Math with Sean

2020-03-28

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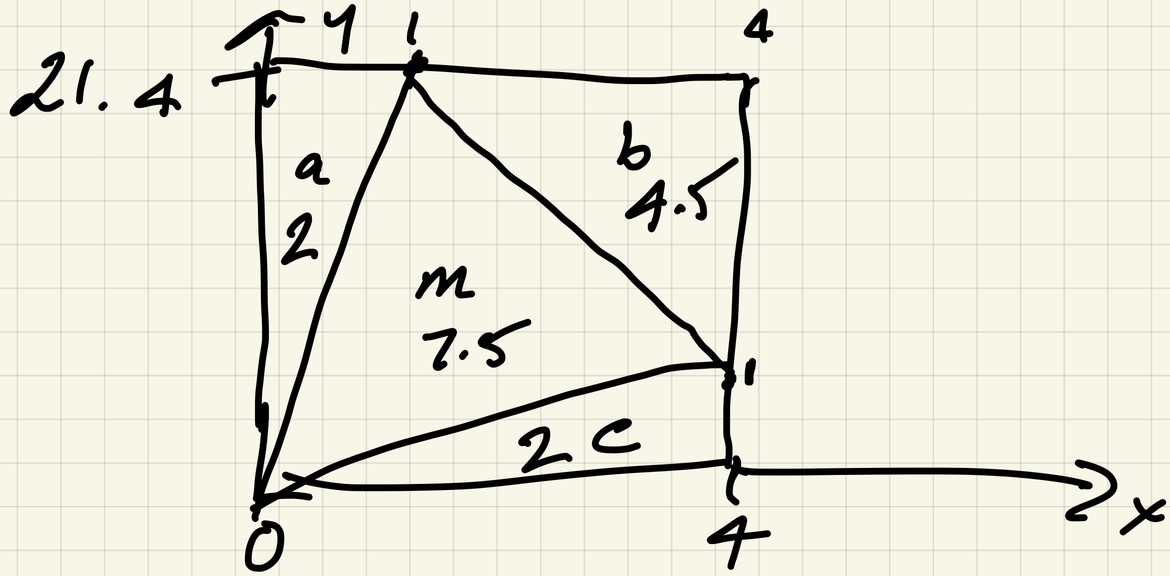
2015 Pascal



