UNIVERSITY OF SAN FRANCISCO

Master of Science in Analytics

Classification

Machine Learning 1

Classification

- Response (Y) is qualitative — i.e. an enumerated class
- Examples:
  - Which candidate will win the 2016 presidential election?
  - What language is the following: 狗不喜欢吃蔬菜
- Algorithm grab bag
  - Now: Logistic Regression, Linear Discriminant Analysis (QDA), K Nearest Neighbours
  - Later: generalised additive models, trees, random forests, SVM
- Why not modify regression?
  - Yes, it's possible to enumerate classes and perform regression
  - But it's inadvisable
    - Ordering of classes may not be "natural"
    - Regression may predict value outside enumerated range

Logistic Regression

- Given features, determine the probability that Y belongs to one of the defined categories
- Shares same theory as linear regression:
  - *$p(X) = \beta_0 + \beta_1 X$*
  - Use ***maximum likelihood*** b/c probabilities must be in interval [0 .. 1]:

$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1 + e^{\beta_0+\beta_1 X}}$$

  - Maximum likelihood is a manipulation of (log) odds / SLR basis:
    - Odds

$$\frac{p(X)}{1 - p(X)} = e^{\beta_0+\beta_1 X}$$

    - Log odds

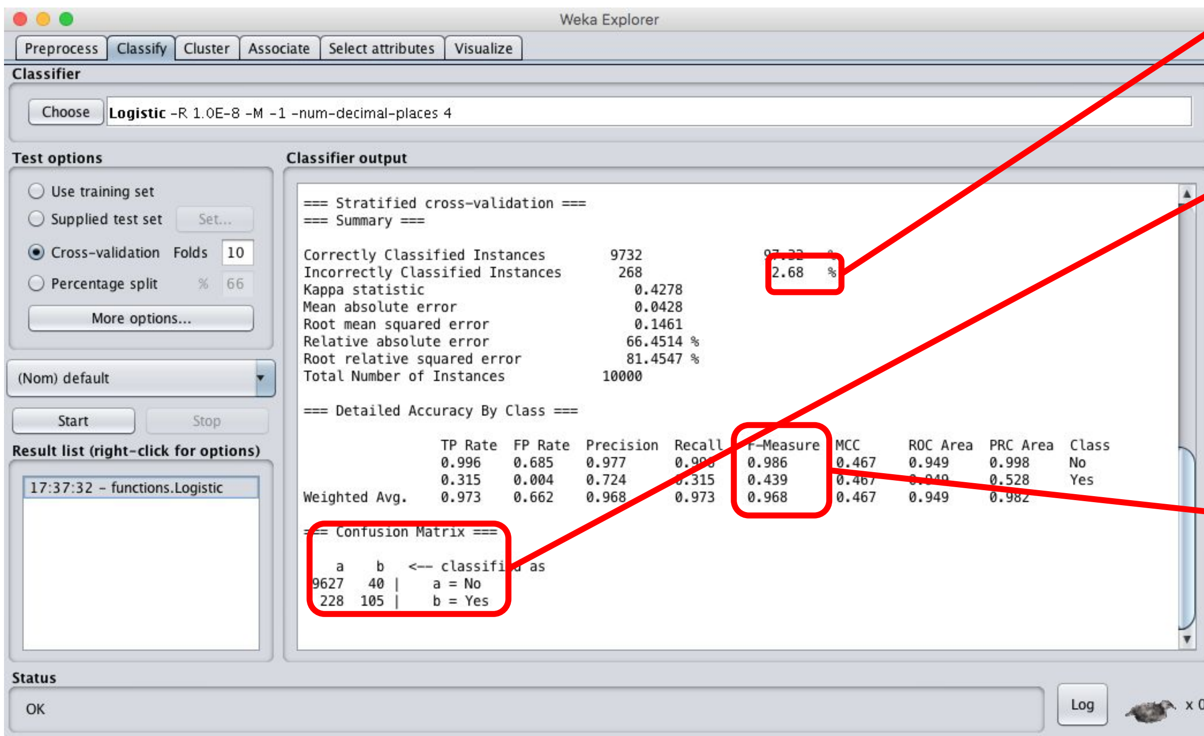
$$\log\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X.$$

Example: Default data

- Data from ISLR dataset, including a person's:
  - Default status ("Yes" or "No")
  - Student status ("Yes" or "No")
  - Balance
  - Income
- Unbalanced data set
  - Total = 10K instances
  - Most people (96.67%) do not default (3.33% baseline error rate)
  - Bimodal income amounts
- Bayesian

Implementation in weka

1) Coerce the data into ARFF format (eg. Default)
2) In weka:
   a) Explorer > [Preprocess] > Open file…
   b) [Classify] > [Choose] (function)
   c) Test options (test set / Cross-validation)



Default dataset

2.68% classification error

```
=== Confusion Matrix ===
     a    b   <-- classifie
9627    40  |  a = No
 228   105  |  b = Yes
```

