

# Algoritmos y Estructuras de Datos

## Practica 3

### 3.1. Precondicion mas debil en SmallLang

#### Ejercicio 1

a)  $def(a + 1) \equiv def(a) + def(1) \equiv True$

b)  $def(a/b) \equiv def(a) \wedge (def(b) \wedge b \neq 0) \equiv b \neq 0$

c)  $def(\sqrt{a/b}) \equiv def(a) \wedge (def(b) \wedge b \neq 0) \wedge \frac{a}{b} \geq 0 \equiv b \neq 0 \wedge \frac{a}{b} \geq 0$

d)  $def(A[i] + 1) \equiv def[A] \wedge def(i) \wedge 0 \leq i < |A| \equiv 0 \leq i < |A|$

e)  $def(A[i + 2]) \equiv (def(A) \wedge def(i)) \wedge -2 \leq i < |A| - 2 \equiv -2 \leq i < |A|$

f)  $def(0 \leq i \leq |A| \wedge_L A[i] \geq 0) \equiv (def(A) \wedge def(i)) \wedge i \neq |A| \equiv i \neq |A|$

**nota:** en f) hay que poner el rango, no el  $\neq$

#### Ejercicio 2

a)

$$wp(\mathbf{a}:=\mathbf{a}+1, \mathbf{b}:=\mathbf{a}/2, b \geq 0) \equiv wp(\mathbf{a}:=\mathbf{a}+1, wp(\mathbf{b}:=\mathbf{a}/2, b \geq 0)) \quad (1)$$

$$\equiv wp(\mathbf{a}:=\mathbf{a}+1, def(a/2) \wedge Q_{a/2}^b) \quad (2)$$

$$\equiv wp(\mathbf{a}:=\mathbf{a}+1, a/2 \geq 0) \quad (3)$$

$$\equiv (def(a) \wedge (a + 1)/2 \geq 0) \quad (4)$$

$$\equiv (a + 1)/2 \geq 0 \equiv a + 1 \geq 0 \equiv a \geq -1 \quad (5)$$

b)

$$wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, \mathbf{b}:=\mathbf{a}*\mathbf{a}, b \neq 2) \equiv wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, wp(\mathbf{b}:=\mathbf{a}*\mathbf{a}, b \neq 2)) \quad (1)$$

$$\equiv wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, def(a) \wedge Q_{a*a}^b) \quad (2)$$

$$\equiv wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, def(a) \wedge a * a \neq 2) \quad (3)$$

$$\equiv (def(a) \wedge 0 \leq i < |A|) \wedge_L Q_{A[i]+1}^a \quad (4)$$

$$\equiv (0 \leq i < |A| \wedge_L (A[i] + 1) * (A[i] + 1) \neq 2) \quad (5)$$

$$\equiv (0 \leq i < |A| \wedge_L A[i]^2 + 2 * A[i] + 1 \neq 2) \quad (6)$$

c)

$$wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, \mathbf{a}:=\mathbf{b}*\mathbf{b}, a \geq 0) \equiv wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, wp(\mathbf{a}:=\mathbf{b}*\mathbf{b}, a \geq 0)) \quad (1)$$

$$\equiv wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, def(b) \wedge b * b \geq 0) \quad (2)$$

$$\equiv wp(\mathbf{a}:=\mathbf{A}[\mathbf{i}]+\mathbf{1}, b * b \geq 0) \quad (3)$$

$$\equiv (def(A) \wedge def(i) \wedge 0 \leq i < |A| \wedge b * b \geq 0) \quad (4)$$

$$\equiv (0 \leq i < |A| \wedge b * b \geq 0) \quad (5)$$

d)

$$wp(\mathbf{a}:=\mathbf{a}-\mathbf{b}; \mathbf{b}:=\mathbf{b}+\mathbf{a}, a \geq 0 \wedge b \geq 0) \equiv wp(\mathbf{a}:=\mathbf{a}-\mathbf{b}, wp(\mathbf{b}:=\mathbf{b}+\mathbf{a}, a \geq 0 \wedge b \geq 0)) \quad (1)$$

$$\equiv wp(\mathbf{a}:=\mathbf{a}-\mathbf{b}, def(b) \wedge def(a) \wedge (a \geq 0 \wedge b + a \geq 0)) \quad (2)$$

