Homework 5

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Problem 1.0:
We emulate a TM with FRTM as follows:
1) Seach Symbol: We use two ☻ to determine what the character on the left is.
2) The start state: We write the input onto the tape and mark the first one with ☻.

☻					
x_1	x_2	x_3	x_4	...	x_n

3) Move Right: First remove ☻ from current position then move right and write ☻ in new position.

For example:

	☻				
x_1	x_2	x_3	x_4	...	x_n

4) Move Left: We will determine the character on the left by guessing. We will move left and write ☻ to the first element.

For example:

☻			☻		
x_1	x_2	x_3	x_4	...	x_n

We move the head right and check if the next element has a ☻. If not move the head to the left again and begin to move right until the leftmost ☻ is encountered. We remove the it and move right, writing ☻ in the element under the new position.

For example:

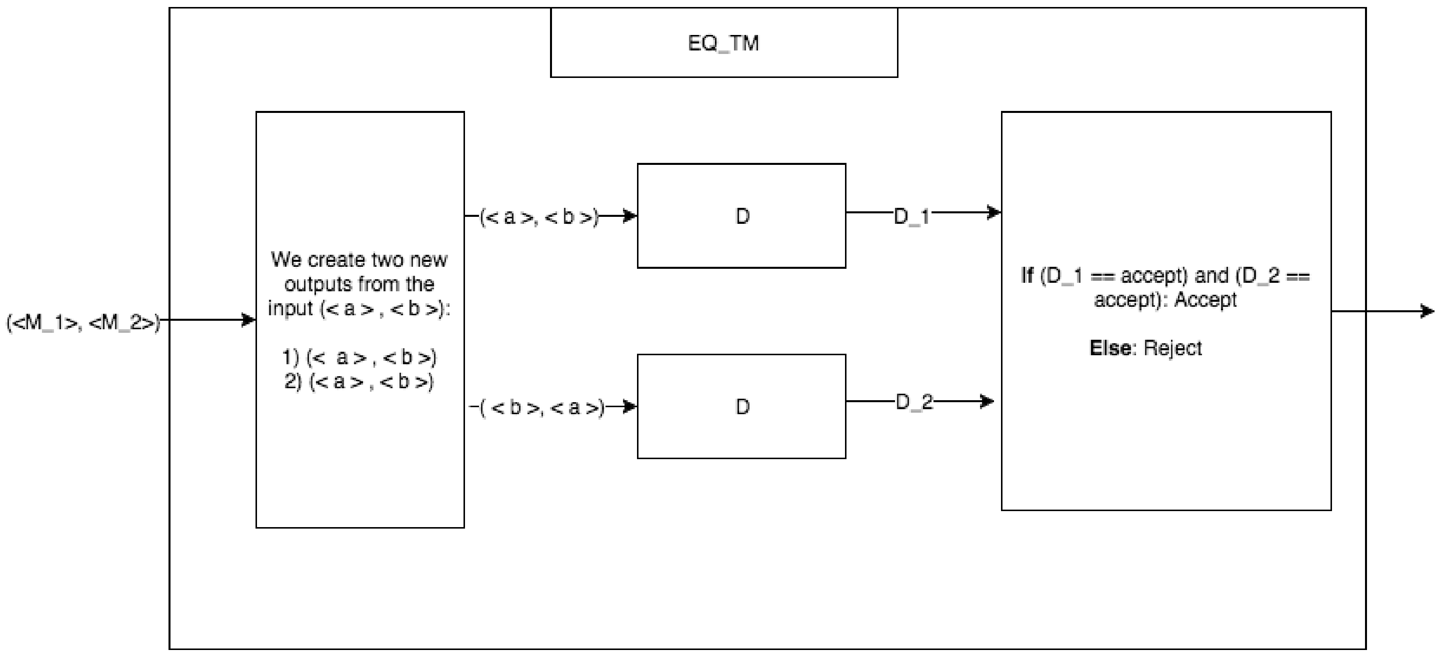
	☻		☻		
x_1	x_2	x_3	x_4	...	x_n

We continue to do this until we have two consecutive ☻. Once this is true we remove the ☻ under the head and go left except this time we stop when we reach ☻. This is the element to the left.

Problem 2.0:

Proof

Suppose $Subset_{TM}$ is decidable $\implies \exists\ D$ decides $Subset_{TM}$. We build a turing machine to decide $EQ_{TM} = \{(< M_1 >, < M_2 >) \mid M_1\ and\ M_2\ are\ TMs\ and\ L(M_1) = L(M_2)\}$



Theorem 5.4 states that EQ_{TM} is not decidable $\Rightarrow\Leftarrow$. Therefore $Subset_{TM}$ is not decidable.

