Homework 4

Feb 19, 2017

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Problem 1.0:

a)

$L_1 = \{0^a 1^b \mid a \text{ divides } b\}$ is equivalent to $L_1 = \{0^a 1^{am} \mid m \geq 0\}$.

Proof.
Towards contradiction:
- Assume L_1 is CF.
- By pumping lemma $\exists$ pumping length p.
- Let $s = 0^p 1^{pm}$ s.t. $|s| \geq p$.
- By pumping lemma $0^p 1^{pm} = xyzuv$

1) $|yzu| \leq p$
2) $|yu| > 0$

Cases:
1) $yzu = 0^k$, $1 \leq k \leq p$. Then $xy^2zu^2v = 0^{p+\alpha}1^{pm}$

$$\frac{a}{b} = m \tag{1.1}$$
$$\frac{pm}{p+\alpha} = m \tag{1.2}$$
$$pm = pm + m\alpha \tag{1.3}$$
$$m\alpha = 0 \tag{1.4}$$

α cannot be zero.
2) $yzu = 1^\alpha$ then $xzv = 0^p 1^{pm-\alpha}$

$$\frac{a}{b} = m \tag{1.5}$$
$$\frac{pm - \alpha}{p} = m \tag{1.6}$$
$$pm - \alpha = pm \tag{1.7}$$
$$\alpha = 0 \tag{1.8}$$

Problem 1.0 continued on next page...

α cannot be zero.
3) $yzu = 0^{\alpha}1^{\beta}$, $1 \leq \alpha + \beta \leq p$ then $xy^2zv^2u = 0^{p+\alpha}1^{pm+\beta}$

$$\frac{a}{b} = m \tag{1.9}$$

$$\frac{pm+\beta}{p+\alpha} = m \tag{1.10}$$

$$pm + \beta = pm + m\alpha \tag{1.11}$$

$$\beta = m\alpha \tag{1.12}$$

□

b)

Problem 2.0:

Problem 3.0: