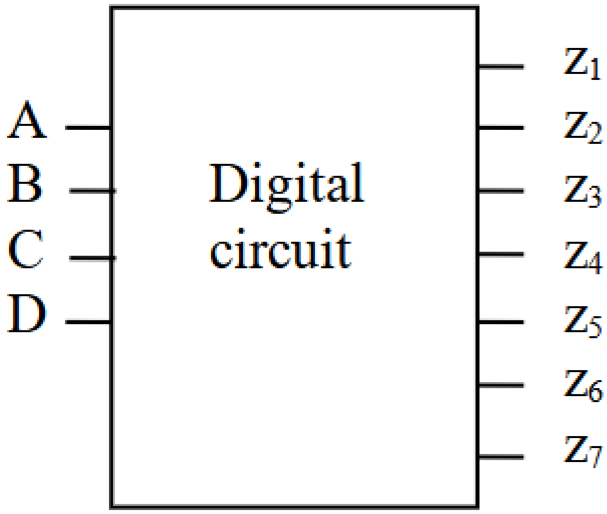
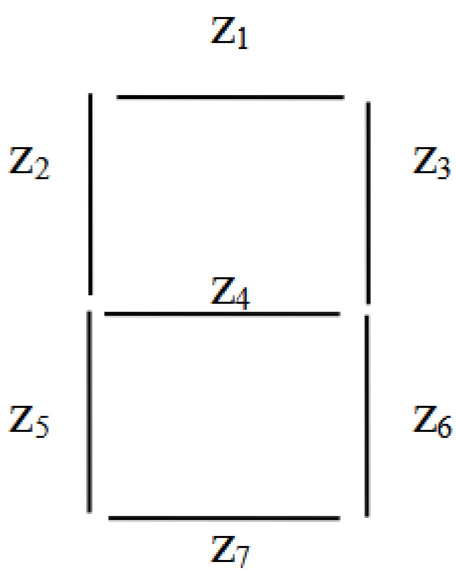
An application of Karnaugh Maps

A digital circuit is needed to control a seven-segment LED display of decimal digits, as shown below. The circuit has four inputs (A, B, C & D), which provide the 4-bit binary code used in packed decimal representation (010 = 0000, 110 = 0001, … 910 = 1001). The seven output Boolean expressions define which LED lights will be activated to display a given decimal digit. Note that some combinations of inputs and outputs are not needed and, consequently, are marked with a ‘d’ (don’t care).





Truth Table for digital circuit:

Input				Output						
A	B	C	D	z_1	z_2	z_3	z_4	z_5	z_6	z_7
0	0	0	0	1	1	1	0	1	1	1
0	0	0	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0	1

Input				Output						
0	0	1	1	1	0	1	1	0	1	1
0	1	0	0	0	1	1	1	0	1	0
0	1	0	1	1	1	0	1	0	1	1
0	1	1	0	1	1	0	1	1	1	1
0	1	1	1	1	0	1	0	0	1	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	0
1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d
1	1	0	0	d	d	d	d	d	d	d
1	1	0	1	d	d	d	d	d	d	d
1	1	1	0	d	d	d	d	d	d	d
1	1	1	1	d	d	d	d	d	d	d

Find the simplified SOP (Sum Of Products) expression for each of the outputs, z_2 through z_7. (The expression for z_1 has already been done.) Using the truth table on the 1st page, you will need to fill in the tables for z_5 through z_7. Since we don't care what output results in those squares occupied by 'd', you may treat them as a 1 or 0 - whichever leads to a simpler SOP expression.

z_1:

AB/CD	00	01	11	10
00	1		1	1

AB/CD	00	01	11	10
01		1	1	1
11	d	d	d	d
10	1	1	d	d

- $z_1 = A + C + BD + \overline{BD}$

z_2:

AB/CD	00	01	11	10
00	1			
01	1	1		1
11	d	d	d	d
10	1	1	d	d

- $z_2 = B\overline{C} + A\overline{C} + \overline{CD} + B\overline{D}$

z_3:

AB/CD	00	01	11	10
00	1	1	1	1
01	1		1	
11	d	d	d	d
10	1	1	d	d

- $z_3 = \overline{B} + \overline{CD} + CD$

z_4:

AB/CD	00	01	11	10
00			1	1
01	1	1		1
11	d	d	d	d
10	1	1	d	d

- $z_4 = B\overline{C} + A + \overline{B}C + C\overline{D}$

Recall, for the next 3 outputs you need to retrieve the 1's from the truth table and insert them in the tables.

z_5:

AB/CD	00	01	11	10
00	1			1
01				1
11	d	d	d	d
10	1		d	d

- $z_5 = C\overline{D} + \overline{BD}$

z_6:

AB/CD	00	01	11	10
00	1	1	1	
01	1	1	1	1
11	d	d	d	d
10	1	1	d	d

- $z_6 = B + D + \overline{C}$

z_7:

AB/CD	00	01	11	10
00	1		1	1
01		1		1
11	d	d	d	d
10	1		d	d

- $z_7 = \overline{BD} + \overline{B}C + B\overline{C}D + C\overline{D}$