Math with Sean

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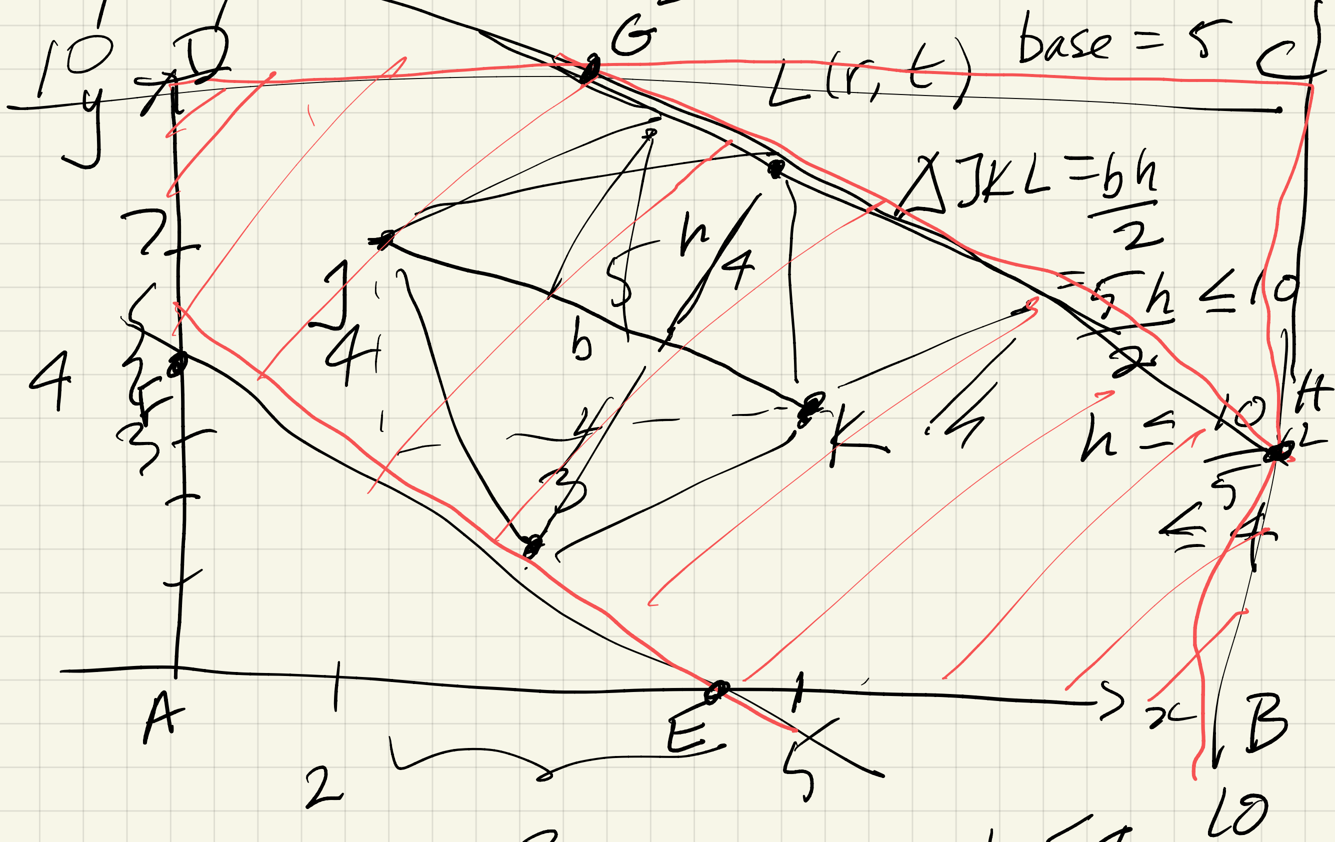