$m + a + b + c = 4 \times 4 = 16$

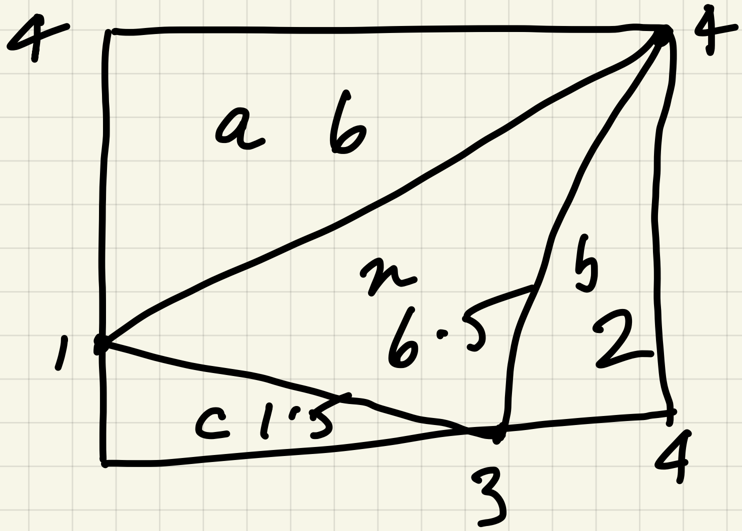
$a = \frac{4 \times 1}{2} = 2$

$b = \frac{3 \times 3}{2} = \frac{9}{2} = 4.5$

$c = \frac{4 \times 1}{2} = 2$

$a + b + c = 8.5$

$m = 16 - 8.5 = 7.5$



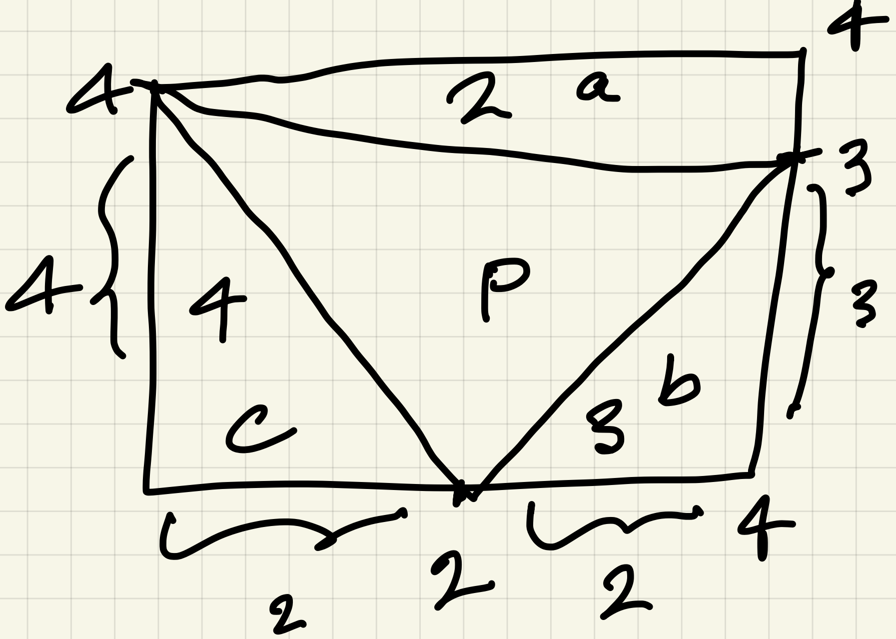
$a = \frac{3 \times 4}{2} = 6$

$b = \frac{4 \times 1}{2} = 2$

$c = \frac{3 \times 1}{2} = 1.5$

$a + b + c = 9.5$

$n = 16 - 9.5$
$= 6.5$



$$a = \frac{4 \times 1}{2} = 2$$

$$b = \frac{3 \times 2}{2} = 3$$

$$c = \frac{4 \times 2}{2} = 4$$

$$a + b + c = 2 + 3 + 4 = 9$$

$$P = 16 - 9 = 7$$

$$m = 7.5 \quad n = 6.5 \quad P = 7$$

$$n < P < m$$

$$\boxed{D}$$ ✓

23.

