Quiz1_Solutions

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Matrix Exercises

M1. (2 points) Evaluate $\mathbb{AB}$ when

$$\mathbb{A} = \begin{bmatrix} -2 & 1 \\ 1 & 1 \\ -2 & 1 \end{bmatrix} \mathbb{B} = \begin{bmatrix} -2 & -2 \\ 0 & 0 \end{bmatrix}$$

$$\mathbb{AB} = \begin{bmatrix} 4 & 4 \\ -2 & -2 \\ 4 & 4 \end{bmatrix}$$

M2. (2 points) For $\mathbb{A}$ as above, write down $\mathbb{A}^T$

$$\mathbb{A} = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

M3. (2 points) For $\mathbb{B}$ as above, find $\mathbb{B}^{-1}$ if it exists. If $\mathbb{B}^{-1}$ doesn't exist, explain how you know this.

(1 point) $\mathbb{B}^{-1}$ does not exist (1 point) $\mathbb{B}^{-1}$ does not exist because $\frac{1}{0-0}$ is undefined.

Sumation exercises

S1. (3 points) A basic exercise.

Calculate $\sum_{i=k}^{k+4}(i+3)$, where k is a whole number. Your answer should depend on k.

(2 points) $(k+3)+(k+4)+(k+5)+(k+6)+(k+7)$ (1 point) $5k+25$

S2. (3 points) An example involving sums of squares and products.

Let $\mathbf{1} = (1, 1, \ldots, 1)$ and $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be two vectors treated as $\mathbf{n} \times \mathbf{1}$ matrices. Use $\sum$ notation to evaluate teh matrix prodcut $\mathbf{1}^T\mathbf{x}$

$$\sum_{i=1}^{n} x_i$$

R exercises

R1. Using rep() and matrix()

(3 points) Which of the following is the output of matrix(c(rep(0, times = 4), rep(1, times = 4)), ncol = 2)?

$$\text{(a)} \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}; \text{(b)} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}; \text{(c)} \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}; \text{(d)} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

A

R2. Manipulating vectors and matrices in R.

(3 points) Suppose x is a matrix in R. Which of the following is NOT equivalent to x?

(a). t(t(x)) (b). X %% *matrix(1,ncol(X)) (c).* $X1$ (d). X %*% diag(ncol(X))

B

Fitting a linear model by least squares

F1. Fitting a linear model by least squares.

```r
library(faraway)
data("sat")
head(sat)
```

```
##            expend ratio salary takers verbal math total
## Alabama     4.405  17.2 31.144      8    491  538  1029
## Alaska      8.963  17.6 47.951     47    445  489   934
## Arizona     4.778  19.3 32.175     27    448  496   944
## Arkansas    4.459  17.1 28.934      6    482  523  1005
## California  4.992  24.0 41.078     45    417  485   902
## Colorado    5.443  18.4 34.571     29    462  518   980
```

(2 points) Which of the following would produce the design matrix $\mathbb{X}$ for the model lm(sat ~ ratio + expend, data = sat).

(a) A <- matrix(rep(1, length(ratio)), ratio, expend)
(b) A <- matrix(1, ratio, expend)
(c) A <- cbind(rep(1, length(ratio)), ratio, expend)
(d) A <- cbind(1, ratio, expend)
(e) A <- cbind(ratio, expend)

C.

F2.(4 points) Consider our kicker data from homework 3.

```r
data_nfl <- read.csv("https://ionides.github.io/401w18/hw/hw03/FieldGoals2003to2006.csv",header = TRUE,
head(data_nfl)
```

```
##             Name Yeart Teamt FGAt  FGt Team.t.1. FGAtM1 FGtM1 FGAtM2 FGtM2
## 1 Adam Vinatieri  2003    NE   34 73.5        NE     30  90.0     NA    NA
## 2 Adam Vinatieri  2004    NE   33 93.9        NE     34  73.5     30  90.0
## 3 Adam Vinatieri  2005    NE   25 80.0        NE     33  93.9     34  73.5
## 4 Adam Vinatieri  2006   IND   19 89.4        NE     25  80.0     33  93.9
## 5    David Akers  2003   PHI   29 82.7       PHI     34  88.2     NA    NA
## 6    David Akers  2004   PHI   32 84.3       PHI     29  82.7     34  88.2
```

Recall that we built the model $y_i = mx_i + c_1 z_{i,1} + c_2 z_{i,2} + \cdots + c_{19} z_{i,19} + e_i$ where where x_i is FGtM1 and $z_{i,1}$ takes the value 1 when row i of the data corresponds to kicker 1 (i.e., for i=1,2,3,4) and 0 otherwise. Write the design matrix of the model. (You do not need to include specific values for x_i.)

$$\begin{bmatrix} x_1 & 1 & 0 & \dots & 0 \\ x_2 & 1 & 0 & \dots & 0 \\ x_3 & 1 & 0 & \dots & 0 \\ x_4 & 1 & 0 & \dots & 0 \\ x_5 & 0 & 1 & \dots & 0 \\ x_6 & 0 & 1 & \dots & 0 \\ x_7 & 0 & 1 & \dots & 0 \\ x_8 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ x_n & 0 & 0 & \dots & 1 \end{bmatrix}$$

Points were allocated as follows:

(2 points) x_i column

(1 point) for $z_{i,j}$'s correctly writen

(1 point) for x_i's and $z_{i,j}$'s correctly labeled/described somewhere

(-1 point) for including 1 column for the intercept.