Sensitivity / Specificity

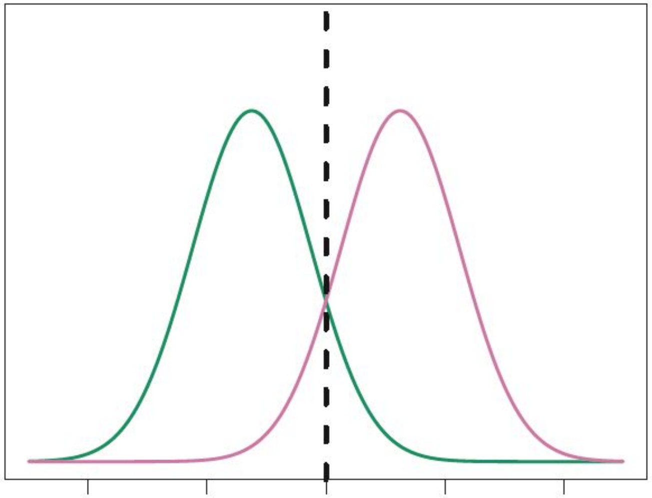
```
F-Measure
0.986    ...    No
0.439    ...    Yes
```

Linear Discriminant Analysis

- Useful when:
  - Number of classes, K ≥ 1
  - Classes are well-separated
  - Distribution of each class is approximately normal
- Bayesian basis:

$$\Pr(Y = k|X = x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)}$$

  - $\boldsymbol{\pi}_k$: Prior probability
  - $f_k(X) = Pr(X{=}x\,|\,Y{=}k)$: Density function
  - Intuitively, looking for a class separator:



Implementation in scikit-learn

1) Import data
   a) If necessary, split data into train, test sets
2) Coerce data into:
   a) test_x, train_x = List-of-lists / numpy matrix: all features
   b) test_y, test_y = List / numpy vector: all targets

```
# 3) LDA

from sklearn.discriminant_analysis import LinearDiscriminantAnalysis

prior_vector = [0.2, 0.8]

algo = LinearDiscriminantAnalysis (priors=prior_vector)    # "priors" vector is optional
algo.fit (train_x, train_y)
hypotheses = algo.predict (test_x)
```

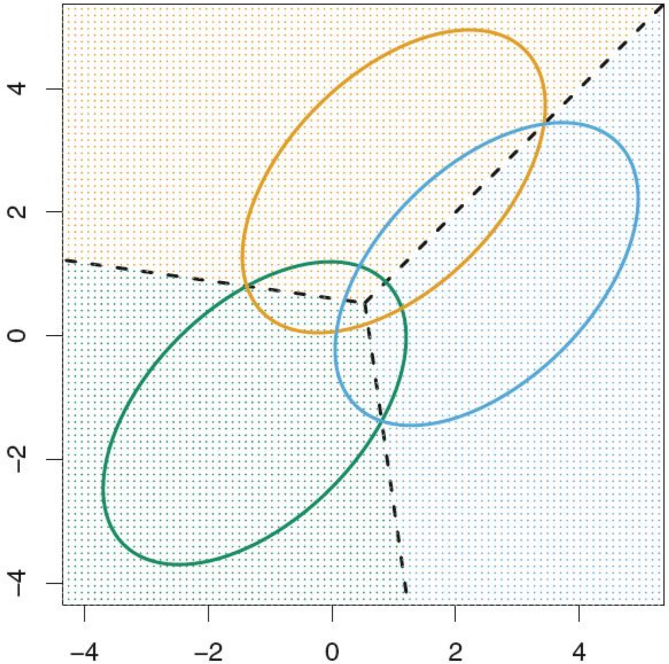
4) Perform analysis on hypotheses

LDA Decision Boundary

- Class separator is formally called "Decision Boundary"

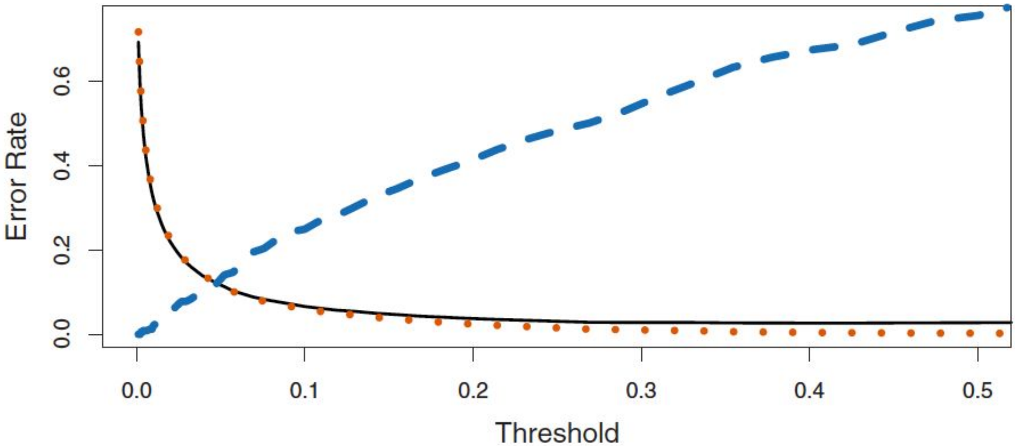
$$\delta_k(x) = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

