Algoritmos y Estructuras de Datos

Practica 3

3.1. Precondicion mas debil en SmallLang

Ejercicio 1

a) $def(a+1) \equiv def(a) + def(1) \equiv True$

b) $def(a/b) \equiv def(a) \land (def(b) \land b \neq 0) \equiv b \neq 0$

c) $def(\sqrt{a/b}) \equiv def(a) \land (def(b) \land b \neq 0) \ \land \dfrac{a}{b} \geq 0 \equiv b \neq 0 \land \dfrac{a}{b} \geq 0$

d) $def(A[i]+1) \equiv def[A] \land def(i) \land 0 \leq i < |A| \equiv 0 \leq i < |A|$

e) $def(A[i+2] \equiv (def(A) \land def(i)) \land -2 \leq i < |A| - 2 \equiv -2 \leq i < |A|$

f) $def(0 \leq i \leq |A| \land_L A[i] \geq 0) \equiv (def(A) \land def(i)) \land i \neq |A| \equiv i \neq |A|$

nota: en f) hay que poner el rango, no el $\neq$

Ejercicio 2

a)

$$\begin{aligned}
wp(\mathbf{a{:=}a{+}1}, \mathbf{b{:=}a/2}, b \geq 0) &\equiv wp(\mathbf{a{:=}a{+}1}, wp(\mathbf{b{:=}a/2}, b \geq 0)) && (1)\\
&\equiv wp(\mathbf{a{:=}a{+}1}, def(a/2) \land Q^{b}_{a/2}) && (2)\\
&\equiv wp(\mathbf{a{:=}a{+}1}, a/2 \geq 0) && (3)\\
&\equiv (def(a) \land (a+1)/2 \geq 0 && (4)\\
&\equiv (a+1)/2 \geq 0 \equiv a+1 \geq 0 \equiv a \geq -1 && (5)
\end{aligned}$$

b)

$$wp(\textbf{a:=A[i]+1}\ , \textbf{b:=a*a}\ , b \neq 2) \equiv wp(\textbf{a:=A[i]+1}\ , wp(\textbf{b:=a*a}\ , b \neq 2)) \quad (1)$$
$$\equiv wp(\textbf{a:=A[i]+1}\ , def(a) \wedge Q^b_{a*a})) \quad (2)$$
$$\equiv wp(\textbf{a:=A[i]+1}\ , def(a) \wedge a * a \neq 2)) \quad (3)$$
$$\equiv (def(a) \wedge 0 \leq i < |A|) \wedge_L Q^a_{A[i]+1} \quad (4)$$
$$\equiv (0 \leq i < |A| \wedge_L (A[i]+1) * (A[i]+1) \neq 2) \quad (5)$$
$$\equiv (0 \leq i < |A| \wedge_L A[i]^2 + 2 * A[i] + 1 \neq 2) \quad (6)$$

c)

$$wp(\textbf{a:=A[i]+1}\ , \textbf{a:=b*b}\ , a \geq 0) \equiv wp(\textbf{a:=A[i]+1}\ , wp(\textbf{a:=b*b}\ , a \geq 0)) \quad (1)$$
$$\equiv wp(\textbf{a:=A[i]+1}\ , def(b) \wedge b * b \geq 0)) \quad (2)$$
$$\equiv wp(\textbf{a:=A[i]+1}\ , b * b \geq 0)) \quad (3)$$
$$\equiv (def(A) \wedge def(i) \wedge 0 \leq i < |A| \wedge b * b \geq 0) \quad (4)$$
$$\equiv (0 \leq i < |A| \wedge b * b \geq 0) \quad (5)$$

d)

$$wp(\textbf{a:=a-b}; \textbf{b:=b+a}, a \geq 0 \wedge b \geq 0) \equiv wp(\textbf{a:=a-b}, wp(\textbf{b:=b+a}, a \geq 0 \wedge b \geq 0)) \quad (1)$$
$$\equiv wp(\textbf{a:=a-b}, def(b) \wedge def(a) \wedge (a \geq 0 \wedge b + a \geq 0))) \quad (2)$$
$$\equiv wp(\textbf{a:=a-b}, (a \geq 0 \wedge (b + a) \geq 0))) \quad (3)$$
$$\equiv (def(a) \wedge def(b) \wedge (a - b) \geq 0 \wedge (b + (a - b)) \geq 0) \quad (4)$$
$$\equiv ((a - b) \geq 0 \wedge a \geq 0) \quad (5)$$
$$\equiv (a \geq b \wedge a \geq 0) \quad (6)$$