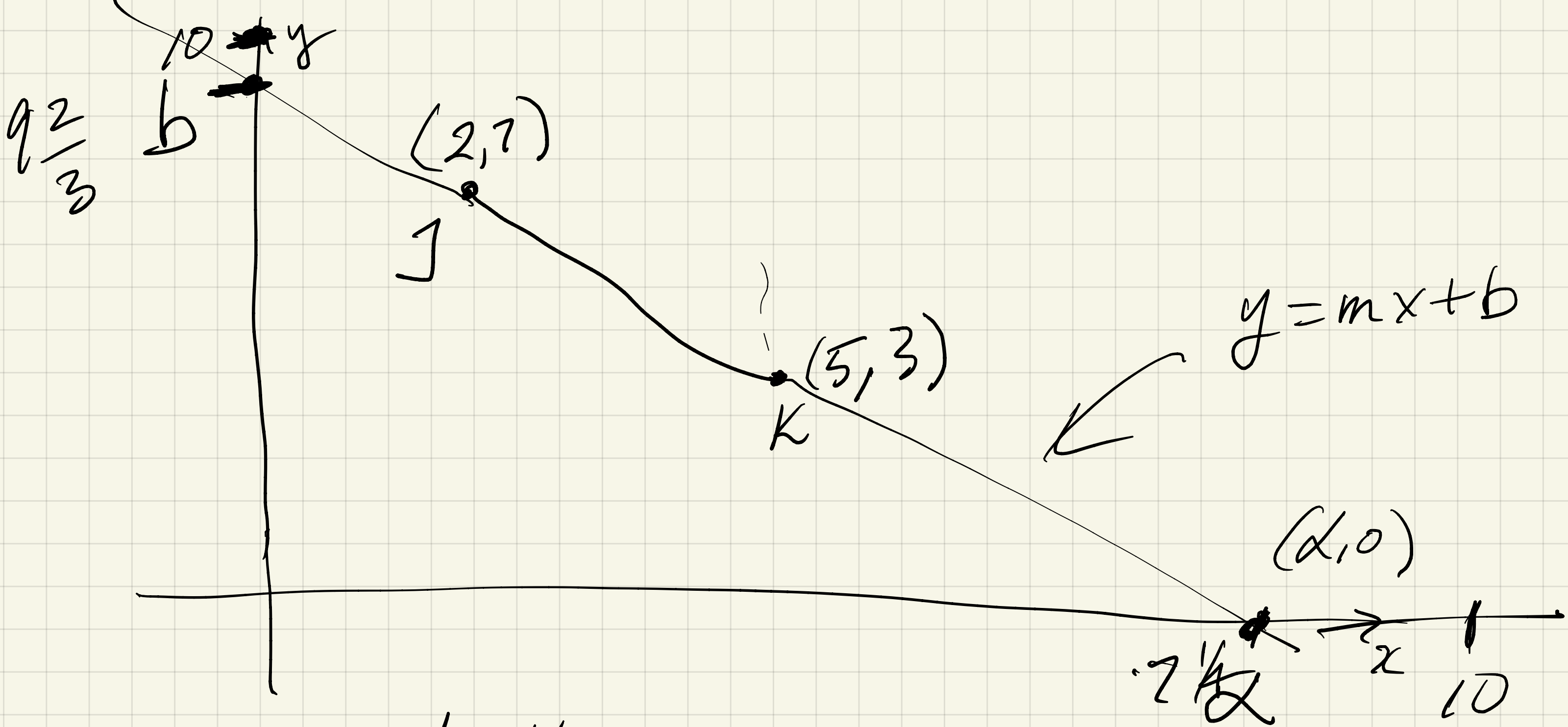
Cayley 2021 #25 $\qquad \Delta JKL \le 10$



