MSAN 593 - Homework 2

Andre Guimaraes Duarte

July 24, 2016

TODO: reference histograms in the text

Question 1

1.1

1.1.1

```
times <- 10000000
myRunifVec <- runif(times, 4, 6)
hist(myRunifVec,
     main = paste("Histogram of ", length(myRunifVec), "\n random variables ~ U(4, 6)",
                  sep = ""), xlab = "x", freq = F)
```

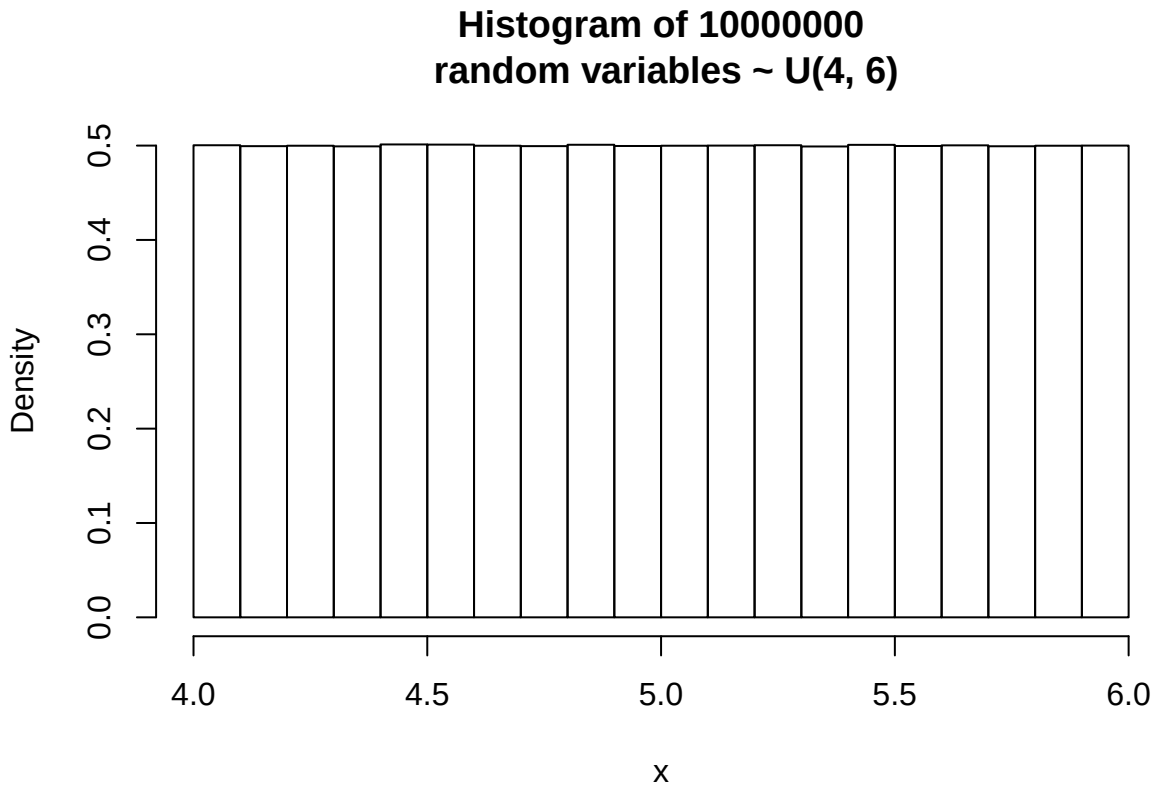

Figure 1: Histogram of 10,000,000 random variables ~ U(4, 6)

The histogram in figure 1 is the histogram of a $\sim U(4, 6)$ distribution with 10000000 random variables.

1.1.2

```
samples <- 100000
myRunifSample <- sample(myRunifVec, samples)
hist(myRunifSample,
     main = paste("Histogram of ", length(myRunifSample),
                  "\n random variables sampled from a ~ U(4, 6)",
                  sep = ""), xlab = "x", freq = F)
```

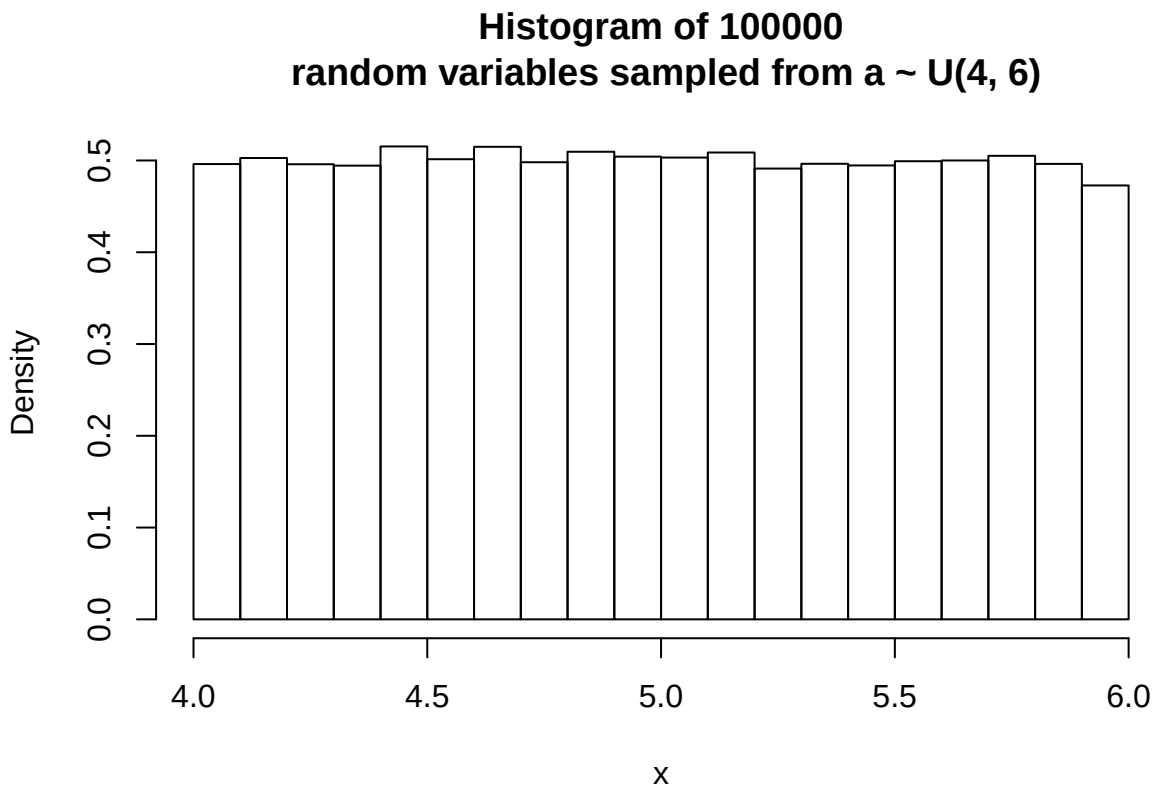

Figure 2: Histogram of a sample of 100,000 random elements of a ~U(4, 6) distribution

The histogram in figure 2 is very similar to the population distribution 1. Sampling elements from a uniform distribution maintains its original distribution.