Ejercicio 3

$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$

a)

$$
\begin{aligned}
wp(\mathbf{A[i]{:=}0}, Q) &\equiv wp(setAt(A,i,0)[j], Q) &(1)\\
&\equiv (def(A) \wedge def(i) \wedge_L 0 \leq i < |A| \wedge_L Q^A_{setAt(A,i,0)}) &(2)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A,i,0)[j] \geq 0)) &(3)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge A[i] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) &\\
& &(4)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge 0 \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) &(5)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)((0 \leq j < |A| \wedge_L j \neq i) \rightarrow_L A[j] \geq 0))) &(6)\\
& &(7)
\end{aligned}
$$

b)

$$
\begin{aligned}
wp(\mathbf{A[i+2]{:=}0}, Q) &\equiv wp(setAt(A,i+2,0)[j], Q) &(1)\\
&\equiv (def(A) \wedge def(i+2) \wedge_L 0 \leq i+2 < |A| \wedge_L Q^A_{setAt(A,i+2,0)}) &(2)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A,i+2,0)[j] \geq 0)) &(3)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i+2 \wedge 0 \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) &\\
& &(4)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)((0 \leq j < |A| \wedge_L j \neq i+2) \rightarrow_L A[j] \geq 0))) &(5)
\end{aligned}
$$

c)

$$
\begin{aligned}
wp(\mathbf{A[i+2]{:=}\text{-}1}, Q) &\equiv wp(setAt(A,i+2,-1)[j], Q) &(1)\\
&\equiv (def(A) \wedge def(i+2) \wedge_L 0 \leq i+2 < |A| \wedge_L Q^A_{setAt(A,i+2,-1)}) &(2)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A,i+2,-1)[j] \geq 0)) &(3)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i+2 \wedge -1 \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) &\\
& &(4)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L \mathit{False} \wedge (j \neq i \wedge A[j] \geq 0))) &(5)\\
&\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L \mathit{False})) &(6)\\
&\equiv (0 \leq i+2 < |A| \wedge_L \mathit{False}) &(7)\\
&\equiv \mathit{False} &(8)
\end{aligned}
$$

Nota: deberia haber considerado la lista vacia de manera que si se cumpla el segundo disyunto a partir de (5)

d)

$$
\begin{aligned}
wp(\textbf{A[i]:=2*A[i]}, Q) &\equiv wp(setAt(A, i, 2 * A[i])[j], Q) && (1)\\
&\equiv (def(A) \wedge def(i) \wedge_L 0 \leq i < |A| \wedge_L Q^A_{setAt(A,i,2*A[i])}) && (2)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i, 2 * A[i])[j] \geq 0)) && (3)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge 2 * A[i] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) && (4)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge A[i] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) && (5)\\
&\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \geq 0)) && (6)
\end{aligned}
$$

e)

$$
\begin{aligned}
wp(\textbf{A[i]:=A[i-1]}, Q) &\equiv wp(setAt(A, i, A[i-1])[j], Q) && (1)\\
&\equiv (def(A) \wedge def(i) \wedge def(A[i-1] \wedge_L 0 \leq i-1 < |A| \wedge_L Q^A_{setAt(A,i,A[i-1])}) && (2)\\
&\equiv (def(A) \wedge def(i) \wedge 0 \leq i-1 < |A| \wedge_L 0 \leq i-1 < |A| \wedge_L Q^A_{setAt(A,i,A[i-1])}) && (3)\\
&\equiv (1 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i, A[i-1])[j] \geq 0)) && (4)\\
&\equiv (1 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge A[i-1] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) && (5)\\
&\equiv (1 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \wedge j \neq i \rightarrow_L A[j] \geq 0)) && (6)
\end{aligned}
$$

