Nonlinear and Noisy State Estimation with Extended Kalman Physics-Informed Bayesian Neural Network Filter

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Abstract

Vehicles of interest, including satellites, automobiles, and Unmanned Aerial Vehicles (UAVs), typically display nonlinear dynamics in real-world applications. Due to inherent nonlinearity of these objects, traditional physics based methods struggle to accurately predict states along their trajectories, particularly when only noisy and sparse observations are available to a model (Zhan and Wan, 2006). With incorporation of linearization approximation techniques, traditional methods can achieve better results but still may contain significant bias (Ludvigson and Paxson, 2001) and while plain neural networks can approximate complex, nonlinear relationships, they often lack capability to generalize robustly (Ramadan, Toler, and Anitescu, 2024). Physics-Informed Neural Networks (PINNs), a class of neural networks that embed physical laws into the architecture, offer a promising hybrid alternative by enhancing model robustness with true knowledge of the relevant physics (Brevi, Mandarino, and Prati, 2024). This paper proposes a novel approach for state estimation along nonlinear UAV trajectories, leveraging noisy sensor measurement data. We augment training of a standard Bayesian Neural Network (BNN) with physics informed components and integrate model prediction with an Extended Kalman Filter (EKF) update model, resulting in a hybrid framework referred to as *EKPIBNNF*. We demonstrate that *EKPIBNNF* achieves superior state estimation accuracy on unseen, high-noise sensor data relative to a basic EKF and BNN.

1 Introduction

Accurate state estimation of nonlinear dynamical systems is vital in fields ranging from aerospace engineering to financial modeling (de Curtò and de Zarzà, 2024). In order to best design tracking and control systems to deal with the natural dynamics of an object of interest, it is vital have an accurate estimate of where the object currently is and where it is heading (Avzayesh, Abdel-Hafez, AlShabi, and Gadsden, 2021). In real-life scenarios we face additional dimensions to the problem including noise uncertainty on the sensor being used to make observations, lack of available observations, and sparse information within the observation itself. For this paper, we define state estimation as the problem of estimating an object's state given previous and live sensor measurements of that state, an understanding of the general uncertainty of the sensor being used to make the measurements, and general knowledge of the underlying physical laws that dictate the object's motion. Conventional techniques, such as the Kalman filter, have achieved notable success but often depend on strict model assumptions and can be sensitive to higher sensor noise (de Curtò and de Zarzà, 2024). Artificial Intelligence (AI) as a discipline has rapidly progressed in the past decade, showcasing particular success of data-driven methods which now serve as a solution suite within various domains (de Curtò and de Zarzà, 2024). However, pure data driven models can fall short in applications where training data is limited or not indicative of the real life scenario. These models are often criticized for fitting too closely to the training data and consequently performing poorly on unseen data - akin to merely just "memorizing" what questions will be tested (Ramadan et al., 2024).

PINNs, a hybrid class of data-driven neural networks, involve embedding physical laws into the neural network framework - typically through customization of a loss function (Wang, Li, He, and Wang, 2022) . The idea is to strengthen the relationship between the observed data and the surrounding physics to help the model grasp essential physical principles, enabling more precise and broadly applicable predictions. Since the introduction of their framework in 2017, PINNs have recently gained traction showing promising application in computation fluid dynamics (Shengze Cai, 2023), controller design (Chen, Yang, Chen, Yan, Zeng, and Dai, 2023), and vibration analysis (Carrasco Ramírez, 2021). I additionally want to make a note that PINNs are a step forward in the direction of achieving "hybrid AI" - AI that blends data driven and true computational logic intelligence (Urankar, 2023). In other words, by incorporating a PINN-like architecture into our neural network, we can confidently state that our model has learned some true understanding of the physical world, as we humans know it, progressively through experience. In this paper, we extend a standard PINN approach to include Bayesian Linear Layers and a conclusive EKF update step on NN prediction to reach a final estimate and uncertainty (*EKPIBNNF*).