1.1.3

```
# How to make it more efficient for 100,000??
n <- 10
unifSampleMean_2 <- replicate(n, {
  mean(sample(myRunifVec, 2))
})
hist(unifSampleMean_2)
```

The histogram in figure 3 is not similar to the population distribution shown in figure 1. In fact, we can start to see a bell-shaped distribution in this figure.

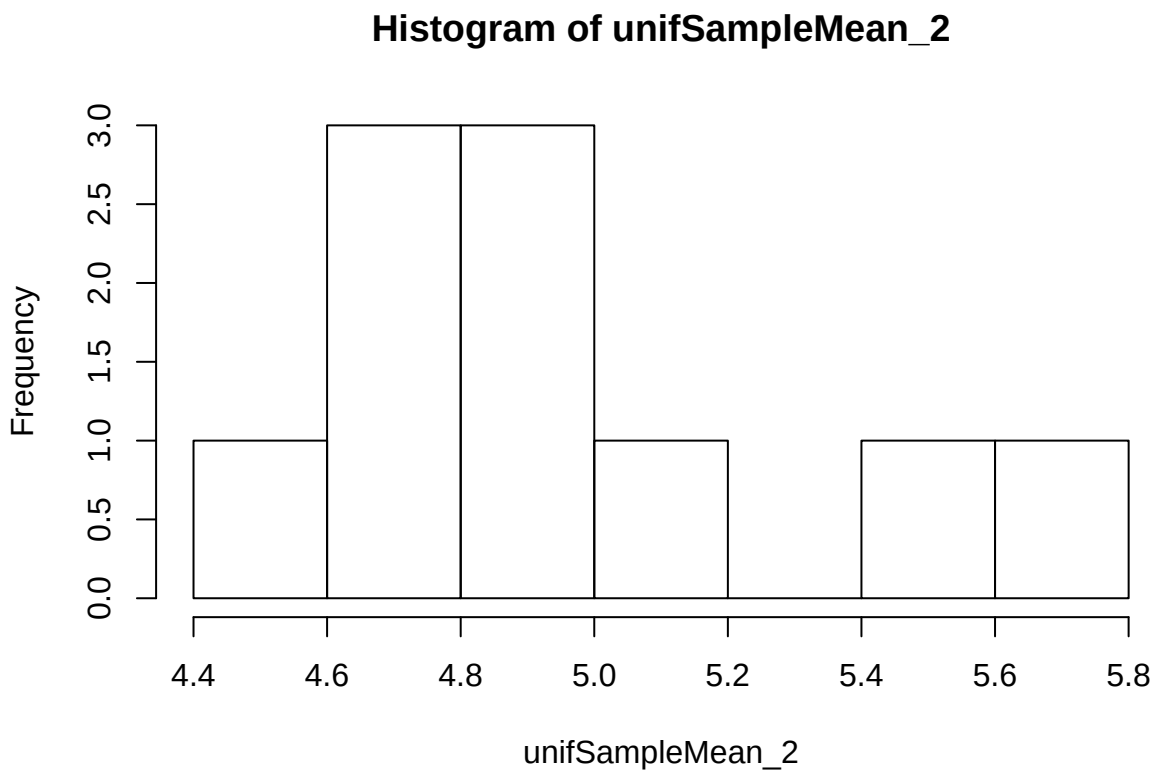

Figure 3: Histogram of a sample of 100,000 random means of two elements of a ~U(4, 6) distribution

1.1.4

```
unifSampleMean_5 <- replicate(n, {
  mean(sample(myRunifVec, 5))
})
hist(unifSampleMean_5)
```

The histogram in figure 4 is different from the population distribution in figure 1. There are more observations around 5, and less toward the tails.

1.1.5

```
unifSampleMean_10 <- replicate(n, {
  mean(sample(myRunifVec, 10))
})
hist(unifSampleMean_10)
```

The histogram in figure 5 is different from the population distribution in figure 1. This one is bell-shaped and seems to be symmetrical around the value 5.

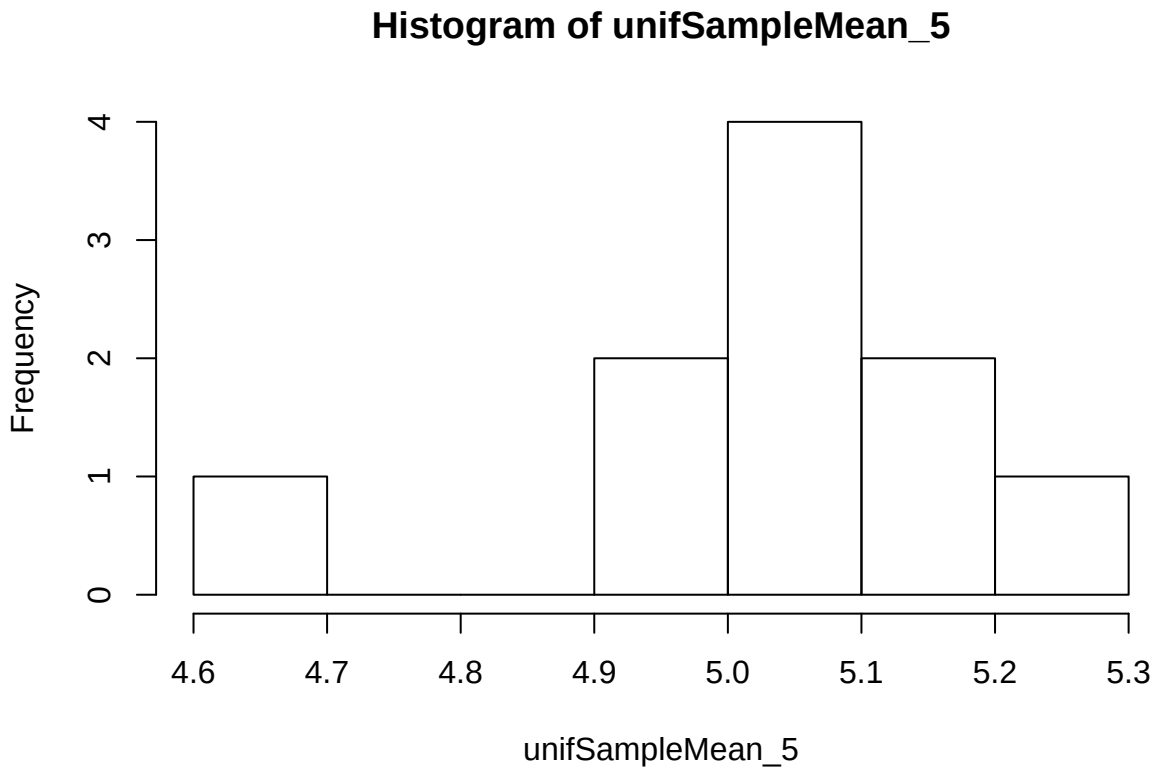

Figure 4: Histogram of a sample of 100,000 random means of five elements of a ~U(4, 6) distribution

