Homework 1

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Problem 1.0:

a

Let $M = (Q, \Sigma, \delta, q_0^{'}, F)$ be a DFA that accepts w. We construct an NFA $N = (Q^{'}, \Sigma, \delta^{'}, q_0, F^{'})$ that also accepts w.

Construct N :
1. $Q^{'} = Q$
2. $q_0^{'} = q_0$
3. $\delta^{'}(q_i, w_i) = \{\delta(q_i, w_i)\}$ a unit set.
4. $F^{'} = F$

From this contruction we emulate M exactly since our delta function only uses one of the possible paths an NFA could take. After the finishing the computation of w there is only one accept state and only one path.

b

Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA that accepts w. We construct a DFA $M = (Q^{'}, \Sigma, \delta^{'}, q_0^{'}, F^{'})$ that also accepts w. Construct M :
1. $Q^{'} = \mathcal{P}(Q)$
2. $\delta^{'}(R, w_i) = \epsilon - closure(\bigcup_{r \in R} \delta(r, w_i))$
3. $q_0 = \epsilon - closure(\{q_0\})$
4. $F^{'} = \{a \in Q^{'} \mid \ |a \in F| > \frac{|a|}{3}\}$

Problem 2.0:
We $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that accepts L. We construct an NFA M' that accepts L_R

Constructing M' :
1. $Q' = Q$
2. $\Sigma = \Sigma$
3. $\delta'(q_i, a \in \Sigma) = \{q_j \mid \delta(q_j, a \in \Sigma) = q_i\}$
4. $q_0' \in F$
5. $F' = q_0$

Claim: w is accepted by M $\iff$ w^R is accepted by M'

Proof.
($\Leftarrow$) Let $w = w_1 w_2 \dots w_m \in L$. Machine M starts at q_0 and $\forall\ i \geq 0$, $q_{i+1} = \delta(q_i, w_i)$. When we reach w_n, $\delta(q_{n-1}, w_n) \in F$.

($\Rightarrow$) Let $w^R = w_n w_{n-1} \dots w_1 \in L_R$. By construction Machine M' starts at $q_0' \in F$, the accepting states of M. Also by construciton of the delta function, $\delta'(q_i, a \in \Sigma)$, we reversed the paths of Machine M s.t $\forall\ i \leq n$, $\delta'(q_i, a)$ will send us to a state q_j where $\delta(q_j, a) = q_i$. When we reach w_1, $\delta(q_1, w_1) \in F' = q_0$, the start state of M. □

Problem 3.0:
We $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that accepts L. We construct an NFA M' that accepts L_{alt}

Constructing M' :
1. $Q' = Q$
2. $\Sigma = \Sigma$
3. $\delta'(q_i, a \in \Sigma) = \{q_j \mid \delta(q_{i+1}, b \in \Sigma) = q_j\}$
4. $q_0' = q_0$
5. $F' = \{q_{n-2}, q_n\}$

Claim: w is accepted by M $\iff$ w_{alt} is accepted by M'

Proof.
($\Leftarrow$) Let $w = w_1 w_2 \dots w_m \in L$. Machine M starts at q_0 and $\forall\ i \geq 0$, $q_{i+1} = \delta(q_i, w_i)$. When we reach w_n, $\delta(q_{n-1}, w_n) \in F$

($\Rightarrow$) Let $w_{alt} = w_1 w_3 w_5 \dots w_m \in L_{alt}$. By construction machine M' starts at q_0. Also by construction of $\delta'(q_i, a \in \Sigma)$ we skip every other state into a state q_j s.t $\delta(q_{i+1}, b \in \Sigma) = q_j$. When we process w_m we will be in one of the accepting states q_{n-2} when m is odd and q_n when m is even. □

Problem 3.0 continued on next page. . .

Problem 4.0:
We $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA that accepts L. We construct an NFA $M^{'}$ that accepts $L_{\frac{1}{2}-}$

Constructing $M^{'}$:
1. $Q^{'} = Q \times Q$
2. $\Sigma = \Sigma$
3. $\delta^{'}(q_i, a \in \Sigma) = \begin{cases} \delta(q_i, a) \\ q_j \mid \exists\ \delta(q_j, b \in \Sigma) = q_i \end{cases}$
4. $q_0^{'} = \{(q_0, q_i) \mid q_i \in F\}$
5. $F^{'} = \{(q_i, q_j) \mid i = j\}$

Claim: w is accepted by M $\iff$ $w_{\frac{1}{2}-}$ is accepted by $M^{'}$

Proof. ($\Leftarrow$) Let $w = w_1 w_2 \ldots w_m \in L$. Machine M starts at q_0 and $\forall\ i \geq 0$, $q_{i+1} = \delta(q_i, w_i)$. When we reach w_n, $\delta(q_{n-1}, w_n) \in F$

($\Rightarrow$) Let $w_{\frac{1}{2}-} = w_1 w_2 \ldots w_{m/2}\ inL_{\frac{1}{2}-}$. By construction machine $M^{'}$ starts at $\{(q_0, q_i) \mid q_i \in F\}$. So we run two machines the original machine M and a modified version of M in reverse. By construction of $\delta^{'}$ □