$c \leq 35$

$0 < a < b < c \leq 35$

$$6ab = c^2$$

$$2 \cdot 3 \cdot ab = c^2 = c \cdot c$$

c^2 is even

$$c = 2^x 3^y 5^z \cdots$$

$$c^2 = 2^{2x} 3^{2y} 5^{2z} \cdots$$

c is even $c = 2n$

c is a multiple of 3 $c = 3m$

$\Rightarrow C = 6N$ for some N

$c \leq 35$

$c = 6N \leq 35$ $N \leq \left\lfloor \frac{35}{6} \right\rfloor = 5$

$C = 6N \qquad 1 \le N \le 5$

$6ab = c^2 = (6N)^2$

$6ab = 36 \cdot N^2$

$ab = 6N^2 \qquad 1 \le N \le 5$

$N=1 \quad \underline{ab = 6} \quad \{1, 2, 3, 6\} \quad C = 6$

$0 < a < b$

$N=1$

a	b	ab
~~1~~	~~6~~	6
2	3	

2 solutions

1

$N = 2 \qquad ab = 6N^2 = 24$

$24 \Rightarrow \underbrace{1, 2, 3, 4}, 6 \cdots$

$c = 6N$
$= 12$

N	a	$<$ b
2	~~1~~	~~24~~
	~~2~~	~~12~~
	3	8
	4	6

4

2

$ab = 6N^2 \quad N = 3$

$= 54 = 2 \cdot 3 \cdot 3 \cdot 3$

$\underbrace{2 \cdot 3^3}_{}$

$C = 6N$
$= 18$

$a = \underline{2^p \cdot 3^q}$

$p = 0, 1, 2$

$q = 0, 1, 2, 3, 4$

$\boxed{ab = 54}$

2^p p	3^q q	a	b
0	0	$\cancel{1}$	$\cancel{54}$
	1	$\cancel{2^0 \cdot 3^1 = 3}$	$\cancel{18}$
	2	$\cancel{2^0 \cdot 3^2 = 9}$	6
	3	$2^0 \cdot 3^3 = 27$	2
1	0	$\cancel{2^1 \cdot 3^0 = 2}$	27
	1	$2^1 \cdot 3^1 = 6$	9
	2	$2^1 \cdot 3^2 = 18$	3
	3	$2^1 \cdot 3^3 = 54$	1

~~4 good~~

~~2~~

1

$N=4 \quad ab = 6\cdot N^2 = 6\cdot 16$
$= 96$

$96 = 2\cdot 3\cdot 4\cdot 4$
$= 2\cdot 3\cdot 2\cdot 2\cdot 2\cdot 2$
$= 2^5\cdot 3^1$

$c = 6N$
$= 24$

$\underbrace{1, 2, 4, 8, 16, \cancel{32}}_{6}$ $\quad$ $\underbrace{1, 3}_{2}$

$ab = 96$

$24 \mid 96$

12 values of $a \mid 96$
12 values of $b \mid 96$

6 solutions

$N=5 \quad ab = 6 \cdot N^2$

$$= 6 \cdot 5^2$$

$$= 6 \cdot 25$$

$$= 150$$

$$150 = 2^1 \times 3^1 \times 5^2$$

$$\underbrace{\begin{matrix}1\\2\end{matrix}}_{2} \quad \underbrace{\begin{matrix}1\\3\end{matrix}}_{2} \quad \underbrace{\begin{matrix}1\\5\\25\end{matrix}}_{3}$$

$$2 \times 2 \times 3 = 12$$

6 solution

N	# sol.
1	2
2	4
3	4
4	6
5	6
	22

$$\boxed{22}$$

$N=4 \qquad c = 6N = 24$

$ab = 6N^2 = 96 = 2^5 \cdot 3^1$

$6 \times 2 = 12$

$ab < 24$

2^x	3^y	a	b
~~1~~	~~1~~	~~1~~	~~96~~
~~2~~	~~1~~	~~2~~	~~48~~
~~4~~	~~1~~	~~4~~	~~24~~
8	1	8	12
~~16~~	~~1~~	~~16~~	~~6~~
~~32~~	~~1~~	~~32~~	~~3~~
~~1~~	~~3~~	~~3~~	~~32~~
2	3	6	16
~~4~~	~~3~~	~~12~~	~~8~~
~~8~~	~~3~~	~~24~~	~~4~~
~~16~~	~~3~~	~~48~~	~~2~~
~~32~~	~~3~~	~~96~~	~~1~~