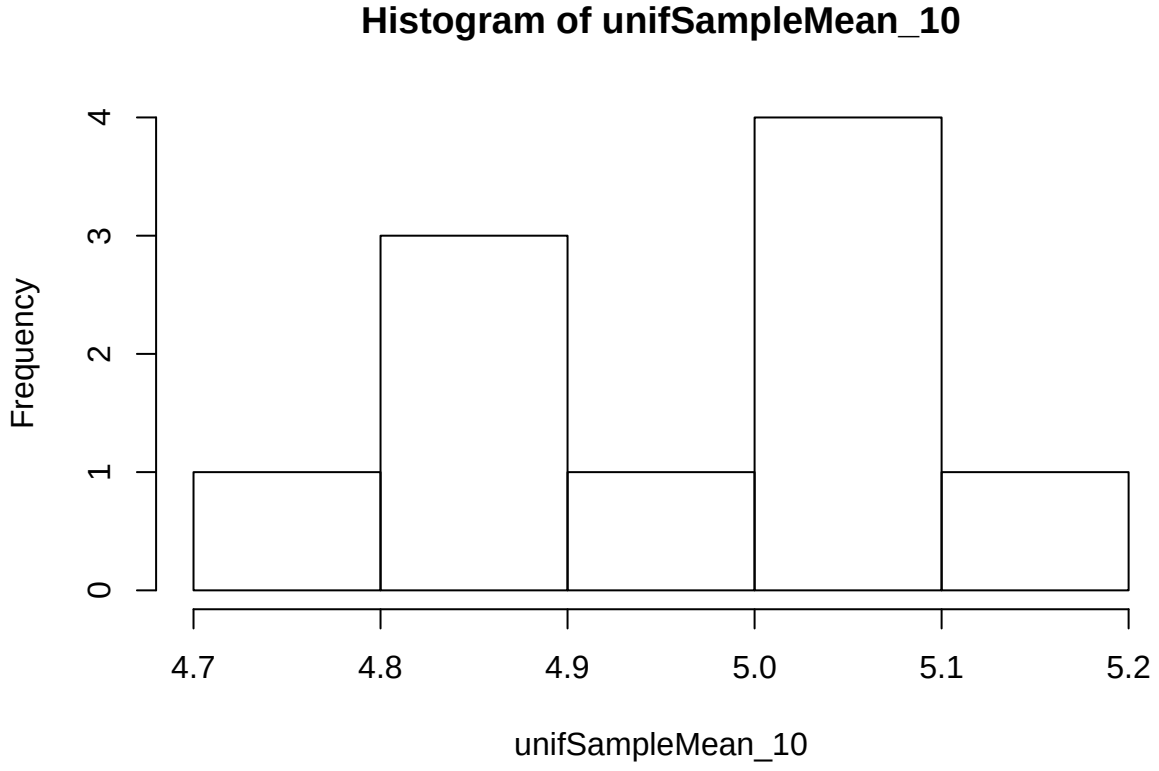

Figure 5: Histogram of a sample of 100,000 random means of ten elements of a ~U(4, 6) distribution

1.1.6

```
unifSampleMean_30 <- replicate(n, {
  mean(sample(myRunifVec, 30))
})
hist(unifSampleMean_30)
```

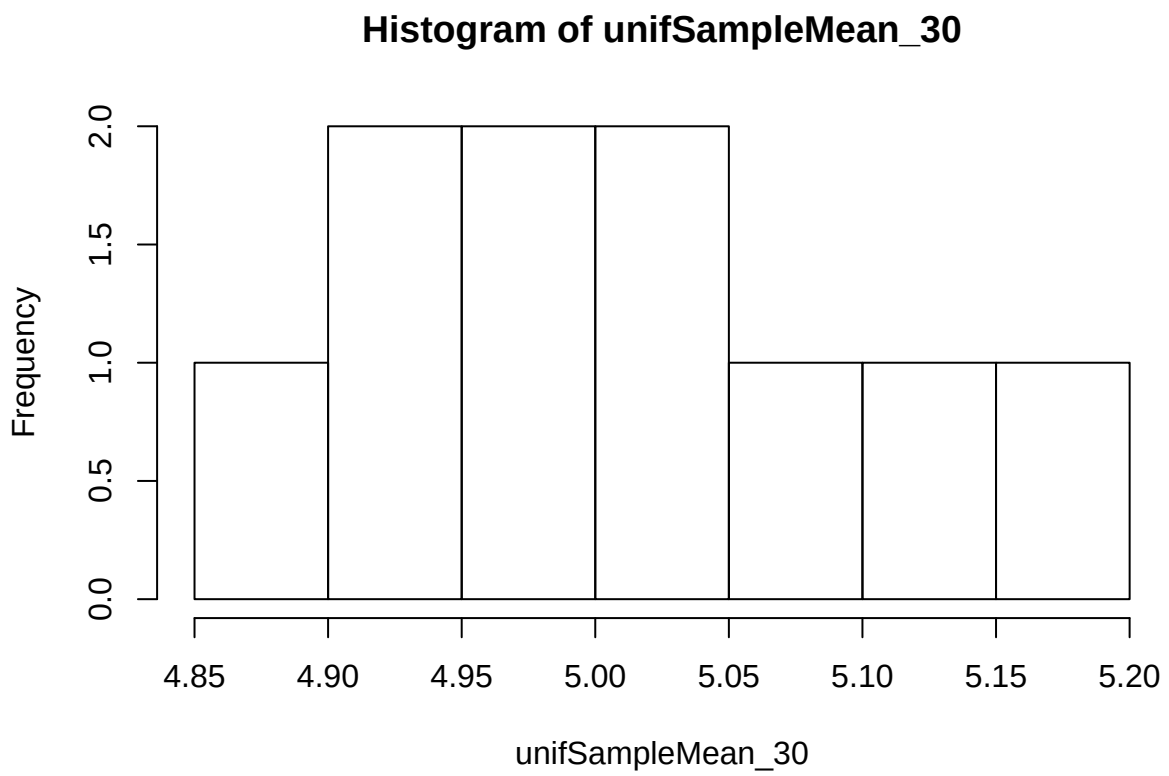

Figure 6: Histogram of a sample of 100,000 random means of thirty elements of a ~U(4, 6) distribution

The histogram in figure 6 is different from the population distribution in figure 1. It looks a lot like a normal distribution centered around 5.

1.2

1.2.1

```
myRexpVec <- rexp(times, 0.5)
hist(myRexpVec,
     main = paste("Histogram of ", length(myRexpVec), "\n random variables ~ Exp(0.5)",
                  sep = ""), xlab = "x", freq = F)
```

Figure 7 shows the histogram of a $\sim Exp(0.5)$ distribution with 10000000 random variables.

