

# <sup>1</sup> cosmo-numba: B-modes and COSEBIs computations accelerated by Numba

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## Software

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<sup>10</sup> Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).  
<sup>11</sup> Cosmo-numba facilitate the computation of E-/B-modes decomposition using two methods.  
<sup>12</sup> One of them is the Complete Orthogonal Sets of E-/B-mode Integrals (COSEBIs) as presented  
<sup>13</sup> in P. Schneider et al. (2010). The COSEBIs rely on very high precision computation requiring  
<sup>14</sup> more than 80 decimal numbers. P. Schneider et al. (2010) propose an implementation  
<sup>15</sup> using mathematica. cosmo-numba make use of combination of sympy and mpmath to reach the  
<sup>16</sup> required precision. This python version enable an easier integration in cosmology pipeline and  
<sup>17</sup> facilitate the null tests.

<sup>18</sup> This software package also include the computation of the pure-mode correlation functions  
<sup>19</sup> presented in Peter Schneider et al. (2022). Those integrals have less constraints than the  
<sup>20</sup> COSEBIs but having a fast computation is necessary to computing the covariance matrix. One  
<sup>21</sup> can also include use those correlation function for cosmological inference in which case the  
<sup>22</sup> multiple call to the likelihood will also require a fast implementation.

## COSEBIs

<sup>23</sup> The COSEBIs are defined as:

$$E_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) + T_{n,-}(\theta) \xi_-(\theta)]; \quad (1)$$

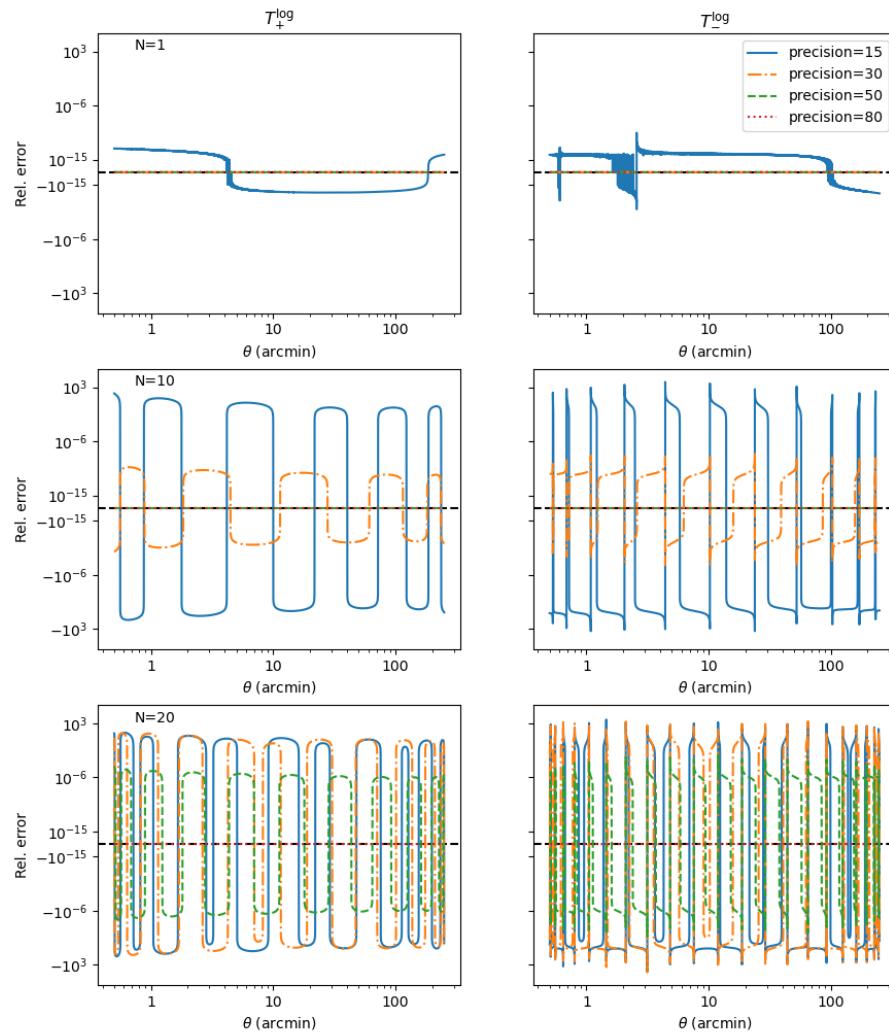
$$B_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) - T_{n,-}(\theta) \xi_-(\theta)]; \quad (2)$$

<sup>25</sup> where  $\xi_\pm(\theta)$  are the shear correlation functions, and  $T_{n,\pm}$  are the weight functions for the  
<sup>26</sup> mode  $n$ . The complexity is in the computation of reside in the computation of the weight  
<sup>27</sup> functions. Cosmo-numba include do the computation of the weight functions in logarithmic  
<sup>28</sup> scale defined by:

