




cosmo-numba: B-modes and COSEBIs computations accelerated by Numba

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DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

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Submitted: 01 January 1970

Published: unpublished

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Summary

Cosmic shear important probe. B-modes computation as null test This software propose at the same time a user friendly interface and fast computation for E-/B-mode decomposition.

Statement of need

Cosmo-numba facilitate the computation of E-/B-modes decomposition using two methods. One of them is the Complete Orthogonal Sets of E-/B-mode Integrals (COSEBIs) as presented in P. Schneider et al. (2010). The COSEBIs rely on very high precision computation requiring more than 80 decimal numbers. P. Schneider et al. (2010) propose an implementation using mathematica. cosmo-numba make use of combination of sympy and mpmath to reach the required precision. This python version enable an easier integration in cosmology pipeline and faciliate the null tests.

This software package also include the computation of the pure-mode correlation functions presented in Peter Schneider et al. (2022). Those integrals have less constraints than the COSEBIs but having a fast computation is necessary to computing the covariance matrix. One can also include use those correlation function for cosmological inference in which case the multiple call to the likelihood will also require a fast implementation.

COSEBIs

The COSEBIs are defined as:

$$E_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) + T_{n,-}(\theta) \xi_+(\theta)]; \quad (1)$$

$$B_n = \frac{1}{2} \int_0^\infty d\theta \theta [T_{n,+}(\theta) \xi_+(\theta) - T_{n,-}(\theta) \xi_+(\theta)]; \quad (2)$$

where $\xi_\pm(\theta)$ are the shear correlation functions, and $T_{n,\pm}$ are the weight functions for the mode n . The complexity is in the computation of reside in the computation of the weight functions. Cosmo-numba include do the computation of the weight functions in logarithmic scale defined by:

$$T_{n,+}^{\log}(\theta) = t_{n,+}^{\log}(z) = N_n \sum_{j=0}^{n+1} \bar{c}_{nj} z^j; \quad (3)$$

where $z = \log(\theta/\theta_{\min})$, N_n is the normalization for the mode n , and \bar{c}_{jn} are defined iteratively from Bessel functions (we refer the readers to P. Schneider et al. (2010) for more details).

We have validated our implementation against the original version in Mathematica from P. Schneider et al. (2010). In figure Figure 1 we show the impact of the precision going from 15 decimals, which correspond to the precision one could achieve using float64, up to 80, the precision used in the original implementation. We can see that classic float64 precision would not be sufficient and with a precision of 80 our code recover exactly the results from the original implementation. Similarly, the impact on the COSEBIs is shown in figure Figure 2.

