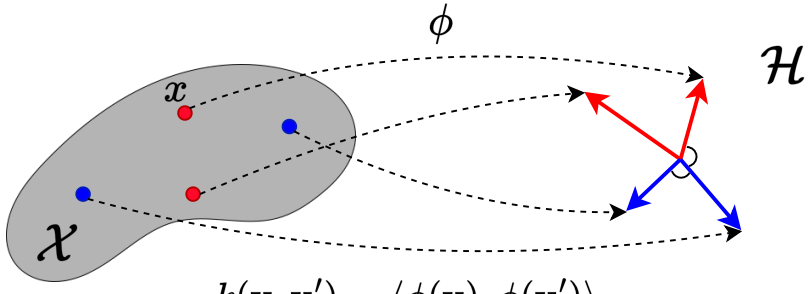


$$k(\mathbf{x}, \mathbf{x}') = \langle \phi(\mathbf{x}), \phi(\mathbf{x}') \rangle_{\mathcal{H}}$$

Shift-invariant Kernels

$$k(\mathbf{x} - \mathbf{x}') = \langle \phi(\mathbf{x}), \phi(\mathbf{x}') \rangle_{\mathcal{H}} \approx \mathbf{z}(\mathbf{x}')^\top \mathbf{z}(\mathbf{x}')$$

Bochner's Theorem

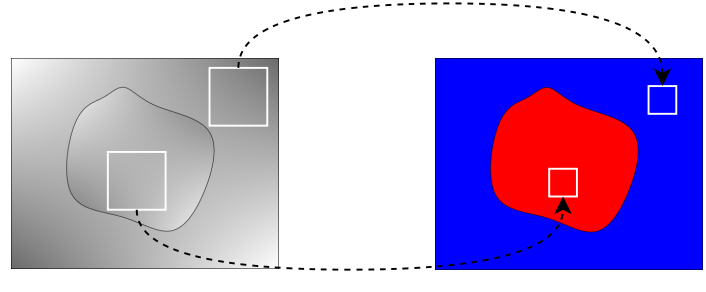
$$k(\mathbf{x} - \mathbf{x}') = \int_{\mathbb{R}^{\tilde{Q}}} p(\boldsymbol{\omega}) \exp(i\boldsymbol{\omega}^\top (\mathbf{x} - \mathbf{x}')) d\boldsymbol{\omega} = \mathbb{E}_{\boldsymbol{\omega}} \{ \exp(i\boldsymbol{\omega}^\top \mathbf{x}) \exp(-i\boldsymbol{\omega}^\top \mathbf{x}) \}$$

Q-samples

$$\boldsymbol{\omega} \sim p(\boldsymbol{\omega})$$
$$b \sim \mathrm{U}(0, 2\pi)$$

$$z(\mathbf{x}) = \sqrt{\frac{2}{Q}} \left[\cos(\boldsymbol{\omega}_1^\top \mathbf{x} + b_1), \ldots, \cos(\boldsymbol{\omega}_Q^\top \mathbf{x} + b_Q) \right]^\top$$

Localities and translation equivariance



$$\mathbf{F}_l = z(\mathbf{F}_{l-1}) = \cos\left(\frac{\mathbf{W}_l}{\Delta_l} \otimes \mathbf{F}_{l-1} + \mathbf{b}_l \right)$$