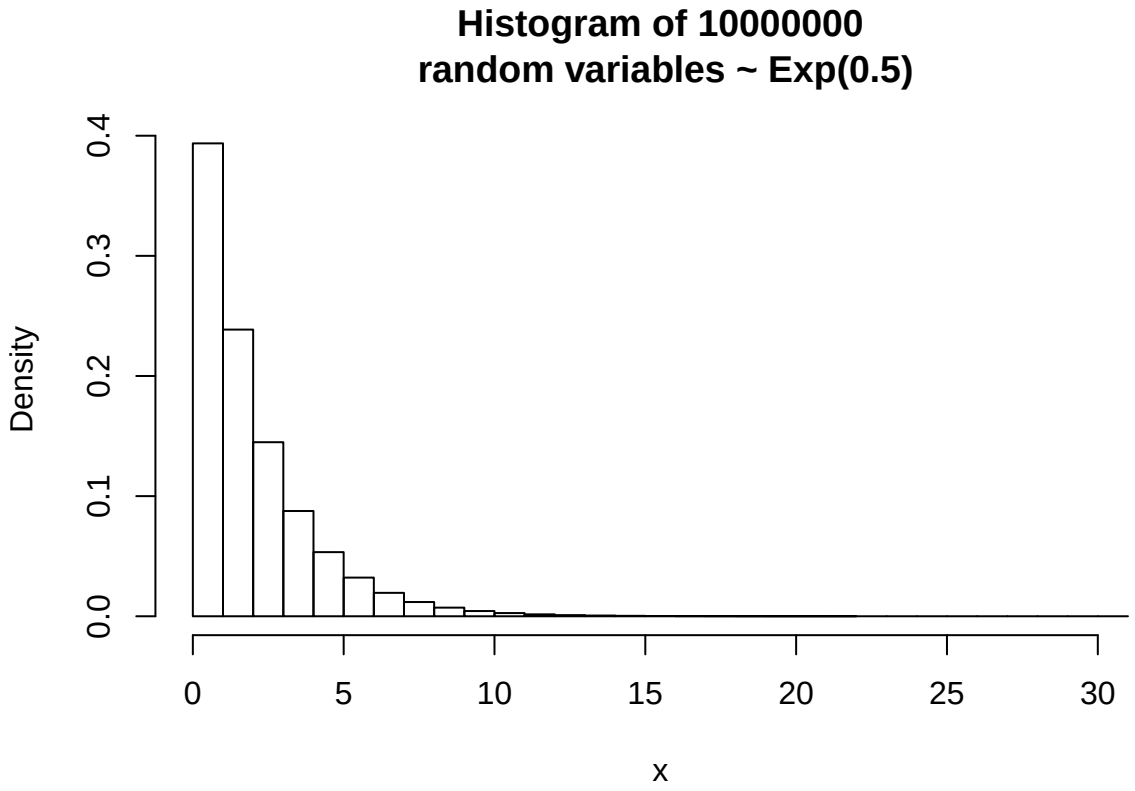

Figure 7: Histogram of 10,000,000 random variables ~Exp(0.5)

1.2.2

```
myRexpSample <- sample(myRexpVec, samples)
hist(myRexpSample,
     main = paste("Histogram of ", length(myRexpSample),
                  "\n random variables sampled from a ~ Exp(0.5)",
                  sep = ""), xlab = "x", freq = F)
```

The histogram in figure 8 is very similar to the population distribution 7. Sampling elements from a negative exponential distribution maintains its original distribution.

1.2.3

```
expSampleMean_2 <- replicate(n, {
  mean(sample(myRexpVec, 2))
})
hist(expSampleMean_2)
```

Figure 9 shows the histogram is slightly more skewed to the right than the population distribution in figure 7.

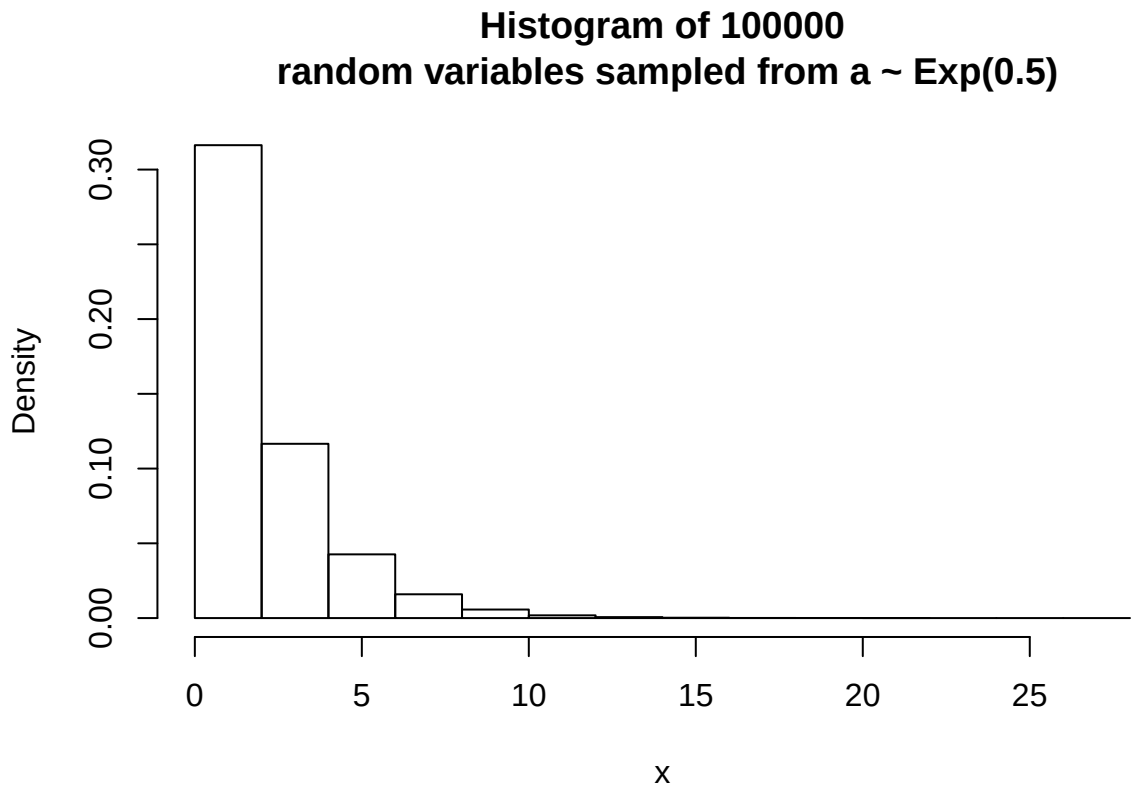

Figure 8: Histogram of a sample of 100,000 random elements of a ~Exp(0.5) distribution

