

# <sup>1</sup> cosmo-numba: B-modes and COSEBIs computations accelerated by Numba

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## Software

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<sup>15</sup> <sup>16</sup> <sup>17</sup> <sup>18</sup> <sup>19</sup>

## <sup>6</sup> Summary

<sup>7</sup> Cosmic shear important probe. B-modes computation as null test This software propose at  
<sup>8</sup> the same time a user friendly interface and fast computation for E-/B-mode decomposition.

## <sup>9</sup> Statement of need

<sup>10</sup> The E-/B-mode composition for cosmic shear poses a significant computational challenge given  
<sup>11</sup> the need for high precision (required to integrate oscillatory functions over a large integration  
<sup>12</sup> range and achieve accurate results) and speed. Cosmo-numba meets this need, facilitating the  
<sup>13</sup> computation of E-/B-modes decomposition using two methods. One of them is the Complete  
<sup>14</sup> Orthogonal Sets of E-/B-mode Integrals (COSEBIs) as presented in P. Schneider et al. (2010).  
<sup>15</sup> The COSEBIs rely on very high precision computation requiring more than 80 decimal places.  
<sup>16</sup> P. Schneider et al. (2010) propose an implementation using mathematica. cosmo-numba uses  
<sup>17</sup> a combination of sympy and mpmath to reach the required precision. This python version  
<sup>18</sup> enables an easier integration within cosmological inference pipelines, which are commonly  
<sup>19</sup> python-based, and facilitates the null tests.

<sup>20</sup> This software package also enables the computation of the pure-mode correlation functions  
<sup>21</sup> presented in Peter Schneider et al. (2022). Those integrals are less numerically challenging  
<sup>22</sup> than the COSEBIs, but having a fast computation is necessary for computing the covariance  
<sup>23</sup> matrix. One can also use those correlation functions for cosmological inference, in which case  
<sup>24</sup> the large number of calls to the likelihood function will also require a fast implementation.

## <sup>25</sup> COSEBIs

<sup>26</sup> The COSEBIs are defined as:

$$E_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) + T_{n,-}(\theta) \xi_-(\theta)], \quad (1)$$

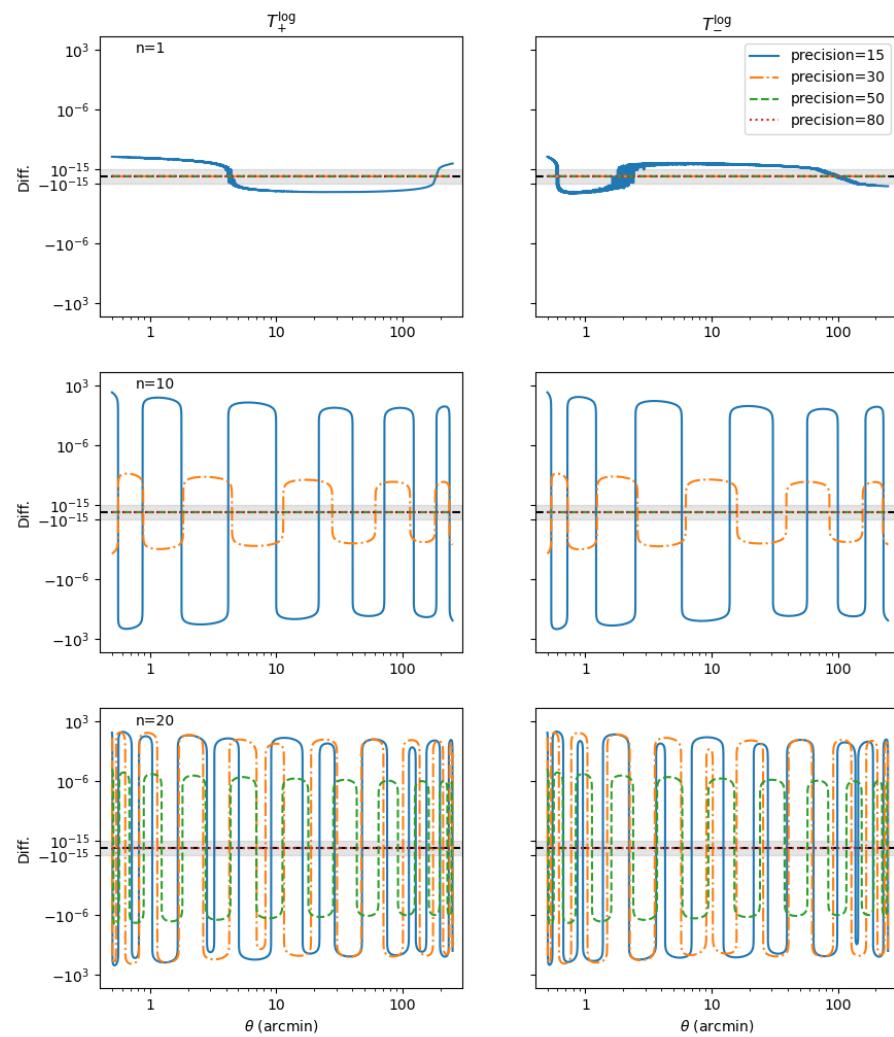
$$B_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) - T_{n,-}(\theta) \xi_-(\theta)]; \quad (2)$$

