

Math with Sean

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Arthur Ryman



1997 Cayley Contest

#15

$$20^{50} = 2^{50} \times 10^{50}$$

5

$$(xy)^n = x^n \cdot y^n$$

$$(xy)^n = \underbrace{xy \cdot xy \cdot \dots \cdot xy}_n \quad \left[x^n \cdot x^m = x^{n+m} \right]$$

$$= \underbrace{x \cdot x \cdot \dots \cdot x}_n \cdot \underbrace{y \cdot y \cdot \dots \cdot y}_n$$

$$= x^n \cdot y^n$$

$$50^{20} = (5 \times 10)^{20} = 5^{20} \cdot 10^{20}$$

$$20^{50} \cdot 50^{20} = 2^{50} \cdot 10^{50} \cdot 5^{20} \cdot 10^{20}$$

$$= 2^{50} \cdot 5^{20} \cdot 10^{50} \cdot 10^{20}$$

$$= 2^{30} \cdot 2^{20} \cdot 5^{20} \cdot 10^{70}$$

$$= 2^{30} \cdot 10^{20} \cdot 10^{70}$$

$$= 2^{30} \cdot 10^{90}$$

$$(C) = 90$$

$$7^5 \cdot 2^5 = (7 \cdot 2)^5$$
$$= 14^5$$

#18 $x > 0, y > 0, z > 0$ integers

$$\frac{30}{7} = x + \left(\frac{1}{y + \frac{1}{z}} \right) \quad 0 < \frac{1}{y + \frac{1}{z}} \leq 1$$

$\frac{30}{7} = (4 \cancel{+} \frac{2}{7})$

$$30 = 28 + 2$$

$$0 < \frac{1}{y + \frac{1}{z}} < 1$$

$$\frac{30}{7} = \frac{28 + 2}{7} = 4 + \frac{2}{7}$$

$$x = 4 \quad \frac{1}{y + \frac{1}{z}} = \frac{2}{7} \quad y + \frac{1}{z} = \frac{7}{2} = 3\frac{1}{2}$$

$$y =$$

$x, y, z > 0$ integers.

$x \geq 1, y \geq 1, z \geq 1$

$$0 < \frac{1}{z} \leq 1$$

$$y + \frac{1}{z} > 1$$

$$0 < \frac{1}{y + \frac{1}{z}} \leq 1$$

$$\frac{30}{7} = 4 \cdot \frac{2}{7} = x + \frac{1}{y + \frac{1}{z}}$$
$$x = 4 \left[\frac{\frac{1}{y + \frac{1}{z}}}{\frac{2}{7}} = \frac{2}{7} \right]$$

$$y + \frac{1}{z} = \frac{7}{2} = 3\frac{1}{2}$$

$$y = 3 \quad z = 2$$

$$x + y + z = 4 + 3 + 2 = 9$$

(B)

#19

$$x^2 y z^3 = 7^4$$

$$x y^2 = 7^5$$

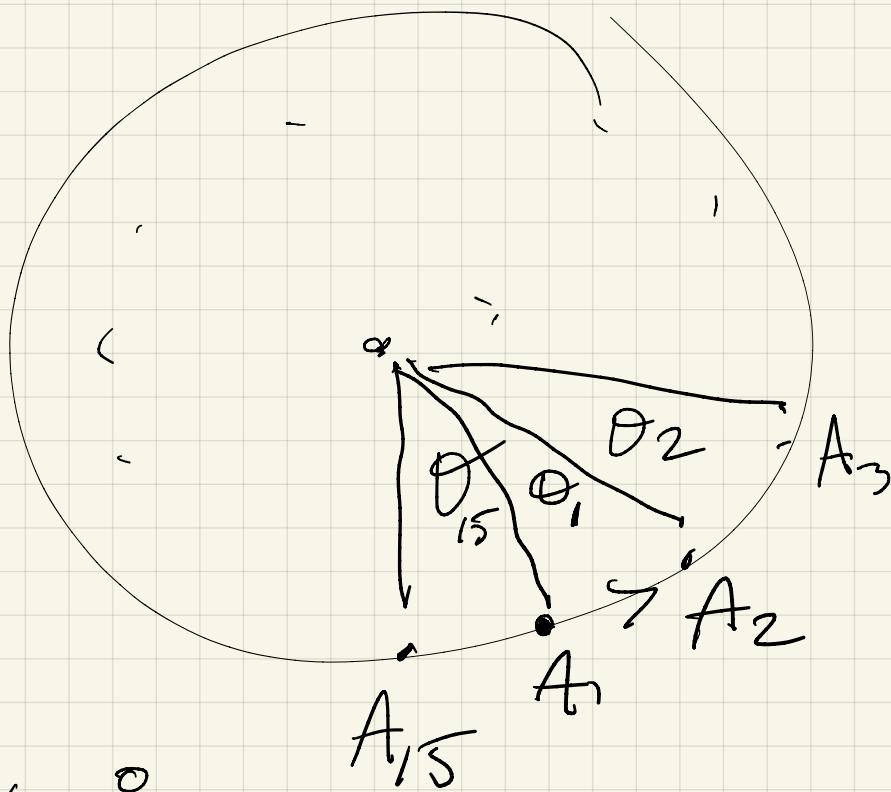
$$\underbrace{x^2 y z^3 \cdot x y^2}_{x^3 y^3 z^3} = 7^4 \cdot 7^5 = 7^9$$