Ejercicio 4

a) $S \equiv \textbf{if}(a < 0)\ b := a\ \textbf{else}\ b := -a\ \textbf{endif}$

$Q \equiv (b = -|a|)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv (def(B) \wedge_L (B \wedge wp(S1, Q)) \vee (\neg B \wedge wp(S2, Q) && (1)\\
&\equiv def(a) \wedge_L (a < 0 \wedge (def(a) \wedge Q^b_a) \vee (a \geq 0 \wedge (def(a) \wedge Q^b_-a) && (2)\\
&\equiv (a < 0 \wedge a = -|a|) \vee (a \geq 0 \wedge -a = -|a|) && (3)\\
&\equiv (a < 0 \wedge \mathit{True}) \vee (a \geq 0 \wedge \mathit{True}) && (4)\\
&\equiv (a < 0) \vee (a \geq 0) && (5)\\
&\equiv \mathit{True} && (6)
\end{aligned}
$$

b) $S \equiv \textbf{if}(a < 0)\ b := a\ \textbf{else}\ b := -a\ \textbf{endif}$

$Q \equiv (b = |a|)$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv (def(B) \wedge_L (B \wedge wp(S1, Q)) \vee (\neg B \wedge wp(S2, Q) && (1)\\
&\equiv def(a) \wedge_L (a < 0 \wedge (def(a) \wedge Q^b_a) \vee (a \geq 0 \wedge (def(a) \wedge Q^b_-a) && (2)\\
&\equiv (a < 0 \wedge a = |a|) \vee (a \geq 0 \wedge -a = |a|) && (3)\\
&\equiv (a < 0 \wedge \mathit{False}) \vee (a \geq 0 \wedge \mathit{False}) && (4)\\
&\equiv \mathit{False} \vee \mathit{False} && (5)\\
&\equiv \mathit{False} && (6)
\end{aligned}
$$

c) $S \equiv \textbf{if}\ (i > 0)\ s[i] := 0\ \textbf{else}\ s[0] := 0\ \textbf{endif}$

$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$wp(\mathbf{S}, Q) \equiv (def(i) \wedge_L (i > 0 \wedge wp(S1, Q)) \vee (i \geq 0 \wedge wp(S2, Q)) \quad (1)$$

$$\equiv (i > 0 \wedge def(S) \wedge def(i) \wedge 0 \leq i < |s| \wedge Q^{s[i]}_{setAt(S,i,0)}) \vee \quad (2)$$

$$(i \leq 0 \wedge def(S) \wedge Q^{s[i]}_{s[0]}) \quad (3)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L setAt(s, i, 0) \geq 0)) \vee \quad (4)$$

$$(i \leq 0 \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[0] \geq 0)) \quad (5)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L (i = j \wedge s[i] \geq 0) \wedge (i \neq j \wedge s[j] \geq 0)) \vee \quad (6)$$

$$(i \leq 0 \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L 0 \geq 0)) \quad (7)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L (i = j \wedge True) \wedge (i \neq j \wedge s[j] \geq 0)) \vee \quad (8)$$

$$(i \leq 0 \wedge True) \quad (9)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \geq 0)) \quad (10)$$

d) $S \equiv \textbf{if } (i > 1)\ s[i] := s[i-1] \textbf{ else } s[i] := 0 \textbf{ endif}$
$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j-1])$