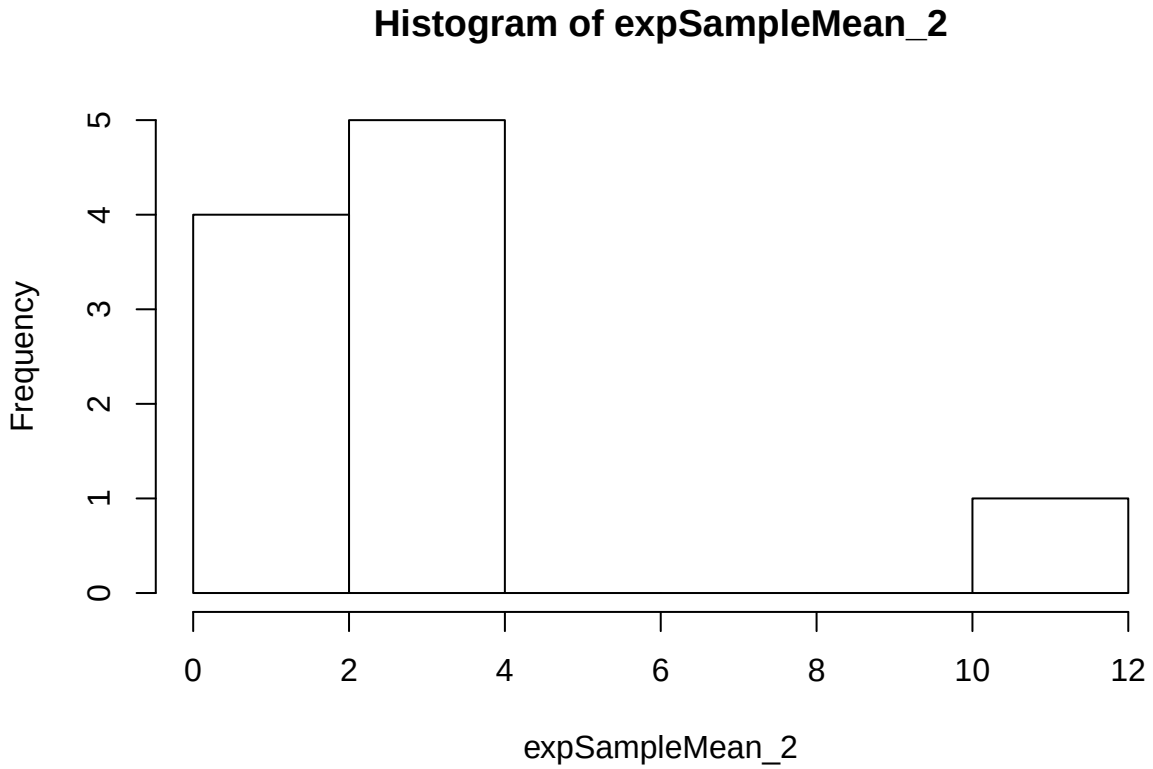

Figure 9: Histogram of a sample of 100,000 random means of two elements of a ~Exp(0.5) distribution

1.2.4

```
expSampleMean_5 <- replicate(n, {
  mean(sample(myRexpVec, 5))
})
hist(expSampleMean_5)
```

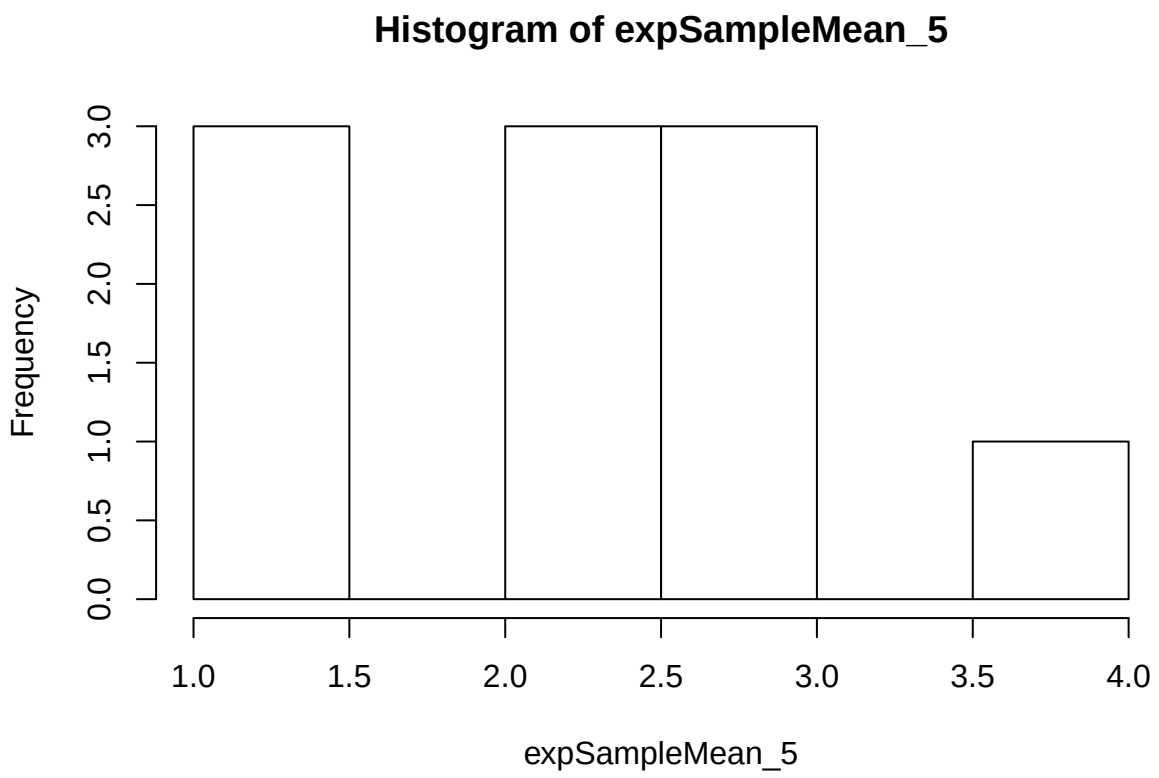

Figure 10: Histogram of a sample of 100,000 random means of five elements of a ~Exp(0.5) distribution

The histogram when we take the mean of five elements from the original population, shown in figure 10, shows more skewness to the right than the histogram of the population distribution in figure 7.

1.2.5

```
expSampleMean_10 <- replicate(n, {
  mean(sample(myRexpVec, 10))
})
hist(expSampleMean_10)
```

By averaging ten elements, the histogram in figure 11 is starting to look a lot like a Gamma distribution.

