

# Performance Comparison of Pulse Pair and 2-Step Prediction Approach to the Doppler Estimation.

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**Abstract** - This work concerns the use of terrestrial pulsed Doppler weather radar echo to estimate windshear hazards disastrous for airplane approach and landing. Two different methods are used to estimate the received Doppler spectrum moments. The second central moment which is the variance of the wind velocity is called width or windshear. One method is in the time domain, while the other is in the frequency domain. These simulations which lead to comparison of the results assumes that the signal portions of the spectrum are approached with Gaussian shape.

**Keywords:** weather radar signal processing, Doppler spectrum estimates, spectral moment estimation.

## I. INTRODUCTION

Despite of the enormous advances in airport control infrastructures, the windshear remains a real problem for safe takeoff and landing and air traffic as a whole. Windshear may be defined as a difference in wind speed and/or direction between two points in the atmosphere. It can affect the aircraft air speed during takeoff and landing in disastrous ways. It is often associated with an additional hazard known as turbulence.

The windshear phenomenon especially present at low altitude (**Low-Altitude Wind Shear**) is even a severe problem to consider since micro bursts remain one of the most undesirable and dangerous weather factors for flight control safety.

The last two decades have seen the emergence of weather radars with terrestrial Doppler Effect (**TDWR**). These types of radar are used for the detection and estimation of various atmospheric base parameters including this phenomenon (wind velocity, width, direction, etc...) . [1]-[2].

This study based on signal processing techniques considers the spectral characteristics of atmospheric radar echoes estimation using fast algorithms in order to minimise processing time and data storage capacity.

Thus, the main aim is to compare the estimate of the spectral moments and the analysis of the spectrum of received Doppler radar representing a wind microburst. The estimation will be done by using two methods: one in the time domain (indirect approach) using the complex autocorrelation function and the other is in the frequency domain (direct approach) using the power spectral density PSD modelling.

The computer simulation of the turbulence echo of the wind is based on the assumption of a random process with Gaussian distribution of the power spectrum [3], [4], and [5].

## II. THEORETICAL BACKGROUND

The radar data, repeated at regular time intervals is referred to as a volume scan. The reflectivity measured in dBZ often contained in weather Doppler radar signal may include precipitation and wind information. The received Doppler spectrum may be shape depicted in figure 1.

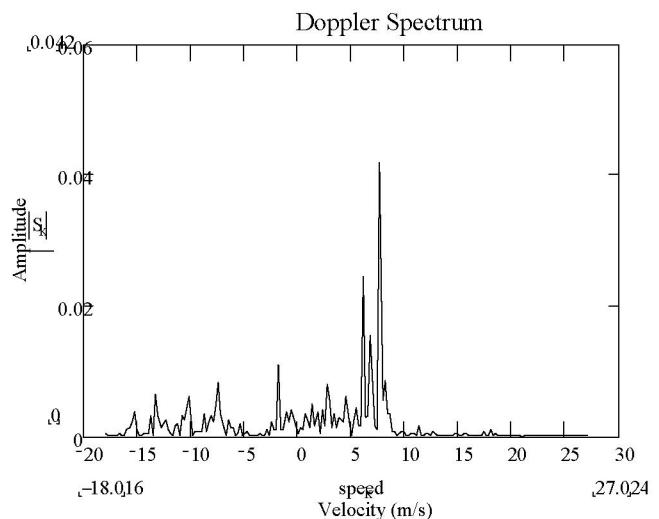


Fig1. The received Doppler spectrum for one Range cell

The Doppler radar output gives in phase and quadrature components forming the complex echo signal  $Z(I, Q)$ . The power spectral density is the Fourier transform of the autocorrelation function  $R_{ZZ}(\tau)$  [7], [8]

$$S_Z(f) = \Im\{R_{ZZ}(\tau)\} \quad (2.1)$$

The Doppler spectrum represents the power spectral density of the received signal. Therefore the total echo power regardless of noise or the density function moment of order zero is [6], [7]:

$$P = \int S(v) dv \quad (2.2)$$

The mean velocity or the first normalised moment [7]:

$$\bar{v} = \frac{1}{P} \int v S(v) dv \quad (2.3)$$

The velocity width or windshear is the square root of the second normalised central moment [7]:

