

¹ cosmo-numba: B-modes and COSEBIs computations accelerated by Numba

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¹¹ Cosmo-numba facilitate the computation of E-/B-modes decomposition using two methods.
¹² One of them is the Complete Orthogonal Sets of E-/B-mode Integrals (COSEBIs) as presented
¹³ in P. Schneider et al. (2010). The COSEBIs rely on very high precision computation requiring
¹⁴ more than 80 decimal numbers. P. Schneider et al. (2010) propose an implementation
¹⁵ using mathematica. cosmo-numba make use of combination of sympy and mpmath to reach the
¹⁶ required precision. This python version enable an easier integration in cosmology pipeline and
¹⁷ facilitate the null tests.

¹⁸ This software package also include the computation of the pure-mode correlation functions
¹⁹ presented in Peter Schneider et al. (2022). Those integrals have less constraints than the
²⁰ COSEBIs but having a fast computation is necessary to computing the covariance matrix. One
²¹ can also include use those correlation function for cosmological inference in which case the
²² multiple call to the likelihood will also require a fast implementation.

COSEBIs

²³ The COSEBIs are defined as:

$$E_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) + T_{n,-}(\theta) \xi_-(\theta)]; \quad (1)$$

$$B_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) - T_{n,-}(\theta) \xi_-(\theta)]; \quad (2)$$

²⁵ where $\xi_\pm(\theta)$ are the shear correlation functions, and $T_{n,\pm}$ are the weight functions for the
²⁶ mode n . The complexity is in the computation of reside in the computation of the weight
²⁷ functions. Cosmo-numba include do the computation of the weight functions in logarithmic
²⁸ scale defined by:

$$T_{n,+}^{\log}(\theta) = t_{n,+}^{\log}(z) = N_n \sum_{j=0}^{n+1} \bar{c}_{nj} z^j; \quad (3)$$

29 where $z = \log(\theta/\theta_{\min})$, N_n is the normalization for the mode n , and \bar{c}_{jn} are defined iteratively
30 from Bessel functions (we refer the readers to P. Schneider et al. (2010) for more details).

31 We have validating our implementation against the original version in Mathematica from P.
32 Schneider et al. (2010). In figure Figure 1 we show the impact of the precision going from 15
33 decimals, which correspond to the precision one could achieve using float64, up to 80, the
34 precision used in the original implementation. We can see that classic float64 precision would
35 not be sufficient and with a precision of 80 our code recover exactly the results from the original
36 implementation. Similarly, the impact on the COSEBIs is shown in figure Figure 2.

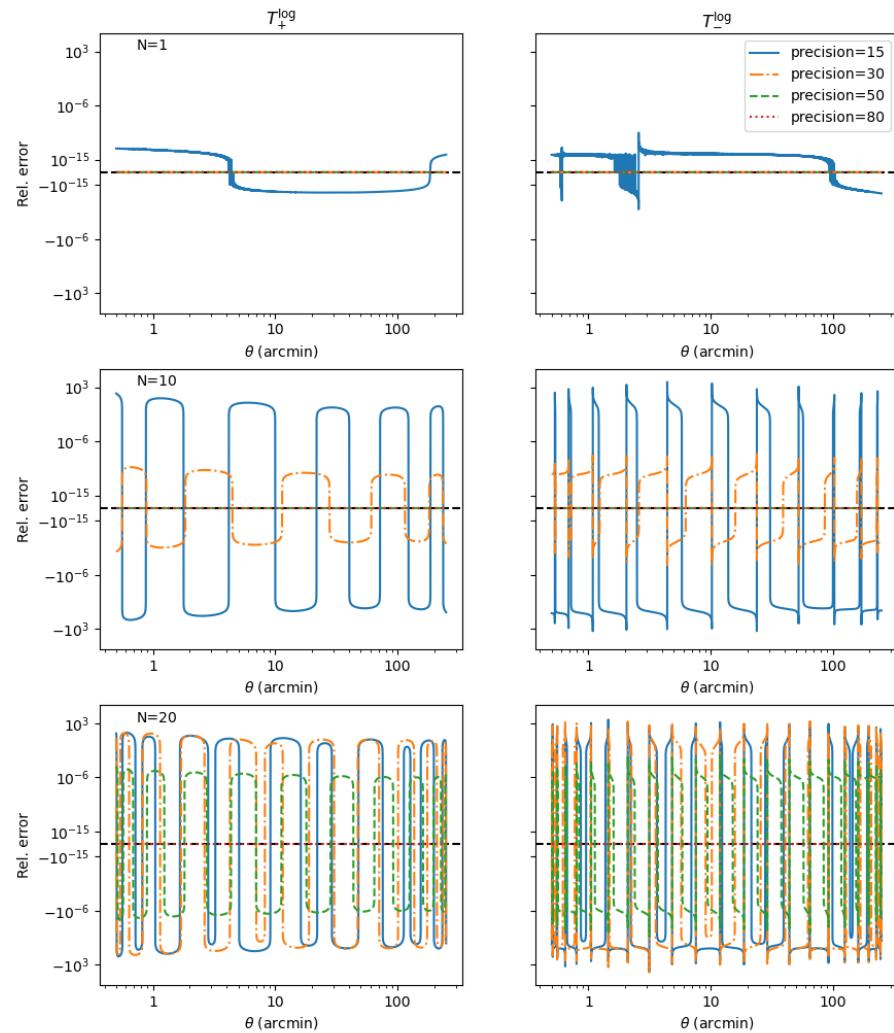


Figure 1: In this figure we show the impact of the precision in the computation of the weight functions T_{\pm}^{\log} . For comparison, a precision of 15 corresponds to what would be achieved using numpy float64. The relative error is computed with respect to the original mathematica implementation presented in P. Schneider et al. (2010).