need to find the region where $h \le 4$

R = 10×10 square − 2 triangles

$100 - \Delta AEF - \Delta CGH$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-7}{5-2} = \frac{-4}{3}$$

at (5,3) $\quad 3 = -\frac{4}{3} \times 5 + b$

$$3 = -\frac{20}{3} + b$$

$$9 = 3b - 20$$

$$29 = 3b$$

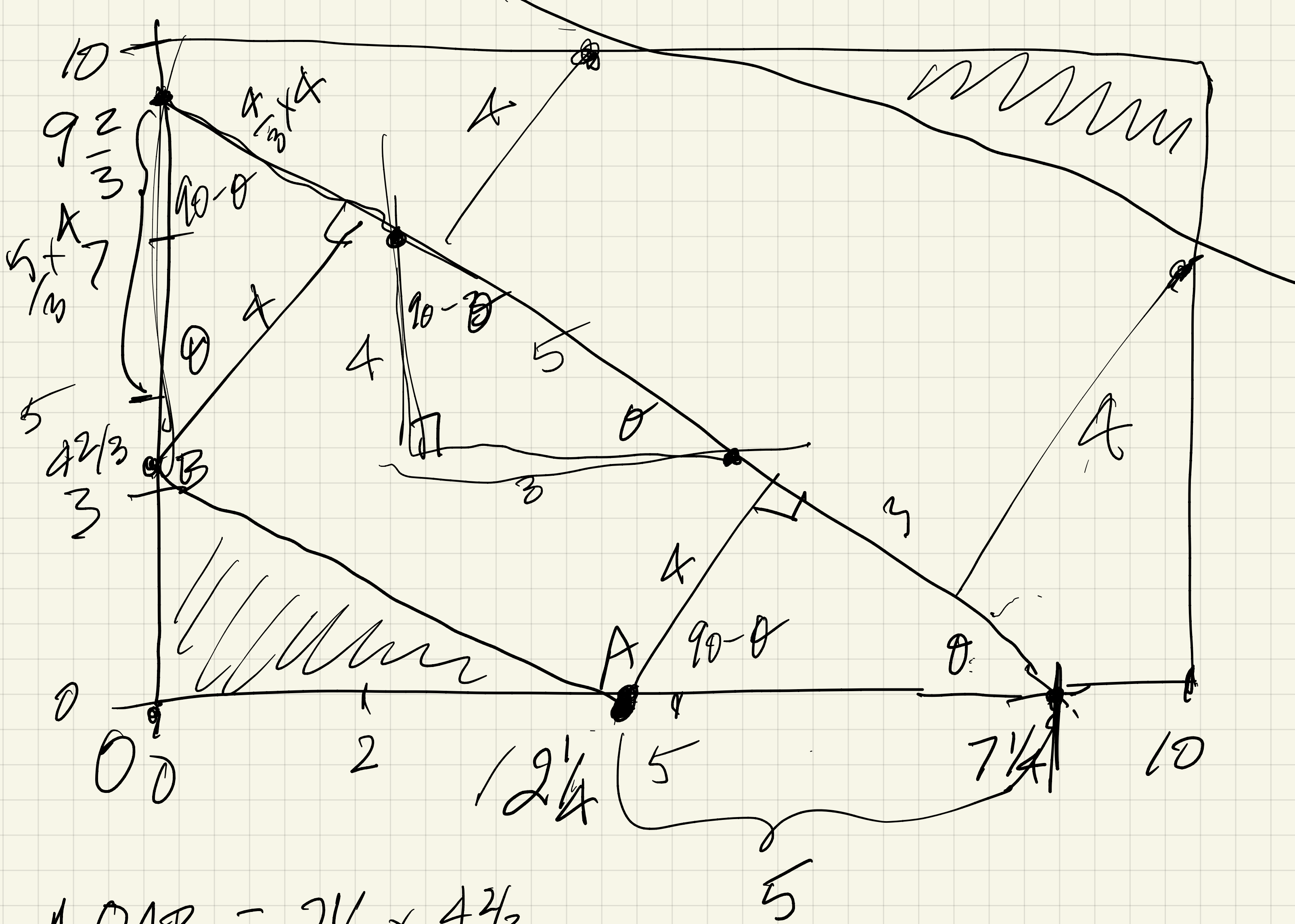
$$b = \frac{29}{3} = 9\frac{2}{3}$$

$$0 = m\alpha + b$$

$$= -\frac{4}{3}\alpha + \frac{29}{3}$$

$$\frac{4}{\cancel{3}}\alpha = \frac{29}{\cancel{3}}$$

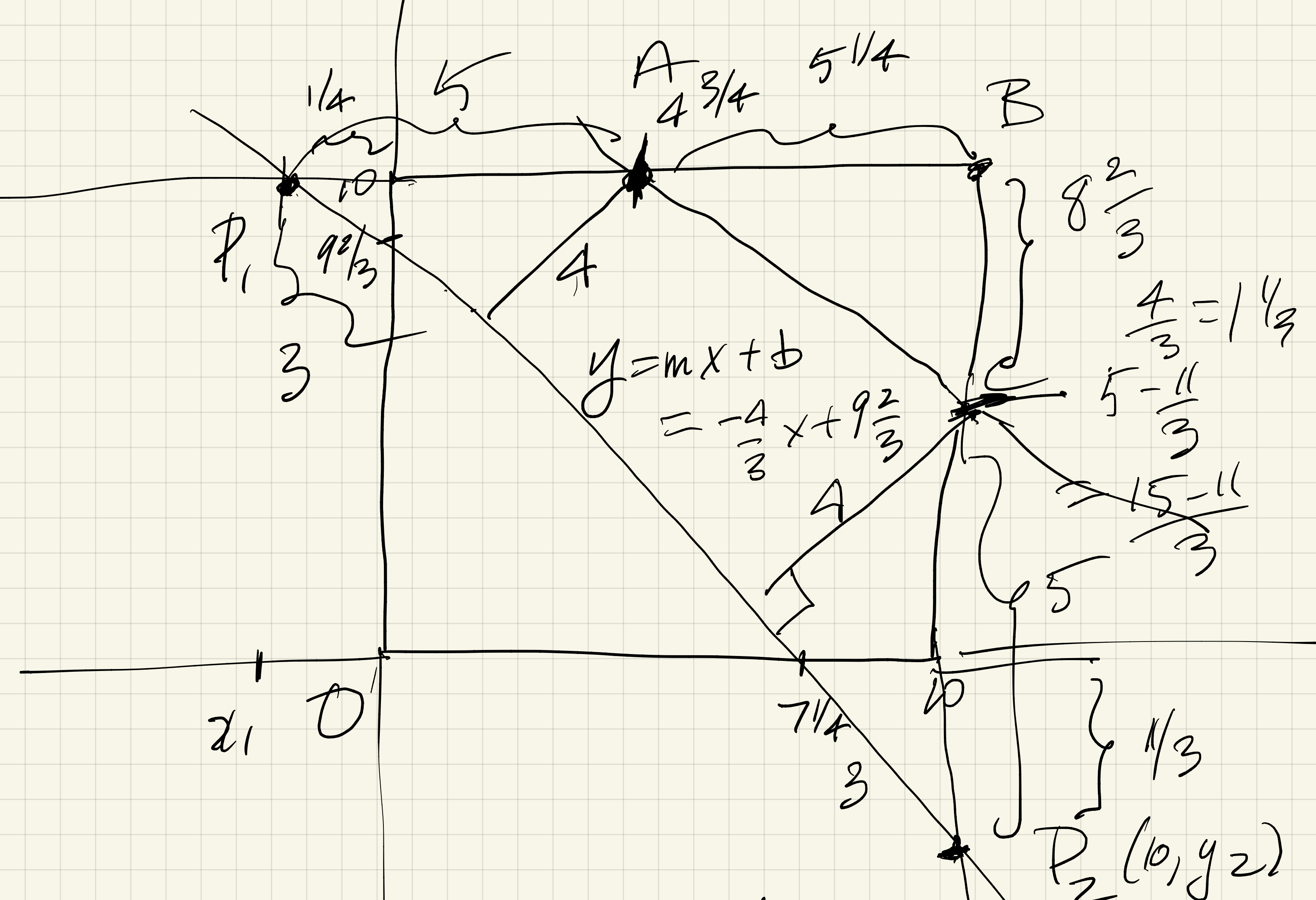
$$\alpha = \frac{29}{4} = 7\frac{1}{4} < 10$$



