

MSAN 502 - Homework 2

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My previous version of the **eliminate** algorithm was built around a big **for** loop (*for every row in matrix A, perform elimination on next rows...*). While this works fine for perfect scenarios, it was not versatile enough to bypass temporary failures. In order to account for them, and therefore allow permutations, I had to modify my code into a big **while** loop (*while we haven't reached the bottom of matrix A, perform elimination. If we can't, try permutation*). My **Eliminate.py** file is commented well enough (in my humble opinion) for someone to understand what every line is doing.

Some big changes comparing to the previous version is that my algorithm now accepts non-square matrices (although it is still not entirely useful for this week's assignment). $n \times n$ cases are handled fairly well in my tests. I have a few checks in place as well, to make sure that everything is set up correctly in order for my algorithm to work without hiccups (A and b must be instances of **np.array**, their dimensions must match, etc).

My code solves $Ax = b$ when there is a unique solution, even if permutations of rows are needed. Print statements (original matrix, reduced matrix, solution vector, etc) have been commented out in order to run the complexity problem below, but can be de-commented as needed.

My algorithm runs into a permanent failure if the **solvability check** fails. This means that, at one point in the algorithm, we reached a row where we have 0 on the left side of the equation and $c \neq 0$ on the right side, which is an impossibility. By calculating the number of pivots and comparing to the number of columns, I also have the number of special solutions.

If there are no special solutions, then it means there is only one solution. In the other case, there are infinitely many solutions. Again, print statements may have been commented out in order to run the complexity problem. In both of these cases, my algorithm runs to the end. If there is only one solution, backwards substitution finds the unique solution. I haven't yet integrated the scenario of infinitely many solutions (finding the particular and special solutions).

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Time complexity of my implementation of this algorithm can be found in **complexity.py**. I run my algorithm ten times to solve $Ax = b$, where A is a $n \times n$ matrix with random noise for $n = 2^k$, where $k \in [2; 12]$. I save the average run time for each n .

Then, I use **complexity_plot.py** to plot *log time* as a function of *log n*. The resulting plot is shown in figure 1.

We can see that there seems to be a linear correlation between *log time* and *log n*. The slope of the most linear part (between $n = 128$ and $n = 4096$), the slope is $\frac{\Delta y}{\Delta x} = 2.05$. Therefore, empirically, the run time complexity of my implementation of the elimination algorithm is $O(n^2)$. This is actually not that bad!

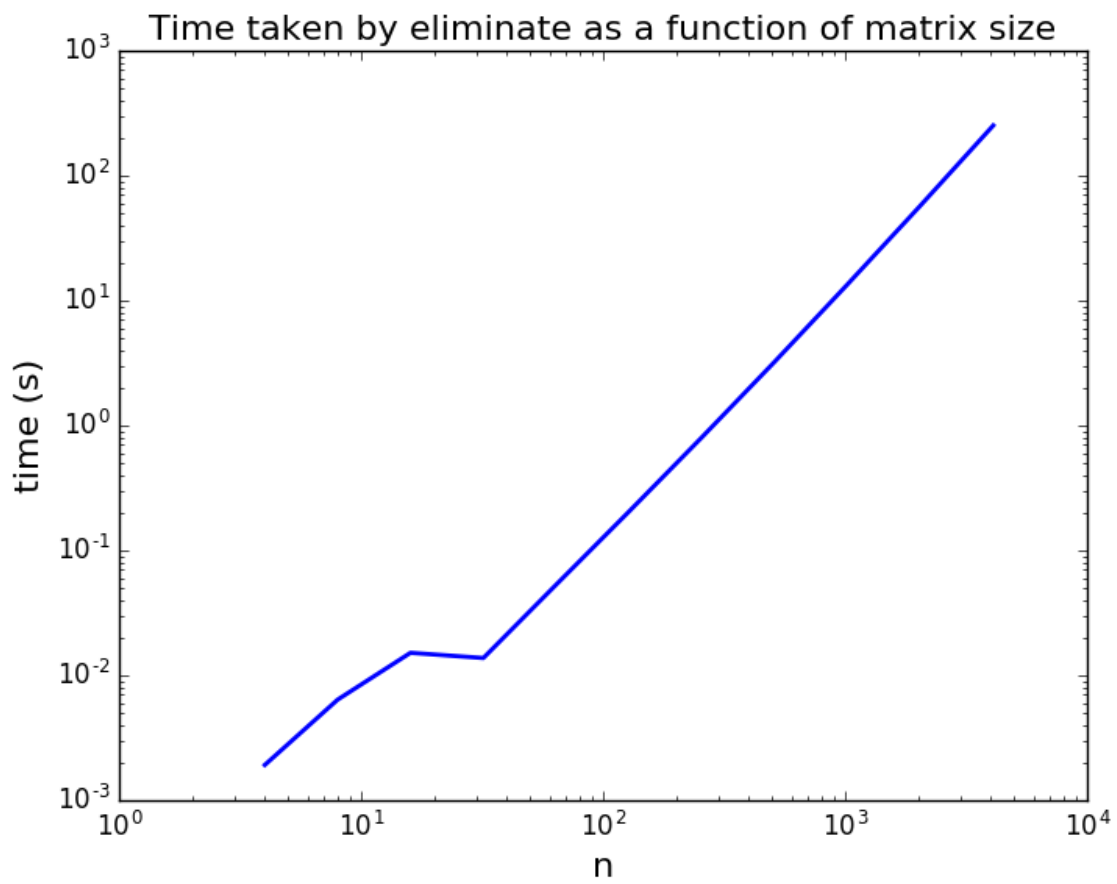


Figure 1: Time complexity of eliminate algorithm