Primero hago la wp entre S1 y Q

$$wp(S1, Q) \equiv ((def(S) \wedge def(i) \wedge def(S[i-1])) \wedge_L 0 \leq i < |s|) \wedge_L Q^{s[i]}_{setAt(s,i,s[i-1])}) \quad (1)$$

$$\equiv (def(S) \wedge 0 \leq i-1 < |s|) \wedge_L 0 \leq i < |s|) \wedge_L Q^{s[i]}_{setAt(s,i,s[i-1])}) \quad (2)$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)(1 \leq j < |s| \rightarrow_L \quad (3)$$

$$setAt(s, i, s[i-1])[j] = setAt(s, i, s[i-1])[j-1]))$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)(1 \leq j < |s| \rightarrow_L \quad (4)$$

$$(i = j \wedge s[i-1] = setAt(s, i, s[i-1])[j-1]) \wedge (i \neq j \wedge s[j] = setAt(s, i, s[i-1])[j-1]))$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)(1 \leq j < |s| \rightarrow_L (i = j \wedge s[i-1] = s[i-1]) \wedge (i \neq j \wedge s[j] = s[j-1])$$

$$(5)$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1]) \quad (6)$$

$$(7)$$

Luego veo la wp de la consigna

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv (def(i) \wedge_L (i > 1 \wedge wp(S1, Q)) \vee (i \leq 1 \wedge wp(S2, Q))) && (1)\\
&\equiv ((i > 1 \wedge ((1 \leq i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1]))\vee && (2)\\
&(i \leq 1 \wedge (def(S) \wedge def(i) \wedge 0 \leq i < |s| \wedge Q^{s[i]}_{setAt(s,i,0)}))))\\
&\equiv ((1 < i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1]))\vee && (3)\\
&(i = 0 \wedge Q^{s[i]}_{setAt(s,i,0)}) \wedge (i = 1 \wedge Q^{s[i]}_{setAt(s,i,0)}))\\
&\equiv ((1 < i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1]))\vee && (4)\\
&(i = 0 \wedge (\forall j : Z)(1 \leq j < |s| \rightarrow_L setAt(s,i,0)[j] = setAt(s,i,0)[j-1]))\wedge\\
&(i = 1 \wedge (\forall j : Z)(1 \leq j < |s| \rightarrow_L setAt(s,i,0)[j] = setAt(s,i,0)[j-1]))\\
&\equiv ((1 < i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1]))\vee && (5)\\
&(i = 0 \wedge (\forall j : Z)(1 \leq j < |s| \rightarrow_L (s[j] = s[j-1]))\wedge\\
&(i = 1 \wedge (\forall j : Z)(1 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] = s[j-1]))
\end{aligned}
$$

e) $S \equiv$ **if** $(s[i] < 0)$ $s[i] := -s[i]$ **else** *skip* **endif**

$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$

$$
\begin{aligned}
wp(\mathbf{S}, Q) &\equiv (def(s) \wedge def(i) \wedge 0 \leq i < |s|) \wedge_L ((s[i] < 0 \wedge wp(S1, Q) \vee (s[i] \geq 0 \wedge wp(S2, Q)) && (1)\\
&\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (setAt(s, i, -s[i]) \wedge Q^{s[i]}_{-s[i]}) \vee (s[i] \geq 0 \wedge 0 \leq i < |s| \wedge_L s[i] \geq 0) && (2)\\
&\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L (0 \leq i < |s| \wedge_L setAt(s, i, -s[i])[i] \geq 0))\\
& && (3)\\
&\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (0 \leq i < |s| \wedge_L setAt(s, i, -s[i])[i] \geq 0)) && (4)\\
&\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (0 \leq i < |s| \wedge_L -s[i] \geq 0)) && (5)\\
&\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (0 \leq i < |s| \wedge_L s[i] \leq 0)) && (6)\\
&\equiv (0 \leq i < |s|) && (7)\\
& && (8)
\end{aligned}
$$

f) $S \equiv$ **if** $(s[i] > 0)$ $s[i] := -s[i]$ **else** *skip* **endif**

$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$

$$wp(\mathbf{S},Q) \equiv (def(s) \wedge def(i) \wedge 0 \leq i < |s|) \wedge_L ((s[i] > 0 \wedge wp(S1,Q) \vee (s[i] \leq 0 \wedge wp(S2,Q)) \quad (1)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] > 0 \wedge (setAt(s,i,-s[i]) \wedge Q^{s[i]}_{-s[i]}) \vee (s[i] \leq 0 \wedge (0 \leq j < |s| \wedge_L s[j] \geq 0)) \quad (2)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] > 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L setAt(s,i,-s[i])[j] \geq 0) \vee \quad (3)$$