$$\triangle OAB = \frac{2\tfrac{1}{4} \times 4\tfrac{2}{3}}{2}$$

$$= \frac{\frac{9}{4} \times \frac{14}{3}}{2} = \frac{\cancel{9}^{3} \times \cancel{14}^{7}}{4 \times \cancel{3} \times \cancel{2}} = \frac{21}{4} = 5\frac{1}{4}$$

$$= 5.25$$



$P_1 = (x_1, 10)$

$10 = -\frac{4}{3}x_1 + 9\frac{2}{3}$

$\frac{1}{3} = -\frac{4}{3}x_1$

$x_1 = -\frac{1}{4}$

$P_2(10, y_2)$

$y_2 = -\frac{4}{3}10 + 9\frac{2}{3}$

$= -\frac{40}{3} + \frac{29}{3}$

$= -\frac{11}{3}$

$$\Delta ABC = \frac{5\frac{1}{4} \times 8\frac{2}{3}}{2} = \frac{\cancel{21}^{7} \times \cancel{26}^{13}}{\cancel{2} \times \cancel{3} \times 4} = \frac{7 \times 13}{4}$$

$$R = 100 - \frac{21}{4} - \frac{91}{4}$$

$$= \frac{400 - 21 - 91}{4}$$

$$\begin{array}{r} 400 \\ -21 \\ \hline 379 \\ -91 \\ \hline 288 \end{array}$$

$$= \frac{288}{4}$$

$$= \frac{144}{2}$$

$$= \boxed{72} \quad \text{area}$$

$$72 = \frac{300 + a}{40 - b}$$

$$\begin{array}{r} 72 \\ 40 \\ \hline 2880 \end{array}$$

