Student ID: ____________________

Collaborators: ____________________

CS181 Winter 2017 – Problem Set 2

Due Thursday, February 2, 11:59 pm

- Please write your student ID **and the names of anyone you collaborated with** in the spaces provided and attach this sheet to the front of your solutions. **Please do not include your name anywhere since the homework will be blind graded.**
- An extra credit of **5%** will be granted to solutions written using LaTeX. Here is one place where you can create LaTeXdocuments for free: `https://www.sharelatex.com/`. The link also has tutorials to get you started. There are several other editors you can use.
- If you are writing solutions by hand, please write your answers in a neat and readable hand-writing.
- Always explain your answers. When a proof is requested, you should provide a rigorous proof.
- 20% of the points will be given if your answer is "I don't know". However, if instead of writing "I don't know" you write things that do not make any sense, no points will be given.
- The homework is expected to take anywhere between 8 to 14 hours. You are advised to start early.
- Submit your homework in drop box A1 in BH 2432 or online on the course webpage on CCLE. You can also hand it in at the end of any class before the deadline.

> Note: *All questions in the problem sets are challenging; you should not expect to know how to answer any question before trying to come up with innovative ideas and insights to tackle the question. If you want to do some practice problems before trying the questions on the problem set, we suggest trying problems 1.17 and 1.23 from the book. Do not turn in solutions to problems from the book.*

1. **(20 points)** Let L_1 and L_2 be languages and define

$$\text{shuffle}(L_1, L_2) = \{x_1\, y_1\, x_2\, y_2\, \ldots\, x_n\, y_n \mid x_1\, \ldots\, x_n \in L_1, y_1\, \ldots\, y_n \in L_2\}.$$

   (a) **(10 points)** Show that if the language L_1 is not regular and L_2 is any language then the languages shuffle(L_1, L_2) and shuffle$(L_1, \overline{L_2})$ cannot both be regular.

   (b) **(10 points)** Show that if L_1 and L_2 are regular languages then shuffle(L_1, L_2) is regular.

   Hint: For (a) recall closure properties of regular languages.

2. (**40 points**) In this problem we investigate the limits of the Pumping Lemma as it was stated in class and look for an alternative that remedies one of these shortcomings.

   (a) **(10 points)** Let L_1 be the language

$$L_1 = \{a^i b^p \mid i \geq 0 \text{ and } p \text{ is a prime}\}.$$

   Prove that the language $L_2 = b^* \cup L_1$ satisfies the conditions of the Pumping Lemma. I.e. show that there exists a $p \in \mathbb{N}$ such that for every word $w \in L_2$ with $|w| \geq p$ we can write $w = xyz$ such that $|xy| \leq p$, $|y| > 0$, and for every $i \geq 0$, $xy^iz \in L_2$.

   (b) **(20 points)** Prove the following generalization of the Pumping Lemma:
   Let L be a regular language. There exists a $p \in \mathbb{N}$ such that for every $w \in L$ and every partition of w into $w = xyz$ with $|y| \geq p$ there are strings a, b, c such that $y = abc$, $|b| > 0$, and for all $i \geq 0$, $xab^icz \in L$.

   (c) **(10 points)** Prove that the language L_2 is not regular.

3. (**40 points**) For a language L over alphabet Σ, we define

$$L_{\frac{1}{3}-\frac{1}{3}} = \{xz \in \Sigma^* \mid \exists y \in \Sigma^* \text{ with } |x| = |y| = |z| \text{ such that } xyz \in L\}.$$

   Prove that if L is regular, then $L_{\frac{1}{3}-\frac{1}{3}}$ need not be regular.
   Hint: Consider the language 0^*21^* *and recall closure properties of regular languages*