$$((i = j \wedge s[j] = 0) \wedge (i \neq j \wedge (s[j] \geq 0))$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] > 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge -s[j] \geq 0) \wedge (i \neq j \wedge)) \vee \quad (4)$$

$$((i = j \wedge s[j] = 0) \wedge (i \neq j \wedge (s[j] \geq 0))$$

$$(5)$$

no terminado/

Ejercicio 5

a) **Nombre del problema:** sumarIesimoElem

Programa: $a := a + s[i]$

$$wp(\mathbf{a{+}s[i]}, a = \sum_{j=0}^{i} s[j]) \equiv ((def(a) \wedge def(s) \wedge def(i) \wedge 0 \leq i < |s|) \wedge_L Q^{a}_{s[i]}) \quad (1)$$

$$\equiv ((0 \leq i < |s|) \wedge_L a + s[i] = \sum_{j=0}^{i} s[j]) \quad (2)$$

$$\equiv ((0 \leq i < |s|) \wedge_L a = \sum_{j=0}^{i-1} s[j]) \quad (3)$$

b) **Nombre del programa:** iesimoElemPositivo

Programa: $res := s[i] \geq 0$

$$wp(res := s[i] \geq 0, res = true \leftrightarrow (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0) \equiv (0 \leq i < |s| \wedge_L \quad (1)$$

$$s[i] \geq 0 = true \leftrightarrow (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0))$$

$$\equiv (0 \leq i < |s| \ \wedge_L (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0))$$

$$(2)$$

c) **Nombre del programa:** hastaIesimoElemFibonacci

Programa: $s[i] := fibonacci(i)$

$$wp(s[i] := fibo(i), (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L s[j] = fibo(j))) \equiv (def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L \tag{1}$$

$$(\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L setAt(s, i, fibo(i))[j] = fibo(j))$$

$$\equiv (0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L \tag{2}$$

$$(i = j \wedge fibo(j) = fibo(j) \wedge (i \neq j \wedge s[j] = fibo(j)$$

$$\equiv (0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) \tag{3}$$

$$(0 \leq j < i \wedge i \neq j \rightarrow_L s[j] = fibo(j) \tag{4}$$

Ejercicio 6

a) I) Es incorrecta dado que al elemento s[—s—] para el cual el programa se indefine

II) En el caso de que se tome la lista [0,4,2] y el segundo elemento, es decir, i=1 entonces el programa solo va a entrar a la rama else del if y por lo tanto s[1]=1. Sin embargo 1 esta en el rango de —s— y no cumple que 1 mod 2 = 0.

III) suponiendo que se ingrese la lista [1,4,3,3] entonces la 4ta posicion (indice 3) entrara por la primer rama del if y entonces s[3]=s[3]+6=9. Sin embargo 9 mod 2 = 0 es falso

IV) Con [1,2,3,4] entonces i=3 no cumple $0 \leq i < 2$ por lo tanto hace falsa la precondicion. Sin embargo al ejecutar el programa entra por la rama then y se hace s[3]=4+6=10 y 10 mod 2 = 0 por lo tanto no cumple P pero si Q.

b) //

3.1.1 Ejercicios de parcial

Ejercicio 7

a) ”wp”= Todas las posiciones en rango deben tener valores negativos

$S \equiv$ **if** $(s[i] < 0)$ $s[i] := -s[i]$ **else** $s[i] := 0$ **endif**

$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] > 0)$

$$wp(S,Q) \equiv ((def(s) \wedge (def(i) \wedge 0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge Q^{s[i]}_{setAt(s,i,-s[i])}) \vee (s[i] \geq 0 \wedge Q^{s[i]}_{setAt(s,i,0)}) \tag{1}$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L setAt(s,i,-s[i])[j] > 0)) \vee \tag{2}$$

$(s[i] \geq 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L setAt(s,i,0)[j] > 0))$

$$\equiv (0 \leq i < |s|) \wedge_L \tag{3}$$

$(s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge -s[i] > 0) \wedge (i \neq j \wedge s[j] > 0))) \vee$

$(s[i] \geq 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge 0 > 0) \wedge (i \neq j \wedge s[j] > 0))$

$$\equiv (0 \leq i < |s|) \wedge_L \tag{4}$$

$(s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge s[i] < 0) \wedge (i \neq j \wedge s[j] > 0))) \vee$

$(s[i] \geq 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L \mathit{False} \wedge (i \neq j \wedge s[j] > 0))$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] > 0))) \vee (s[i] \geq 0 \wedge \mathit{False}) \tag{5}$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] > 0))) \vee \mathit{False} \tag{6}$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] > 0))) \tag{7}$$

c) $S \equiv$ **if** $(s[i] \neq 2^i)$ **then**

$s[i] = 2 * s[i-1]$

else

$s[0] = 1$

endif

$Q \equiv \{(\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] = 2^j)\}$

Veo la wp:

$$wp(S,Q) \equiv def(B) \wedge (B \wedge_L wp(S1,Q)) \vee (\neg B \wedge_L wp(S2,Q)) \tag{1}$$

$$\equiv def(i) \wedge def(s) \wedge 0 \leq i < |s| \wedge (s[i] \neq 2^i \wedge_L wp(S1,Q)) \vee (s[i] = 2^i \wedge_L wp(S2,Q)) \tag{2}$$

$$\equiv 0 \leq i < |s| \wedge (s[i] \neq 2^i \wedge_L wp(S1,Q)) \vee (s[i] = 2^i \wedge_L wp(S2,Q)) \tag{3}$$

Ahora veo ambas wp por separado:

$$
\begin{aligned}
wp(S1,Q) &\equiv def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge 0 \leq i-1 < |s| \wedge Q^{s[j]}_{setAt(s,i,2*s[i-1])} && (1)\\
&\equiv 1 \leq i < |s| \wedge (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j) \wedge (i = j \wedge 2*s[i-1] = 2^i) && (2)\\
&\equiv 1 \leq i < |s| \wedge (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j) \wedge (i = j \wedge s[i-1] = 2^{i-1}) && (3)\\
&\equiv 1 \leq i < |s| \wedge (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j) && (4)
\end{aligned}
$$

(Vale True el segundo disy dado que si $i = j$ entonces $i - 1 \neq j$ y el primer disy hace a esto True)

$$
\begin{aligned}
wp(S2,Q) &\equiv def(S) \wedge 0 \leq 0 < |s| \wedge Q^{s[j]}_{setAt(s,0,1)} && (1)\\
&\equiv (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge (0 = j \wedge s[0] = 2^0) && (2)\\
&\equiv (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge (0 = j \wedge 1 = 1) && (3)\\
&\equiv (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge (0 = j \wedge True) && (4)\\
&\equiv (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge True && (5)\\
&\equiv (\forall j:\mathbb{Z})((0 < j < |s|) \rightarrow_L s[j] = 2^j) && (6)
\end{aligned}
$$

Ahora reemplazo las wp halladas en la original

$$
\begin{aligned}
&\equiv 0 \leq i < |s| \wedge (s[i] \neq 2^i \wedge_L 1 \leq i < |s| \wedge (\forall j:\mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j)) \vee && (1)\\
&(s[i] = 2^i \wedge_L (\forall j:\mathbb{Z})((0 < j < |s|) \rightarrow_L s[j] = 2^j))
\end{aligned}
$$

d) $S \equiv$ **if** $(i \mod 3 = 0)$ **then**

$s[i] = s[i] + 6$

else

$s[i] = 1$

endif

$Q \equiv (\forall j:\mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \mod 3 = 0)$

Veo la wp:

$$
wp(S,Q) \equiv def(i) \wedge ((i \mod 3 = 0 \wedge wp(S1,Q)) \vee (i \mod 3 \neq 0 \wedge wp(S2,Q))) \qquad (1)
$$

