MSAN 504 — Probability/Statistics — Summer 2016
Quiz One Solutions

1. Consider the random experiment of rolling two fair dice. Let X be the maximum of the two results. What is the probability mass function associated to X?

The probability mass function of the random variable X just tells us how much probability the random variable X charges (or "loads") on every real number. The following table enumerates the action of p_X:

x	1	2	3	4	5	6
$p(x)$	1/36	3/36	5/36	7/36	9/36	11/36

with $p(x) = 0$ otherwise.

[4 points] Discretion is largely left to the grader, who is encouraged to give partial credit for attempts that indicate some comprehension of the question being posed. Evidence of some limited understading comes, for example, from a grid of the values that might be realized independently by the two dice, as well as the sum of the two realizations at each corresponding location in the grid. One point should be deducted with the student forgets to include the "otherwise" proviso. The student need not argue or note that the resulting function is a true probability mass function.

2. Let X be a continuous random variable with cumulative distribution function $F(x)$. What properties must F have in order to be a cumulative distribution function?

In class, we mentioned a number of properties that the distribution function F must have. In particular, $\lim_{x\longrightarrow\infty} F(x) = 1$, $\lim_{x\longrightarrow-\infty} F(x) = 0$, F must be non-decreasing, and F must be right-continuous. The first three properties together mean that $0 \leq F(x) \leq 1$ for every $x \in \mathbb{R}$, and so this is not really a free-standing property. What does it mean for F to be right-continuous? It means that $F(t+) = F(t) = \lim_{s\longrightarrow t^+} F(s)$ for every $t \in \mathbb{R}$.

[6 points] One point for the limit at minus infinity, one point for the limit at plus infinity, two points for monotonicity, and two points for right-continuity. The student may introduce redundant conditions provided that all necessary conditions for a cumulative distribution function are present. Partial credit is left to the discretion of the grader. It is not necessary for the student to rigorously define what it means for a function to be right-continuous at a point or every point.

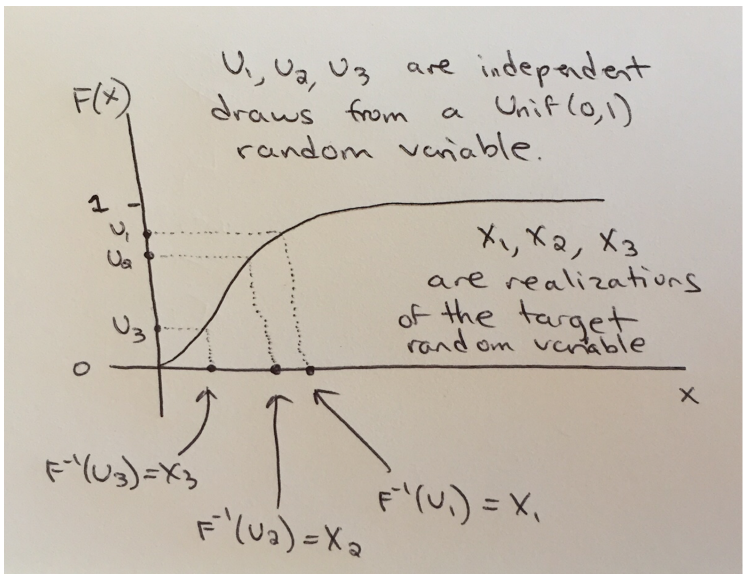
3. Let X be a discrete random variable. Prove that $\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

Recall that $\mathrm{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$. This expression is, in turn, equal to $\mathbb{E}[X^2 - 2X\mathbb{E}[X] + \mathbb{E}[X]^2]$. By linearity of the expected value operator, this expression reduces to $\mathbb{E}[X^2] - 2\mathbb{E}[X]\mathbb{E}[X] + \mathbb{E}[X]^2$. The result then follows.

[5 points] One point for correct foiling. Two points for correct application of the linearity property of expectation. Two points for a final and fully simplified answer.

4. Let X be a continuous random variable with a cumulative distribution function called F. Suppose that you want to use the Method of Inverse Transformation to simulate realizations of X. Draw and label a picture that explains why the Method of Inverse Transformation works. Write a brief explanation of how the method works to go along with your picture.

We (independently) sample uniform (over the unit interval) random variates. The unit interval is the range of every cumulative distance function. By passing these random variates through the inverse of the cumulative distribution function of the target random variable, we generate realizations of it. A very good picture that illustrates this concept might look like the following:



[5 points] Partial credit is largely left to the discretion of the grader. However, at least one point should be taken off if there is no connection between the uniform random variate and the y-axis. At least one other point should be taken off if there is no visualization — say, for example, a line drawn back from the y-axis to the function and then again to the x-axis – of the action of the inverse function. Finally, a point should be taken off if there is no indication (by way of drawing something to the x-axis) that we get something that is regarded as a realization of the target random variable. (**Note:** The student's picture does not have to be as good as the picture in the solutions.)