To compute the optimal score strategy? pick some ordering on P.

Assume a total ordering on P

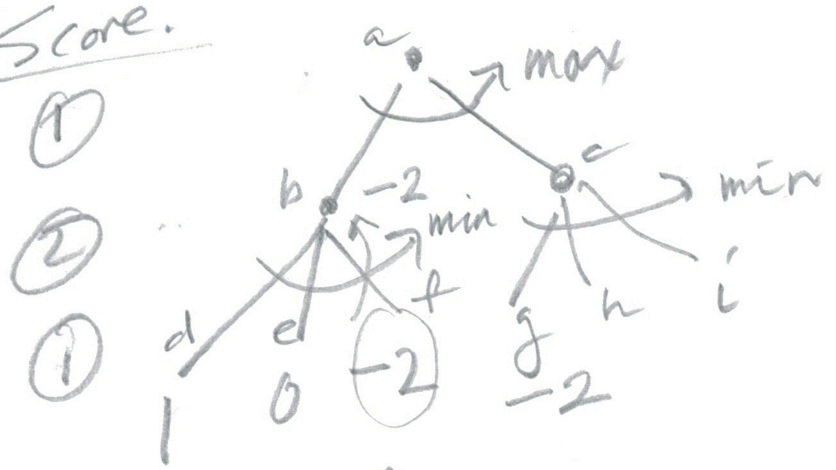
Then let optimal-strategy(P)

$$= \min_{q \mid (p,q) \in \text{moves} \wedge \text{optimal score}(p) = \text{optimal-score}(q)} q$$

Note that we can stop if optimal-score(q) is the min or max, depending on the player since we can't improve on that.

Alpha-beta pruning:

This is a way to limit that search. i.e. the computation of the optimal score.



Compute a → compute b = −2

compute c → compute g

$s(g) = -2$

$s(c) = \min_{x \in \{g, h, i\}} s(x) \leq -2$

$\Rightarrow \max\{s(b), s(c)\}$

$= \max\{-2, s(2)\}$

$= -2$ since $s(2) \leq -2$

therefore skip computation of h + i.

Proof obligation:

I gave an axiomatic description of optimal-score. We need to show that it exists and is unique.
But this is beyond the spec as written.
I need to define the set of all optimal score functions and show that it has a unique member.

Z allows "loose" specs.
An axiomatic description does not necessarily uniquely define the object.

2021-08-04

Prove something!

Try to prove that some sorting algorithm produces a sorted array.

1. Specify what it means for an array to be a sorted version of another array.
   - Look a integer sequences.

   A sequence is sorted if its elements are in ascending order...

Ascending
$$s : \operatorname{seq} \mathbb{Z}$$
$$\forall i, j : \operatorname{dom} s \mid i < j \bullet s(i) \leq s(j)$$

Permutation
$$P :$$

$n + (n-1) + \cdots + 1$

$n(n+1)/2$

$ascending = \{ s : LIST \mid (\forall i, j : dom\ s \mid i < j \cdot s(i) \leq s(j) \}$

$ascending1 = \{ s : LIST \mid (\forall i, j : dom\ s \mid i < \#dom \cdot s(i) \leq s(i+1) \}$

1. Goal $ascending = ascending1$

unfold $=$ $ascending \subseteq ascending1 \wedge ascending1 \subseteq ascending$

1.1 Goal $ascending \subseteq ascending1$

unfold $\subseteq$ $s \in ascending \Rightarrow s \in ascending1$

~~hyp~~ assume $s \in ascending$

unfold $s \in LIST \wedge (\forall i, j : dom\ s \mid i < j \cdot s(i) \leq s(j)$

intro $s \in LIST$

$\forall i, j : dom\ s \mid i < j \cdot s(i) \leq s(j)$

intro $i \in dom\ s$

$j \in dom\ s$

$i < j$

$s(i) \leq s(j)$

unfold $j \in 1 .. \#s$

$j \leq \#s$

$\Rightarrow$ $i < \#s$

$\Rightarrow$ $i + 1 \leq \#s$

$\Rightarrow$ $i \geq 1$

$\Rightarrow$

Lemma $\langle\rangle \in \text{ascending}1$ } recursive
$\langle\rangle \in \text{ascending}$ } proof.

$i, j : \text{dom } s \mid i < j$
$i : \text{dom } s \mid i < \#s \Rightarrow$ def $j = i+1$
$\Rightarrow i < j$
$\Rightarrow$ ~~$j \in$ do~~
$1 \le j \le \#s$
$\Rightarrow j : \text{dom } S$
$s(i) < s(j) = s(i+1)$

$i : \text{dom } s \mid i < \#s$

Prove $\text{ascending}1 \subseteq \text{ascending}$

A

$s : \text{ascending}1$

$\forall i : \text{dom } s \mid i < \#s \cdot s(i) \le s(i+1)$

$j : \text{dom } s \mid i < j$

if $j = i+1$ then $s(i) \le s(j)$

else if $j = i+2$ then

$s(i) \le s(i+1)$

and $s(i+1) \le s(i+2)$

so $s(i) \le s(i+2)$

so $s(i) \le s(j)$

$P(k) = s(i) \le s(i+k)$ true for $k=0$, $k=1$.

assume true for $k \le n$

then $s(i) \le s(i+n)$

but by def of ascending1 $s(i+n) \le s(i+n+1)$

so $s(i) \le s(i+n+1)$

Therefore $P(k)$ is true for all k in range.

s : ascending1

$\#s = 0 \Rightarrow s \in$ ascending?

ie $s = \langle\rangle \in$ ascending.

$\#s = 1 \Rightarrow s \in$ ascending

$n : \mathbb{N} \mid n > 1$

$\#s = n \Rightarrow s \in$ ascending Assume.

Assume.

$\#s = n$

$\#s = n + 1$ Assume.

~~$s = r \frown \langle x \rangle$~~

$r =$ head(s) } requires proof

show $r \in$ ascending1

$\#r = n$

so $r \in$ ascending.

$x =$ tail(s) $r(n) \le x$

$r(\#r) \le x \le r(\#r) = r(n)$

but $r(i) \le r(j)$ for all $i \le n$

$r(i) \le x$. $s(i) \le s(n+1) = x$

$r(n) \le x$ $i < j$

$\forall i, j \mid i < j$

so