1.2.6

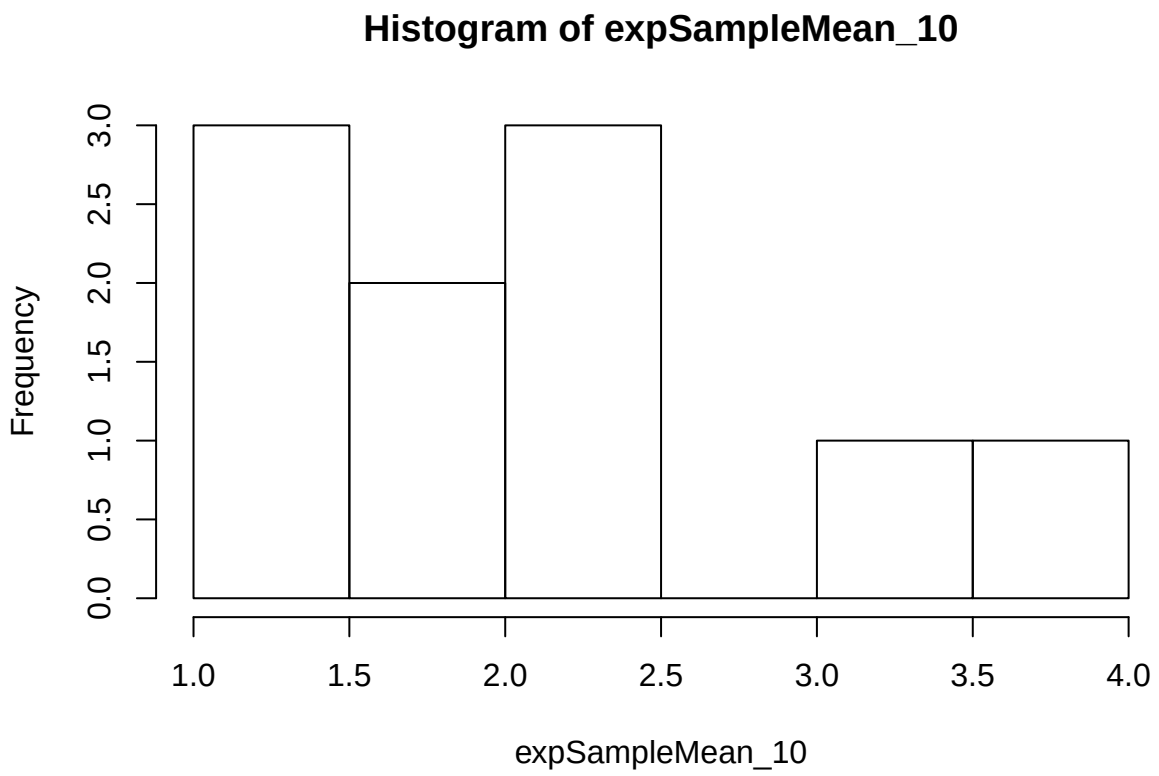

Figure 11: Histogram of a sample of 100,000 random means of ten elements of a ~Exp(0.5) distribution

```
expSampleMean_30 <- replicate(n, {
  mean(sample(myRexpVec, 30))
})
hist(expSampleMean_30)
```

In figure 12, we see the histogram when we take the average of 30 elements from the original negative exponential population shown in figure 7. The distribution is now very different, and looks like a Gamma distribution.

1.3

1.3.1

```
times <- 5000000
myBdist <- c(rnorm(times, -3, 1), rnorm(times, 3, 1))
hist(myBdist, main="Bimodal histogram of ~N(-3, 1) and ~N(3, 1)", xlab="x", freq = F)
```

1.3.2

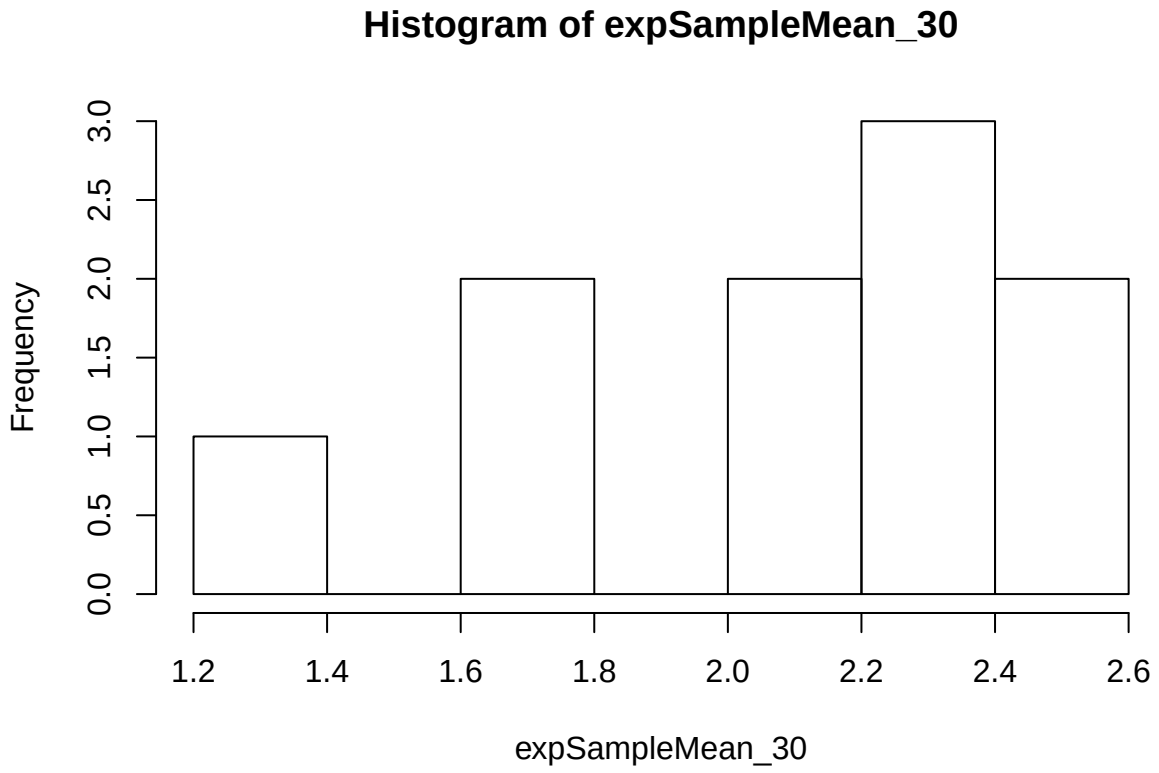

Figure 12: Histogram of a sample of 100,000 random means of thirty elements of a ~Exp(0.5) distribution