$$\sigma_v^2 = \frac{1}{P} \int (v - \bar{v})^2 S(v) dv \quad (2.4)$$

The Doppler spectrum  $S(v)$  may be scaled in velocity as  $S(v)$  since the relationship between velocity and the Doppler frequency shift with  $\lambda$  being the wavelength is: [6]

$$v = \left( \frac{\lambda}{2} \right) f \quad (2.5)$$

In the same manner, the relationship between the velocity width also called the windshear and the standard deviation of the Doppler spectrum is [6], [7] :

$$w = \left( \frac{\lambda}{2} \right) \sigma_f \quad (2.6)$$

### III. ESTIMATION ALGORITHMS

#### A. Time Domain

This approach is based on the estimate of the complex autocorrelation function of the radar signal [8]. A complex and stationary random process representing the time series of the radar signal (I: in phase and Q: quadratic components) sampled at the pulse repetition time  $T_s$  can be written as:

$$Z(kT_s) = I(kT_s) + jQ(kT_s) \quad (3.1)$$

If the signals considered are statistically independent, then the autocorrelation function can be written as:

$$R_{ZZ}(T_s) = \frac{1}{m} \sum_{k=0}^{m-1} Z^*(kT_s) Z((k+1)T_s) \quad (3.2)$$

Where  $m$  is the number of pulses considered. Assuming prior estimation of the noise power  $N$  the total power can be determined by [8]:

$$\hat{P} = R_{ZZ}(0) - N \quad (3.3)$$

The total power or order zero moment is also estimated by [8]:

$$\hat{P} = \frac{1}{m} \sum_{k=1}^m |Z(kT_s)|^2 - N \quad (3.4)$$

The estimate of the wind velocity mean and shear are given by [8]-[9]-[10]:

$$\hat{v}_{PP} = \frac{\lambda}{4\pi T_s} \arg[R_{ZZ}(T_s)] \quad (3.5)$$

$$\hat{w}_{PP}^2 = \frac{\lambda^2}{8\pi^2 T_s^2} \left[ 1 - \frac{R_{ZZ}(T_s)}{\hat{P}} \right] \quad (3.6)$$

The subscript PP stands for pulse pair.

#### A.1. Pulse Pair Algorithm

The above estimators can be simplified by considering mathematical approximation methods. Indeed, the autocorrelation function of  $Z(kT_s)$  at lag  $\tau$  expressed in the polar form is [9]-[10]:

$$R_{ZZ}(\tau) = h(\tau) \exp(jg(\tau)) \quad (3.7)$$

These two functions  $h$  and  $g$  can be expanded in McLaurin's series. The coefficients of these series can be expressed in terms of central moment function of the spectral average power [9]-[10]. For the discrete case in presence of an additive white Gaussian noise of total power  $N$  it leads to:

$$h(k) = N \delta(k) + P \left\{ 1 - \frac{M_2 k^2}{2!} + \frac{M_4 k^4}{4!} - \dots \right\} \quad (3.8)$$

And

$$g(k) = k\mu - \frac{M_3 k^3}{3!} + \frac{(M_5 - 10M_2 M_3) k^5}{6!} + \dots \quad (3.9)$$

With  $P$  being the total power contained in the spectrum  $S(\omega)$  over the interval  $[-\pi, \pi]$ . The average wind velocity is given by [10]:

$$\mu = \frac{1}{P} \int_{-\pi}^{\pi} \omega S(\omega) d\omega \quad (3.10)$$

The term  $M_n$  refers to the nth central moment which is expressed by:

$$M_n = \frac{1}{P} \int_{-\pi}^{\pi} (\omega - \mu)^n S(\omega) d\omega \quad (3.11)$$

The moment of order  $n$  if the autocorrelation is known may be also given by:

$$M_n = \frac{R_{ZZ}^{[n]}(0)}{(j2\pi)^n} \quad (3.12)$$

Where the term  $[n]$  refers to the derivative operator.

Solving these equations, by considering the autocorrelation lags zero and one and only two terms in the amplitude

series expansion and only one term in the phase series expansion leads to three equations as follows:

$$\hat{P} = h(0) - N \quad (3.13)$$

$$\hat{v}_{PP} = g(1) \quad (3.14)$$

And the wind velocity variance is:

$$M_2 = 2 \left[ 1 - \frac{h(1)}{P_0} \right] \quad (3.15)$$

The set of expressions (6), (7) and (8) is known as the pulse pair (PP) algorithm developed by Rummller in 1968, [11]-[12] which assumes prior estimation of noise power  $N$ . Thus the PP algorithm seems to be a convenient estimator of the wind velocity mean and variance.

