Homework 2

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Problem 1.0:

$$Q = \begin{pmatrix} \hat{\beta} \\ \hat{Y} \end{pmatrix} = \begin{pmatrix} (X'X)^{-1}X'Y \\ (I-H)Y \end{pmatrix} \tag{1.1}$$

$$Q = LY \tag{1.2}$$

$$Var(Q) = Var(LY) = LVar(Y)L' \tag{1.3}$$

$$= \sigma^2 LL' \tag{1.4}$$

$$= \sigma^2 \begin{pmatrix} (X'X)^{-1}X' \\ (I-H) \end{pmatrix} \begin{pmatrix} (X'X)^{-1}X' & (I-H) \end{pmatrix} \tag{1.5}$$

$$= \sigma^2 \begin{pmatrix} (X'X)^{-1} & (X'X)^{-1}X'(I-H) \\ (I-H)X(X'X)^{-1} & (I-H) \end{pmatrix} \tag{1.6}$$

$$Var(Q) = \sigma^2 \begin{pmatrix} (X'X)^{-1} & 0 \\ 0 & (I-H) \end{pmatrix} \tag{1.7}$$

Therefore $\hat{Y}$ and e are independent.

Problem 2.0:

Problem 3.0:

Problem 3.0 continued on next page...

We have that $S^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$ and $S_e^2 = \frac{e'e}{n-k-1} = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{n-k-1}$

$$R_a^2 = \frac{S^2 - s_e^2}{S^2} \tag{3.8}$$

$$= \frac{n-1}{\sum_{i=1}^{n}(y_i - \bar{y})^2}\left[\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} - \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-k-1} + \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{n-k-1}\right] \tag{3.9}$$

$$= \frac{n-1}{\sum_{i=1}^{n}(y_i - \bar{y})^2}\left[\frac{(n-k-1)\sum_{i=1}^{n}(y_i - \bar{y})^2 - (n-1)\sum_{i=1}^{n}(y_i - \bar{y})^2 + (n-1)\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{(n-1)(n-k-1)}\right] \tag{3.10}$$

$$= \left[\frac{(n-k-1) - (n-1) + (n-1)\frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}}{n-k-1}\right] \tag{3.11}$$

$$= 1 - \frac{(n-1) - (n-1)R^2}{n-k-1} \tag{3.12}$$

$$\boxed{R_a^2 = 1 - \frac{n-1}{n-k-1}(1 - R^2)} \tag{3.13}$$

Problem 4.0:

Problem 5.0:

$$(Y - Xc)'(Y - Xc) - (Y - X\hat{\beta})'(Y - X\hat{\beta}) \tag{5.14}$$

$$Y'Y - Y'Xc - c'X'Y + c'X'Xc - Y'Y - Y'Y + Y'X\hat{\beta} + \hat{\beta}'X'Y - \hat{\beta}'X'X\hat{\beta} \tag{5.15}$$

$$\tag{5.16}$$

Problem 5.0 continued on next page. . .

We use the normal equations to substitute for $Y'X$ and $X'Y$.

$$-\hat{\beta}'X'Xc - c'X'X\hat{\beta} + c'X'Xc + \hat{\beta}X'X\hat{\beta} + \hat{\beta}'X'X\hat{\beta} - \hat{\beta}X'X\hat{\beta} \tag{5.17}$$

$$c'X'Xc - \hat{\beta}'X'Xc - c'X'X\hat{\beta} + \hat{\beta}'X'X\hat{\beta} \tag{5.18}$$

$$(c'X'X - \hat{\beta}'X'X)(c - \hat{\beta}) \tag{5.19}$$

$$(c' - \hat{\beta}')X'X(c - \hat{\beta}) \tag{5.20}$$

$$\boxed{(c - \hat{\beta})'X'X(c - \hat{\beta})} \tag{5.21}$$

Problem 6.0:

a

$$\hat{\beta} = \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = (X'X)^{-1}X'Y \tag{6.22}$$

$$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \tag{6.23}$$

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} 8 \\ -11 \\ 7 \end{pmatrix} \tag{6.24}$$

b

$$X = \begin{pmatrix} 1 & 3 & 1 & -1 & 1 \\ 1 & 4 & 1 & 1 & -1 \\ 1 & 5 & -1 & 1 & 1 \\ 1 & 6 & 0.5 & 0.2 & 0.3 \\ 1 & 8 & 0.8 & 0.1 & 0.1 \\ 1 & 9 & 0.3 & 0.5 & 0.2 \\ 1 & 10 & 0.2 & 0.3 & 0.5 \\ 1 & 13 & 0.1 & 0.6 & 0.3 \end{pmatrix} \tag{6.25}$$

$x_2 + x_3 + x_4 = 1 \implies$ matrix is not full rank and we cannot invert $X'X$.

Problem 7.0: