cosmo-numba: B-modes and COSEBIs computations accelerated by Numba

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Summary

Cosmic shear important probe. B-modes computation as null test This software propose at the same time a user friendly interface and fast computation for E-/B-mode decomposition.

Statement of need

`Cosmo-numba` facilitate the computation of E-/B-modes decomposition using two methods. One of them is the Complete Orthogonal Sets of E-/B-mode Integrals (COSEBIs) as presented in P. Schneider et al. (2010). The COSEBIs rely on very high precision computation requirering more than 80 decimal numbers. P. Schneider et al. (2010) propose an implementation using `mathematica`. `cosmo-numba` make use of combination of `sympy` and `mpmath` to reach the required precision. This python version enable an easier integration in cosmology pipeline and facilate the null tests.

This software package also include the computation of the pure-mode correlation functions presented in Peter Schneider et al. (2022). Those integrals have less constraints than the COSEBIs but having a fast computation is necessary to computing the covariance matrix. One can also include use those correlation function for cosmological inference in which case the multiple call to the likelihood will also require a fast implementation.

COSEBIs

The COSEBIs are defined as:

$$E_n = \frac{1}{2}\int_0^\infty d\theta\theta[T_{n,+}(\theta)\xi_+(\theta) + T_{n,-}(\theta)\xi_+(\theta)], \tag{1}$$

$$B_n = \frac{1}{2}\int_0^\infty d\theta\theta[T_{n,+}(\theta)\xi_+(\theta) - T_{n,-}(\theta)\xi_+(\theta)]; \tag{2}$$

where $\xi_\pm(\theta)$ are the shear correlation functions, and $T_{n,\pm}$ are the weight functions for the mode n. The complexity is in the computation of reside in the computation of the weight functions. `Cosmo-numba` include do the computation of the weight functions in logarithmic scale defined by:

$$T^{\log}_{n,+}(\theta) = t^{\log}_{n,+}(z) = N_n \sum_{j=0}^{n+1} \bar{c}_{nj} z^j; \tag{3}$$

whare $z = log(\theta/\theta_{\min})$, N_n is the normalization for the mode n, and $\bar{c}_{jn}$ are defined iterratively from Bessel functions (we refer the readers to P. Schneider et al. (2010) for morre details).

We have validating our implementation against the original version in `Mathematica` from P. Schneider et al. (2010). In figure Figure 1 we show the impact of the precsion going from 15 decimals, which correspond to the precision one could achieve using float64, up to 80, the precision used in the original implementation. We can see that classic float64 precision would not be suficient and with a precision of 80 our code recover exactly the results from the orignal implementation. Similarly, the impact on the COSEBIs is shown in figure Figure 2.