$ab = 96$

$c = 24$

2 solutions

$N=5$ $\quad ab = 6 \cdot N^2 = 6 \cdot 25 = 150$

$150 = 2^1 \cdot 3^1 \cdot 5^2$ $\quad c = 6N = 30$

$a < b < 30$

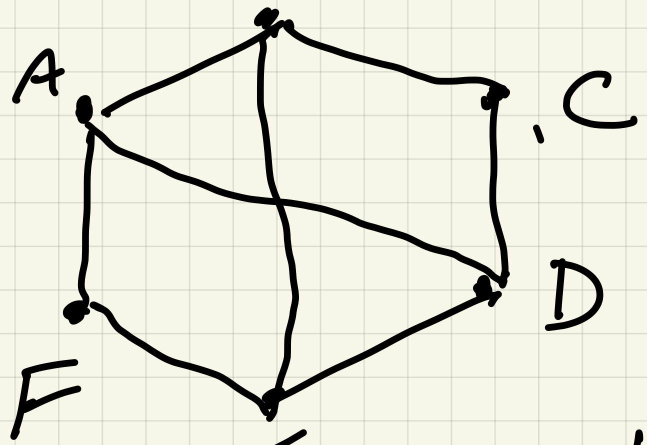
$ab = 150$

2^x	3^y	5^z	a	b	
~~1~~	~~1~~	~~1~~	~~1~~	~~150~~	
		~~5~~	~~5~~	~~30~~	
		~~25~~	~~25~~	~~6~~	
1	3	~~1~~	~~3~~	~~50~~	
		~~5~~	~~15~~	~~10~~	
		~~25~~	~~75~~		
2	1	1	~~2~~	~~75~~	
		5	10	15	✓
		~~25~~	~~50~~		
2	3	1	6	25	✓
		~~5~~	~~30~~		
		~~25~~			

2

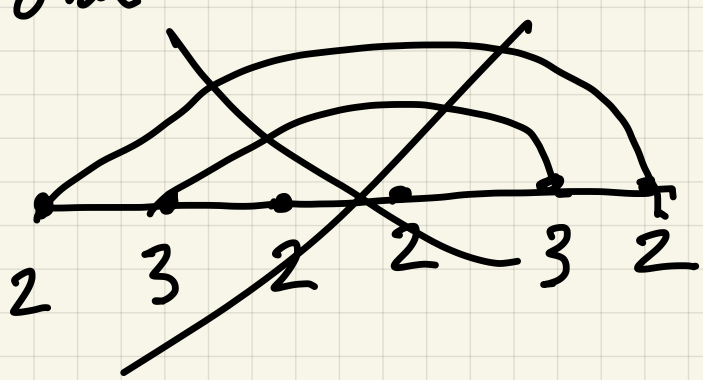
$1+1+2+2+2 = 8$ solutions

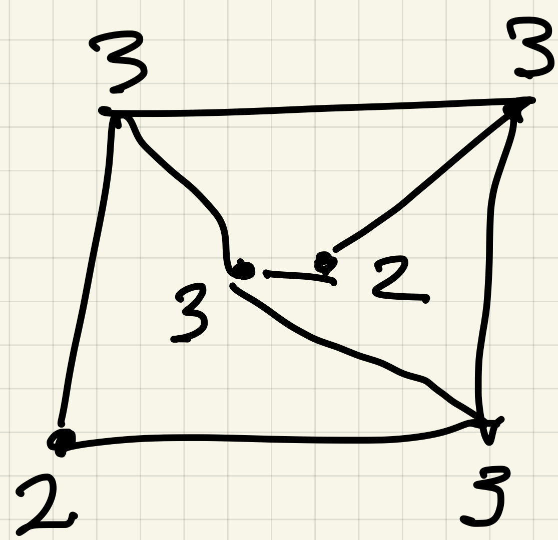
21.

