Homework # 1 Answer Key

1) a) $(E \cap G) \cap F^c$

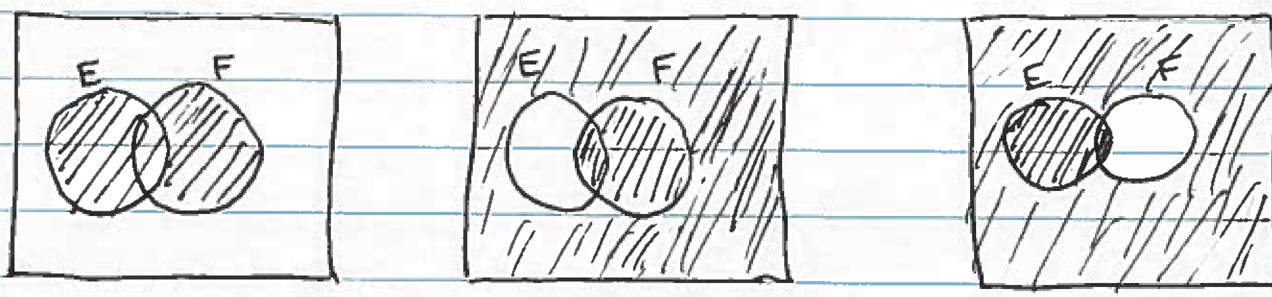
b) $(E \cap F) \cup (E \cap G) \cup (F \cap G) \cup (E \cap F \cap G)$

c) $[E \cap (F \cup G)^c] \cup [F \cap (E \cup G)^c] \cup$

$[G \cap (E \cup F)^c]$

2) This can be readily seen from a venn diagram illustration :

$(E \cup F)$ $\qquad$ $(E^c \cup F)$ $\qquad$ $(E \cup F^c)$



So, we see that the intersection of all of these is just $E \cap F$. You could also prove this algebraically.

3) i) We know that $P(S) = 1$ from the first axiom. Also, we know that $E \cup E^c = S$. So, $P(E \cup E^c) = 1$. But we also know that $E \cap E^c = \emptyset \Rightarrow E$ and E^c are mutually exclusive. It follows from the finite

3) i) continued...

generalization of the third axiom that

$$P(E \cup E^c) = P(E) + P(E^c)$$

So, $P(E) + P(E^c) = 1$

$\Rightarrow P(E^c) = 1 - P(E)$ $\square$

ii) we know that $\emptyset = S^c$. Using the result from (i), we have that $P(\emptyset) = 1 - P(S)$
$= 1 - 1 = 0$ $\square$

4) (a). We already have the first and second axioms directly from construction.
To prove the third axiom, we only need to consider the events $\{H\}$ and $\{T\}$.
We just need to show that

$$P(\{H\} \cup \{T\}) = P(\{H\}) + P(\{T\}) \quad (*)$$

Well, $P(\{H\} \cup \{T\}) = P(S) = 1$ and

$$P(\{H\}) + P(\{T\}) = p + 1 - p = 1.$$

So $(*)$ holds ✓

(b) If $P(\{T\}) = 1 - \frac{p}{2}$, we see that $(*)$ above will not hold. So Axiom 3 does not hold.

5) a) $Pr(\text{passes all 3 exams}) = P(F \cap S \cap T)$

$$= P(T \mid F \cap S)\, P(F \cap S)$$
$$= P(T \mid F \cap S)\, P(S \mid F)\, P(F)$$

$$= 0.7\,(0.8)(0.9) = 0.504$$

Above, the second & third lines are due to conditioning.

b) We want $P(S^c \mid (F \cap S \cap T)^c) =$

$$\frac{P(S^c \cap (F \cap S \cap T)^c)}{1 - P(F \cap S \cap T)} = \frac{P((S^c \cap F^c) \cup (S^c) \cup (S^c \cap T^c))}{0.496}$$

Note that if she does not pass an exam, she is not allowed to take the next one. So the previous value is equal to:

$$= \frac{P(F^c \cup S^c \cup S^c)}{0.496}$$

$$= \cancel{P(F^c) +} \frac{P((F \cap S)^c)}{0.496}$$

$$= \frac{1 - P(F \cap S)}{0.496} = \frac{1 - P(S \mid F)\, P(F)}{0.496}$$

$$= \frac{1 - (0.8)(0.9)}{0.496} = 0.565$$

6) a) To check for independence, we can check the following condition:

$$P(E_1 \cap F) = P(E_1)\,P(F) \qquad (*)$$

Well, $P(E_1 \cap F) = P(\{4,2\}) = \frac{1}{36}$

Also,

$$P(E_1) = P(\{1,5\} \cup \{2,4\} \cup \{3,3\} \cup \{4,2\} \cup \{5,1\}) = \frac{5}{36}$$

$$P(F) = \frac{1}{6}$$

So, $P(E_1)\,P(F) = \frac{5}{216} \neq \frac{1}{36}$ so no these events are not independent.

b) We follow the above strategy.

$$P(E_2 \cap F) = P(\{4,3\}) = \frac{1}{36}$$

$$P(F) = \frac{1}{6}$$

$$P(E_2) = P(\{1,6\} \cup \{2,5\} \cup \{3,4\} \cup \{4,3\} \cup \{5,2\} \cup \{6,1\}) = \frac{6}{36} = \frac{1}{6}$$

So, $P(E_2)\,P(F) = \frac{1}{36} \overset{\checkmark}{=} P(E_2 \cap F)$

Thus, these event are independent.

Note: this problem is interesting! Changing an event slightly can lead to independence!

7) $P(\text{breast cancer} \mid \text{pos. result}) =$

$$\frac{P(\text{pos. result} \mid \text{breast cancer})\, P(\text{breast cancer})}{P(\text{pos. result} \mid \text{cancer})\, P(\text{cancer}) + P(\text{pos. result} \mid \text{no cancer}) \cdot P(\text{no cancer})}$$

$$= \frac{0.90\,(0.008)}{0.90\,(0.008) + 0.07\,(0.992)} = 0.0939$$

This is just a simple application of Bayes rule!

Notes about Computational Problems:

① For the Monty Hall problem, you actually have a $2/3$ chance of winning if you switch doors! This is counter-intuitive to many folks, but is easily seen with simulation.

② It's also amazing to see that you only need 23 people in a room to have over a 50 % chance of a repeated birthdays!