UAVs tend to exhibit nonlinear motion by nature and are expected to grow significantly in investment within the aerospace industry due to their relative inexpensiveness and efficiency in maneuverability (Research, 2024). Like any other aerospace vehicle, UAV motion is also constrained by different physical laws. In this paper we describe the *EKPIBNNF* approach and examine how performative it is to the state estimation problem for a given UAV dataset. Section 2 provides a brief overview of related work in this field. Section 3 outlines our methodology for abstracting the state estimation problem, along with the methods and metrics used in the comparison study. Section 4 presents the results of the performance evaluation, followed by a discussion of these results and key takeaways in Sections 5 and 6, respectively.

2 Related Work

A lot of related research has been conducted tangent to this topic. For example, there has been work done on the potential for deep learning, physics-based methods in replacing traditional linear solvers - particular in the use case of the Poisson equation (Markidis, 2021). PINNs has also been extended to the nonlinear characterization problem, where its utility in tracking newly formulated space debris after in elastic collision in Low-Earth Orbit has been investigated (Harsha M, 2024). Other researchers reach further out to Geostationary orbit and utilize PINNs for orbit determination which is a nonlinear trajectory generation problem in itself (Scorsoglio, D'Ambrosio, Campbell, Furfaro, and Reddy, 2024). Perhaps the most related work is a novel integrated PINNs and Unscented Kalman Filter (UKF) approach proposed to provide an alternative solution to the state estimation problem (de Curtò and de Zarzà, 2024).

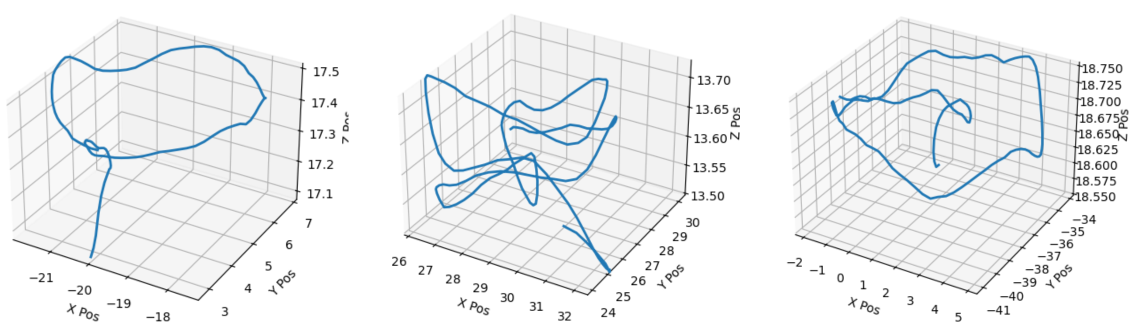
This paper certainly derives its inspiration from a few of the source examples depicted above, however also distinguishes itself in its purpose to provide an inferential insight on the strengths and weaknesses of the novel *EKPIBNNF* approach with respect to traditional approaches across various abstracted modeled sensor error and training data availability scenarios.

3 Methodology

In this section, we describe our approach to modeling the general state estimation problem, provide an overview of the different algorithms used in this comparison study - including *EKPIBNNF* - and outline the key experiments and evaluation metrics of interest.

3.1 Modeling Strategy

We utilize the *Synthetic-UAV-Flight-Trajectories* dataset from Hugging Face, which comprises of over 5000 random UAV trajectories collected over 20 hours of synthetic flight time (riotu lab, 2024). It is intended for experimentation of data-driven AI algorithms for trajectory prediction applications. Sample trajectories are show in *Figure 1* and are observantly nonlinear.