$$\equiv wp(\mathbf{a}:=\mathbf{a}-\mathbf{b}, (a \geq 0 \wedge (b + a) \geq 0)) \quad (3)$$

$$\equiv (def(a) \wedge def(b) \wedge (a - b) \geq 0 \wedge (b + (a - b)) \geq 0) \quad (4)$$

$$\equiv ((a - b) \geq 0 \wedge a \geq 0) \quad (5)$$

$$\equiv (a \geq b \wedge a \geq 0) \quad (6)$$

### Ejercicio 3

$$Q \equiv (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \geq 0)$$

a)

$$wp(\mathbf{A}[i] := 0, Q) \equiv wp(setAt(A, i, 0)[j], Q) \quad (1)$$

$$\equiv (def(A) \wedge def(i) \wedge_L 0 \leq i < |A| \wedge_L Q_{setAt(A, i, 0)}^A) \quad (2)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i, 0)[j] \geq 0)) \quad (3)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge A[i] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (4)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge 0 \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (5)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)((0 \leq j < |A| \wedge_L j \neq i) \rightarrow_L A[j] \geq 0)) \quad (6)$$

$$(7)$$

b)

$$wp(\mathbf{A}[i+2] := 0, Q) \equiv wp(setAt(A, i+2, 0)[j], Q) \quad (1)$$

$$\equiv (def(A) \wedge def(i+2) \wedge_L 0 \leq i+2 < |A| \wedge_L Q_{setAt(A, i+2, 0)}^A) \quad (2)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i+2, 0)[j] \geq 0)) \quad (3)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i+2 \wedge 0 \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (4)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)((0 \leq j < |A| \wedge_L j \neq i+2) \rightarrow_L A[j] \geq 0)) \quad (5)$$

c)

$$wp(\mathbf{A}[i+2] := -1, Q) \equiv wp(setAt(A, i+2, -1)[j], Q) \quad (1)$$

$$\equiv (def(A) \wedge def(i+2) \wedge_L 0 \leq i+2 < |A| \wedge_L Q_{setAt(A, i+2, -1)}^A) \quad (2)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i+2, -1)[j] \geq 0)) \quad (3)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i+2 \wedge -1 \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (4)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L False \wedge (j \neq i \wedge A[j] \geq 0))) \quad (5)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L False)) \quad (6)$$

$$\equiv (0 \leq i+2 < |A| \wedge_L False) \quad (7)$$

$$\equiv False \quad (8)$$

**Nota:** debería haber considerado la lista vacía de manera que si se cumpla el segundo disyunto a partir de (5)

d)

$$wp(\mathbf{A}[i] := 2 * \mathbf{A}[i], Q) \equiv wp(setAt(A, i, 2 * A[i])[j], Q) \quad (1)$$

$$\equiv (def(A) \wedge def(i) \wedge_L 0 \leq i < |A| \wedge_L Q_{setAt(A, i, 2 * A[i])}^A) \quad (2)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i, 2 * A[i])[j] \geq 0)) \quad (3)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge 2 * A[i] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (4)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge A[i] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (5)$$

$$\equiv (0 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L A[j] \geq 0)) \quad (6)$$

e)

$$wp(\mathbf{A}[i] := \mathbf{A}[i-1], Q) \equiv wp(setAt(A, i, A[i-1])[j], Q) \quad (1)$$

$$\equiv (def(A) \wedge def(i) \wedge def(A[i-1]) \wedge_L 0 \leq i-1 < |A| \wedge_L Q_{setAt(A, i, A[i-1])}^A) \quad (2)$$

$$\equiv (def(A) \wedge def(i) \wedge 0 \leq i-1 < |A| \wedge_L 0 \leq i-1 < |A| \wedge_L Q_{setAt(A, i, A[i-1])}^A) \quad (3)$$

$$\equiv (1 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L setAt(A, i, A[i-1])[j] \geq 0)) \quad (4)$$

$$\equiv (1 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \rightarrow_L (j = i \wedge A[i-1] \geq 0) \wedge (j \neq i \wedge A[j] \geq 0))) \quad (5)$$

$$\equiv (1 \leq i < |A| \wedge_L (\forall j : Z)(0 \leq j < |A| \wedge j \neq i \rightarrow_L A[j] \geq 0)) \quad (6)$$

