13) 1.

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + n & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ①

$$T(n-1) + n$$

paso ②

$$\left[T(n-2) + (n-1)\right] + n =$$

$$= T(n-2) + (n-1) + n = \quad T(n-2) + 2n - 1$$

paso ③

$$\left[T(n-3) + (n-2)\right] + 2n - 1$$

$$= T(n-3) + n - 2 + 2n + 1$$

$$= T(n-3) + 3n - 3$$

paso (i) $T(n-i) + i.n - \sum_{j=0}^{i-1} j =$

$= T(n-i) + i.n - \frac{(i-1).((i-1)+1)}{2}$

caso base

$n-i = 1$

$n = 1+i$

$n-1 = i$

reemplazo i

$T(n-(n-1)) + (n-1).n - \frac{((n-1)-1).(((n-1)-1)+1)}{2} =$

$= T(1) + (n-1).n - \left(\frac{n^2-3n+2}{2}\right) =$

$= 2 + n^2 - n - \left(\frac{n^2-3n+2}{2}\right) =$

$= 2 + n^2 - n - \left(\frac{n^2-3n+2}{2}\right) =$

$= \frac{4 + 2n^2 - 2n - n^2 + 3n - 2}{2} =$

$= \frac{2 + n^2 + n}{2} =$

$= 1 + \frac{n^2}{2} + \frac{n}{2} =$

$= \boxed{n^2 . \frac{1}{2} + n . \frac{1}{2} + 1}$

$O(n^2)$

calculo orden

ler termino

$$n^2 \frac{1}{2} <= n^2$$

orden

$$n^2 <= n^2$$

mult a ambos por $\frac{1}{2}$

$$n^2 \frac{1}{2} <= n^2 \frac{1}{2}$$

se mantiene desigualdad con $c_1 = \frac{1}{2}$ para todo $n \geq 0$

2do termino

$$n \frac{1}{2} <= n^2$$

orden

$$n <= n^2$$

mult a ambos por $\frac{1}{2}$

$$n \frac{1}{2} <= n^2 \frac{1}{2}$$

se mantiene desigualdad, con $c_2 = \frac{1}{2}$ para todo $n \geq 0$.

3ro

$c = 1$ $n \geq 1$

calculo c y no para todo $T(n)$

$$T(n) <= c_1 n^2 + c_2 n^2$$

$$T(n) <= (c_1 + c_2) n^2$$

$$T(n) <= \left(\frac{1}{2} + \frac{1}{2} + 1\right) n^2$$

$$T(n) <= 2 n^2$$

$T(n) <= O(n^2)$ con $c = 2$ para todo $n \geq 1$.

b)

$$T(n) = \begin{cases} 2 & , n = 1 \\ T(n-1) + \frac{n}{2} & , n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso 1

$$T(n-1) + \frac{n}{2}$$

paso 2

$$\left[T(n-2) + \frac{n-1}{2} \right] + \frac{n}{2} =$$

$$= T(n-2) + \frac{n-1}{2} + \frac{n}{2}$$

$$= T(n-2) + \frac{2n-1}{2}$$

paso 3

$$\left[T(n-3) + \frac{n-2}{2} \right] + \frac{2n-1}{2} =$$

$$= T(n-3) + \frac{3n-3}{2}$$

paso 4

$$\left[T(n-4) + \frac{n-3}{2} \right] + \frac{3n-3}{2}.$$

$$= T(n-4) + \frac{4n-6}{2}$$

paso i

$$T(n-i) + \frac{in}{2} - \sum_{j=0}^{i-1} j \cdot \frac{1}{2} =$$

$$= T(n-i) + \frac{in}{2} - \frac{1}{2} \cdot \left(\frac{(i-1) \cdot ((i-1)+1)}{2} \right) =$$

$$= T(n-i) + \frac{in}{2} - \left(\frac{i^2 - i}{4} \right) =$$

$$= T(n-i) + \frac{in}{2} + \frac{i - i^2}{4}$$

caso base

$$n - i = 1$$

$$n = 1 - i$$

$$n - 1 = i$$

reemplazo i

$$T(n-(n-1)) + \frac{(n-1) n}{2} + \frac{(n-1) - (n-1)^2}{4} =$$

$$= T(1) + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= 2 + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= \frac{\cancel{8} + \cancel{2}n^2 - \cancel{2n} + n - \cancel{1} - \cancel{n^2} + \cancel{2n} - \cancel{1}}{}$$

$$\frac{6 + n^2 + n}{4} =$$

$$= \frac{3}{2} + n^2 \cdot \frac{1}{4} + n \cdot \frac{1}{4} = T(n)$$

$O(n^2)$

$T(n) <= O(n^2)$ con $c = 2$ para todo $n \geq 1$.

