

Homework 5

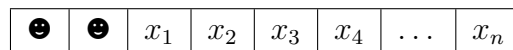
March 3, 2017

(804501476)

Problem 1.0:

We emulate a TM with FRTM as follows:

1) **The start state:** We add two ☹ and then copy the input after.



2) **Move left:** After moving to the left-end of the tape. Move right until we see second ☹ then move right once.

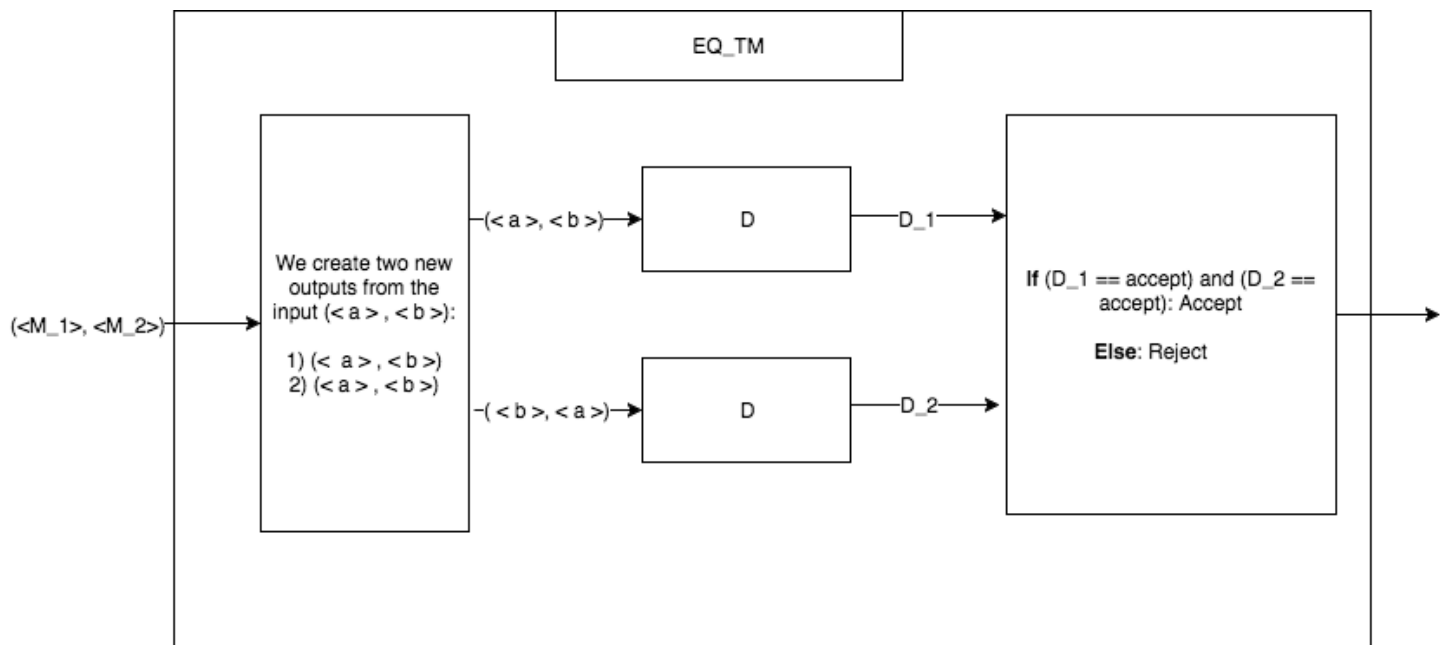
3) **Move right:** If first time moving right, just move right. Else:

- Move left and then move right until we see ☹ and delete it.
- Move right until we find the next ☹. Then more move right twice.
- Write ☹ and move right twice.

Problem 2.0:

Proof

Suppose $Subset_{TM}$ is decidable $\implies \exists D$ decides $Subset_{TM}$. We build a turing machine to decide $EQ_{TM} = \{(\langle M_1 \rangle, \langle M_2 \rangle) \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$



Theorem 5.4 states that EQ_{TM} is not decidable $\Rightarrow \Leftarrow$. Therefore $Subset_{TM}$ is not decidable.