## Ejercicio 4

a)  $S \equiv \mathbf{if}(a < 0) \ b := a \ \mathbf{else} \ b := -a \ \mathbf{endif}$

$$Q \equiv (b = -|a|)$$

$$wp(\mathbf{S}, Q) \equiv (def(B) \wedge_L (B \wedge wp(S1, Q)) \vee (\neg B \wedge wp(S2, Q)) \quad (1)$$

$$\equiv def(a) \wedge_L (a < 0 \wedge (def(a) \wedge Q_a^b) \vee (a \geq 0 \wedge (def(a) \wedge Q_{-a}^b)) \quad (2)$$

$$\equiv (a < 0 \wedge a = -|a|) \vee (a \geq 0 \wedge -a = -|a|) \quad (3)$$

$$\equiv (a < 0 \wedge True) \vee (a \geq 0 \wedge True) \quad (4)$$

$$\equiv (a < 0) \vee (a \geq 0) \quad (5)$$

$$\equiv True \quad (6)$$

b)  $S \equiv \mathbf{if}(a < 0) \ b := a \ \mathbf{else} \ b := -a \ \mathbf{endif}$

$$Q \equiv (b = |a|)$$

$$wp(\mathbf{S}, Q) \equiv (def(B) \wedge_L (B \wedge wp(S1, Q)) \vee (\neg B \wedge wp(S2, Q)) \quad (1)$$

$$\equiv def(a) \wedge_L (a < 0 \wedge (def(a) \wedge Q_a^b) \vee (a \geq 0 \wedge (def(a) \wedge Q_{-a}^b)) \quad (2)$$

$$\equiv (a < 0 \wedge a = |a|) \vee (a \geq 0 \wedge -a = |a|) \quad (3)$$

$$\equiv (a < 0 \wedge False) \vee (a \geq 0 \wedge False) \quad (4)$$

$$\equiv False \vee False \quad (5)$$

$$\equiv False \quad (6)$$

c)  $S \equiv \mathbf{if} \ (i > 0) \ s[i] := 0 \ \mathbf{else} \ s[0] := 0 \ \mathbf{endif}$

$$Q \equiv (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

$$wp(\mathbf{S}, Q) \equiv (def(i) \wedge_L (i > 0 \wedge wp(S1, Q)) \vee (i \geq 0 \wedge wp(S2, Q))) \quad (1)$$

$$\equiv (i > 0 \wedge def(S) \wedge def(i) \wedge 0 \leq i < |s| \wedge Q_{setAt(S, i, 0)}^{s[i]}) \vee \quad (2)$$

$$(i \leq 0 \wedge def(S) \wedge Q_{s[0]}^{s[i]}) \quad (3)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L setAt(s, i, 0) \geq 0)) \vee \quad (4)$$

$$(i \leq 0 \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L s[0] \geq 0)) \quad (5)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L (i = j \wedge s[i] \geq 0) \wedge (i \neq j \wedge s[j] \geq 0))) \vee \quad (6)$$

$$(i \leq 0 \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L 0 \geq 0)) \quad (7)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)(0 \leq j < |s| \rightarrow_L (i = j \wedge True) \wedge (i \neq j \wedge s[j] \geq 0))) \vee \quad (8)$$

$$(i \leq 0 \wedge True) \quad (9)$$

$$\equiv (0 < i < |s| \wedge (\forall j : Z)((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \geq 0)) \quad (10)$$

d)  $S \equiv \mathbf{if} (i > 1) \ s[i] := s[i - 1] \ \mathbf{else} \ s[i] := 0 \ \mathbf{endif}$

$$Q \equiv (\forall j : Z)(1 \leq j < |s| \rightarrow_L s[j] = s[j - 1])$$

**Primero hago la wp entre S1 y Q**

$$wp(S1, Q) \equiv ((def(S) \wedge def(i) \wedge def(S[i - 1])) \wedge_L 0 \leq i < |s| \wedge_L Q_{setAt(s, i, s[i - 1])}^{s[i]}) \quad (1)$$