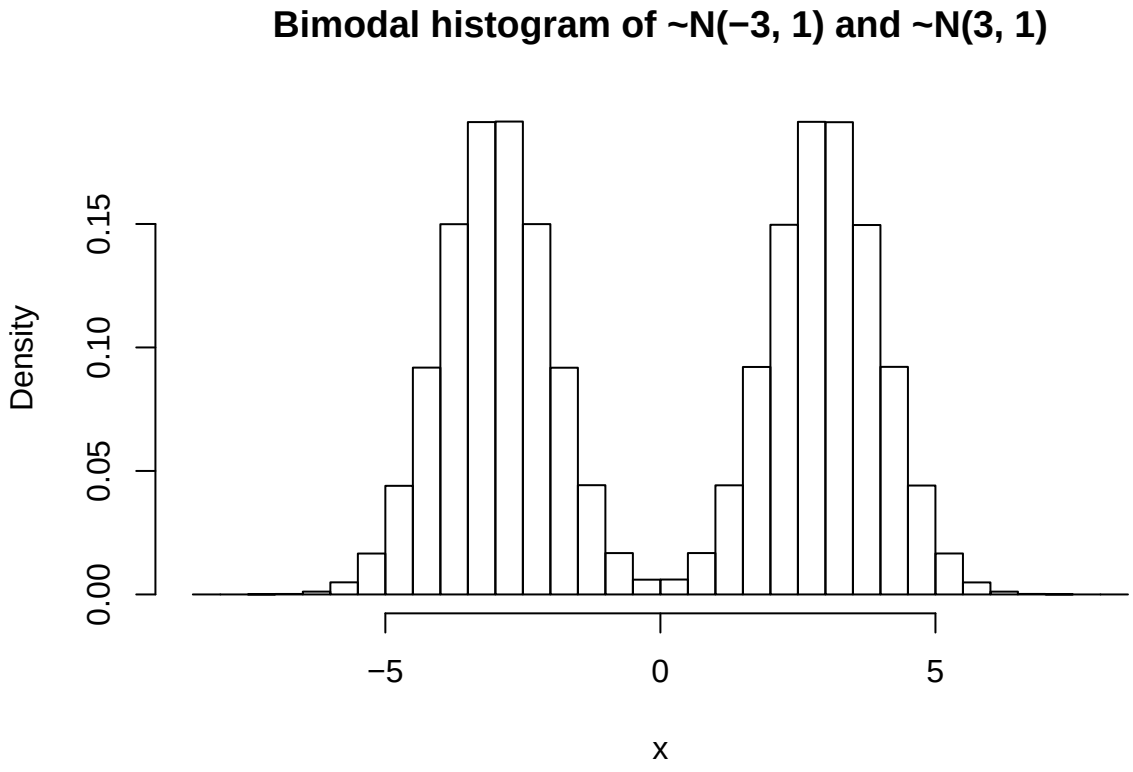

Figure 13: Histogram of 5,000,000 random variables from a ~N(-3, 1) and 5,000,000 random variables from a ~N(3, 1)

```r
myBdist_5 <- replicate(n, {
  mean(sample(myBdist, 5))
})
hist(myBdist_5)
```

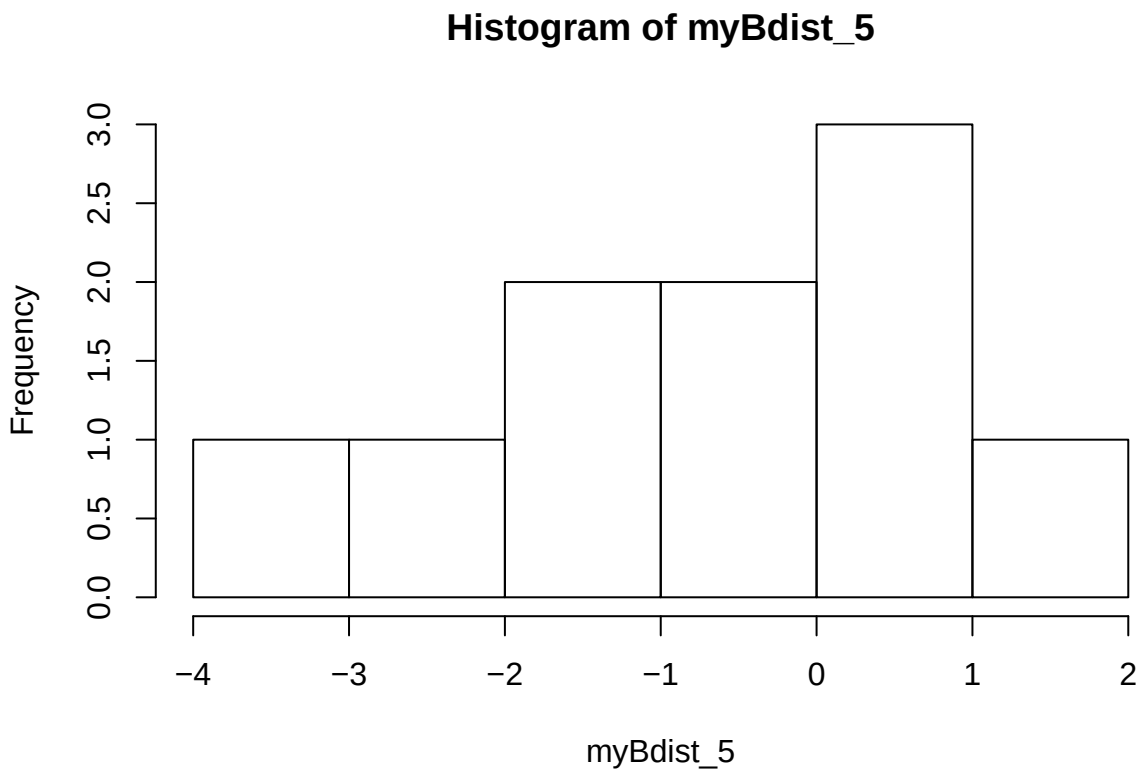

Figure 14: Histogram of a sample of 100,000 random means of five elements from a ~N(-3, 1) and a ~N(3, 1) distributions

1.3.3

```r
myBdist_10 <- replicate(n, {
  mean(sample(myBdist, 10))
})
hist(myBdist_10)
```

```r
myBdist_20 <- replicate(n, {
  mean(sample(myBdist, 20))
})
hist(myBdist_20)
```

```r
myBdist_30 <- replicate(n, {
  mean(sample(myBdist, 30))
})
hist(myBdist_30)
```

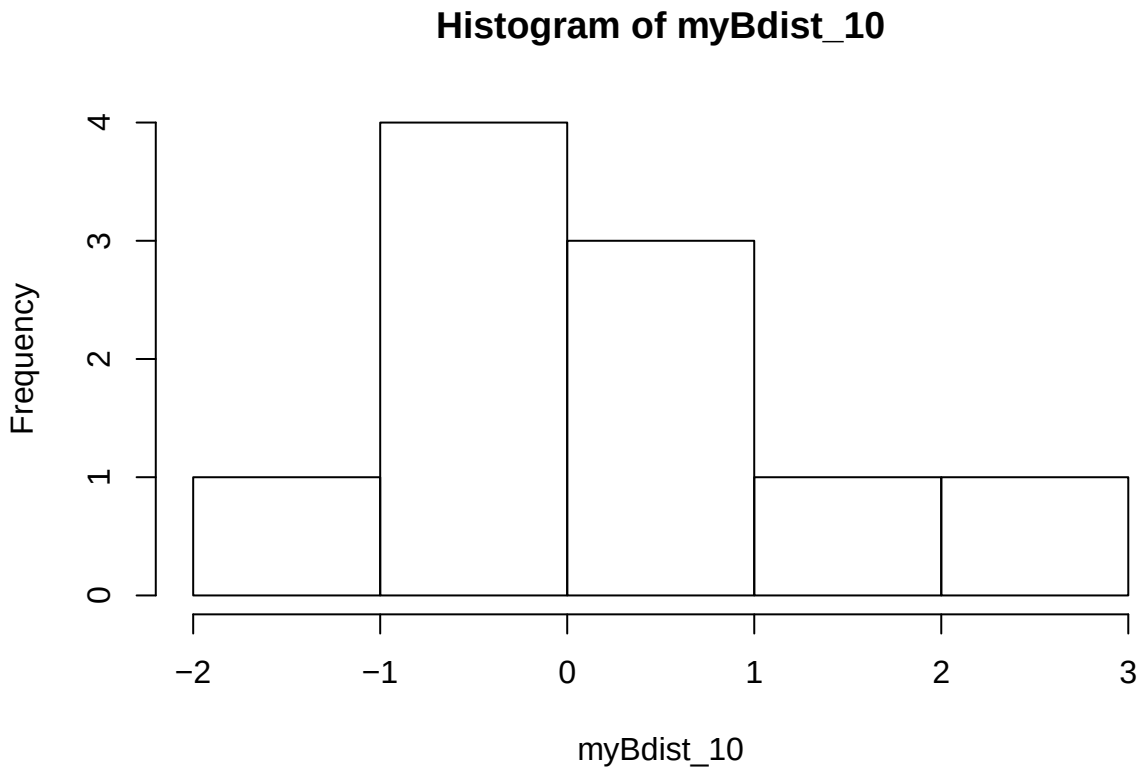

Figure 15: Histogram of a sample of 100,000 random means of ten elements from a ~N(-3, 1) and a ~N(3, 1) distributions

