

$$1) (x, y) \in \mathbb{R}^2 \times \{-1, +1\}$$

$$X = (x_1, x_2), \quad x_1, x_2 \in [0, 1]$$

$$x_1 \sim U(0, 1), \quad x_2 \sim g(x_2) = 3x_2^2 \quad \text{for } 0 \leq x_2 \leq 1$$

$$\eta(x_1, x_2) = \frac{x_1 + x_2}{2}$$

$$a) \phi^*(x_0) = \arg \max_j (P(Y=j | X=x_0))$$

$$= \arg \max_{j \in \{-1, +1\}} (P(Y=+1 | X=x_0), P(Y=-1 | X=x_0))$$

$$= \boxed{\arg \max_{j \in \{-1, +1\}} (\eta(x_1, x_2), 1 - \eta(x_1, x_2))}$$

The decision boundary is such that $\eta(x_1, x_2) = 1 - \eta(x_1, x_2)$

$$\Leftrightarrow \frac{x_1 + x_2}{2} = 1 - \frac{x_1 + x_2}{2}$$

$$\Leftrightarrow x_1 + x_2 = 2 - x_1 - x_2$$

$$\Leftrightarrow \boxed{x_2 = 1 - x_1}$$

b) We have $f(x_1) = 1$ and $g(x_2) = 3x_2^2$. Also, $x_1 \perp x_2$

$$\text{then } P(x) = h(x_1, x_2) = f(x_1)g(x_2) = \boxed{3x_2^2}$$