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Fall 2016 MSAN 604: Quiz 1

Thursday October 27, 2016

Question 1

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(a) Define what it means for a time series $\{X_t\}$ to be *strictly* stationary.

$\{X_t\}$ is a strictly stationary time series if the joint distribution of $X_{t_1}, X_{t_2}, \ldots X_{t_n}$ is the same as that for $X_{t_1+h}, X_{t_2+h}, \ldots, X_{t_n+h}$ where $t_1, \ldots, t_n, n, h \in \mathbb{Z}$. That is, the joint distribution is preserved under time shifts.

(b) A time series $\{X_t\}$ is said be *weakly* stationary if it satisfies three properties. What are they? Explain your answer carefully.

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- $E(X_t^2) < \infty$ for all t
- $\mu_x(t) = E(X_t)$ is independent of t i.e., $E(X_t) = \mu_x \; \forall t$.
- $\gamma_x(t, t+h) = Cov(X_t, X_{t+h})$ is independent of t i.e., $Cov(X_t, X_{t+h}) = \gamma_x(h)$ for all t.

(c) Suppose $\{\varepsilon_t\} \sim IID(0, \sigma^2)$. Show that $\{\varepsilon_t\}$ is weakly stationary.

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- $E(\varepsilon_t) = 0$ for all t
- $E(\varepsilon_t^2) = Var(\varepsilon_t) = \sigma^2 < \infty$
- $Cov(\varepsilon_t, \varepsilon_{t+h}) = E(\varepsilon_t \varepsilon_{t+h})$

$$= \begin{cases} E(\varepsilon_t^2) = \sigma^2 & \text{if } h = 0 \\ 0 & \text{otherwise} \end{cases}$$

doesn't depend on t

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Question 2

Consider the stationary time series $X_t = \varepsilon_t + \theta\varepsilon_{t-2}$, where $\{\varepsilon_t\} \sim WN(0, \sigma^2)$

3 (a) Calculate $\gamma_X(h)$, the lag h autocovariance function of $\{X_t\}$.

$$\gamma_X(h) = Cov(X_t, X_{t+h}) = Cov(\varepsilon_t + \theta\varepsilon_{t-2}, \varepsilon_{t+h} + \theta\varepsilon_{t+h-2})$$

$$= Cov(\varepsilon_t, \varepsilon_{t+h}) + \theta Cov(\varepsilon_t, \varepsilon_{t+h-2}) + \checkmark$$

$$\theta Cov(\varepsilon_{t-2}, \varepsilon_{t+h}) + \theta^2 Cov(\varepsilon_{t-2}, \varepsilon_{t+h-2})$$

$$= \begin{cases} \sigma^2 + \theta^2\sigma^2 & \text{if } h = 0 \quad \checkmark \\ \theta\sigma^2 & \text{if } h = \pm 2 \quad \checkmark \\ 0 & \text{o.w.} \end{cases}$$

1 (b) Calculate $\rho_X(h)$, the lag h autocorrelation function of $\{X_t\}$.

$$\rho_X(h) = Corr(X_t, X_{t+h}) = \frac{\gamma_X(h)}{\gamma_X(0)}$$

$$= \begin{cases} 1 & \text{if } h = 0 \\ \dfrac{\theta}{1+\theta^2} & \text{if } h = \pm 2 \\ 0 & \text{o.w.} \end{cases} \quad \checkmark$$

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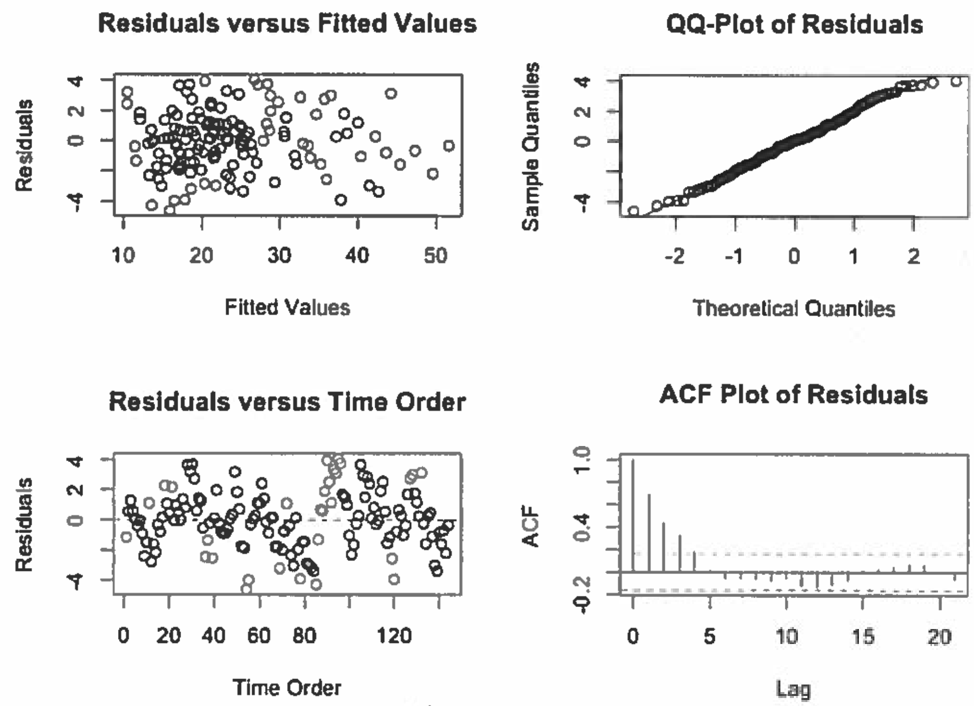
Question 3

Suppose we fit the classically decomposed time series model ($X_t = m_t + s_t + \varepsilon_t$) using ordinary least squares (OLS) regression. After having fit this model, we decide to perform diagnostics on the residuals to ensure the OLS assumptions are met. Relevant plots of these residuals are shown below. Based on these plots, comment on whether the OLS assumptions are met. If they are not, in your own words, describe the effect this will have on the validity of the model for forecasting.



OLS Assumptions: $\varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$ where σ^2 is constant

- Residuals vs. Fitted Values plot indicates a fairly random scattering of points with constant amplitude. The lack of a "funnel-shape" suggests σ^2 is indeed constant.
- The QQ-plot, being a fairly straight line, indicates the the residual quantiles match closely with the theoretical Normal quantiles, suggesting the normality assumption is reasonable
- The ACF plot of the residuals indicates significant autocorrelation at lags 1, 2, 3, suggesting that residuals separated in time by 1, 2, 3 timepoints are significantly correlated, violating the independence assumption. Thus predictions and prediction intervals associated with this model may not be valid.