- Calculation is tractable for p > 1 & k > 2

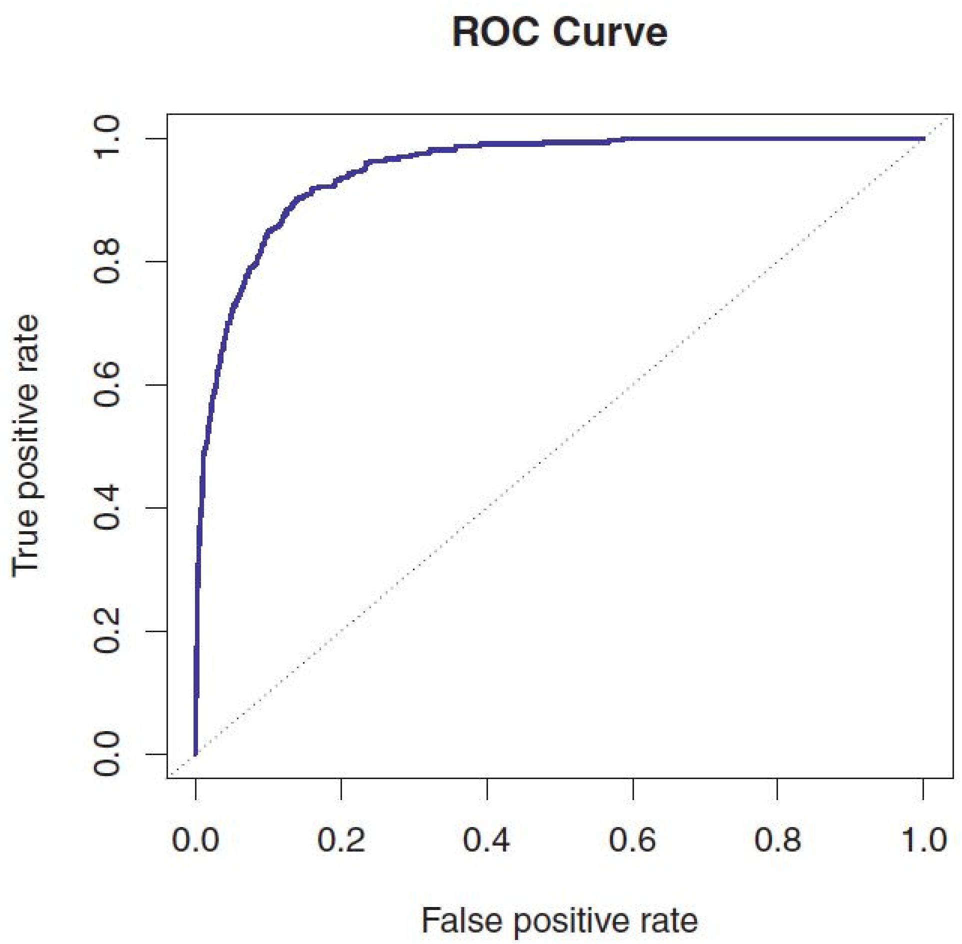


When you care about minorities...

- Sometimes minority performance is more important:
  - Disease detection
  - Credit card fraud
- Many functions increase performance this way
  - Assigns observations to class with highest posterior probability
  - Default 50% (or higher) will be assigned to majority class
  - Can change that threshold; see improvement in confusion matrix



Making the tradeoffs visible



- ROC curve
  - Shows both errors for all possible thresholds
  - Defines AUC = .95?
  - Curve ideally hugs top left corner

Non-linear classification

- Quadratic Discriminative Analysis
  - Relaxes assumptions:
    - Each class has a unique (Gaussian) distribution (μ_k, Σ_k)
    - Assigns label which is the max of:

$$\begin{aligned}\delta_k(x) &= -\frac{1}{2}(x-\mu_k)^T \boldsymbol{\Sigma}_k^{-1}(x-\mu_k) + \log \pi_k \\ &= -\frac{1}{2}x^T \boldsymbol{\Sigma}_k^{-1} x + x^T \boldsymbol{\Sigma}_k^{-1}\mu_k - \frac{1}{2}\mu_k^T \boldsymbol{\Sigma}_k^{-1}\mu_k + \log \pi_k\end{aligned}$$

  - Quadratic?
    - Note "*x*" is quadratic function
    - Class boundaries may be non-linear
- K Nearest Neighbours
  - Assigns label according to majority (plurality) of neighbours
  - Can produce highly non-linear boundaries