<sup>28</sup> where  $\xi_\pm(\theta)$  are the shear correlation functions, and  $T_{n,\pm}$  are the weight functions for the  
<sup>29</sup> COSEBI mode  $n$ . The complexity is in the computation of the weight functions. Cosmo-numba  
<sup>30</sup> carries out the computation of the weight functions in a logarithmic scale defined by:

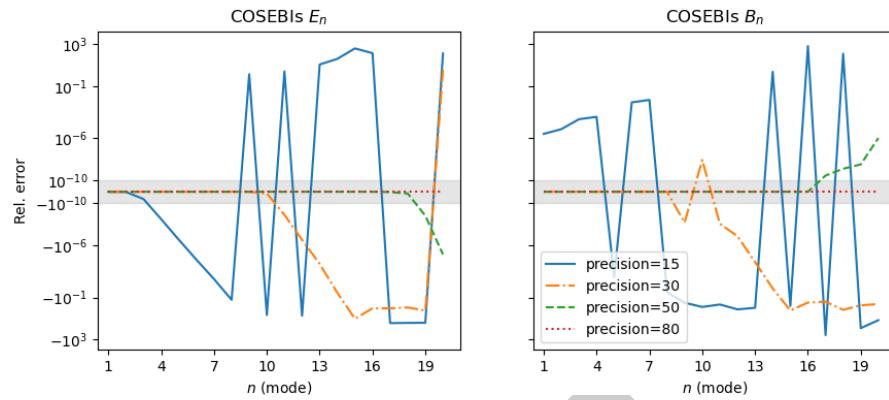
$$T_{n,+}^{\log}(\theta) = t_{n,+}^{\log}(z) = N_n \sum_{j=0}^{n+1} \bar{c}_{nj} z^j; \quad (3)$$

<sup>31</sup> where  $z = \log(\theta/\theta_{\min})$ ,  $N_n$  is the normalization for the mode  $n$ , and  $\bar{c}_{jn}$  are defined iteratively  
<sup>32</sup> from Bessel functions (we refer the readers to P. Schneider et al. (2010) for more details).

<sup>33</sup> We have validating our implementation against the original version in Mathematica from P.  
<sup>34</sup> Schneider et al. (2010). In Figure 1 we show the impact of the precision going from 15 decimal  
<sup>35</sup> places, which corresponds to the precision one could achieve using float64, up to 80 decimal  
<sup>36</sup> places, the precision used in the original Mathematica implementation. We can see that classic  
<sup>37</sup> float64 precision would not be sufficient, and with a precision of 80 our code exactly recovers  
<sup>38</sup> the results from the original implementation. Similarly, the impact on the COSEBIs is shown  
<sup>39</sup> in Figure 2.