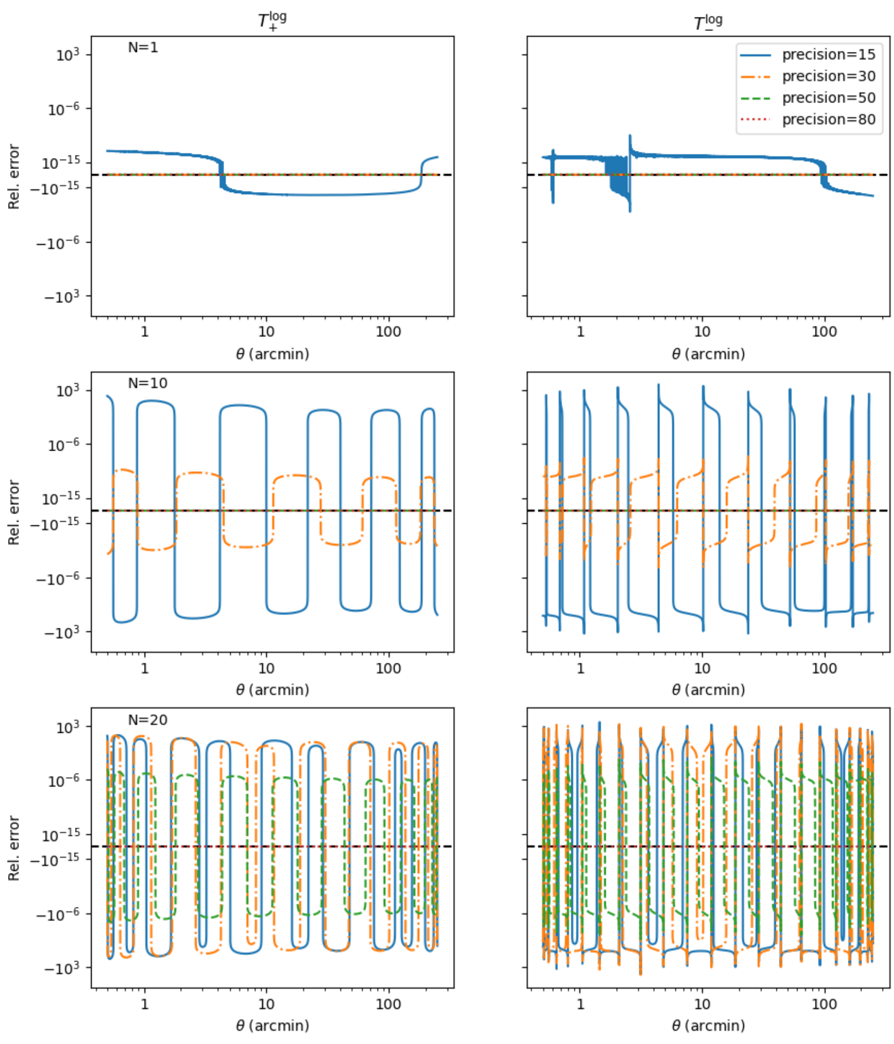

Figure 1: In this figure we show the impact of the precision in the computation of the weight functions $T_{\pm}^{\log}$. For comparion, a precision of 15 correspond to what would be achieve using `numpy` float64. The relative error is computed with respect to the orignal mathematica implementation presented in P. Schneider et al. (2010).

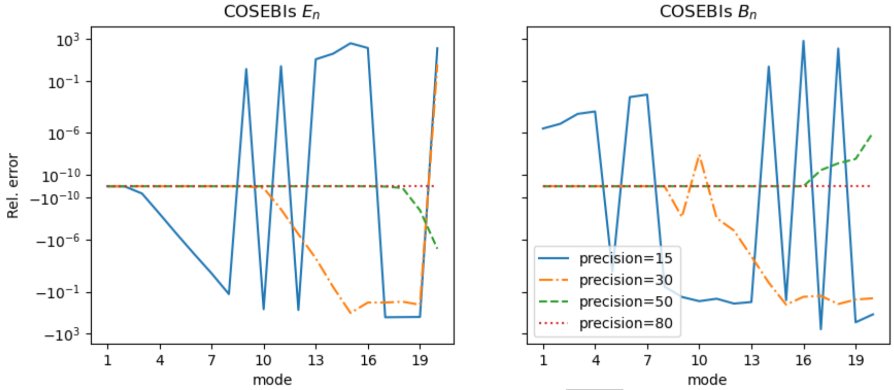

Figure 2: Same as figure Figure 1 for the COSEBIs E- and B-mode.

COSEBIs can also be defined from the power spectrum as:

$$E_n = \int_0^\infty \frac{d\ell\ell}{Z\pi} P_E(\ell) W_\ell; \tag{4}$$

$$B_n = \int_0^\infty \frac{d\ell\ell}{Z\pi} P_B(\ell) W_\ell; \tag{5}$$

where $P_{E/B}(\ell)$ is the power spectrum of E- and B-modes and $W_n(\ell)$ are the filter functions which can be computed from $T_{n,+}$ as:

$$W_n(\ell) = \int_{\theta_{min}}^{\theta_{max}} d\theta\theta T_{n,+}(\theta) J_0(\ell\ell); \tag{6}$$

with $J_0(\ell\ell)$ the 0-th order Bessel function. The Equation 6 is an Hankel transform of order 0. It can be computed using the `FFTLog` algorithm presented in Hamilton (2000) implemented here in `Numba`. The Figure 3 shows the comparison between the COSEBIs computed from $\xi_\pm(\theta)$ and from $C_{E/B}(\ell)$. We can see that the COSEBIs E-modes agrees very well but the B-modes are more stable when computed from the $C(\ell)$ space.