3. $$T(n) = \begin{cases} 1 & n = 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n} & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ① $$2T\left(\frac{n}{4}\right) + \sqrt{n}$$

paso ② $$2\left[2T\left(\frac{n}{4^2}\right) + \sqrt{\frac{n}{4}}\right] + \sqrt{n} =$$

$$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{\frac{n}{4}} + \sqrt{n} =$$

$$= 4T\left(\frac{n}{4^2}\right) + \cancel{2}\frac{\sqrt{n}}{\cancel{2}} + \sqrt{n} =$$

$$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{n}$$

paso ③ $$4\left[2T\left(\frac{n}{4^3}\right) + \sqrt{\frac{n}{4^2}}\right] + 2\sqrt{n} =$$

$$= 8T\left(\frac{n}{4^3}\right) + 4\sqrt{\frac{n}{16}} + 2\sqrt{n} =$$

$$= 8T\left(\frac{n}{4^3}\right) + \cancel{4}\frac{\sqrt{n}}{\cancel{4}} + 2\sqrt{n} =$$

$$= 8T\left(\frac{n}{4^3}\right) + 3\sqrt{n}$$

paso (i)

$$2^i \, T\left(\frac{n}{4^i}\right) + i\sqrt{n}$$

caso base

$$\frac{n}{4^i} = 1$$

$$n = 4^i$$

$$\log_4(n) = i$$

$$n = (2^2)^i$$

$$n = 2^{2*i}$$

$$n = (2^i)^2 \Rightarrow \sqrt[2]{(2^i)^2}$$

$$\sqrt{n} = 2^i$$

reemplazo i

$$\sqrt{n} \; T\left(\frac{n}{4^{\log_4(n)}}\right) + \log_4(n)\sqrt{n} =$$

$$= \sqrt{n} + T(1) + \log_4(n)\sqrt{n} =$$

$$= \sqrt{n} + 1 + \log_4(n).\sqrt{n}$$

$$O\left(\log_4(n).\sqrt{n}\right)$$

$T(n) <= O(\log_4(n).\sqrt{n})$ con $c=3$ para todo $n \geq 2$

4.

$$T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{2}\right) + c & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso (1)

$$2T\left(\frac{n}{2}\right) + c$$

paso 2

$$2\left[2T\left(\frac{n}{2^2}\right)+c\right]+c=$$

$$=4T\left(\frac{n}{2^2}\right)+2c+c=$$

$$=4T\left(\frac{n}{2^2}\right)+3c$$

paso i

$$2^i\,T\left(\frac{n}{2^i}\right)+\left(2^i-1\right).c$$

caso base

$$\frac{n}{2^i}=1$$

$$n=2^i$$

$$\log_2(n)=i$$

reemplazo i

$$n.T\left(\frac{n}{2^{\log_2(n)}}\right)+\left(n-1\right).c=$$

$$=n.\,T(1)+nc-c_1=$$

$$=n+nc_1-c_1=T(n)$$

$O(n)$

$T(n)$ <= $O(n)$ con $c=c_1+1$ para todo $n\geq 1$

5.

$$T(n) = \begin{cases} 1 & n = 1 \\ 4T\left(\frac{n}{2}\right) + n^2 & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso 1

$$4T\left(\frac{n}{2}\right) + n^2$$

paso 2

$$4\left[4T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right] + n^2 =$$

$$= 16T\left(\frac{n}{2^2}\right) + \cancel{4}\frac{n^2}{\cancel{2^2}} + n^2 =$$

$$= 16T\left(\frac{n}{2^2}\right) + 2n^2$$

paso 3

$$16\left[4T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right] + 2n^2 =$$

$$= 64T\left(\frac{n}{2^3}\right) + \cancel{16}\frac{n^2}{\cancel{16}} + 2n^2 =$$

$$= 64T\left(\frac{n}{2^3}\right) + 3n^2$$

paso i

$$4^i T\left(\frac{n}{2^i}\right) + i n^2$$

Escaneado con CamScanner

caso base.

$$\frac{n}{2^i} = 1$$

si $n = 2^i$
entonces ¿

$$n^2 = (2^i)^2$$

$$(2^i)^2 = (2^2)^i = 4^i$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$n^2 + \left(\frac{n}{2^{\log_2(n)}} \right) + \log_2(n) . n^2 =$$

$$= n^2 . T(1) + \log_2(n) . n^2 =$$

$$= n^2 + \log_2(n) . n^2$$

$O(\log_2(n) . n^2)$

$T(n) <= O(\log_2(n) . n^2)$ con $c = 2$ para todo $n >= 2$