A Note on Vector Autoregression

Why is it called *vector* autoregression? As mentioned in class we ran write the VAR(p) system of equations in a vector/matrix notation. Recall that for r endogenous variables these equations look as follows:

$$Y_{1,t} = c_1 + \sum_{i=1}^{p} \phi_{11,i} Y_{1,t-i} + \sum_{i=1}^{p} \phi_{12,i} Y_{2,t-i} + \cdots + \sum_{i=1}^{p} \phi_{1r,i} Y_{r,t-i} + \varepsilon_{1,t}$$

$$Y_{2,t} = c_2 + \sum_{i=1}^{p} \phi_{21,i} Y_{1,t-i} + \sum_{i=1}^{p} \phi_{22,i} Y_{2,t-i} + \cdots + \sum_{i=1}^{p} \phi_{2r,i} Y_{r,t-i} + \varepsilon_{2,t}$$

$$\vdots$$

$$Y_{r,t} = c_r + \sum_{i=1}^{p} \phi_{r1,i} Y_{1,t-i} + \sum_{i=1}^{p} \phi_{r2,i} Y_{2,t-i} + \cdots + \sum_{i=1}^{p} \phi_{rr,i} Y_{r,t-i} + \varepsilon_{r,t}$$

To simplify this, we can define the $r\times 1$ vector $\boldsymbol{Y}_t = \left(Y_{1,t}, Y_{2,t}, \dots, Y_{r,t}\right)^T$ to be the vector of endogenous variable observations at time point t. Thus at every time point we observe a *vector* of values as opposed to a single value. Using this notation, we compactly write the VAR(p) systems of equations using vector/matrix notation as follows:

$$\boldsymbol{Y}_t = \boldsymbol{c} + A_1 \boldsymbol{Y}_{t-1} + A_2 \boldsymbol{Y}_{t-2} + \cdots + A_r \boldsymbol{Y}_{t-r} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{c} = (c_1, c_2, \dots, c_r)$ is an $r\times 1$ vector of constants, $\boldsymbol{\varepsilon}_t = \left(\varepsilon_{1,t}, \varepsilon_{2,t}, \dots, \varepsilon_{r,t}\right)^T$ is an $r\times 1$ vector of white noise error terms and each A_k $(k = 1,2,\dots,r)$ is an $r\times r$ matrix of coefficients given by:

$$A_k = \begin{bmatrix} \phi_{11,k} & \phi_{12,k} & \cdots & \phi_{1r,k} \\ \phi_{21,k} & \phi_{22,k} & \cdots & \phi_{2r,k} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{r1,k} & \phi_{r2,k} & \cdots & \phi_{rr,k} \end{bmatrix}$$