cosmo-numba: B-modes and COSEBIs computations accelerated by Numba

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Summary

Cosmic shear important probe. B-modes computation as null test This software propose at the same time a user friendly interface and fast computation for E-/B-mode decomposition.

Statement of need

`Cosmo-numba` facilate the computation of E-/B-modes decomposition using two methods. One of them is the Complete Orthogonal Sets of E-/B-mode Integrals (COSEBIs) as presented in P. Schneider et al. (2010). The COSEBIs rely on very high precision computation requirering more than 80 decimal numbers. P. Schneider et al. (2010) propose an implementation using `mathematica`. `cosmo-numba` make use of combination of `sympy` and `mpmath` to reach the required precision. This python version enable an easier integration in cosmology pipeline and facilate the null tests.

This software package also include the computation of the pure-mode correlation functions presented in Peter Schneider et al. (2022). Those integrals have less constraints than the COSEBIs but having a fast computation is necessary to computing the covariance matrix. One can also include use those correlation function for cosmological inference in which case the multiple call to the likelihood will also require a fast implementation.

COSEBIs

The COSEBIs are defined as:

$$E_n = \frac{1}{2}\int_0^\infty d\theta\theta[T_{n,+}(\theta)\xi_+(\theta) + T_{n,-}(\theta)\xi_+(\theta)] \tag{1}$$

$$B_n = \frac{1}{2}\int_0^\infty d\theta\theta[T_{n,+}(\theta)\xi_+(\theta) - T_{n,-}(\theta)\xi_+(\theta)] \tag{2}$$

where $\xi_\pm(\theta)$ are the shear correlation functions, and $T_{n,\pm}$ are the weight functions for the mode n. The complexity is in the computation of reside in the computation of the weight functions. `Cosmo-numba` include do the computation of the weight functions in logarithmic scale defined by:

$$T_{n,+}^{\log}(\theta) = t_{n,+}^{\log}(z) = N_n \sum_{j=0}^{n+1} \bar{c}_{nj} z^j, \tag{3}$$

whare $z = log(\theta/\theta_{\min})$, N_n is the normalization for the mode n, and $\bar{c}_{jn}$ are defined iterratively from Bessel functions (we refer the readers to P. Schneider et al. (2010) for morre details).

We have validating our implementation against the original version in `Mathematica` from P. Schneider et al. (2010). In figure Figure 1 we show the impact of the precsion going from 15 decimals, which correspond to the precision one could achieve using float64, up to 80, the precision used in the original implementation. We can see that classic float64 precision would not be suficient and with a precision of 80 our code recover exactly the results from the orignal implementation. Similarly, the impact on the COSEBIs is shown in figure Figure 2.

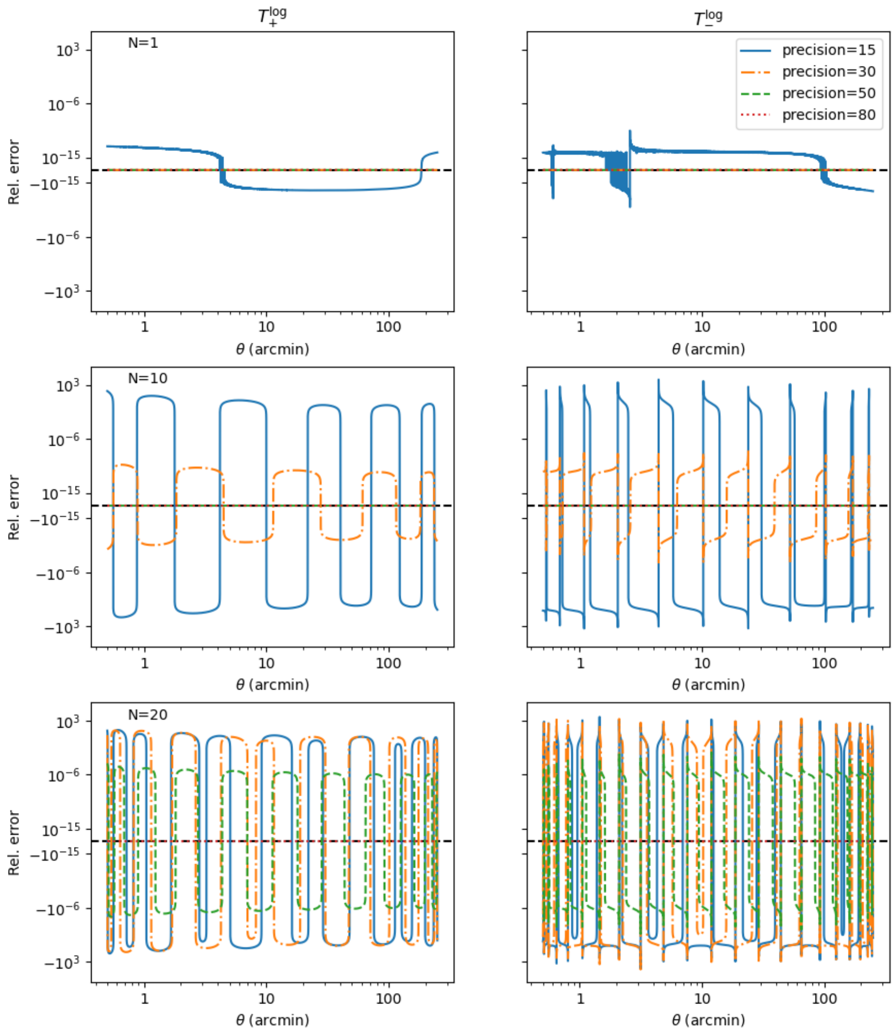

Figure 1: In this figure we show the impact of the precision in the computation of the weight functions $T_{\pm}^{\log}$. For comparion, a precision of 15 correspond to what would be achieve using `numpy` float64. The relative error is computed with respect to the orignal mathematica implementation presented in P. Schneider et al. (2010).