**Figure 1:** In this figure we show the impact of the precision in the computation of the weight functions  $T_{\pm}^{\log}$ . For comparison, a precision of 15 corresponds to what would be achieved using numpy float64. The relative error is computed with respect to the original Mathematica implementation presented in P. Schneider et al. (2010).



**Figure 2:** Same as figure [Figure 1](#) for the COSEBIs E- and B-mode.

<sup>40</sup> COSEBIs can also be defined from the power spectrum as:

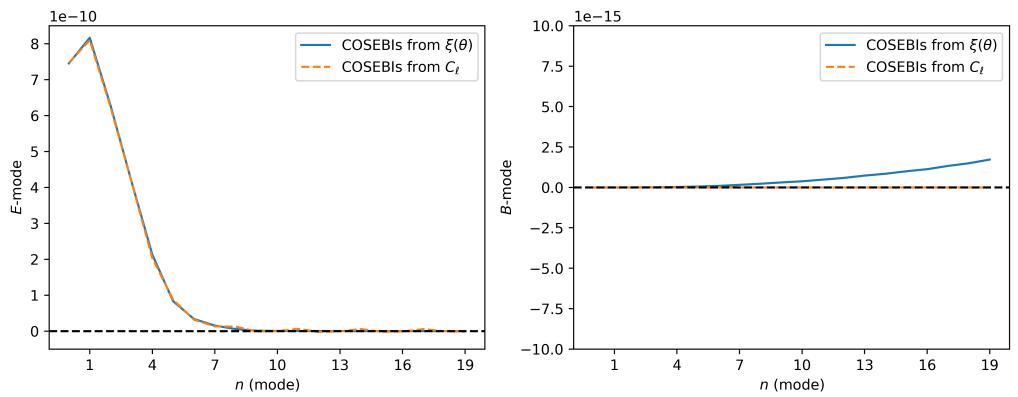
$$E_n = \int_0^\infty \frac{d\ell \ell}{Z\pi} P_E(\ell) W_n(\ell); \quad (4)$$

$$B_n = \int_0^\infty \frac{d\ell \ell}{Z\pi} P_B(\ell) W_n(\ell); \quad (5)$$

<sup>42</sup> where  $P_{E/B}(\ell)$  is the power spectrum of E- and B-modes and  $W_n(\ell)$  are the filter functions  
<sup>43</sup> which can be computed from  $T_{n,+}$  as:

$$W_n(\ell) = \int_{\theta_{\min}}^{\theta_{\max}} d\theta \theta T_{n,+}(\theta) J_0(\ell\theta); \quad (6)$$

<sup>44</sup> with  $J_0(\ell\theta)$  the 0-th order Bessel function. The [Equation 6](#) is a Hankel transform of order 0.  
<sup>45</sup> It can be computed using the FFTLog algorithm presented in [Hamilton \(2000\)](#) implemented  
<sup>46</sup> here in Numba. [Figure 3](#) shows the comparison between the COSEBIs computed from  $\xi_{\pm}(\theta)$   
<sup>47</sup> and from  $C_{E/B}(\ell)$ . We can see that the COSEBI E-modes agree very well but the B-modes  
<sup>48</sup> are more stable when computed from the  $C(\ell)$  space.