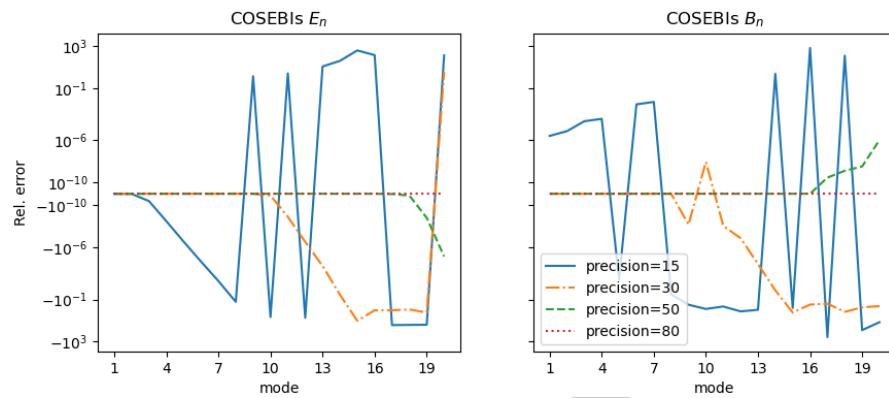
$$T_{n,+}^{\log}(\theta) = t_{n,+}^{\log}(z) = N_n \sum_{j=0}^{n+1} \bar{c}_{nj} z^j; \quad (3)$$

29 where  $z = \log(\theta/\theta_{\min})$ ,  $N_n$  is the normalization for the mode  $n$ , and  $\bar{c}_{jn}$  are defined iteratively  
30 from Bessel functions (we refer the readers to P. Schneider et al. (2010) for more details).

31 We have validating our implementation against the original version in Mathematica from P.  
32 Schneider et al. (2010). In figure Figure 1 we show the impact of the precision going from 15  
33 decimals, which correspond to the precision one could achieve using float64, up to 80, the  
34 precision used in the original implementation. We can see that classic float64 precision would  
35 not be sufficient and with a precision of 80 our code recover exactly the results from the original  
36 implementation. Similarly, the impact on the COSEBIs is shown in figure Figure 2.



**Figure 1:** In this figure we show the impact of the precision in the computation of the weight functions  $T_{\pm}^{\log}$ . For comparison, a precision of 15 corresponds to what would be achieved using numpy float64. The relative error is computed with respect to the original Mathematica implementation presented in P. Schneider et al. (2010).



**Figure 2:** Same as figure [Figure 1](#) for the COSEBIs E- and B-mode.

<sup>37</sup> COSEBIs can also be defined from the power spectrum as:

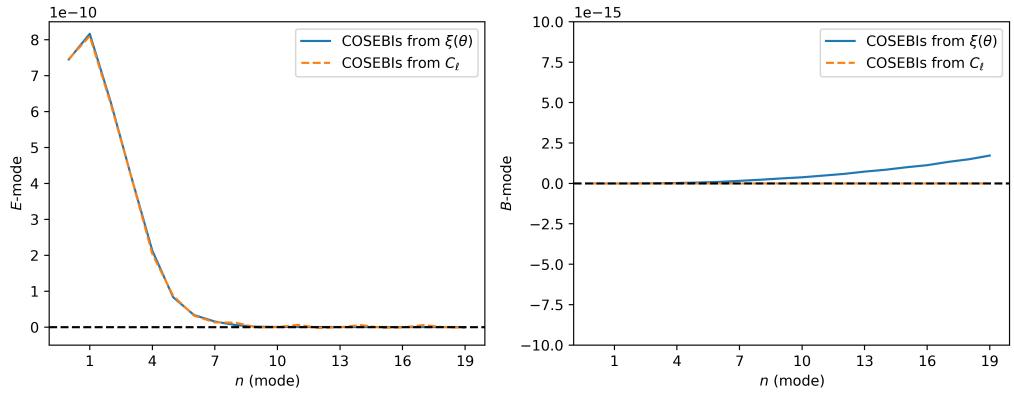
$$E_n = \int_0^\infty \frac{d\ell \ell}{Z\pi} P_E(\ell) W_\ell; \quad (4)$$

$$B_n = \int_0^\infty \frac{d\ell \ell}{Z\pi} P_B(\ell) W_\ell; \quad (5)$$

<sup>39</sup> where  $P_{E/B}(\ell)$  is the power spectrum of E- and B-modes and  $W_n(\ell)$  are the filter functions  
<sup>40</sup> which can be computed from  $T_{n,+}$  as:

$$W_n(\ell) = \int_{\theta_{min}}^{\theta_{max}} d\theta \ell T_{n,+}(\theta) J_0(\ell \theta); \quad (6)$$

<sup>41</sup> with  $J_0(\ell \theta)$  the 0-th order Bessel function. The [Equation 6](#) is an Hankel transform of order 0.  
<sup>42</sup> It can be computed using the FFTLog algorithm presented in [Hamilton \(2000\)](#) implemented  
<sup>43</sup> here in Numba. The [Figure 3](#) shows the comparison between the COSEBIs computed from  
<sup>44</sup>  $\xi_{\pm}(\theta)$  and from  $C_{E/B}(\ell)$ . We can see that the COSEBIs E-modes agrees very well but the  
<sup>45</sup> B-modes are more stable when computed from the  $C(\ell)$  space.



**Figure 3:** Comparison of the COSEBIs E- and B-mode computed from  $\xi_{\pm}(\theta)$  and  $C_{E/B}(\ell)$ .

## <sup>46</sup> Pure-Mode Correlation Functions

<sup>47</sup> In this section we look into the computation of the pure-mode correlation functions as defined  
<sup>48</sup> in Peter Schneider et al. (2022). There are defined as follow:

$$\xi_+^E = \frac{1}{2} \left[ \xi_+ + \xi_- + \int_{\theta}^{\theta} \frac{d\theta}{\theta} \xi_-(\theta) \left( 4 - \frac{12\theta^2}{\theta^2} \right) \right] \quad (7)$$

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<sup>51</sup> support from Kathryn Johnston during the genesis of this project.

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