### B. Frequency Domain

#### B.1. Two step prediction ARMA ( $n, 1$ ) Algorithm

In order to improve the results of the estimates of the wind velocity mean and variance, another spectral method is proposed. It consists of the evaluation of the power spectral density (PSD) of the Doppler spectrum by an autoregressive model with variable average ARMA ( $n, 1$ ) [13].

This method is based on finding a particular solution which maximizes the entropy function in a stable all-poles model of order  $n$ . The maximization of the entropy function is equivalent to the maximization of the minimum mean square error (MMSE) related to the predictors with one step [13]

If  $X(nT)$  is a stochastic process of discrete time, null, and stationary average in the broad sense, its autocorrelation function is given by:

$$R_{XX}(k) = E[X(nT); X^*((n+k)T)] \quad (3.16)$$

$$k = 0, 1, 2, \dots, \infty$$

It can be shown according to [13] that the existence of two factors of Wiener which maximize the minimum mean square error (MMSE) is associated to a predictor with two steps. This predictor is compatible with the autocorrelations given by  $R_0, R_1, R_2, \dots, R_n$  and having a positive Toeplitz matrix. For convenience  $R_{XX}(k)$  is written  $R_k$ .

From [13] these Wiener's factors turn out to be stable filters ARMA ( $n, 1$ ), given by:

$$B_2(z) = \frac{\alpha + \beta z}{\sum_{k=0}^n g_k z^k} \quad (3.17)$$

Where

$$\alpha = \pm \frac{1}{2} \left[ \sqrt{(a+b)^2 - 1} + \sqrt{(a-b)^2 - 1} \right] \quad (3.18)$$

and

$$\beta = \pm \frac{1}{2} \left[ \sqrt{(a+b)^2 - 1} - \sqrt{(a-b)^2 - 1} \right] \quad (3.19)$$

With

$$a = -2bR \cos\left(\frac{\varphi}{3}\right) \quad (3.20)$$

And

$$b = \frac{a_0}{a_n} \quad (3.21)$$

So the Doppler spectrum is predicted close with the formula:

$$g_k = a_{k-1}b + a_k a - a_{n-k+1}, \quad k = 0, 1, 2, \dots, n. \quad (3.22)$$

Coefficients  $a_0, a_1, \dots, a_n$  are given by the following equation:

$$P_n(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n \quad (3.23)$$

And the  $P_n(z)$ , satisfies Levinson's recursion.

The signs of  $\alpha$  and  $\beta$  parameters are chosen so as to satisfy equation:

$$\alpha\beta = ab \quad (3.24)$$

Furthermore  $R$  and  $\varphi$  are given in equations, respectively.

$$R = -sng\left(\frac{a_1}{a_0}\right) \cdot \sqrt{\frac{2}{3} \left( 1 + \frac{1}{b^2} + \frac{a_1^2}{2a_0^2} \right)} \quad (3.25)$$

With

$$\cos\varphi = \frac{q}{2R^3} = \frac{\left| \frac{a_1}{a_0} \right| \left( 1 - \frac{I}{b^2} \right)}{\left[ \frac{2}{3} \left( 1 + \frac{1}{b^2} + \frac{a_1^2}{2a_0^2} \right) \right]^{3/2}} \quad (3.26)$$

As noticed rather, two choices of the parameter  $a$ , has in the equations cited induce two real limit functions and two Wiener's admissible factors.

The factors of Wiener prove to be ARMA ( $n, 1$ ) stable filters. The calculation algorithm of the spectrum (i.e. a two predictor steps) is given by the flow chart of Figure 2.