$$72 \times (40 - b) = 300 + a$$

$$2880 - 72 = 300 + a$$

$$72 = \frac{300+a}{40-b} = \frac{288}{4}$$

$$300+a = n \; 288$$

$n=2$ $\quad 300+a = 2 \times 288$

$$a = 2 \times 288 - 300$$

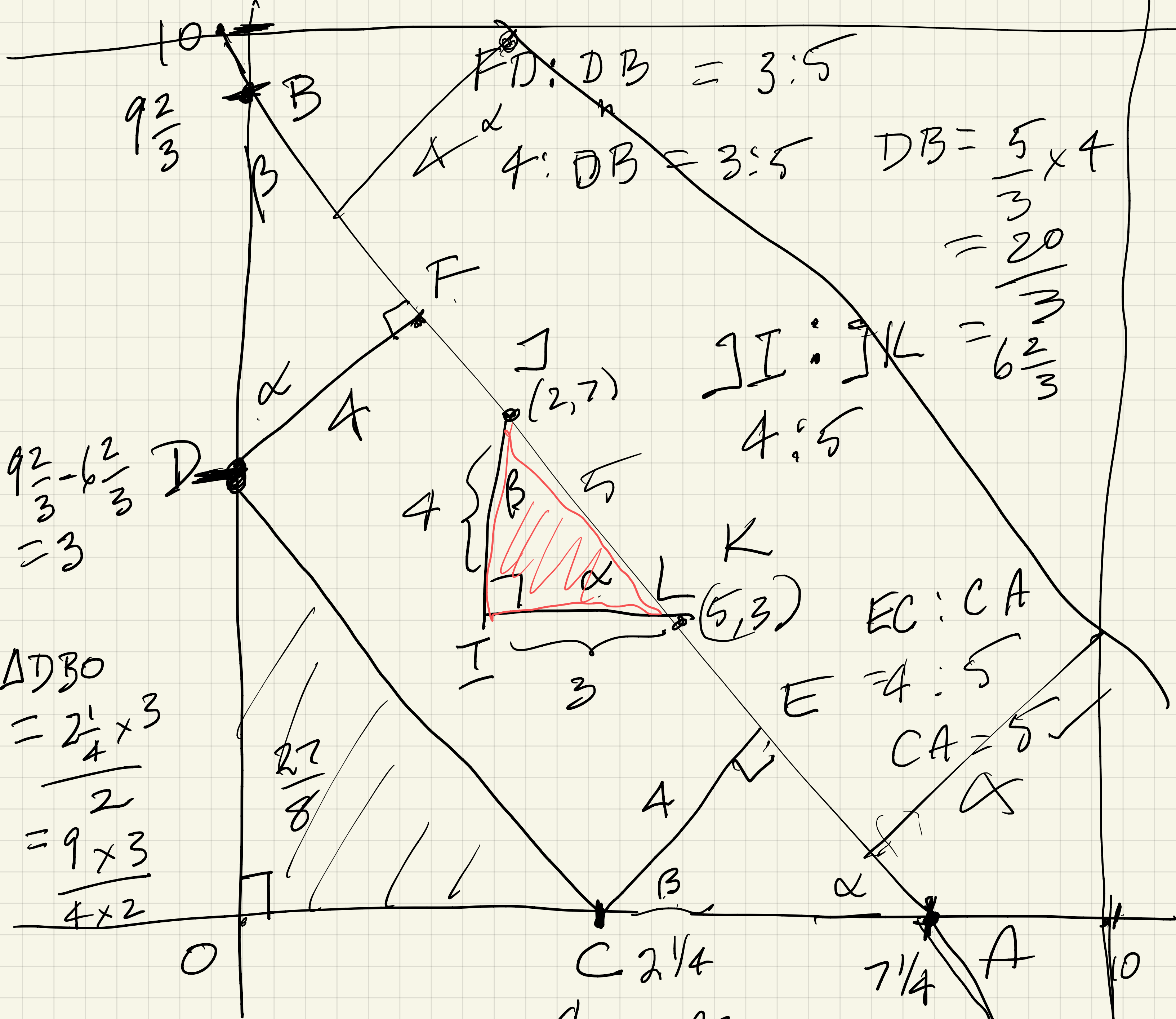
$$= 576 - 300 = 276$$

$$40-b = 2 \cdot 4 = 8$$

$$40 - 8 = b = 32$$

$$a+b = 276 + 32$$

$$= 308$$



L : $y = mx + b = -\frac{4}{3}x + 9\frac{2}{3}$

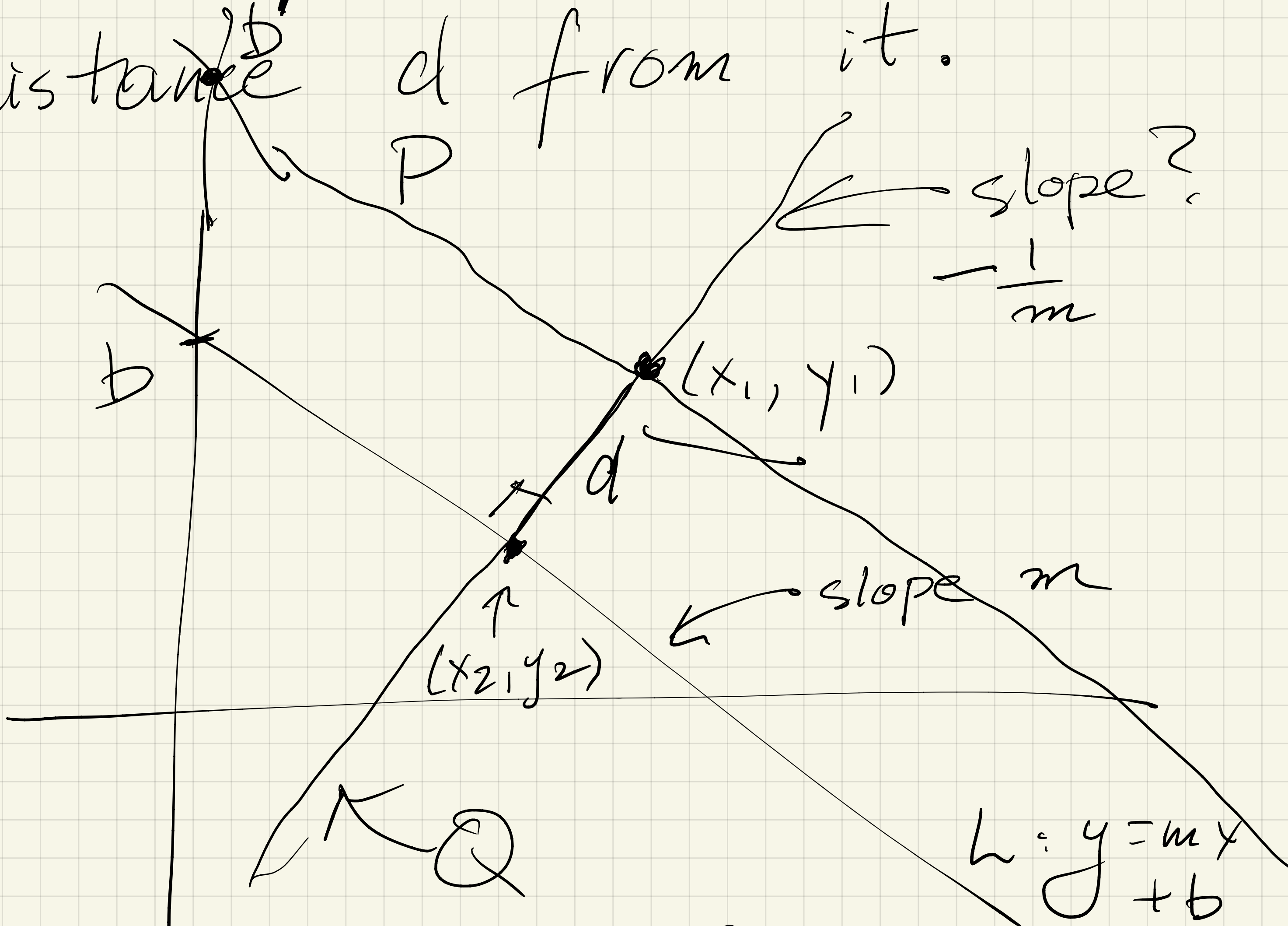
ΔOAB base $= 7\frac{1}{4}$ $\frac{h}{b} = \frac{4}{3}$