$$\equiv (def(S) \wedge 0 \leq i - 1 < |s| \wedge_L 0 \leq i < |s| \wedge_L Q_{setAt(s, i, s[i - 1])}^{s[i]}) \quad (2)$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)(1 \leq j < |s| \rightarrow_L \quad (3)$$

$$setAt(s, i, s[i - 1])[j] = setAt(s, i, s[i - 1])[j - 1]))$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)(1 \leq j < |s| \rightarrow_L \quad (4)$$

$$(i = j \wedge s[i - 1] = setAt(s, i, s[i - 1])[j - 1]) \wedge (i \neq j \wedge s[j] = setAt(s, i, s[i - 1])[j - 1]))$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)(1 \leq j < |s| \rightarrow_L (i = j \wedge s[i - 1] = s[i - 1]) \wedge (i \neq j \wedge s[j] = s[j - 1])) \quad (5)$$

$$\equiv ((1 \leq i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j - 1]) \quad (6)$$

$$(7)$$

Luego veo la wp de la consigna

$$wp(\mathbf{S}, Q) \equiv (def(i) \wedge_L (i > 1 \wedge wp(S1, Q)) \vee (i \leq 1 \wedge wp(S2, Q))) \quad (1)$$

$$\equiv ((i > 1 \wedge ((1 \leq i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1])) \vee \quad (2)$$

$$(i \leq 1 \wedge (def(S) \wedge def(i) \wedge 0 \leq i < |s| \wedge Q_{setAt(s, i, 0)}^{s[i]}))) \quad (3)$$

$$\equiv ((1 < i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1])) \vee \quad (4)$$

$$(i = 0 \wedge Q_{setAt(s, i, 0)}^{s[i]}) \wedge (i = 1 \wedge Q_{setAt(s, i, 0)}^{s[i]})$$

$$\equiv ((1 < i < |s|) \wedge_L (\forall j : Z)((1 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = s[j-1])) \vee \quad (5)$$

$$(i = 0 \wedge (\forall j : Z)(1 \leq j < |s| \rightarrow_L (s[j] = s[j-1]))) \wedge$$

$$(i = 1 \wedge (\forall j : Z)(1 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] = s[j-1]))$$

e)  $S \equiv \text{if } (s[i] < 0) \ s[i] := -s[i] \ \text{else skip} \ \text{endif}$

$$Q \equiv 0 \leq i < |s| \wedge_L s[i] \geq 0$$

$$wp(\mathbf{S}, Q) \equiv (def(s) \wedge def(i) \wedge 0 \leq i < |s|) \wedge_L ((s[i] < 0 \wedge wp(S1, Q)) \vee (s[i] \geq 0 \wedge wp(S2, Q))) \quad (1)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (setAt(s, i, -s[i]) \wedge Q_{-s[i]}^{s[i]}) \vee (s[i] \geq 0 \wedge 0 \leq i < |s| \wedge_L s[i] \geq 0)) \quad (2)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L (0 \leq i < |s| \wedge_L setAt(s, i, -s[i])[i] \geq 0))) \quad (3)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (0 \leq i < |s| \wedge_L setAt(s, i, -s[i])[i] \geq 0)) \quad (4)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (0 \leq i < |s| \wedge_L -s[i] \geq 0)) \quad (5)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (0 \leq i < |s| \wedge_L s[i] \leq 0)) \quad (6)$$

$$\equiv (0 \leq i < |s|) \quad (7)$$

$$(8)$$

f)  $S \equiv \text{if } (s[i] > 0) \ s[i] := -s[i] \ \text{else skip} \ \text{endif}$

$$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \geq 0)$$

$$wp(\mathbf{S}, Q) \equiv (def(s) \wedge def(i) \wedge 0 \leq i < |s|) \wedge_L ((s[i] > 0 \wedge wp(S1, Q) \vee (s[i] \leq 0 \wedge wp(S2, Q))) \quad (1)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] > 0 \wedge (setAt(s, i, -s[i]) \wedge Q_{-s[i]}^{s[i]}) \vee (s[i] \leq 0 \wedge (0 \leq j < |s| \wedge_L s[j] \geq 0))) \quad (2)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] > 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L setAt(s, i, -s[i])[j] \geq 0) \vee \quad (3)$$