(a) Trajectory 1 (b) Trajectory 2 (c) Trajectory 3

Figure 1: Example UAV Trajectories

Within each trajectory, we are provided with a 3D position point in the XYZ frame with kilometer units (km), and a timestamp that corresponds to each point. For the purposes of modeling our study, we consider these data-points as the **true** position state of the drone at any time. Consequently, we will use $\bar{x_t}$ to refer to the true and full position state of the drone at time t (Equation 1). Since we have position and time along the entire truth trajectory, we differentiated between sequential data points to derive the true instantaneous velocity at each timestamp t, which was added to our row observation information (Equation 2).

$$\text{True Position State at time } t: \quad \bar{x_t} = \begin{bmatrix} x_t & y_t & z_t \end{bmatrix} \tag{1}$$

$$\text{True Velocity State at time } t: \quad \bar{v_t} = (\bar{x_{t+1}} - \bar{x_t})/\triangle t \tag{2}$$

Finally, to simulate measurements we would receive from a real-life sensor, we added Gaussian noise to each of our true states to generate a "measurement" state for both position and velocity (Equation 3, Equation 4). The σ noise factor is also included in our raw observation information for each time step t, assuming we would have prior knowledge of sensor specifications before training the model. Additionally, σ is parameterized in our experiment noise to demonstrate how the *EKPIBNNF* outperforms the EKF in higher noise scenarios.

$$\text{Measured Position State at time } t: \quad \tilde{x_t} = \bar{x_t} + \mathcal{N}(0, \sigma_n) \tag{3}$$

$$\text{Measured Velocity State at time } t: \quad \tilde{v_t} = \bar{v_t} + \mathcal{N}(0, \sigma_n/1000) \tag{4}$$

While each model approach for our experiments are distinct in nature, we ensure to design our experiment such that the input feature vector to each model (Equation 5), and the resulting output at a given time step remain the exact same for each algorithm *f*, (Equation 6). This way, we are able to evaluate the *EKPIBNNF*, with respect to our baseline algorithms, in a consistent, unbiased manner.

$$\text{Feature Vector: } \mathbf{z_t} = \begin{bmatrix} \tilde{x}_t & \tilde{x}_{t-1} & \tilde{v}_{t-1} & \sigma_{x_t} & \sigma_{v_t} & \Delta t \end{bmatrix} \tag{5}$$

$$\text{Estimated Position State at time } t: \quad \hat{x_t} = f(z_t) \tag{6}$$

In essence, the general question of this paper is how well can *EKPIBNNF* perform at reducing the error between our estimated position state, $\hat{x_t}$ (Equation 6), and our truth position state, $\bar{x_t}$ (Equation 1) given the information present in the feature state (Equation 5) across varying magnitudes of Gaussian simulated sensor noise relative to a baseline physics-based approach (EKF) and a baseline data-driven approach (BNN).

3.2 Algorithms

3.2.1 Baseline Extended Kalman Filter Implementation

The EKF is considered a de facto method to facilitate the state estimation of a non-linear dynamical system (Barrau and Bonnabel, 2018). It is an extension of the original Kalman Filter that utilizes discrete linearization about the current mean and covariance estimate of a dynamical system (Wan, 2006). To establish a baseline performance measure for our study, we employ a standard EKF that couples a state space and observation model to conduct a prediction and update step of the state estimate using noisy measurement data. While the EKF tends to produce solutions relatively quick being a purely physical solution, it can often become unstable in highly nonlinear and noisy systems, yielding bias due to the native shortcomings of discrete linearization (Zhan and Wan, 2006). We thus proceeded to set EKF as a baseline method to measure performance across scenarios of varying sensor uncertainties. The hypothesis was that the *EKPIBNNF* will generally outperform EKF in state accuracy more commonly in higher sensor noise data experiments. Below, we describe the general equations used to model our EKF and break the process into predict and update steps (Equation 7),(Equation 8), (Equation 9), (Equation 10) (Addison, 2020).

