MSAN 504 — Probability/Statistics — Summer 2016
Homework Four

1. Suppose that two fair dice have been tossed and the total of their top faces is found to be divisible by 5. What is the probability that both of them have landed 5?

2. Suppose that 15% of the population of a country are unemployed women, and a total of 25% are unemployed. What percect of the unemployed are women?

3. One of the cards of an ordinary deck of 52 cards is lost. What is the probability that a random card drawn from this deck is a spade?

4. On a multiple-choice exam with four choices for each question, a student either knows the answer to a question or marks it at random. If the probability that he or she knows the correct answer is 2/3, what is the probability that an answer that was marked correctly was not marked randomly?

5. A judge is 65% sure that a suspect has committed a crime. During the course of the trial, a witness convinces the judge that there is an 85% chance that the criminal is left-handed. If 23% of the population if left-handed and the suspect is also left-handed, with this new information, how certain should the judge be of the guilt of the suspect?

6. Let A, B, and C be three events. Prove that

$$\begin{aligned}\mathbb{P}(A \cup B \cup C) = {} & \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) \\ & - \mathbb{P}(A \cap B) - \mathbb{P}(B \cap C) - \mathbb{P}(A \cap C) \\ & + \mathbb{P}(A \cap B \cap C).\end{aligned}$$

7. Eleven chairs are numbered one through eleven. Four girls and seven boys sit on these chairs at random. What is the probability that chair five is occupied by a boy?

8. Let A and B be two events. Prove that $\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$.

9. Which of the following statements is true? If a statement is true, prove it. If a statement is false, give a counterexample. (a) If $\mathbb{P}(A)+\mathbb{P}(B)+\mathbb{P}(C) = 1$, then the events A, B, and C are mutually exclusive. (b) If $\mathbb{P}(A \cup B \cup C) = 1$, then A, B, and C are mutually exclusive events.

10. From an ordinary deck of 52 cards, we draw cards at random and without replacement until only cards of one suit are left. Find the probability that the cards left are all spades.

11. Suppose that we have two identical boxes: box 1 and box 2. Box 1 contains 5 red balls and 3 blue balls. Box 2 contains 2 red balls and 4 blue balls. A box is selected at random and exactly one ball is drawn from the box. (a) What is the probability that the ball is blue? (b) Given that a selected ball is blue, what is the probability that it came from the first box?

12. You are selling a product in an area where 30% of the people live in the city. The rest live in the suburbs. Twenty percent of the city dwellers (urbanites) use your product. Ten percent of the suburbanites use your product. What percentage of the people using your product are city dwellers?

13. Research the concepts of **sensitivity** and **specificity**. In the context of the null hypothesis "This person is not HIV positive" and the alternate hypothesis "This person is HIV positive," discuss what would happen if a test produces a type I and type II error. Then, discuss what the sensitivity and specificity of that test must mean.

14. After a robbery, a thief jumps into a taxi and disappears. An eyewitness on the crime scene is telling the police that the cab is yellow. In order to make sure that this testimony is worth something, the assistant district attorney makes a Bayesian analysis of the situation. After some research, he comes up with the following information: (1) In that particular city, 80% of taxis are black and 20% of taxis are yellow; and (2) Eyewitnesses are not always reliable and from past experience, it is expected that an eyewitness is 80% accurate. In other words, he will identify the color of a taxi accurately (yellow or black) eight out of ten times. What is the probability that the cab was really yellow, given that it was reported as being yellow?

15. By conditioning on whether or not the first card is an ace or not, compute the probability that the second card in a well-shuffled deck is an ace.

16. We have two urns. The first has four red balls and six green balls, and the second has six red balls and four green balls. We toss a fair coin. If the coin comes up heads, we pick a ball at random from the first hat. If the coin comes up tails, we pick a ball at random from the second hat. What is the probability of getting a red ball?

17. **A Bayesian Estimator.** Let Y be the sum of the observations of a random sample from a Poisson distribution with mean θ. Let the prior probability density function of θ be gamma with parameters α and β. (a) Find the posterior probability density function of θ, given that $Y = y$. (b) Find the Bayesian estimator, given your answer for part (a). (c) Show that the estimator you found in part (b) is a weighted average of the maximum likelihood estimator y/n and the prior mean $\alpha\beta$, with respective weights of $n/(n+1/\beta)$ and $(1/\beta)/(n+1/\beta)$.