Ahora veo ambas wp por separado:

$$wp(S1,Q) \equiv def(i) \land 0 \leq i < |s| \land_L Q^{s[j]}_{setAt(s,i,s[i]+6)} \quad (1)$$

$$\equiv 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})((0 \leq j < |s| \land i \neq j) \rightarrow_L s[j] \mod 3 = 0) \land (i = j \land s[i] + 6 \mod 3 = 0) \quad (2)$$

$$\equiv 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})((0 \leq j < |s| \land i \neq j) \rightarrow_L s[j] \mod 3 = 0) \land (i = j \land s[i] \mod 3 = 0) \quad (3)$$

$$\equiv 0 \leq i < |s| \land_L s[i] \mod 3 = 0 \quad (4)$$

$$wp(S2,Q) \equiv def(s) \land def(i) \land 0 \leq i < |s| \land_L Q^{s[j]}_{setAt(s,i,i)} \quad (1)$$

$$\equiv 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})((0 \leq j < |s| \land i \neq j) \rightarrow_L s[j] \mod 3 = 0) \land (i = j \land i \mod 3 = 0) \quad (2)$$

Ahora reemplazo las wp halladas en la original

$$(i \mod 3 = 0 \land wp(S1,Q)) \lor (i \mod 3 \neq 0 \land wp(S2,Q)) \quad (1)$$

$$\equiv (i \mod 3 = 0 \land 0 \leq i < |s| \land_L s[i] \mod 3 = 0) \lor \quad (2)$$

$$(i \mod 3 \neq 0 \land 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})((0 \leq j < |s| \land i \neq j) \rightarrow_L s[j] \mod 3 = 0) \land (i = j \land i \mod 3 = 0))$$

$$\equiv (i \mod 3 = 0 \land 0 \leq i < |s| \land_L s[i] \mod 3 = 0) \lor \quad (3)$$

$$(i \mod 3 \neq 0 \land 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})((0 \leq j < |s| \land i \neq j) \rightarrow_L s[j] \mod 3 = 0) \land False)$$

$$\equiv (i \mod 3 = 0 \land 0 \leq i < |s| \land_L s[i] \mod 3 = 0) \lor False \quad (4)$$

$$\equiv (i \mod 3 = 0 \land 0 \leq i < |s| \land_L s[i] \mod 3 = 0) \quad (5)$$

e) $S \equiv$ **if** $(i \mod 2 = 0)$ **then**

$s[i] = 2 * s[i]$

else

$s[0] = 3$

endif

$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \mod 2 = 0)$

Veo la wp:

$$wp(S,Q) \equiv def(i) \wedge ((i \mod 2 = 0 \wedge wp(S1,Q)) \vee (i \mod 2 \neq 0 \wedge wp(S2,Q))) \tag{1}$$

$$\tag{2}$$

Ahora veo ambas wp:

$$wp(S1,Q) \equiv def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L Q^{s[j]}_{setAt(s,i,2*s[i])} \tag{1}$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 2 = 0) \wedge (i = j \wedge 2 * s[i] \mod 2 = 0) \tag{2}$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 2 = 0) \wedge True \tag{3}$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 2 = 0) \tag{4}$$

$$\tag{5}$$

$$wp(S2,Q) \equiv def(s) \wedge 0 \leq 0 < |s| \wedge_L Q^{s[j]}_{setAt(s,0,3)} \tag{1}$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] \mod 2 = 0) \wedge (0 = j \wedge 3 \mod 2 = 0) \tag{2}$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] \mod 2 = 0) \wedge False \tag{3}$$

$$False \tag{4}$$

Reemplazo las wp halladas:

$$def(i) \wedge ((i \mod 2 = 0 \wedge wp(S1,Q)) \vee (i \mod 2 \neq 0 \wedge wp(S2,Q))) \tag{1}$$

$$\equiv (i \mod 2 = 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 2 = 0)) \vee (i \mod 2 \neq 0 \wedge False)) \tag{2}$$

$$\equiv (i \mod 2 = 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 2 = 0)) \vee False \tag{3}$$

$$\equiv (i \mod 2 = 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 2 = 0)) \tag{4}$$