Predict Step:

$$\text{Estimated State: } \hat{\mathbf{x}}_{t|t-1} = f(\hat{\mathbf{x}}_{t-1|t-1}, \mathbf{u}_{t-1}) \tag{7}$$

$$\text{Uncertainty: } \mathbf{P}_{t|t-1} = \mathbf{F}_{t-1}\mathbf{P}_{t-1|t-1}\mathbf{F}_{t-1}^{\top} + \mathbf{Q}_{t-1} \tag{8}$$

Where:

- $\hat{\mathbf{x}}_{t|t-1}$ is the predicted state estimate.
- $\mathbf{P}_{t|t-1}$ is the predicted error covariance.
- $f(\cdot)$ is the nonlinear state transition function.
- $\mathbf{F}_{t-1} = \frac{\partial f}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{t-1|t-1}, \mathbf{u}_{t-1}}$ is the Jacobian of $f(\cdot)$ with respect to $\mathbf{x}$.
- $\mathbf{Q}_{t-1}$ is the process noise covariance.

Update Step:

$$\text{Updated State: } \hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t \left(\mathbf{z}_t - h(\hat{\mathbf{x}}_{t|t-1})\right) \tag{9}$$

$$\text{Updated Covariance: } \mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_{t|t-1} \tag{10}$$

Where:

- $\mathbf{K}_t$ is the Kalman gain.
- $\mathbf{z}_t$ is the measurement at step t.
- $h(\cdot)$ is the nonlinear measurement function.
- $\mathbf{H}_t = \frac{\partial h}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{t|t-1}}$ is the Jacobian of $h(\cdot)$ with respect to $\mathbf{x}$.

At the convergence of an EKF, we arrive with a final state estimate $\hat{x}_t$, and a corresponding uncertainty for that state estimate P_t.

3.2.2 Baseline Bayesian Neural Network Implementation

Data-driven artificial neural networks have demonstrated superior performance over purely analytical linear solvers in fields such as signal processing (Lapedes and Farber) and biology (Almeida, 2002). However, these models often face challenges with data scarcity and generalization. To show these limitations and highlight the utility of *EKPIBNNF*, we employed a Bayesian Neural Network (BNN) as an additional baseline method. The BNN consisted of torchbnn Bayesian linear layers (Lee, Kim, and Lee, 2022) and used a combined loss function composed of a mean squared error (MSE) and Kullback-Leibler (KL) divergence.

As a stochastic model, the BNN provides uncertainty estimates for its regression outputs through Monte Carlo (MC) sampling during prediction. Since uncertainty quantification is vital for state estimation tasks, the BNN serves as an effective baseline to evaluate the performance of *EKPIBNNF*. Specifically, we compared both methods under experiments with varying amounts of training data and sensor noise. The hypothesis was that the *EKPIBNNF* would generally always outperform the BNN in state accuracy and certainty, due to its added physics informed components.

$$\text{State Estimate: } \hat{x}_t = f(\mathbf{z}_t) + b \tag{11}$$

$$\text{Uncertainty Estimate: } \mathbf{P}_t = f(\mathbf{z}_t) + b \tag{12}$$

3.2.3 Extended Kalman Physics Informed Bayesian Nerual Network Filter

We propose an Extended Kalman Physics-Informed Bayesian Neural Network Filter (*EKPIBNNF*) which incorporates the update observation model of a standard EKF with a physics augmented BNN as a replacement for the EKF state space model (*Figure 2*). The idea is to still train a BNN at the core, but add physics informed analytical methods around the training and prediction process to refine the state estimate.

During training, the model first uses a simple kinematics model to propagate input measurements from time *t-1* to time t (Equation 13). The propagated state, combined with additional features, is trained to predict the true position state using a three-layer BNN (Equation 14). During training, a simple kinematics-based loss term is added to the standard MSE + KL divergence and penalizes the model for incorrect implied displacement predictions based on the true velocity of the UAV (Equation 15). The weighted sum of the losses is then backpropagated and optimized within the BNN (Equation 16).

$$\text{Initial Kinematics State Estimate: } x'_t = \tilde{x}_{t-1} + dt * \tilde{v}_{t-1} \tag{13}$$

$$\text{BNN Estimate: } x''_t = f(x'_t, \mathbf{z}_t) + b \tag{14}$$

$$\text{Kinematics Loss: } L_{\text{Physics}} = dt * (\bar{v}_t - (x''_t - \tilde{x}_{t-1})) \tag{15}$$

$$\text{Total Loss: } L_T = \lambda_1 L_{\text{Physics}} + \lambda_2 (L_{\text{MSE}} + L_{\text{KLD}}) \tag{16}$$

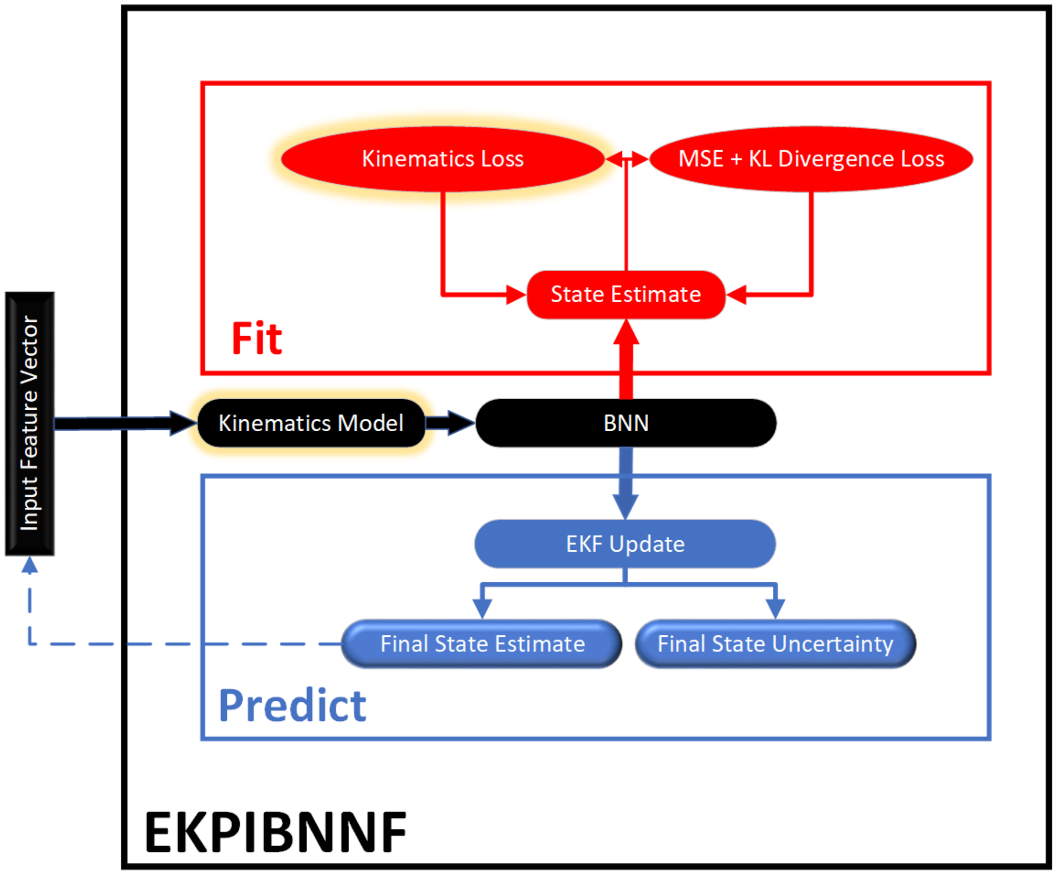


Figure 2: Extended Kalman Physics-Informed Bayesian Neural Network Filter Architecture. Yellow glow shapes represent processes informed by physics.

