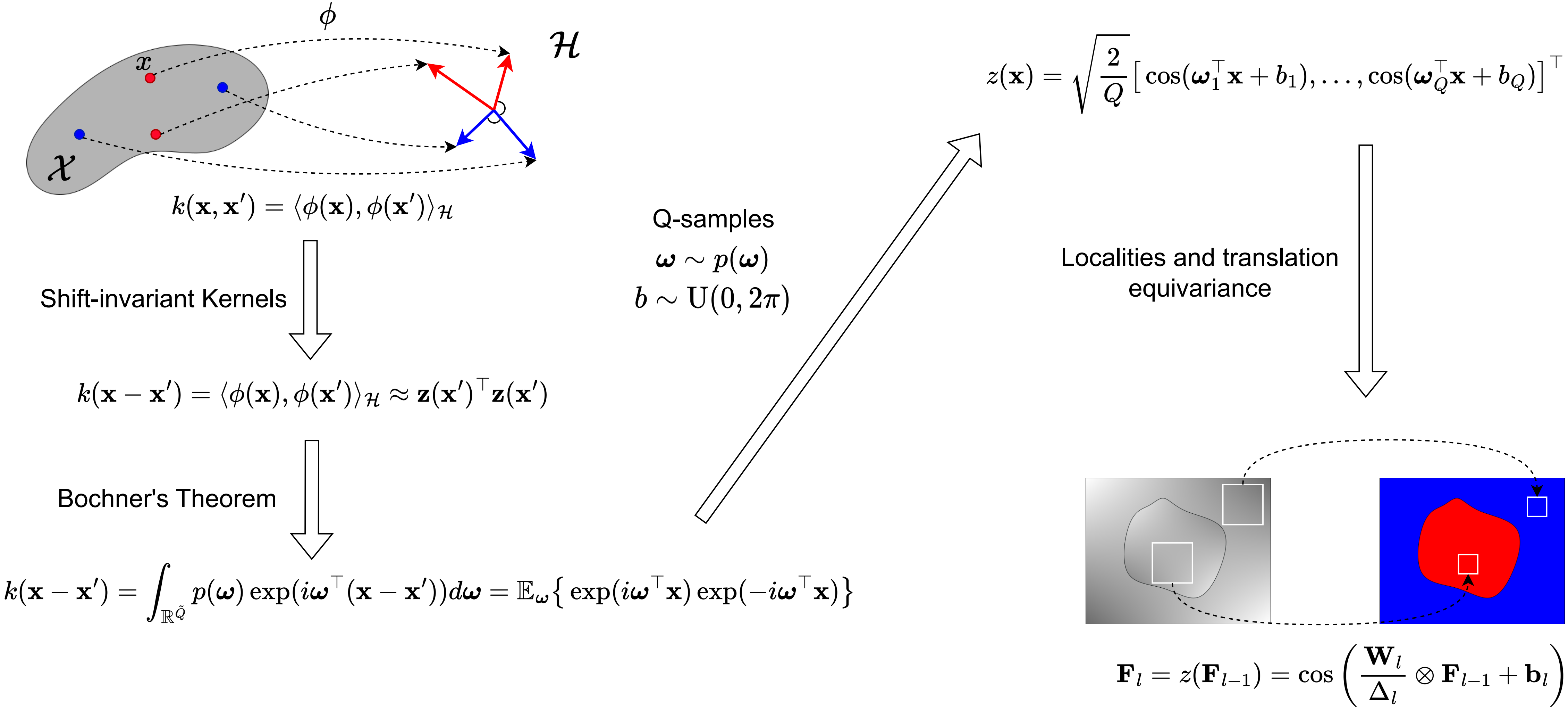
ϕ
$\mathcal{H}$
x
$\mathcal{X}$
$k(\mathbf{x}, \mathbf{x}') = \langle \phi(\mathbf{x}), \phi(\mathbf{x}') \rangle_{\mathcal{H}}$
Shift-invariant Kernels
$k(\mathbf{x} - \mathbf{x}') = \langle \phi(\mathbf{x}), \phi(\mathbf{x}') \rangle_{\mathcal{H}} \approx \mathbf{z}(\mathbf{x}')^\top \mathbf{z}(\mathbf{x}')$
Bochner's Theorem
$k(\mathbf{x} - \mathbf{x}') = \int_{\mathbb{R}^{\tilde{Q}}} p(\boldsymbol{\omega}) \exp(i\boldsymbol{\omega}^\top (\mathbf{x} - \mathbf{x}')) d\boldsymbol{\omega} = \mathbb{E}_{\omega}\{ \exp(i\boldsymbol{\omega}^\top \mathbf{x}) \exp(-i\boldsymbol{\omega}^\top \mathbf{x})\}$
Q-samples
$\boldsymbol{\omega} \sim p(\boldsymbol{\omega})$
$b \sim \mathrm{U}(0, 2\pi)$
$z(\mathbf{x}) = \sqrt{\frac{2}{Q}} [\cos(\boldsymbol{\omega}_1^\top \mathbf{x} + b_1), \ldots, \cos(\boldsymbol{\omega}_Q^\top \mathbf{x} + b_Q)]^\top$
Localities and translation equivariance
$\mathbf{F}_l = z(\mathbf{F}_{l-1}) = \cos\left(\frac{\mathbf{W}_l}{\Delta_l} \otimes \mathbf{F}_{l-1} + \mathbf{b}_l\right)$