Problem 3.0:

(a) We can include L_1^{Diag} into the enumeration as such $(L_1^{Diag}, L_1, L_2, \dots)$ and let α_i be a lexicographical enumeration of strings over $\{0, 1\}$. We construct

$$L_2^{Diag} = \{\bar{x}_{ii} \mid \text{if } x_{ii} = 1\} \quad (3.1)$$

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	...
L_1^{diag}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	...
L_1	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	...
L_2	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	...
L_3	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	...
L_4	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	...
L_5	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	...
L_6	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	...

Claim: L_2^{Diag} is not in the table.

Proof.

Towards contradiction suppose L_2^{Diag} is in the table. Then $\exists i$ s.t $L' = x_{1i}x_{2i}x_{3i}\dots \in L_2^{Diag}$. Look at position x_{ii} . If x_{ii} is in L' then by definition it is not in L_2^{diag} . Therefore L_2^{Diag} is no in $L_i \forall i \geq i$ and $L_2^{diag} \neq L_1^{diag}$. \square

(b)

Properties:

$$1 \ \forall i, j, i \neq j, L_i^{diag} \neq L_j^{diag}$$

$$2 \ \forall j \ l_i^{diag} \neq L_j$$

Using the method in the previous problem we can construct new L_i^{Diag} 's and include them into the enumeration $E = (L_1^{Diag}, L_2^{Diag}, L_3^{Diag}, \dots, L_1, L_2, L_3, \dots)$. We also note that by consruction of E, $\mathcal{L}^{diag} = \{L_1^{diag}, L_2^{diag}, L_3^{diag}, \dots\} \subseteq E$

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	...
L_1^{diag}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	...
L_2^{diag}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	...
L_3^{diag}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	...
...								
L_1	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	...
L_2	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	...
L_3	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	...
L_4	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	...

Claim: \mathcal{L}^{diag} satisfies properties 1) and 2):

Proof.

Base case: Let $E = (L_1, L_2, L_3, \dots)$ in class we showed that L_1^{diag} is not in E and satisfies properties 1) and 2) so we add it $E = (L_1^{diag}, L_1, L_2, L_3, \dots)$.

Inductive Step: Assume properties are true for n , s.t $E = (L_1^{diag}, L_2^{diag}, \dots, L_n^{diag}, L_1, L_2, L_3, \dots)$ holds. Then we can construct L_{n+1}^{diag} , by definition the first $x_{11}x_{22} \dots x_{n-1n-1} = L_n^{diag}$. We add $x_{n+1} = \{\bar{x}_{nn} \mid \text{if } x_{nn} = 1\}$. Now L_{n+1}^{diag} holds for property 2) since L_n^{diag} holds and the addition of x_{n+1} makes it so that $L_{n+1}^{diag} \neq L_n^{diag}$. Therefore L_{n+1}^{diag} is not equal to any language in the table. \square

(c) We can arrange the set E' constructed in the previous problem as such $E' = (L_1, L_1^{diag}, L_2, L_2^{diag}, \dots)$, where we alternate between the languages L_i and the languages L_i^{diag} . We construct $L^{superdiag} = \{\bar{x}_{ii} \mid \text{if } x_{ii} = 1\}$.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	...
L_1	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	...
L_1^{diag}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	...
L_2	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	...
L_2^{diag}	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	...
...

Claim: $L^{superdiag}$ is not in the set.

Proof. Towards contradiction suppose $L^{superdiag} \in E$. Then $\exists i$ s.t $L' = x_{1i}x_{2i}x_{3i} \dots \in L^{superdiag}$. Look at position x_{ii} . If x_{ii} is in L' then by definition it is not in $L^{superdiag} \Rightarrow \Leftarrow$. Therefore $L^{superdiag} \notin E$. \square

(d) We can append $L^{superdiag}$ to set E' of the previous problem and then create $L_2^{superdiag} = \{\bar{x}_{ii} \mid \text{if } x_{ii} = 1\}$.

Claim: $L_2^{superdiag}$ is not in the set.

Proof. Towards contradiction suppose $L_2^{superdiag} \in E'$. Then $\exists i$ s.t $L' = x_{1i}x_{2i}x_{3i} \dots \in L_2^{superdiag}$. Look at position x_{ii} . If x_{ii} is in L' then by definition it is not in $L^{superdiag} \Rightarrow \Leftarrow$. Therefore $L^{superdiag} \notin E'$. \square