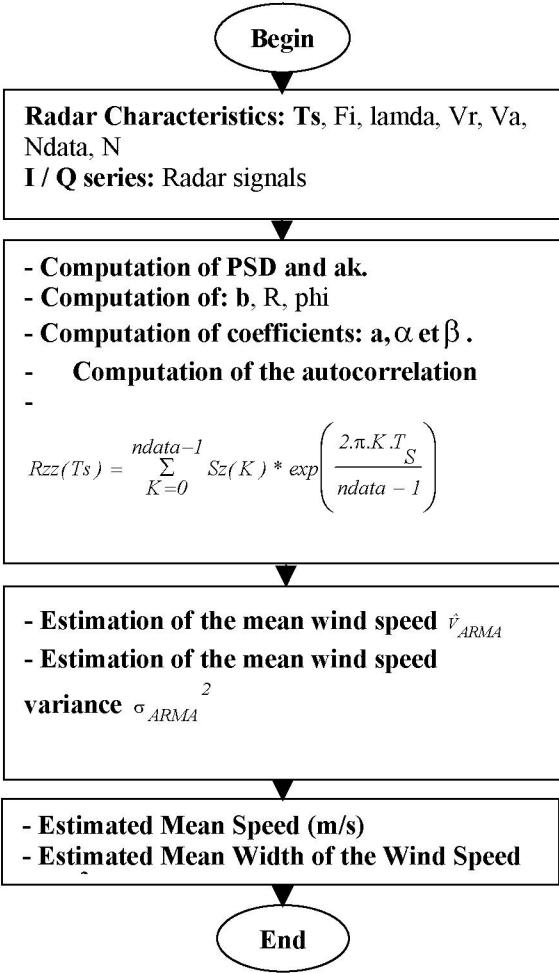


Fig.2 Two steps prediction algorithm

#### IV. RESULTS AND COMMENTS

For simulation purpose, the Radar data entries are:

The actual data used is taken from the Memphis Tennessee WSR-88D (Weather Surveillance-1988 Doppler) operational weather radar in July of 1997. It includes I, Q, Azimuth, Elevation, Prt, Time (UNIX time),

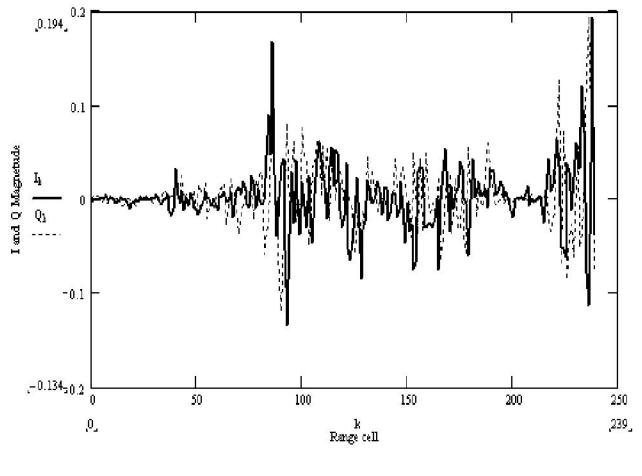
Carrier frequency  $f_c = 5.6\text{GHz}$ , Signal wavelength  $\lambda = 5.35\text{cm}$

Pulse repetition period  $T_s = 0.5\text{ms}$ ,  
Receiver SNR = 10dB,

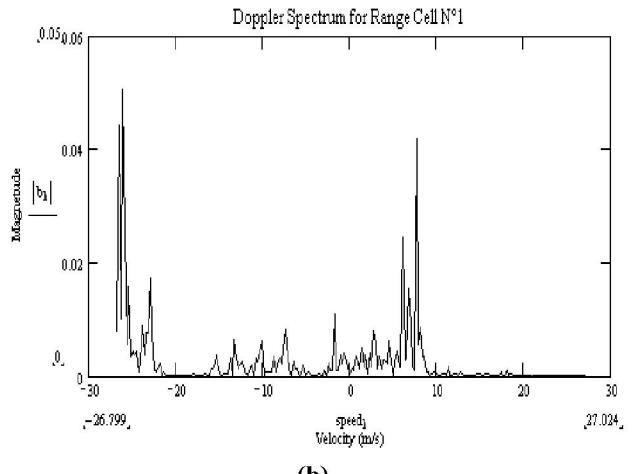
Ambiguous velocity  $v_a = 26.78\text{ m/s}$ ,  
Width of the spectrum  $\Delta f = 1\text{m/s}$ ,

The spectrum of Doppler power as well as the complex signals Z (I, Q), are shown in figure 3.

Data used Z (I, Q) (complex series) are shown in the figure 3.(a) as in phase I and quadrature Q signals and as well as their Doppler spectrum on the figure 3 (b) for the range cell N°1



(a)



(b)

Fig.3 (a) I&Q signals; (b) Spectrum Doppler Weather

The already mentioned methods for the estimate of the spectral characteristics of the weather echo of the windshear led to the results represented in Figures 3 and 4.