At prediction, the model again uses the kinematics model to generate an initial state estimate, which is passed through the trained BNN. The output state estimate is further refined using an EKF update with the current measurement at t (Equation 9), (Equation 10). The resulting state replaces the *t-1* position measurements as inputs for the next state prediction.

The physics-based and intermediate BNN aims to provide a more accurate initial state estimate to the final EKF update. The hypothesis is that this model can behave more robustly in state and uncertainty estimation across high-noise and limited training data scenarios compared to a standalone BNN or EKF.

3.3 Testing

For all experiments, we conducted testing and evaluation over a with-held test set. This set consisted of 509 trajectories that were excluded during the training of any neural network. We conducted three separate experiments of various sensor noise levels with two primary evaluation metrics for each. All neural networks were trained on 4584 trajectories for 10000 epochs and included 3 Bayesian Linear layers, 64 neurons per hidden layer, and an Adam optimizer (Kingma and Ba, 2017).

3.4 Metrics

1. State Estimate Error: Euclidean position error between model estimate and true state. Represents model **accuracy**.

$$e_t = \sqrt{(\hat{x}_1 - \bar{x}_1)^2 + (\hat{x}_2 - \bar{x}_2)^2 + (\hat{x}_3 - \bar{x}_3)^2} \tag{17}$$

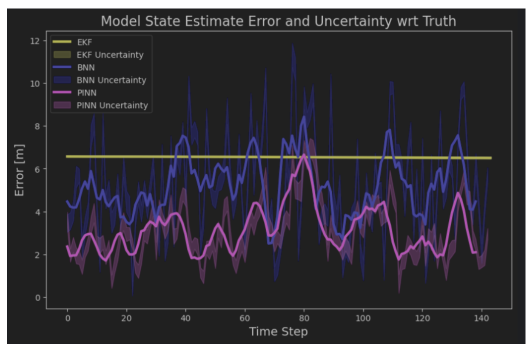
2. State Estimate Uncertainty: Averaged variance across diagonal of output covariance matrix $\mathbf{P}_t$. Represents model **uncertainty**.

$$\sigma_t^2 = \frac{1}{3} \sum \mathrm{diag}(\mathbf{P}_t) \tag{18}$$

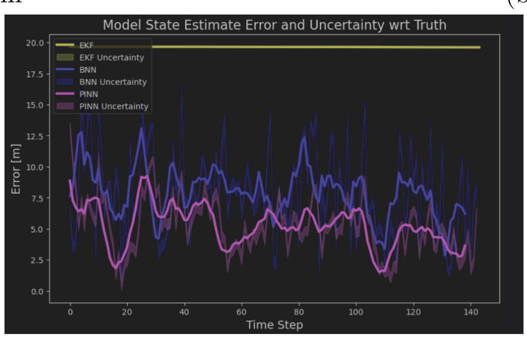
4 Results

As the primary experiment, we vary the Gaussian sensor noise factor, σ_n, to values of 1, 5, and 10 meters. Each NN model is trained and evaluated on the noisy datasets, alongside regular evaluation from the baseline EKF. We illustrate the average model error e_t and uncertainty σ_t across the varying noise levels for sample test trajectories and provide a comparison of overall model accuracy and precision.