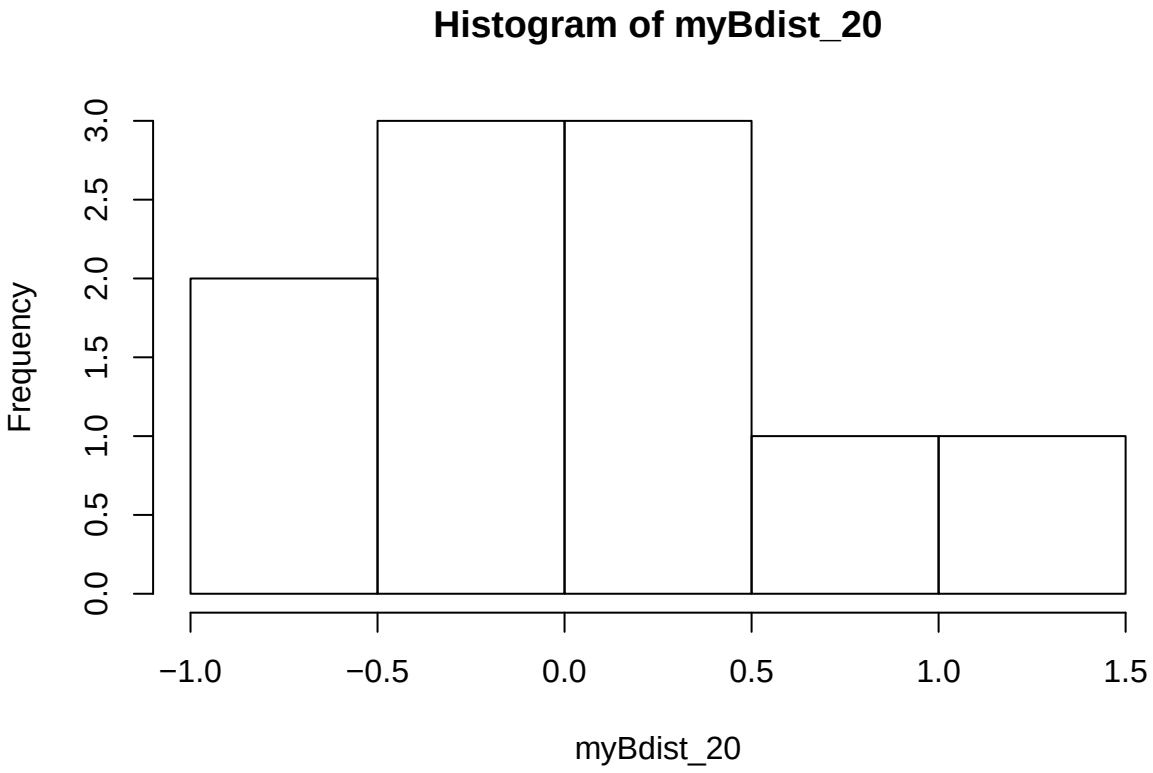

Figure 16: Histogram of a sample of 100,000 random means of twenty elements from a ~N(-3, 1) and a ~N(3, 1) distributions

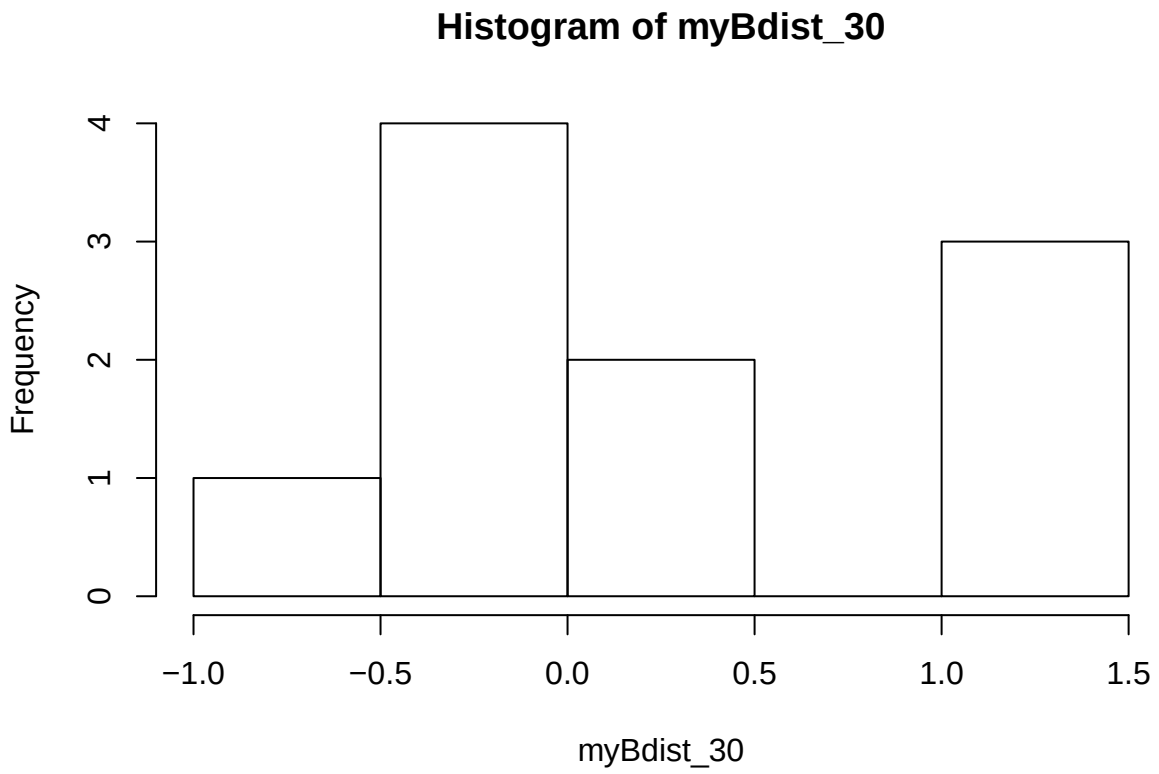

Figure 17: Histogram of a sample of 100,000 random means of thirty elements from a ~N(-3, 1) and a ~N(3, 1) distributions

1.3.4