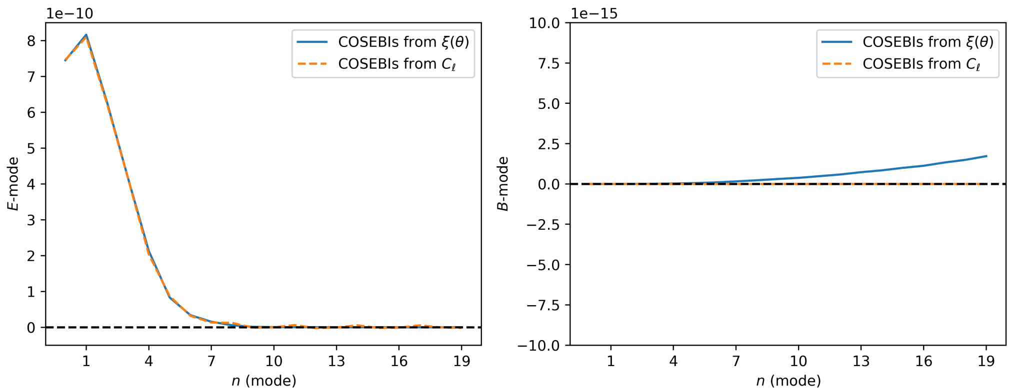

Figure 3: Comparison of the COSEBIs E- and B-mode computed from $\xi_\pm(\theta)$ and $C_{E/B}(\ell)$.

Pure-Mode Correlation Functions

In this section we look into the computation of the pure-mode correlation functions as defined in Peter Schneider et al. (2022). There are defined as follow:

$$\xi_+^E(\vartheta) = \frac{1}{2}\left[\xi_+(\vartheta) + \xi_-(\vartheta) + \int_\vartheta^{\vartheta_{\max}} \frac{d\theta}{\theta}\xi_-(\theta)\left(4 - \frac{12\vartheta^2}{\theta^2}\right)\right] - \frac{1}{2}\left[S_+(\vartheta) + S_-(\vartheta)\right], \quad (7)$$

$$\xi_+^B(\vartheta) = \frac{1}{2}\left[\xi_+(\vartheta) - \xi_-(\vartheta) - \int_\vartheta^{\vartheta_{\max}} \frac{d\theta}{\theta}\xi_-(\theta)\left(4 - \frac{12\vartheta^2}{\theta^2}\right)\right] - \frac{1}{2}\left[S_+(\vartheta) - S_-(\vartheta)\right], \quad (8)$$

$$\xi_-^E(\vartheta) = \frac{1}{2}\left[\xi_+(\vartheta) + \xi_-(\vartheta) + \int_{\vartheta_{\min}}^{\vartheta} \frac{d\theta\theta}{\vartheta^2}\xi_+(\theta)\left(4 - \frac{12\theta^2}{\vartheta^2}\right)\right] - \frac{1}{2}\left[V_+(\vartheta) + V_-(\vartheta)\right], \quad (9)$$

$$\xi_-^B(\vartheta) = \frac{1}{2}\left[\xi_+(\vartheta) - \xi_-(\vartheta) + \int_{\vartheta_{\min}}^{\vartheta} \frac{d\theta\theta}{\vartheta^2}\xi_+(\theta)\left(4 - \frac{12\theta^2}{\vartheta^2}\right)\right] - \frac{1}{2}\left[V_+(\vartheta) - V_-(\vartheta)\right]; \quad (10)$$

where $\xi_\pm(\theta)$ correspond to the shear-shear correlation function. The functions $\S_\pm(\theta)$ and $V_\pm(\theta)$ are themselves defined by integrals and we refer the reader to Peter Schneider et al. (2022) for more details about their definition. By contrast with the computation of the COSEBIs, these integrals are more stable and strainghtforward to compute but still requires some level of precision. This is why we are using the quads method with a 5-th order interpolation. In addtition, as one can see from the equations above, the implementation will require to loop over a range of ϑ. This is why having a fast implementation will be required if one want to use those correlation functions in cosmological inference for instance.

Acknowledgements

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