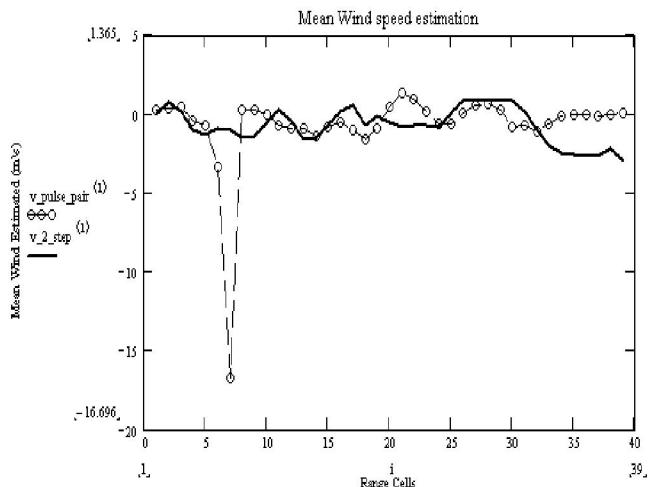


Fig.4 Wind Velocity Mean Estimation

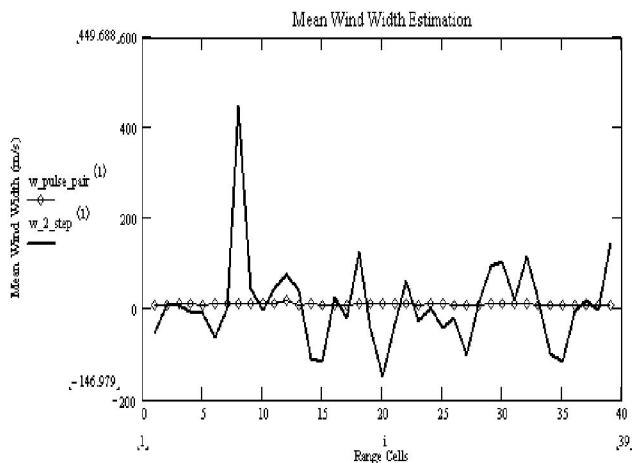


Fig.5 Wind Width Mean Velocity estimation

The time domain estimate PP is a method simple to program considering the autocorrelation function of the complex signals  $Z(iT_s)$  of the echo Radar Doppler received. It is a very fast method, because its execution time is low to the ARMA ( $n, 1$ ) seconds only and it converges at the first iteration (as shown in figure 4).

The estimate made by this method for the mean radial velocity of the Doppler spectrum of the windshear, is very close to the real velocity.

The estimate of the variance  $\sigma_v^2$  and the spectral width  $w$  are definitely weaker compared to those of the ARMA ( $n, 1$ ) method. Its only disadvantage is that the results related to the autocorrelation function are difficult to interpret, contrary to those of the spectral methods which use periodograms, easy to read and interpret [13], [14].

In addition it is seen well that the estimates provided by the algorithm ARMA ( $n, 1$ ) are however similar to those estimated by method PP and non divergent since the values estimated mean velocities oscillate around the close real radial velocity. The only disadvantage of this method lies in the execution time which is long, that can be justified by the number of parameters used for the calculation of the PSD, (see Figure 2).

## V. CONCLUSION

The algorithms used for the estimation of the moments of order zero (power), one (mean velocity), and two (the spectral width) of the Doppler spectrum of the weather disturbance (wind shear) provided results in the expected range.

The pulse-pair method is a popular method used in the Doppler frequency estimation. In the literature it has been shown that, is attractive because it is less computationally intensive [15], [16]. The number of operations required for evaluation of the estimation in (20) is  $O[N]$  while a technique such as using the discrete Fourier transform is  $O[N \log[N]]$ .

The application of the two step prediction algorithm ARMA ( $n, 1$ ) for the calculation of the power spectral density is a considerable contribution, because these results are close to those of the basic algorithm PP. This is made

possible by using the maximization of minimum mean square error MMSE criterion during the estimation of the PSD of the received signal.

Further the results obtained may be improved by using recent techniques of radar signals, such as dual polarization, and Multi-Radar techniques i.e. the use of several sources of information [16].

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