height $= 9\frac{2}{3}$

Check $\frac{9\frac{2}{3}}{7\frac{1}{4}} = \frac{29}{3} \Big/ \frac{29}{4} = \frac{4}{3}$ ✓

Given a line $y = mx + b$

Find the lines that are a distance d from it.



$$y = -\frac{1}{m}x + C$$

$$y_1 = -\frac{x_1}{m} + C$$

$$\boxed{C = y_1 + \frac{x_1}{m}}$$

(x_2, y_2) is on both L & Q

L: $y = mx + b$

Q: $y = -\frac{1}{m}x + c$ $\quad c = y_1 + \frac{x_1}{m}$

L: $y_2 = mx_2 + b$

Q: $y_2 = -\frac{x_2}{m} + c$

$mx_2 + b = -\frac{x_2}{m} + c$

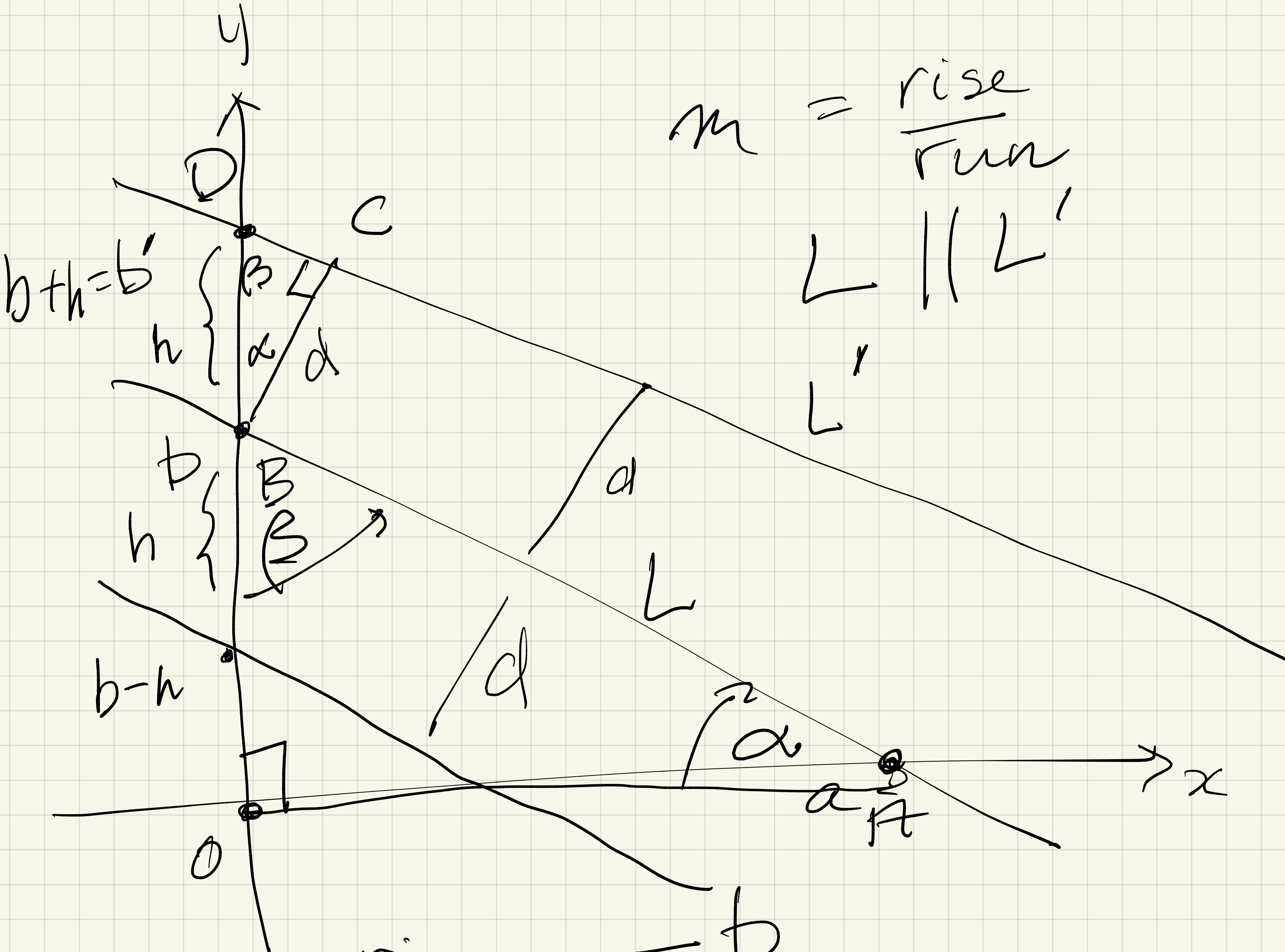
Given $\boxed{m, b, d}$ Find

$mx_2 + b = -\frac{x_2}{m} + y_1 + \frac{x_1}{m}$

P: $y = mx + b'$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Work out b'



$$m = \frac{rise}{run}$$

$$L \parallel L'$$

$$rise = -b$$
$$run = a$$
$$m = \frac{-b}{a}$$

$$\triangle OAB \equiv \triangle CBD$$

$$OB : OA = b : a = CD : CB$$

$$AB = \sqrt{a^2+b^2} \qquad CB : DB = OA : BA$$

$$d : b'-b = a : \sqrt{a^2+b^2}$$

$$CB : DB = OA : BA$$

$$d : b' - b = a : \sqrt{a^2+b^2}$$

$$\frac{b'-b}{d} = \frac{\sqrt{a^2+b^2}}{a}$$

$$b'-b = \frac{d}{a}\sqrt{a^2+b^2}$$

$$b' = b + \frac{d}{a}\sqrt{a^2+b^2}$$

$$b' = b + \pm \frac{d}{a}\sqrt{a^2+b^2}$$

Given $y = mx + b$ + d

then we can compute a.

$$h = \frac{d}{a}\sqrt{a^2+b^2}$$

$$\boxed{y = mx + \underbrace{b \pm h}}$$

Parallel Lines at distance h.

Homework: Solve #25 using algebra. Use the equations of the parallel lines at distance d.