$$((i = j \wedge s[j] = 0) \wedge (i \neq j \wedge (s[j] \geq 0)))$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] > 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge -s[j] \geq 0) \wedge (i \neq j \wedge))) \vee \quad (4)$$

$$((i = j \wedge s[j] = 0) \wedge (i \neq j \wedge (s[j] \geq 0)))$$

$$(5)$$

no terminado/

## Ejercicio 5

a) **Nombre del problema:** sumarIesimoElem

**Programa:**  $a := a + s[i]$

$$wp(\mathbf{a} + \mathbf{s}[i], a = \sum_{j=0}^i s[j]) \equiv ((def(a) \wedge def(s) \wedge def(i) \wedge 0 \leq i < |s|) \wedge_L Q_{s[i]}^a) \quad (1)$$

$$\equiv ((0 \leq i < |s|) \wedge_L a + s[i] = \sum_{j=0}^i s[j]) \quad (2)$$

$$\equiv ((0 \leq i < |s|) \wedge_L a = \sum_{j=0}^{i-1} s[j]) \quad (3)$$

b) **Nombre del programa:** iesimoElemPositivo

**Programa:**  $res := s[i] \geq 0$

$$wp(res := s[i] \geq 0, res = true \leftrightarrow (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0)) \equiv (0 \leq i < |s| \wedge_L \quad (1)$$

$$s[i] \geq 0 = true \leftrightarrow (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0))$$

$$\equiv (0 \leq i < |s| \wedge_L (\forall j : Z)(0 \leq j \leq i \rightarrow_L s[j] \geq 0)) \quad (2)$$

c) **Nombre del programa:** hastaIesimoElemFibonacci

**Programa:**  $s[i] := fibonacci(i)$



$$wp(s[i] := fibo(i), (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L s[j] = fibo(j))) \equiv (def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L \quad (1)$$

$$(\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L setAt(s, i, fibo(i))[j] = fibo(j))$$

$$\equiv (0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})(0 \leq j \leq i \rightarrow_L \quad (2)$$

$$(i = j \wedge fibo(j) = fibo(j) \wedge (i \neq j \wedge s[j] = fibo(j))$$

$$\equiv (0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) \quad (3)$$

$$(0 \leq j < i \wedge i \neq j \rightarrow_L s[j] = fibo(j)) \quad (4)$$

## Ejercicio 6

- a) I) Es incorrecta dado que al elemento  $s[-s-]$  para el cual el programa se indefine
- II) En el caso de que se tome la lista  $[0,4,2]$  y el segundo elemento, es decir,  $i=1$  entonces el programa solo va a entrar a la rama else del if y por lo tanto  $s[1]=1$ . Sin embargo 1 esta en el rango de  $-s-$  y no cumple que  $1 \bmod 2 = 0$ .
- III) suponiendo que se ingrese la lista  $[1,4,3,3]$  entonces la 4ta posicion (indice 3) entrara por la primer rama del if y entonces  $s[3]=s[3]+6=9$ . Sin embargo  $9 \bmod 2 = 1$  es falso
- IV) Con  $[1,2,3,4]$  entonces  $i=3$  no cumple  $0 \leq i < 2$  por lo tanto hace falsa la precondition. Sin embargo al ejecutar el programa entra por la rama then y se hace  $s[3]=4+6=10$  y  $10 \bmod 2 = 0$  por lo tanto no cumple P pero si Q.

b) //

## 3.1.1 Ejercicios de parcial

## Ejercicio 7

- a) "wp"= Todas las posiciones en rango deben tener valores negativos

$$S \equiv \text{if } (s[i] < 0) \ s[i] := -s[i] \ \text{else } s[i] := 0 \ \text{endif}$$

$$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] > 0)$$

$$wp(S, Q) \equiv ((def(s) \wedge (def(i) \wedge 0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge Q_{setAt(s, i, -s[i])}^{s[i]}) \vee (s[i] \geq 0 \wedge Q_{setAt(s, i, 0)}^{s[i]})) \quad (1)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L setAt(s, i, -s[i])[j] > 0)) \vee \quad (2)$$

$$(s[i] \geq 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L setAt(s, i, 0)[j] > 0))$$

