DRRRR}$
RDRRR
RRDRR
RRRDR
RRRRD

5

A DLD B   AB = "DRRRR"

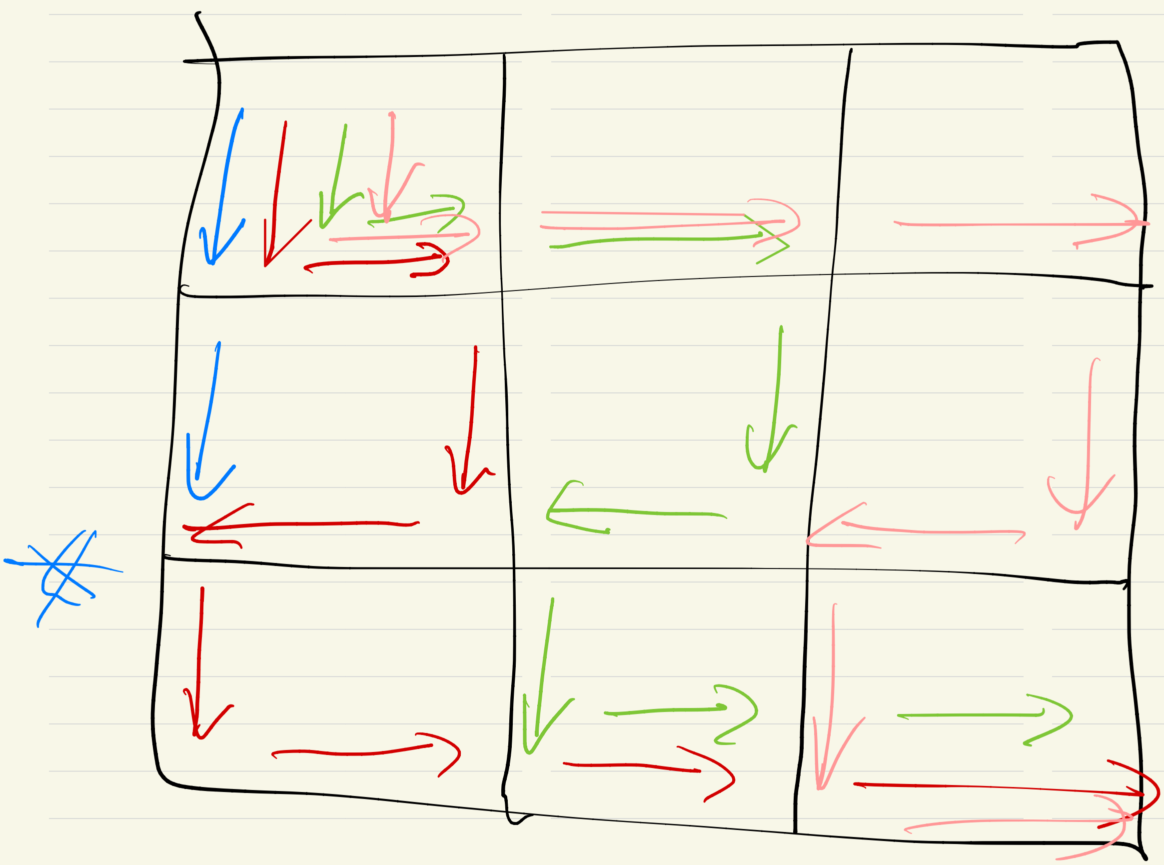
#A

1. D DLD RRRR ✗

(2) DR DLD RRR ✓

(3) DRR DLD RR ✓

(4) DRRR DLD R ✓



1) There is always DLD in the middle.

2) There MUST be at least 1 R in A

3) The path is of length 8 and has 4R, 1L, 3D

#A = 1, 2, 3, 4

AB = 1D, 4R

#A		#B		#
1	= R	4	3R, D	$\binom{4}{3} = 4$
2	$\boxed{1R\ 1D}$	2 $\underline{3}$	3R	1 2
	$\boxed{2R}$	1	2R 1D	3 $\frac{3}{5}$

4 x 4 has minimum path 8

enumerate paths of length 10

Consider paths with one L move.

All paths are like A DLD B

where A has at least 1 R

$ADLDB = \{4D, 1L, 5R\}$

$\# A = \{\underline{1, 2, 3}, 4, 5, 6\}$

$AB = \{2D, 5R\}$

#	A		B		
1	R		$\{2D, 4R\}$	$\binom{6}{4} = \frac{6!}{4!\,2!}$	$= 15$
2	$\{2R\} = 1$		$\{2D, 3R\}$	$\binom{5}{3} = 10 \times 1$	$= 10$
	$\underline{\{R, D\}} = 2$		$\{D, 4R\}$	$\binom{5}{4} = 5 \times 2$	$= 10$
3	$\{3R\} = 1$		$\{2D, 2R\}$	$\binom{4}{2} = 6 \times 1$	$= 6$
	$\{2R, D\} = 3$		$\{1D, 3R\}$	$\binom{4}{3} = 4 \times 3$	$= 12$
	$\{R, 2D\} = 3$		$\{4R\}$	$1 \quad 1 \times 3$	$= 3$