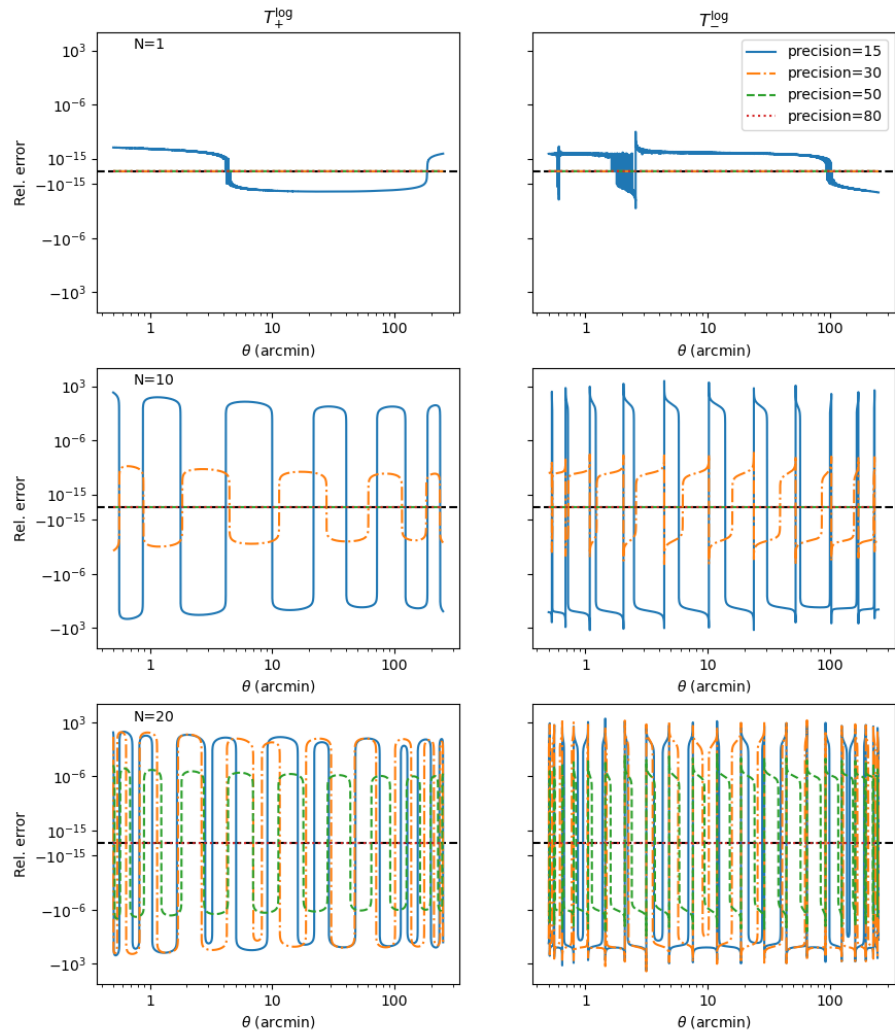


Figure 1: In this figure we show the impact of the precision in the computation of the weight functions T_{\pm}^{\log} . For comparison, a precision of 15 correspond to what would be achieve using numpy float64. The relative error is computed with respect to the original mathematica implementation presented in P. Schneider et al. (2010).

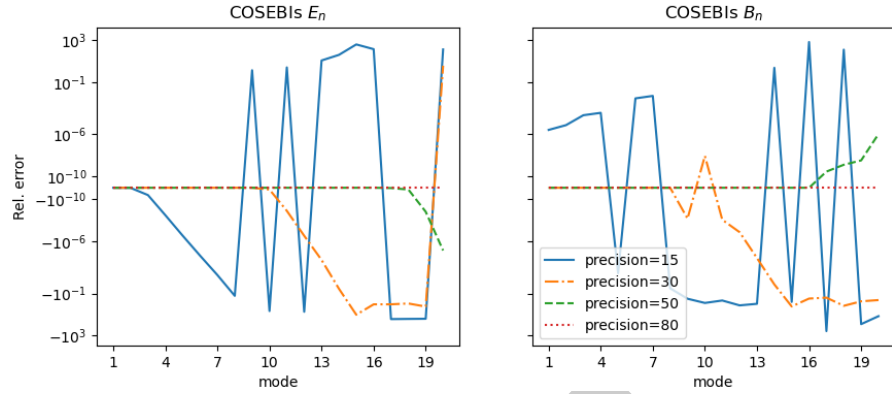


Figure 2: Same as figure Figure 1 for the COSEBs E- and B-mode.

COSEBs can also be defined from the power spectrum as:

$$E_n = \int_0^\infty \frac{d\ell}{Z\pi} P_E(\ell) W_\ell; \quad (4)$$

$$B_n = \int_0^\infty \frac{d\ell}{Z\pi} P_B(\ell) W_\ell; \quad (5)$$

where $P_{E/B}(\ell)$ is the power spectrum of E- and B-modes and $W_n(\ell)$ are the filter functions which can be computed from $T_{n,+}$ as:

$$W_n(\ell) = \int_{\theta_{min}}^{\theta_{max}} d\theta T_{n,+}(\theta) J_0(\ell\ell); \quad (6)$$

with $J_0(\ell\ell)$ the 0-th order Bessel function. The Equation 6 is an Hankel transform of order 0. It can be computed using the FFTLog algorithm presented in Hamilton (2000) implemented here in Numba. The Figure 3 shows the comparison between the COSEBs computed from $\xi_\pm(\theta)$ and from $C_{E/B}(\ell)$. We can see that the COSEBs E-modes agrees very well but the B-modes are more stable when computed from the $C(\ell)$ space.