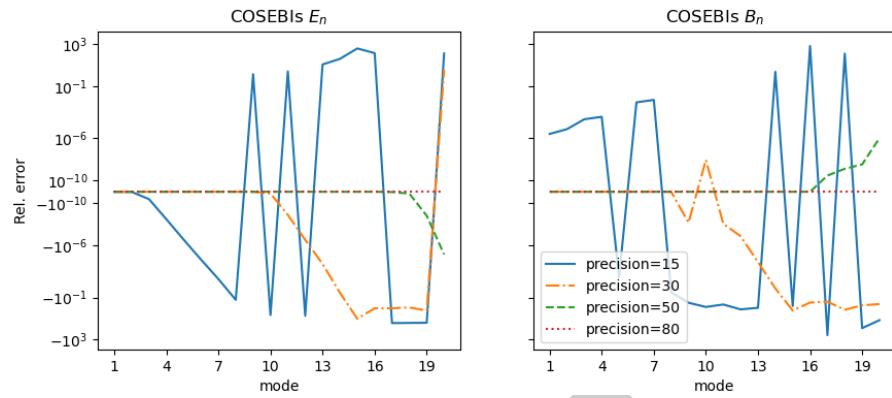


Figure 2: Same as figure [Figure 1](#) for the COSEBIs E- and B-mode.

³⁷ COSEBIs can also be defined from the power spectrum as:

$$E_n = \int_0^\infty \frac{d\ell \ell}{Z\pi} P_E(\ell) W_\ell; \quad (4)$$

$$B_n = \int_0^\infty \frac{d\ell \ell}{Z\pi} P_B(\ell) W_\ell; \quad (5)$$

³⁹ where $P_{E/B}(\ell)$ is the power spectrum of E- and B-modes and $W_n(\ell)$ are the filter functions
⁴⁰ which can be computed from $T_{n,+}$ as:

$$W_n(\ell) = \int_{\theta_{min}}^{\theta_{max}} d\theta \ell T_{n,+}(\theta) J_0(\ell \theta); \quad (6)$$

⁴¹ with $J_0(\ell \theta)$ the 0-th order Bessel function. The [Equation 6](#) is an Hankel transform of order 0.
⁴² It can be computed using the FFTLog algorithm presented in [Hamilton \(2000\)](#) implemented
⁴³ here in Numba. The [Figure 3](#) shows the comparison between the COSEBIs computed from
⁴⁴ $\xi_{\pm}(\theta)$ and from $C_{E/B}(\ell)$. We can see that the COSEBIs E-modes agrees very well but the
⁴⁵ B-modes are more stable when computed from the $C(\ell)$ space.

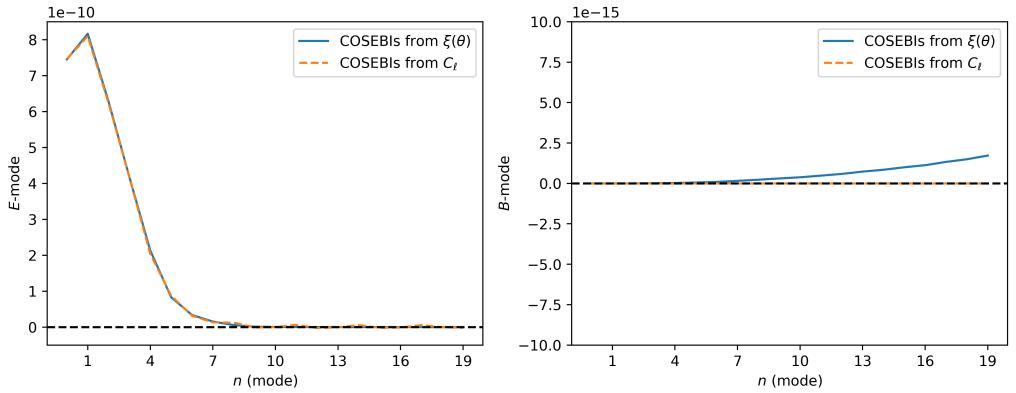


Figure 3: Comparison of the COSEBIs E- and B-mode computed from $\xi_{\pm}(\theta)$ and $C_{E/B}(\ell)$.

⁴⁶ Pure-Mode Correlation Functions

⁴⁷ In this section we look into the computation of the pure-mode correlation functions as defined
⁴⁸ in Peter Schneider et al. (2022). There are defined as follow:

$$\xi_+^E(\vartheta) = \frac{1}{2} \left[\xi_+(\vartheta) + \xi_-(\vartheta) + \int_{\vartheta}^{\vartheta_{\max}} \frac{d\theta}{\theta} \xi_-(\theta) \left(4 - \frac{12\vartheta^2}{\theta^2} \right) \right] - \frac{1}{2} [S_+(\vartheta) + S_-(\vartheta)] \quad (7)$$

$$\xi_+^B(\vartheta) = \frac{1}{2} \left[\xi_+(\vartheta) - \xi_-(\vartheta) - \int_{\vartheta}^{\vartheta_{\max}} \frac{d\theta}{\theta} \xi_-(\theta) \left(4 - \frac{12\vartheta^2}{\theta^2} \right) \right] - \frac{1}{2} [S_+(\vartheta) - S_-(\vartheta)] \quad (8)$$

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⁵² References

- ⁵³ Hamilton, A. J. S. (2000). Uncorrelated modes of the non-linear power spectrum. *Monthly Notices of the Royal Astronomical Society*, 312(2), 257–284. <https://doi.org/10.1046/j.1365-8711.2000.03071.x>
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