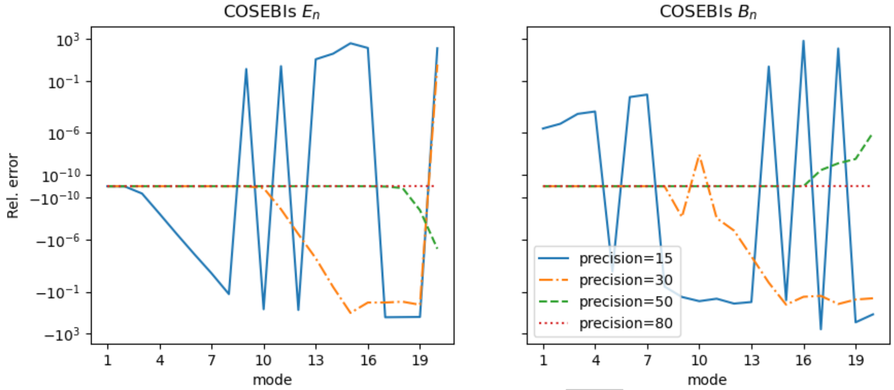

Figure 2: Same as figure Figure 1 for the COSEBIs E- and B-mode.

COSEBIs can also be defined from the power spectrum as:

$$E_n = \int_0^\infty \frac{d\ell\ell}{Z\pi} P_E(\ell) W_\ell; \tag{4}$$

$$B_n = \int_0^\infty \frac{d\ell\ell}{Z\pi} P_B(\ell) W_\ell, \tag{5}$$

where $P_{E/B}(\ell)$ is the power spectrum of E- and B-modes and $W_n(\ell)$ are the filter functions which can be computed from $T_{n,+}$ as:

$$W_n(\ell) = \int_{\theta_{min}}^{\theta_{max}} d\theta\theta T_{n,+}(\theta) J_0(\ell\ell) \tag{6}$$

with $J_0(\ell\ell)$ the 0-th order Bessel function. We can see that Eq. Equation 6 is an Hankel transform. It can be computed using the FFTLog algorithm presented in Hamilton (2000) implemented here in Numba. Figure Figure 3 shows the comparison between the COSEBIs computed from $\xi_\pm(\theta)$ and from $C_{E/B}(\ell)$. We can see that the COSEBIs E-modes agrees very well but the B-modes are more stable when computed from the $C(\ell)$ space.

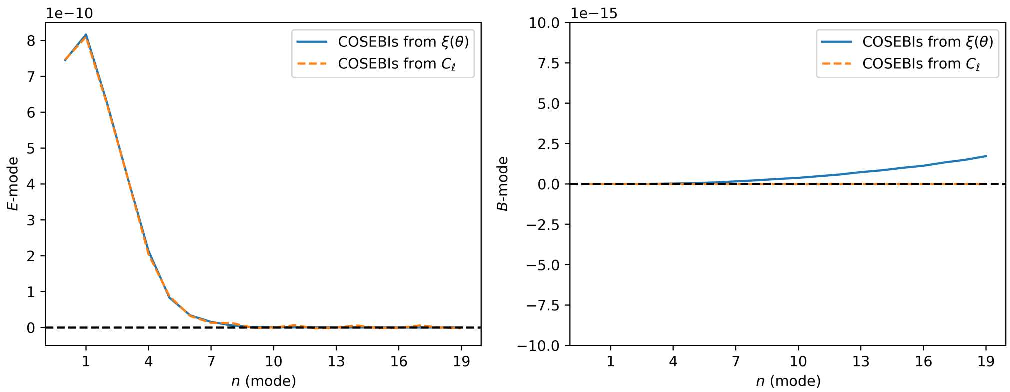

Figure 3: Comparison of the COSEBIs E- and B-mode computed from $\xi_\pm(\theta)$ and $C_{E/B}(\ell)$.

Mathematics

Single dollars ($) are required for inline mathematics e.g. $f(x) = e^{\pi/x}$

Double dollars make self-standing equations:

$$\Theta(x) = \left\{\begin{array}{l} 0\textrm{ if } x < 0\cr 1\textrm{ else} \end{array}\right.$$

You can also use plain LaTeX for equations

$$\hat f(\omega) = \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx \tag{7}$$

and refer to Equation 7 from text.

Citations

Citations to entries in paper.bib should be in rMarkdown format.

If you want to cite a software repository URL (e.g. something on GitHub without a preferred citation) then you can do it with the example BibTeX entry below for (**?**).

For a quick reference, the following citation commands can be used: - `@author:2001` -> "Author et al. (2001)" - `[@author:2001]` -> "(Author et al., 2001)" - `[@author1:2001; @author2:2001]` -> "(Author1 et al., 2001; Author2 et al., 2002)"

Figures

Figures can be included like this: Caption for example figure. and referenced from text using section .

Figure sizes can be customized by adding an optional second parameter: Caption for example figure.

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