(a) $\sigma_t = 1$ m

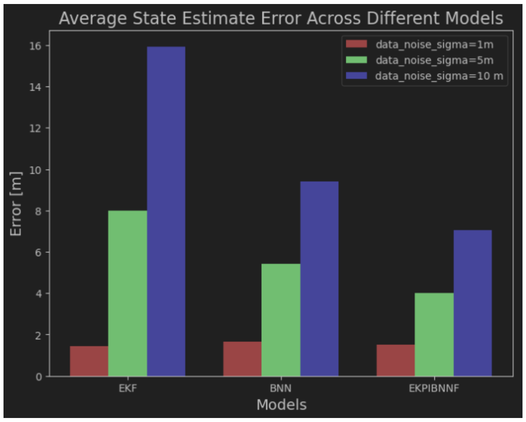


(b) $\sigma_t = 5$ m

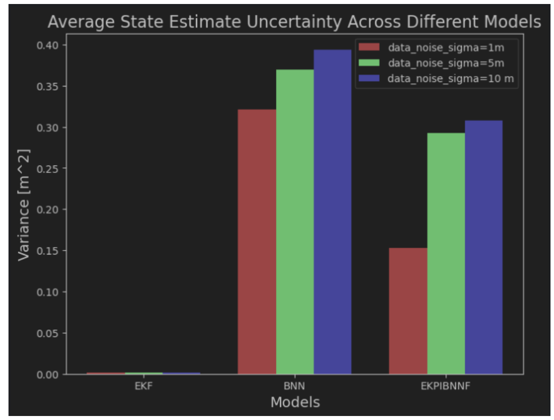


(c) $\sigma_t = 10$ m

Figure 3: Comparison of algorithm state estimate error and uncertainty on a sample truth trajectory across various magnitudes of sensor noise.



(a) State Estimate Accuracy Quantification



(b) State Estimate Uncertainty Quantification

Figure 4: Comparison of average algorithm state estimate error and uncertainty across all 509 test trajectories under various noise conditions.

	$\sigma = 1$		$\sigma = 5$		$\sigma = 10$	
Models	**Error**	**Uncertainty**	**Error**	**Uncertainty**	**Error**	**Uncertainty**
EKF	1.45	0.002	7.97	0.002	15.93	0.001
BNN	1.66	0.32	5.41	0.37	9.39	0.39
EKPIBNNF	1.53	0.15	**4.01**	0.29	**7.047**	0.31

Table 1: Numerical comparison of overall test trajectory error and uncertainty averages.

5 Discussion

Figure 3 illustrates the accuracy of the model state estimates and the associated uncertainty over a single test trajectory under varying levels of simulated sensor noise. Progressing through Figures 3a-c, we observe that the *EKPIBNNF* model begins to produce noticeably lower estimation errors compared to the EKF and BNN models. However, it is important to note the consistency of the EKF estimates relative to the neural network (NN)-based models, which indicates greater precision from the EKF. This precision, while generally advantageous, can be a double-edged sword, when the estimates are significantly biased, as this results in being “confidently wrong”.

Figures 4a and 4b display the state estimate error and uncertainty, respectively, averaged across all test trajectories for each model under different noise conditions. Consistent with the earlier findings, the *EKPIBNNF* demonstrates better relative state estimate accuracy as noise levels increase but exhibits lower precision compared to the baseline EKF.

The numerical results corresponding to Figures 4a-b are presented in Table 1. Here, the red values highlight cases where *EKPIBNNF* outperforms the BNN, while the **bolded** values indicate where *EKPIBNNF* surpasses both the EKF and BNN. Notably, the augmented BNN framework, *EKPIBNNF*, consistently outperforms the baseline BNN across all experiments, demonstrating the benefits of hybridizing a standard NN approach. Furthermore, particularly in high-noise scenarios, the hybrid framework achieves superior estimation accuracy compared to the state-of-the-art EKF, .

6 Conclusion

Conclusively, this paper introduces a novel approach, *EKPIBNNF*, to address the nonlinear state estimation problem in the presence of noisy sensor measurements. We demonstrate that *EKPIBNNF* outperforms both the baseline EKF and BNN models for high-noise scenarios in state estimation accuracy. Such high-noise conditions are more prevalent than commonly assumed and can arise due to unexpected environmental factors, adversarial jamming, or manufacturing defects. Moreover, we demonstrate that *EKPIBNNF* consistently surpasses the baseline BNN in both state estimation accuracy and uncertainty quantification. This highlights the advantages of hybridizing artificial intelligence with principled computational frameworks, resulting in a more robust and reliable state prediction system.

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