Problem 3.0:

(a) We can include L_1^{Diag} into the enumeration as such $(L_1^{Diag}, L_1, L_2, \dots)$ and let α_i be a lexicographical enumeration of strings over $\{0,1\}$. We constructm

$$L_2^{Diag} = \{\bar{x}_{ii} \mid \text{if } x_{ii} = 1\} \tag{3.1}$$

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	$\dots$
L_1^{diag}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	$\dots$
L_1	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	$\dots$
L_2	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	$\dots$
L_3	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	$\dots$
L_4	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	$\dots$
L_5	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	$\dots$
L_6	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	$\dots$

Claim: L_2^{Diag} is not in the table.

Proof.

Towards contradiction suppose L_2^{Diag} is in the table. Then $\exists\ i$ s.t $L' = x_{1i}x_{2i}x_{3i}\dots \in L_2^{Diag}$. Look

Problem 3.0 continued on next page. . .

at position x_{ii}. If x_{ii} is in L' then by definition it is not in L_2^{diag}. Therfore L_2^{Diag} is no in $L_i \ \forall \ i \geq i$ and $L_2^{diag} \neq L_1^{diag}$. □

(b)
Properties:

1 $\forall i, j, \ i \neq j, \ L_i^{diag} \neq L_j^{diag}$

2 $\forall j \ l_i^{diag} \neq L_j$

Using the method in the previous problem we can construct new L_i^{Diag}'s and include them into the enumeration $E = (L_1^{Diag}, L_2^{Diag}, L_3^{Diag}, \ldots, L_1, L_2, L_3, \ldots)$. We also note that by consruction of E, $\mathcal{L}^{diag} = \{L_1^{diag}, L_2^{diag}, L_3^{diag}, \ldots\} \subseteq E$

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	$\ldots$
L_1^{diag}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	$\ldots$
L_2^{diag}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	$\ldots$
L_3^{diag}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	$\ldots$
$\ldots$								
L_1	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	$\ldots$
L_2	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	$\ldots$
L_3	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	$\ldots$
L_4	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	$\ldots$

Claim: $\mathcal{L}^{diag}$ satisfies properties 1) and 2):

Proof.
Base case: Let $E = (L_1, L_2, L_3, \ldots)$ in class we showed that L_1^{diag} is not in E and satisfies properties 1) and 2) so we add it $E = (L_1^{Diag}, L_1, L_2, L_3, \ldots)$.
Inductive Step: Assume properties are true for n, s.t $E = (L_1^{Diag}, L_2^{Diag}, \ldots, L_n^{diag}, L_1, L_2, L_3, \ldots)$ holds. Then we can construct L_{n+1}^{diag}, by definition the first $x_{11}x_{22}\ldots x_{n-1n-1} = L_n^{diag}$. We add $x_{n+1} = \{\bar{x}_{nn} \mid \text{if } x_{nn} = 1\}$. Now L_{n+1}^{diag} holds for property 2) since L_n^{diag} holds and the addition of x_{n+1} makes it so that $L_{n+1}^{diag} \neq L_n^{diag}$ Therefore L_{n+1}^{diag} is not equal to any language in the table. □

(c) We can arrange the set E' constructed in the previous problem as such $E' = (L_1, L_1^{diag}, L_2, L_2^{diag}, \ldots)$, where we alternate between the languages L_i and the languages L_i^{diag}. We construct $L^{superdiag} = \{\bar{x}_{ii} \mid \text{if } x_{ii} == 1\}$.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	$\ldots$
L_1	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	$\ldots$
L_1^{diag}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	$\ldots$
L_2	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	$\ldots$
L_2^{diag}	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

Problem 3.0 continued on next page. . .

Claim: $L^{superdiag}$ is not in the set.

Proof. Towards contradiction suppose $L^{Superdiag} \in E$. Then $\exists\ i$ s.t $L^{'} = x_{1i}x_{2i}x_{3i}\ldots \in L^{superdiag}$. Look at position x_{ii}. If x_{ii} is in $L^{'}$ then by definition it is not in $L^{Superdiag}$ $\Rightarrow\Leftarrow$. Therfore $L^{Superdiag} \notin E$. $\square$

(d) We can append $L^{superdiag}$ to set $E^{'}$ of the previous problem and then create $L_2^{superdiag} = \{\bar{x}_{ii} \mid \text{if } x_{ii} = 1\}$.
Claim: $L_2^{superdiag}$ is not in the set.

Proof. Towards contradiction suppose $L_2^{Superdiag} \in E^{'}$. Then $\exists\ i$ s.t $L^{'} = x_{1i}x_{2i}x_{3i}\ldots \in L_2^{superdiag}$. Look at position x_{ii}. If x_{ii} is in $L^{'}$ then by definition it is not in $L^{Superdiag}$ $\Rightarrow\Leftarrow$. Therfore $L^{Superdiag} \notin E^{'}$. $\square$