Math with Sean

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Cayley 2018 #24

n black sock, $2n$ gold socks +
at least 2 gold socks between
any 2 black socks.

$n=1$ 1 black sock, 2 gold socks

B   G G $n=1$

↑   ↑   ↑ $m=3$

B   B   B

$n=2$ 2 black socks, 4 gold socks

BGGB   2 gold leftover

↑

G

BGGB

↑ 2G's

0 1 2

GGBGGB

1

BGGB

/2G

G B GGB

|1G

GBGGGB,

GBGGBG

2

0,1,2

||

BGGB ↑ 2G

BGGGB ↑ 1G

BGGGGB

BGGBGG

BGGGBG

3

n = 2

m = 6

$n=3$ 3 blacks 6 golds

BGG B GG B 2 G left

6	0	0	0	2
	0	0	1	1
	0	0	2	0
	0	1	0	1
	0	1	1	0
	0	2	0	0
3	1	0	0	1
	1	0	1	0
	1	1	0	0
1	2	0	0	0

$10 \neq 12$

$$= \frac{(n+1)(n+2)}{2}$$

n	m	
1	3	
		$+3$
2	6	
		$+4$
3	10	
		$+5$
4	$= \frac{(4+1)(4+2)}{2} = \frac{5 \times 6}{2} = 15$	

$$m = f(n) = \frac{(n+1)(n+2)}{2}$$

$$f(n+1) - f(n)$$

$$= \frac{((n+1)+1)((n+1)+2)}{2} - \frac{(n+1)(n+2)}{2}$$

$$= \frac{(n+2)(n+3)}{(2)} - \frac{(n+1)(n+2)}{2} \quad x$$

$n=4 \quad 4B \quad 8G$

$$\boxed{B\,G\,G\,B\,G\,G\,B\,G\,G\,B}$$

we use up n B $+2(n-1)$ G

Start with $2n$ G

use up $2(n-1)$ G

so $2n-2(n-1)=2n-2n+2$

$=2$

always have 2 G left over

There are $n+1$ positions to put the golds.

Suppose we put 2 G in one of the positions. How many arrangements do we get?

$5 \Rightarrow n+1$ arrangements.

now suppose no position has 2 G (extra golds)

need 2 different position.

let's number the position

$$\underbrace{1,\ 2,\ 3,\ ---,\ n+1}_{n+1 \text{ positions.}}$$

1, 2, 3, 4, 5

Choose 2 from 5 $\Rightarrow \binom{5}{2}$

(1,2), (1,3), (1,4), (1,5),
(2,3) (2,4), 2,5),
(3,4 ,(3,5),
,(4,5).

$$\binom{5}{2} = \frac{5!}{2!(5-2)!}$$

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\,3!}$$

$$= \frac{5 \times \cancel{4}^{2} \times \cancel{3 \times 2 \times 1}}{\cancel{2} \times 1 \times \cancel{3 \times 2 \times 1}}$$

$$= 10 \text{ (Hurray)}$$

in general $\binom{n+1}{2} = \frac{(n+1)!}{2!\,(n-1)!}$

$$= \frac{(n+1)(n)}{2}$$

in $\boxed{m = (n+1) + \frac{(n+1)n}{2}}$

2G's

1G, 1G

$$m = (n+1) + \frac{(n+1)n}{2}$$

$$= (n+1)\left(1 + \frac{n}{2}\right)$$

$$= (n+1)\left(\frac{2+n}{2}\right)$$

$$= \frac{(n+1)(n+2)}{2}$$

What's smallest value of n for which $m > 1000\,000$?

$$\frac{(n+1)(n+2)}{2} > 1000\,000$$

$$(n+1)(n+2) > 2\,000\,000$$

$$(x+1)(x+2) = 2000\,000$$

$$x^2 + 3x + 2 = 2000\,000$$

$$x^2 + 3x + 2 - 2000\,000 = 0$$

$$x^2 + 3x - 1999\,998 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(-1999\,998)}}{2}$$

$$= \frac{-3 \pm \sqrt{9 + 4(1999998)}}{2}$$

$$x = \frac{\sqrt{9 + 4(1999998)} - 3}{2}$$

$$= \frac{\sqrt{8000\,001} - 3}{2}$$

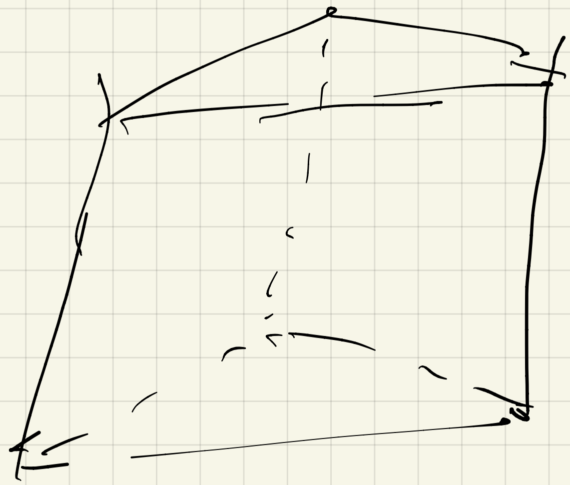
$$= 1412{,}713\ldots$$

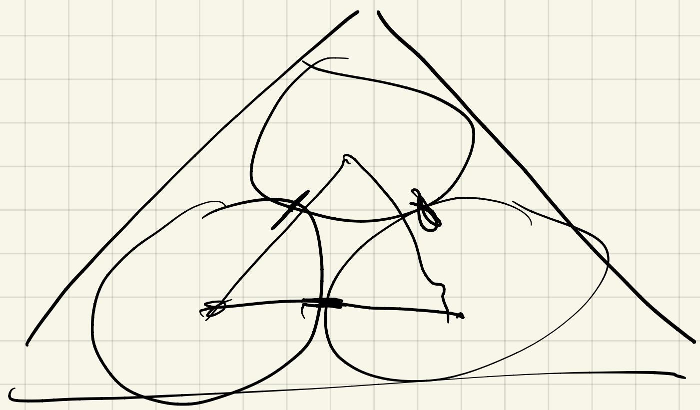
$n = 1413$

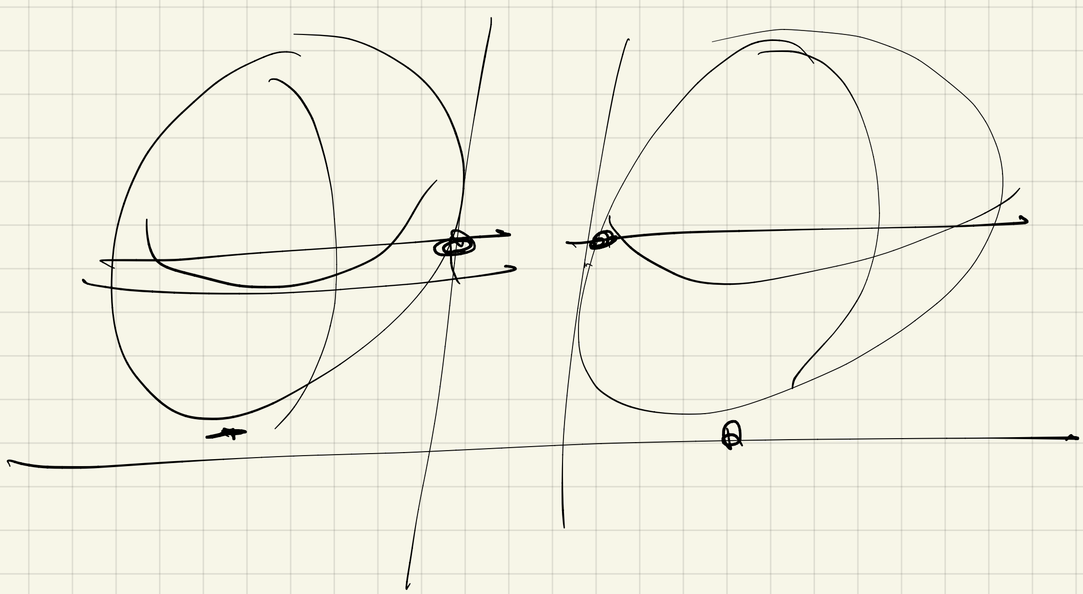
Sum of digits $= 9$ (A)

#23

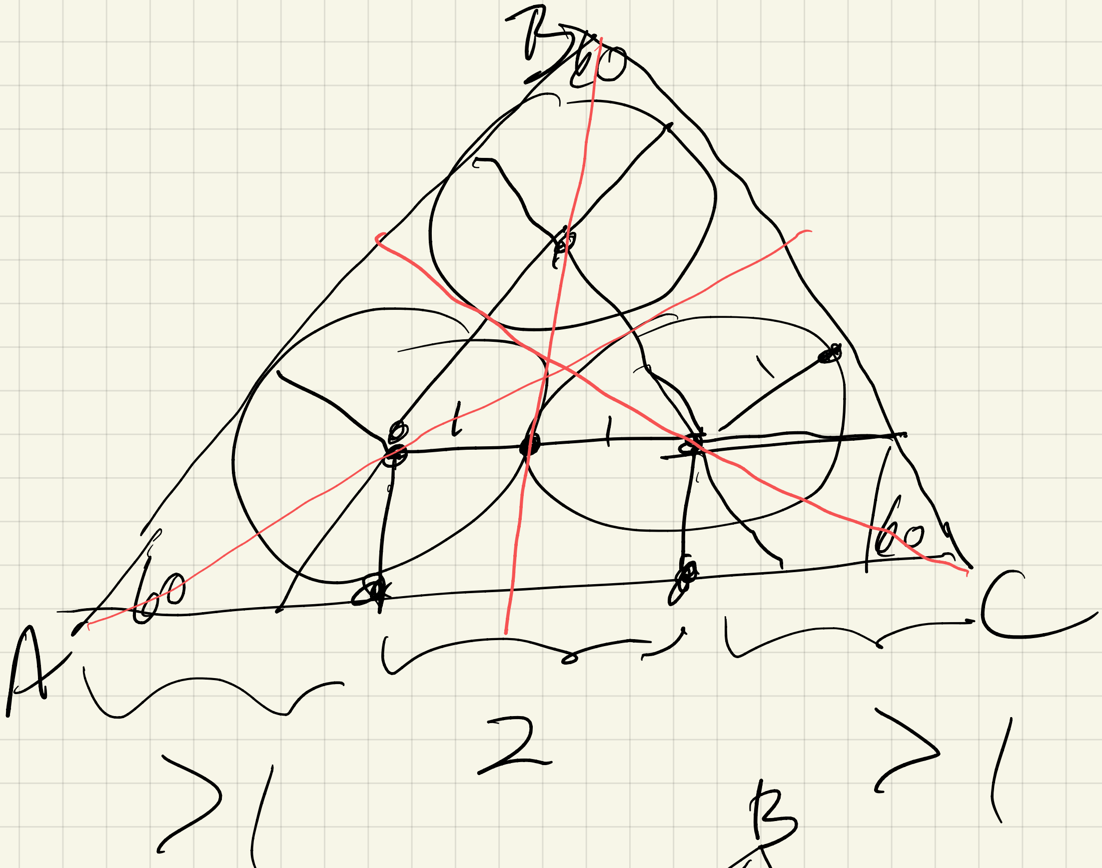
triangular prism

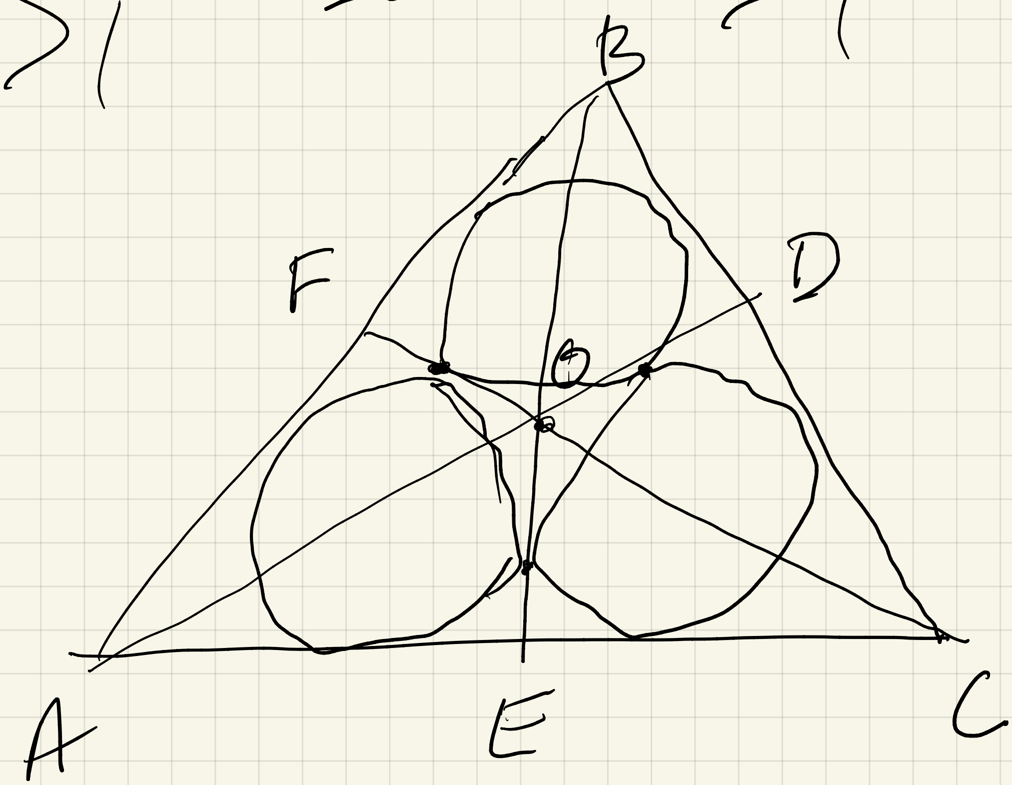






Base - 3 circles of radius 1
touch eachother





$DC = 1 + \sqrt{3}$

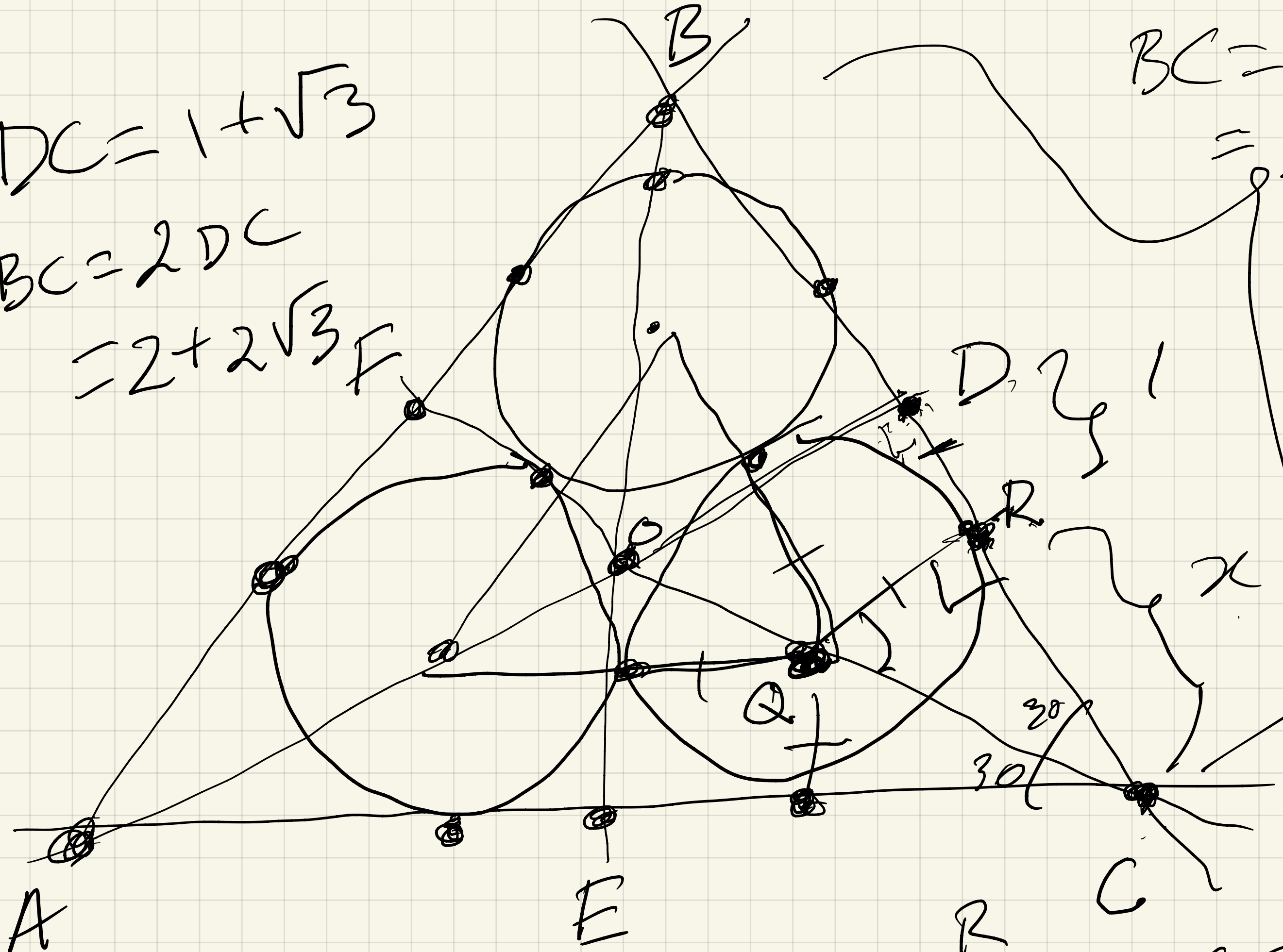
$BC = 2DC$

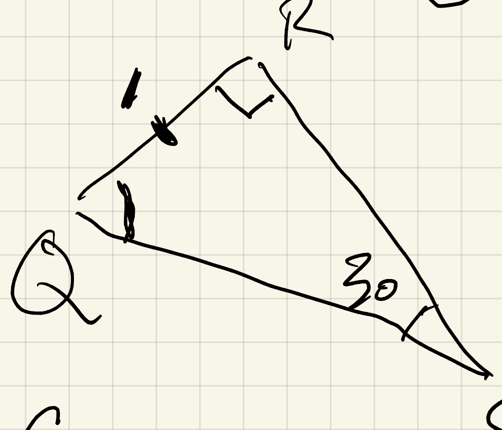
$= 2 + 2\sqrt{3}$