**Figure 3:** Comparison of the COSEBIs E- and B-mode computed from  $\xi_{\pm}(\theta)$  and  $C_{E/B}(\ell)$ .

## 49 Pure-Mode Correlation Functions

50 In this section we describe the computation of the pure-mode correlation functions as defined  
51 in Peter Schneider et al. (2022). There are defined as follow:

$$\xi_+^E(\vartheta) = \frac{1}{2} \left[ \xi_+(\vartheta) + \xi_-(\vartheta) + \int_{\vartheta}^{\vartheta_{\max}} \frac{d\theta}{\theta} \xi_-(\theta) \left( 4 - \frac{12\vartheta^2}{\theta^2} \right) \right] - \frac{1}{2} [S_+(\vartheta) + S_-(\vartheta)], \quad (7)$$

$$52 \quad \xi_+^B(\vartheta) = \frac{1}{2} \left[ \xi_+(\vartheta) - \xi_-(\vartheta) - \int_{\vartheta}^{\vartheta_{\max}} \frac{d\theta}{\theta} \xi_-(\theta) \left( 4 - \frac{12\vartheta^2}{\theta^2} \right) \right] - \frac{1}{2} [S_+(\vartheta) - S_-(\vartheta)], \quad (8)$$

$$\xi_-^E(\vartheta) = \frac{1}{2} \left[ \xi_+(\vartheta) + \xi_-(\vartheta) + \int_{\vartheta_{\min}}^{\vartheta} \frac{d\theta \theta}{\vartheta^2} \xi_+(\theta) \left( 4 - \frac{12\theta^2}{\vartheta^2} \right) \right] - \frac{1}{2} [V_+(\vartheta) + V_-(\vartheta)], \quad (9)$$

$$53 \quad \xi_-^B(\vartheta) = \frac{1}{2} \left[ \xi_+(\vartheta) - \xi_-(\vartheta) + \int_{\vartheta_{\min}}^{\vartheta} \frac{d\theta \theta}{\vartheta^2} \xi_+(\theta) \left( 4 - \frac{12\theta^2}{\vartheta^2} \right) \right] - \frac{1}{2} [V_+(\vartheta) - V_-(\vartheta)]; \quad (10)$$

54 where  $\xi_{\pm}(\theta)$  correspond to the shear-shear correlation function. The functions  $S_{\pm}(\theta)$  and  $V_{\pm}(\theta)$   
55 are themselves defined by integrals and we refer the reader to Peter Schneider et al. (2022)  
56 for more details about their definition. By contrast with the computation of the COSEBIs,  
57 these integrals are more stable and straightforward to compute but still require some level of  
58 precision. This is why we are using the quads method with a 5-th order spline interpolation.  
59 In addition, as one can see from the equations above, the implementation requires a loop over  
60 a range of  $\vartheta$  values. This is why having a fast implementation will be required if one want to  
61 use those correlation functions in cosmological inference.

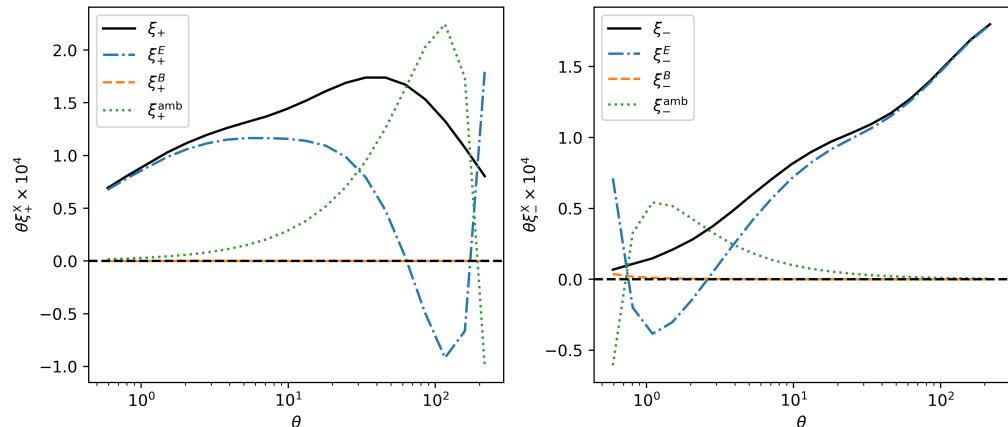


Figure 4: This figure shows the decomposition of the shear-shear correaltion functions in E- and B-modes (and ambiguous mode).

## 62 Acknowledgements

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