$$\equiv (0 \leq i < |s|) \wedge_L \quad (3)$$

$$(s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge -s[i] > 0) \wedge (i \neq j \wedge s[j] > 0))) \vee$$

$$(s[i] \geq 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge 0 > 0) \wedge (i \neq j \wedge s[j] > 0)))$$

$$\equiv (0 \leq i < |s|) \wedge_L \quad (4)$$

$$(s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L (i = j \wedge s[i] < 0) \wedge (i \neq j \wedge s[j] > 0))) \vee$$

$$(s[i] \geq 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L False \wedge (i \neq j \wedge s[j] > 0)))$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] > 0)) \vee (s[i] \geq 0 \wedge False) \quad (5)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] > 0)) \vee False \quad (6)$$

$$\equiv (0 \leq i < |s|) \wedge_L (s[i] < 0 \wedge (\forall j : \mathbb{Z})(0 \leq j < |s| \wedge i \neq j \rightarrow_L s[j] > 0)) \quad (7)$$

c)  $S \equiv \mathbf{if} (s[i] \neq 2^i) \mathbf{then}$

$$s[i] = 2 * s[i - 1]$$

**else**

$$s[0] = 1$$

**endif**

$$Q \equiv \{(\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] = 2^j)\}$$

Veo la wp:

$$wp(S, Q) \equiv def(B) \wedge (B \wedge_L wp(S1, Q)) \vee (\neg B \wedge_L wp(S2, Q)) \quad (1)$$

$$\equiv def(i) \wedge def(s) \wedge 0 \leq i < |s| \wedge (s[i] \neq 2^i \wedge_L wp(S1, Q)) \vee (s[i] = 2^i \wedge_L wp(S2, Q)) \quad (2)$$

$$\equiv 0 \leq i < |s| \wedge (s[i] \neq 2^i \wedge_L wp(S1, Q)) \vee (s[i] = 2^i \wedge_L wp(S2, Q)) \quad (3)$$

Ahora veo ambas wp por separado:

$$wp(S1, Q) \equiv def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge 0 \leq i - 1 < |s| \wedge Q_{setAt(s, i, 2 * s[i-1])}^{s[j]} \quad (1)$$

$$\equiv 1 \leq i < |s| \wedge (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j) \wedge (i = j \wedge 2 * s[i-1] = 2^i) \quad (2)$$

$$\equiv 1 \leq i < |s| \wedge (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j) \wedge (i = j \wedge s[i-1] = 2^{i-1}) \quad (3)$$

$$\equiv 1 \leq i < |s| \wedge (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j) \quad (4)$$

(Vale True el segundo disy dado que si  $i = j$  entonces  $i - 1 \neq j$  y el primer disy hace a esto True)

$$wp(S2, Q) \equiv def(S) \wedge 0 \leq 0 < |s| \wedge Q_{setAt(s, 0, 1)}^{s[j]} \quad (1)$$

$$\equiv (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge (0 = j \wedge s[0] = 2^0) \quad (2)$$

$$\equiv (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge (0 = j \wedge 1 = 1) \quad (3)$$

$$\equiv (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge (0 = j \wedge True) \quad (4)$$

$$\equiv (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] = 2^j) \wedge True \quad (5)$$

$$\equiv (\forall j : \mathbb{Z})((0 < j < |s|) \rightarrow_L s[j] = 2^j) \quad (6)$$

Ahora reemplazo las wp halladas en la original

$$\equiv 0 \leq i < |s| \wedge (s[i] \neq 2^i \wedge_L 1 \leq i < |s| \wedge (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] = 2^j)) \vee \quad (1)$$

$$(s[i] = 2^i \wedge_L (\forall j : \mathbb{Z})((0 < j < |s|) \rightarrow_L s[j] = 2^j))$$

d)  $S \equiv \mathbf{if} (i \bmod 3 = 0) \mathbf{then}$

$$s[i] = s[i] + 6$$

**else**

$$s[i] = 1$$

**endif**

$$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \bmod 3 = 0)$$

Veo la wp:

$$wp(S, Q) \equiv def(i) \wedge ((i \bmod 3 = 0 \wedge wp(S1, Q)) \vee (i \bmod 3 \neq 0 \wedge wp(S2, Q))) \quad (1)$$

