Math With Sean

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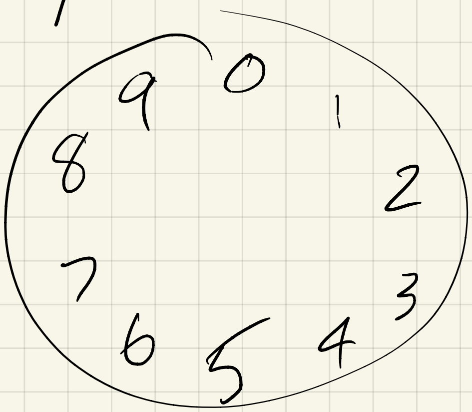
2021-03-27

Cayley 2010 #25



$$f(0) = 0$$
$$f(1) = f(0) + 1^1$$
$$f(2) = f(1) + 2^2$$

mod 10

$$f(n) = f(n-1) + n^n \pmod{10}$$

$$f(n) = \sum_{k=1}^{n} k^k \quad \text{for } n \geq 1$$

Required to find $f(1234)$ mod 10

n	n^n	mod 10
1	$1^1 = 1$	1
2	$2^2 = 4$	4
3	$3^3 = 27$	7
4	$4^4 = 256$	6
5	$5^5 = 3125$	5
6	$6^6 = 46656$	6

$$6^6 \pmod{10} = 6^3 \times 6^3 \pmod{10}$$

$$= 6^3 \pmod{10} \times 6^3 \pmod{10}$$

$$6 \xrightarrow{\times 6} 36 \xrightarrow{\text{mod } 10} 6 \times 6 \rightarrow 36 \rightarrow 6$$

$$20 \xrightarrow{\text{mod } 10} 0 \times 20 = 0 \xrightarrow{\text{mod } 10} 0$$

if $n \equiv 0 \mod 10$ then $n^n \equiv 0 \mod 10$

$$n^n \mod 10 = (n \mod 10)^n$$

$$21 \xrightarrow{\text{mod } 10} 1 \times 21 \xrightarrow{\text{mod } 10} 1$$

if $n \equiv 1 \pmod{10}$ then $n^n \equiv 1 \mod 10$

$$22 \to 2 \times 22 \to \cancel{\times\times} 4 \to 4$$

$$4 \times 2 = 8 \times 2 \to 6 \times 2 \to 2$$

$n \equiv 2 \mod 10$

$$\begin{tikzcd} 2 \arrow[r] & 4 \arrow[r] & 8 \arrow[r] & 6 \arrow[lll, bend left] \end{tikzcd} \quad \text{4 cycle}$$

$n \equiv 3 \mod 10$

$$\begin{tikzcd} 3 \arrow[r] & 9 \arrow[r] & 7 \arrow[r] & 1 \arrow[lll, bend left] \end{tikzcd} \quad \text{4-cycle}$$

$n \equiv 4 \pmod{10}$

$$\begin{tikzcd} 4 \arrow[r] & 6 \arrow[l, bend left] \end{tikzcd}$$ 2-cycle

$n \equiv 5 \pmod{10}$

$$\begin{tikzcd} 5 \arrow[loop below] \end{tikzcd}$$ 1-cycle

$n \equiv 6 \pmod{10}$

$$\begin{tikzcd} 6 \arrow[loop below] \end{tikzcd}$$ 1-cycle

$n \equiv 7 \pmod{10}$

$$\begin{tikzcd} 7 \arrow[r] & 9 \arrow[r] & 3 \arrow[r] & 1 \arrow[lll, bend left] \end{tikzcd}$$ 4-cycle

$n \equiv 8 \pmod{10}$

$$\begin{tikzcd} 8 \arrow[r] & 4 \arrow[r] & 2 \arrow[r] & 6 \arrow[lll, bend left] \end{tikzcd}$$ 4-cycle

$n \equiv 9 \pmod{10}$

$$\begin{tikzcd} 9 \arrow[r] & 1 \arrow[l, bend left] \end{tikzcd}$$ 2-cycle

n consider all n such that
n^n $\quad n(\mathrm{mod}\ 10) = 1$

01 $\rightarrow$ 1

11 $\rightarrow$ 1 $\quad n^n \equiv 1$ since $n \equiv 1 (\mathrm{mod}\ 10)$

21 $\rightarrow$ 1 $\quad 21^{21} \equiv 1 \ \mathrm{mod}\ 10$

1231 $\rightarrow$ 1

~~123~~ $\quad 124 \ \mathrm{mod}\ 10 = 4$

all n with $n \ \mathrm{mod}\ 10 \equiv 1$
contribute 4 to the sum.

$n \equiv 0$ all contribe 0 so ignore

$n \equiv 2 \mod 10$

$n \bmod 4$	n	
2	02	$\rightarrow$ 4
0	12	$\rightarrow$ 6 $\quad 12^{12} \equiv 6 \mod 10$
2	22	$\rightarrow$ 4 $\quad 22^{22} \equiv$
0	32	6
2	42	
0	52	

$$\begin{array}{cccc} 2^9 & 2^{10} & 2^{11} & 12^{12} \\ 2^5 & 2^6 & 2^7 & 2^8 \\ 2^1 & 2^2 & 2^3 & 2^4 \end{array}$$

$$\begin{tikzcd} 2 \arrow[r] & 4 \arrow[r] & 8 \arrow[r] & 6 \arrow[lll, bend left] \end{tikzcd}$$

$$22^{22} \mod 10 \equiv 2^{22} \mod 10$$

$$\begin{tikzcd} \overset{2^1}{2} \arrow[r] & \overset{2^2}{4} \arrow[r] & \overset{2^3}{8} \arrow[r] & \overset{2^4}{6} \arrow[lll, bend left] \end{tikzcd}$$

2^5

$22 \bmod 4 = 2$

$22^{22} \equiv 4 \mod 10$

$2^{22} = 2^2 \times 2^{10} \times 2^{10}$

$4 \times 1024 \times 1024$

$4 \times 4 \times 4 = 16 \times 4 = 4$

$n \equiv 2 \mod 10$

$$\left.\begin{array}{ll} 2^{2} & 4 \\ \boxed{1}2^{12} & 6 \\ 22^{22} & 4 \\ 32^{32} & 6 \\ \vdots & \\ 1232^{1232} & 6 \end{array}\right\}$$

an even number of n that are $\equiv 2 \mod 10$ between 1 & 1234