$BC = 2 + 2\sqrt{3}$

$= 2DC$

$x = \sqrt{3}$



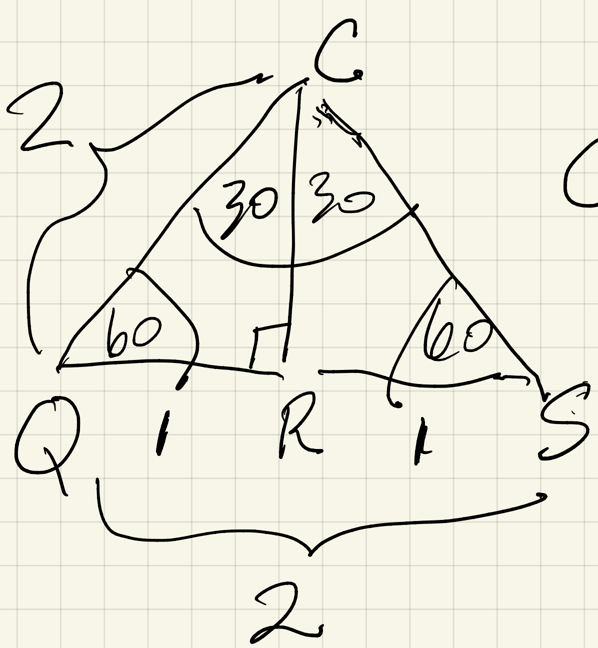


$QR = 1$

$\angle R = 90$

$\angle C = 30$

$\angle Q = 60$



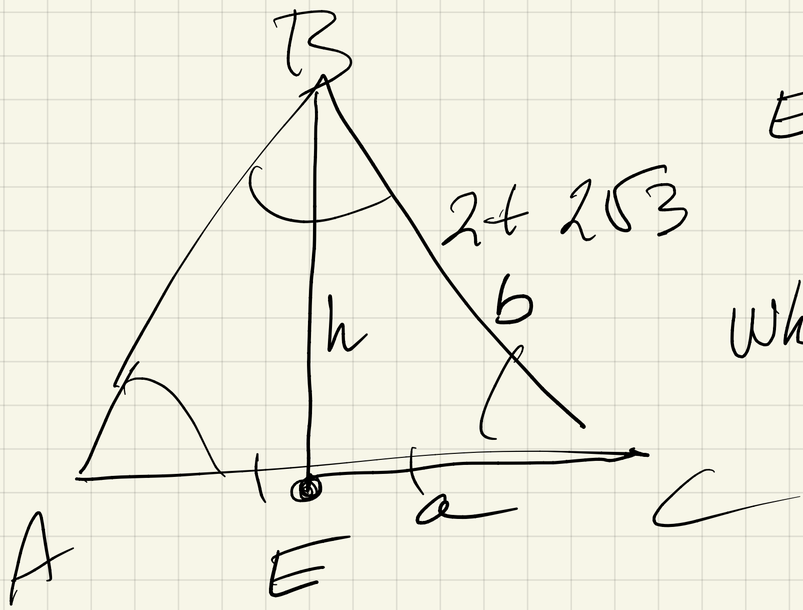
$CR = x$

$\triangle QRC$

$2^2 = 1^2 + x^2$

$4 = 1 + x^2$

$x^2 = 3 \quad x = \sqrt{3}$



Equilateral Δ

What is the area?

$b = 2+\sqrt{12}$

$$\text{area } \Delta ABC = \frac{bh}{2}$$

$$b = 2+2\sqrt{3} = 2+\sqrt{12}$$

$$a = 1+\sqrt{3} = \frac{b}{2}$$

Pythogas for a, h, b in Δ BEC

$$a^2 + h^2 = b^2$$

$$h^2 = b^2 - a^2$$

$$= (2+\sqrt{12})^2 - (1+\sqrt{3})^2$$

= HOMEWORK