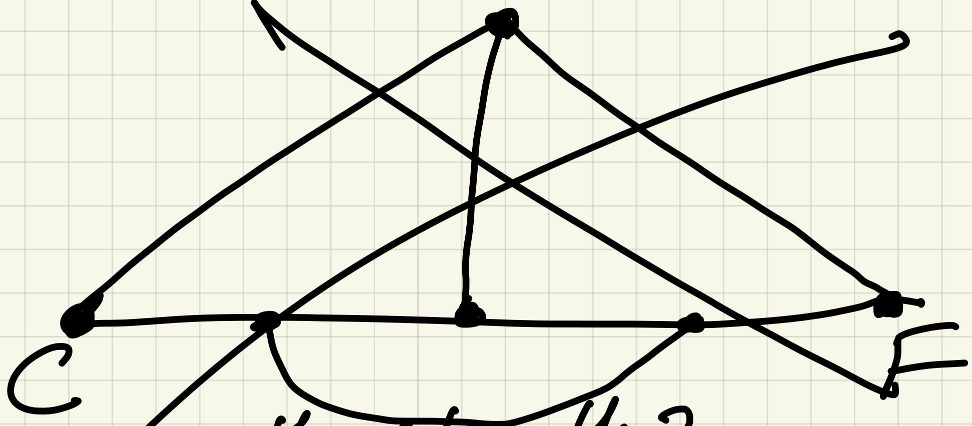


	deg
A	3
B	3
C	2
D	3
E	3
F	2
Total	16

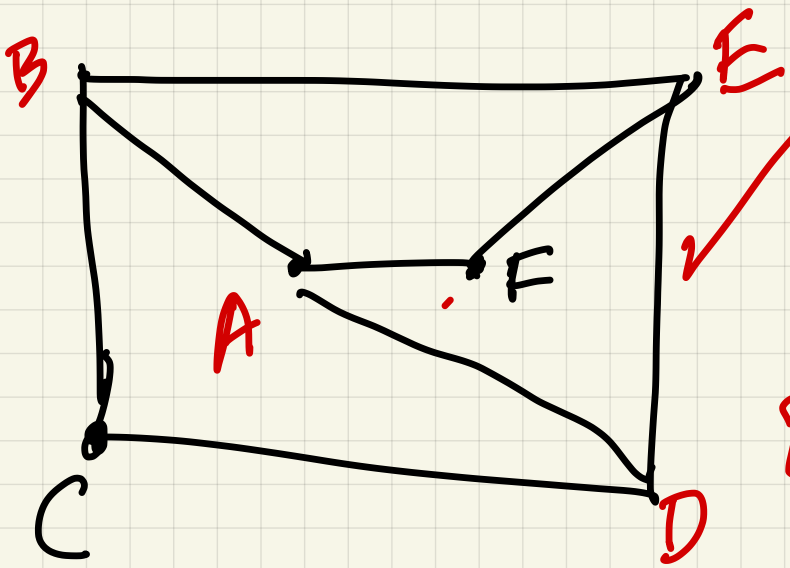
deg	count
2	2
3	4

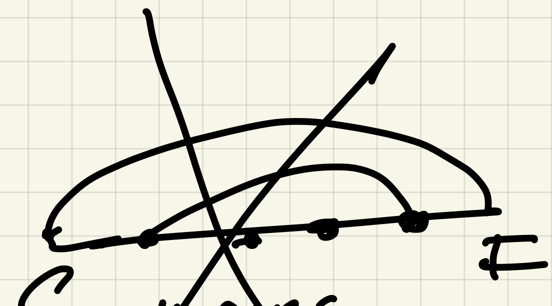






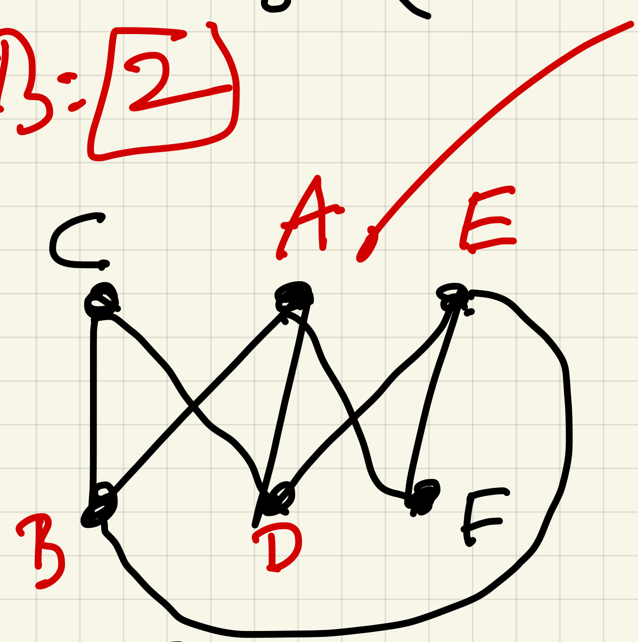
path of length 2

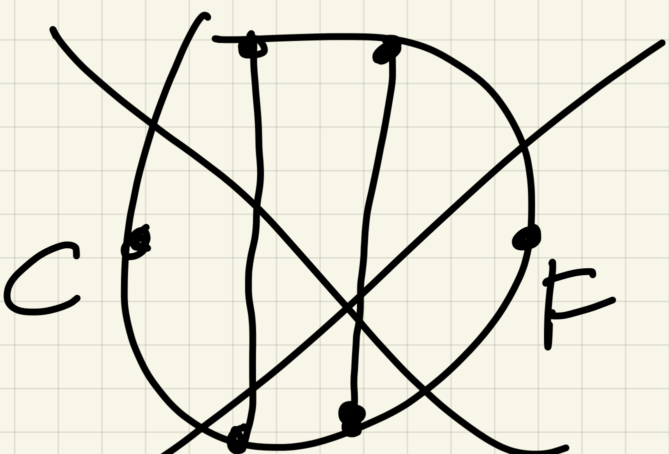




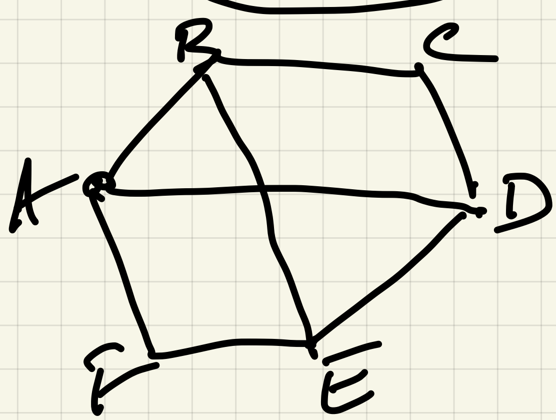
wrong degrees

$B: \boxed{2}$





path of length 4 C → F



25 k = row number

if $V(k) = n$

$W(k) = 2n+1 \Rightarrow W(k)$

$X(k) = 3n+1 \Rightarrow X(k) \bmod 3 = 1$

$Y(k) = 5n+1$

$Z(k) = 7n+1$

$W(j) = \underline{2731} = \underline{V(j) \cdot 2 + 1}$

$V(j) = \frac{2731 - 1}{2} = \frac{2730}{2} = 1365$

$\boxed{2731 \bmod 3 = 1}$

$2731 \bmod 5 = 1$

$\boxed{2731 \bmod 7 = 1}$

$$
\begin{array}{r}
390 \\
7\overline{)2731} \\
\underline{21} \\
63 \\
\underline{63} \\
1