$$= x^3 y^3 z^3 = 7^9 = (7^3)^3$$

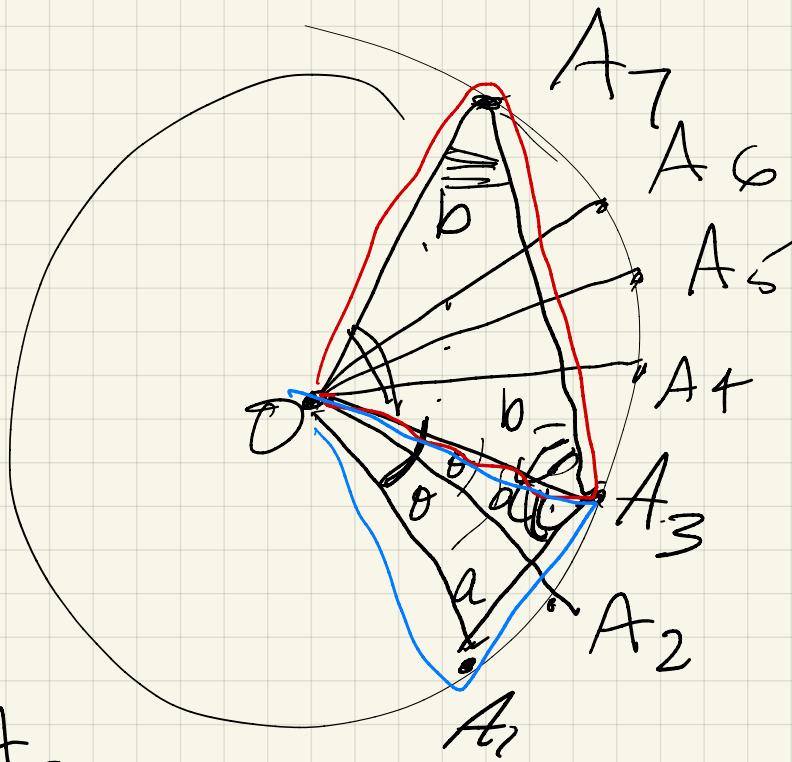
$$x y z = 7^3$$

(C)

#20



$$\theta = \frac{360^\circ}{15}$$



A hand-drawn diagram on lined paper. It features a triangle on the left with blue hatching inside. To its right is a circle. To the right of the circle are two labels: 'A1' above 'A3', both written in black ink.

$$\angle A_1 \circ A_3 = 20$$

$$2\theta + 2a = 180$$

 D A₃ A₇

$$\angle A_3 O A_1 = 40$$

$$40 + 26 = 180$$

$$\begin{cases} 2\theta + 2a = 180 \\ \theta = \frac{360^\circ}{15} \end{cases}$$

$$4\theta + 2b = 180$$

$$\text{Solve for } \psi = a + b$$

$$6\theta + 24 = 360$$

$$24 = 360 - 6\theta$$

$$\begin{aligned}\psi &= 180 - 3\theta \\ &= 180 - 3 \times \frac{360}{15}\end{aligned}$$

$$= 180 - \frac{360}{5}$$

$$= 180 - 72$$

$$= 108$$

(D)

#21 a, b, c are positive integers.

$$\frac{a}{c} + \frac{a}{b} + 1 = 11$$

$$\frac{b}{a} + \frac{b}{c} + 1$$

Find the number of (a, b, c)

such that $\boxed{a + 2b + c \leq 40}$

$$1 \leq a \quad 1 \leq b \quad 1 \leq c$$

$$\frac{a}{c} + \frac{a}{b} + 1 = 11 \quad \left(\frac{b}{a} + \frac{b}{c} + 1 \right)$$

$$\boxed{\frac{ab + ac + bc}{ab} = 11 \quad \left(\frac{bc + ab + ac}{ac} \right)}$$

$$\frac{a}{c} = \frac{ab}{cb} \quad \frac{a}{b} = \frac{ac}{bc} \quad 1 = \frac{bc}{bc}$$

$$\boxed{a(ab + ac + bc) = 11 \cdot b \cdot (bc + ab + ac)}$$

$$\nearrow a = 11b$$

$$\boxed{a + 2b + c \leq 40}$$

$$1 \leq a, 1 \leq b, 1 \leq c$$

$$a = 1/b$$

$$1/b + 2b + c \leq 40$$

$$\underbrace{13b + c \leq 40}$$

$$13b \leq 40 - c$$

$$\Rightarrow b \leq 3$$

$$b = 1, 2, 3$$

$$\text{Case } b=1 \quad c \leq 40 - 13 \cdot b$$

$$c \leq 40 - 13 \cdot 1$$

$$c \leq 40 - 13$$

$$1 \leq c \leq 27 \quad \Rightarrow 27 \text{ sol's}$$

$$b=2$$

$$c \leq 40 - 2b$$

$$c \leq 14$$

$$\Rightarrow 14 \text{ sol's}$$

$$b=3$$

$$c \leq 1$$

$$\Rightarrow 1 \text{ sol}$$

(D)

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