Ahora veo ambas wp por separado:

$$\begin{aligned}
wp(S1, Q) &\equiv def(i) \wedge 0 \leq i < |s| \wedge_L Q_{setAt(s, i, s[i]+6)}^{s[j]} & (1) \\
&\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 3 = 0) \wedge (i = j \wedge s[i] + 6 \mod 3 = 0) & (2) \\
&\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 3 = 0) \wedge (i = j \wedge s[i] \mod 3 = 0) & (3) \\
&\equiv 0 \leq i < |s| \wedge_L s[i] \mod 3 = 0 & (4)
\end{aligned}$$

$$\begin{aligned}
wp(S2, Q) &\equiv def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L Q_{setAt(s, i, i)}^{s[j]} & (1) \\
&\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 3 = 0) \wedge (i = j \wedge i \mod 3 = 0) & (2)
\end{aligned}$$

Ahora reemplazo las wp halladas en la original

$$\begin{aligned}
&(i \mod 3 = 0 \wedge wp(S1, Q)) \vee (i \mod 3 \neq 0 \wedge wp(S2, Q)) & (1) \\
&\equiv (i \mod 3 = 0 \wedge 0 \leq i < |s| \wedge_L s[i] \mod 3 = 0) \vee & (2) \\
&\quad (i \mod 3 \neq 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 3 = 0) \wedge (i = j \wedge i \mod 3 = 0)) \\
&\equiv (i \mod 3 = 0 \wedge 0 \leq i < |s| \wedge_L s[i] \mod 3 = 0) \vee & (3) \\
&\quad (i \mod 3 \neq 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z})((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \mod 3 = 0) \wedge False) \\
&\equiv (i \mod 3 = 0 \wedge 0 \leq i < |s| \wedge_L s[i] \mod 3 = 0) \vee False & (4) \\
&\equiv (i \mod 3 = 0 \wedge 0 \leq i < |s| \wedge_L s[i] \mod 3 = 0) & (5)
\end{aligned}$$

e)  $S \equiv \mathbf{if} (i \mod 2 = 0) \mathbf{then}$

$$s[i] = 2 * s[i]$$

**else**

$$s[0] = 3$$

**endif**

$$Q \equiv (\forall j : \mathbb{Z})(0 \leq j < |s| \rightarrow_L s[j] \mod 2 = 0)$$

Veamos la wp:

$$wp(S, Q) \equiv def(i) \wedge ((i \bmod 2 = 0 \wedge wp(S1, Q)) \vee (i \bmod 2 \neq 0 \wedge wp(S2, Q))) \quad (1)$$

$$(2)$$

Ahora veamos ambas wp:

$$wp(S1, Q) \equiv def(s) \wedge def(i) \wedge 0 \leq i < |s| \wedge_L Q_{setAt(s, i, 2 * s[i])}^{s[j]} \quad (1)$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \bmod 2 = 0) \wedge (i = j \wedge 2 * s[i] \bmod 2 = 0) \quad (2)$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \bmod 2 = 0) \wedge True \quad (3)$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \bmod 2 = 0) \quad (4)$$

$$(5)$$

$$wp(S2, Q) \equiv def(s) \wedge 0 \leq 0 < |s| \wedge_L Q_{setAt(s, 0, 3)}^{s[j]} \quad (1)$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] \bmod 2 = 0) \wedge (0 = j \wedge 3 \bmod 2 = 0) \quad (2)$$

$$\equiv 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge 0 \neq j) \rightarrow_L s[j] \bmod 2 = 0) \wedge False \quad (3)$$

$$False \quad (4)$$

Reemplazo las wp halladas:

$$def(i) \wedge ((i \bmod 2 = 0 \wedge wp(S1, Q)) \vee (i \bmod 2 \neq 0 \wedge wp(S2, Q))) \quad (1)$$

$$\equiv (i \bmod 2 = 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \bmod 2 = 0)) \vee (i \bmod 2 \neq 0 \wedge False) \quad (2)$$

$$\equiv (i \bmod 2 = 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \bmod 2 = 0)) \vee False \quad (3)$$

$$\equiv (i \bmod 2 = 0 \wedge 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) ((0 \leq j < |s| \wedge i \neq j) \rightarrow_L s[j] \bmod 2 = 0)) \quad (4)$$