\end{array}
$$

$$Z(a) = 7V(a) + 1 = 2731$$

$$V(a) = \frac{2730}{7} = \underline{390}$$

W: $390 \bmod 2 = 0 \neq 1$ never appear W

X: $390 \bmod 3 = 0 \neq 1$ 〃

3×130

Y: $390 \bmod 5 = 0 \neq 1$ never Y

Z: $390 \bmod 7 = 5 \neq 1$ newe Z

$$Y(a) = 5V(a) + 1$$
$$= 2731$$
$$V(a) = \frac{2731 - 1}{5}$$
$$= \frac{2730}{5}$$
$$= \boxed{546}$$

$W : 546 \bmod 2 = 0 \neq 1$

$X : 546 \bmod 3 = 0 \neq 1$

$Y : 546 \bmod 5 = 1$ could appear in Y

$Z : 546 \bmod 7 = 0 \neq 1$

Can 546 ever appear in Y?
Assume it appears in row a.

$$546 = 5 \cdot V(a) + 1$$

$$V(a) = \frac{546 - 1}{5}$$

$$= \frac{545}{5}$$

$$= 109$$

109 can appear in V if it never appears in W, X, Y, Z

W: $109 \bmod 2 = 1$ $\Rightarrow$ could appear

X: $109 \bmod 3 = 1$ $\Rightarrow$ "

Y: $109 \bmod 5 = 4 \neq 1$

Z: $109 \bmod 7 = 4 \neq 1$