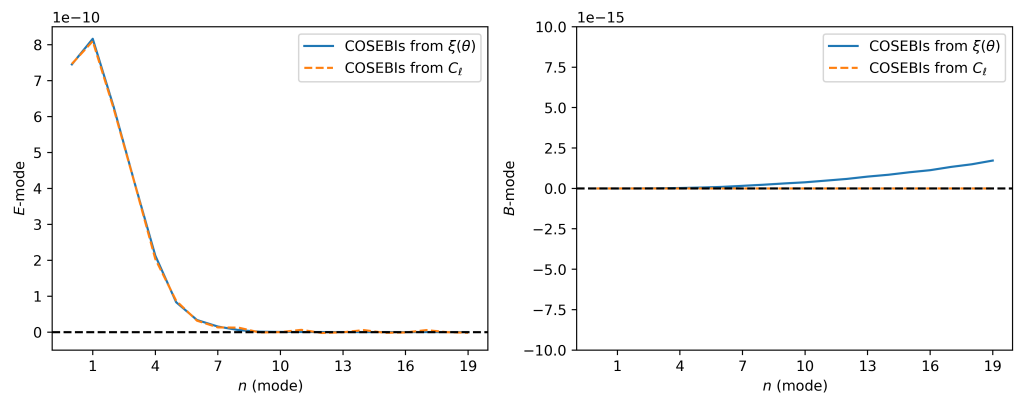


Figure 3: Comparison of the COSEBs E- and B-mode computed from $\xi_\pm(\theta)$ and $C_{E/B}(\ell)$.

Pure-Mode Correlation Functions

In this section we look into the computation of the pure-mode correlation functions as defined in Peter Schneider et al. (2022). There are defined as follow:

$$\xi_+^E(\vartheta) = \frac{1}{2} \left[\xi_+(\vartheta) + \xi_-(\vartheta) + \int_{\vartheta}^{\vartheta_{\max}} \frac{d\theta}{\theta} \xi_-(\theta) \left(4 - \frac{12\vartheta^2}{\theta^2} \right) \right] - \frac{1}{2} [S_+(\vartheta) + S_-(\vartheta)] \quad (7)$$

$$\xi_+^B(\vartheta) = \frac{1}{2} \left[\xi_+(\vartheta) - \xi_-(\vartheta) - \int_{\vartheta}^{\vartheta_{\max}} \frac{d\theta}{\theta} \xi_-(\theta) \left(4 - \frac{12\vartheta^2}{\theta^2} \right) \right] - \frac{1}{2} [S_+(\vartheta) - S_-(\vartheta)] \quad (8)$$

Acknowledgements

We acknowledge contributions from Brigitta Sipocz, Syrtis Major, and Semyeong Oh, and support from Kathryn Johnston during the genesis of this project.

References

- Hamilton, A. J. S. (2000). Uncorrelated modes of the non-linear power spectrum. *Monthly Notices of the Royal Astronomical Society*, 312(2), 257–284. <https://doi.org/10.1046/j.1365-8711.2000.03071.x>
- Schneider, Peter, Asgari, M., Jozani, Y. N., Dvornik, A., Giblin, B., Harnois-Déraps, J., Heymans, C., Hildebrandt, H., Hoekstra, H., Kuijken, K., Shan, H., Tröster, T., & Wright, A. H. (2022). Pure-mode correlation functions for cosmic shear and application to KiDS-1000. *Astronomy & Astrophysics*, 664, A77. <https://doi.org/10.1051/0004-6361/202142479>
- Schneider, P., Eifler, T., & Krause, E. (2010). COSEBIs: Extracting the full e-/b-mode information from cosmic shear correlation functions. *Astronomy and Astrophysics*, 520, A116. <https://doi.org/10.1051/0004-6361/201014235>