13) 1.

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + n & n \geq 2 \end{cases}$$

Suponiendo que $n \geq 2$

paso ①

$$T(n-1) + n$$

paso ②

$$\left[T(n-2) + (n-1)\right] + n =$$

$$= T(n-2) + (n-1) + n = T(n-2) + 2n - 1$$

paso ③

$$\left[T(n-3) + (n-2)\right] + 2n - 1$$

$$= T(n-3) + n - 2 + 2n + 1$$

$$= T(n-3) + 3n - 3$$

paso (i) $T(n-i) + i.n - \sum_{j=i}^{i-1} j =$

$$= T(n-i) + i.n - \frac{(i-1).((i-1)+1)}{2}$$

caso base

$n-1 = 1$

$n = 1+i$

$n-1 = i$

reemplazo i $T(n-(n-1)) + (n-1).n - \frac{((n-1)-1).(((n-1)-1)+1)}{2} =$

$$= T(1) + (n-1).n - \frac{n^2 - 3n + 2}{2} =$$

$$= 2 + n^2 - n - \left(\frac{n^2 - 3n + 2}{2}\right) =$$

$$= 2 + n^2 - n - \frac{n^2 + 3n - 2}{2} =$$

$$= \frac{\cancel{4} + 2n^2 - 2n - n^2 + 3n - \cancel{4}}{2} =$$

$$= \frac{n^2 + n}{2} =$$

$$= \frac{n^2}{2} + \frac{n}{2} =$$

$$= \boxed{n^2 . \frac{1}{2} + n . \frac{1}{2}}$$

Escaneado con CamScanner

$O(n^2)$

calculo orden

$\boxed{\text{1er termino}}$

$$n^2 \frac{1}{2} <= n^2$$

orden

$$n^2 <= n^2$$

mult a ambos por $\frac{1}{2}$

$$n^2 \frac{1}{2} <= n^2 \frac{1}{2}$$

se mantiene desigualdad con $c_1 = \frac{1}{2}$ para todo n.

$\boxed{\text{2do termino}}$

$$n \frac{1}{2} <= n^2$$

orden

$$n <= n^2$$

mult a ambos por $\frac{1}{2}$

$$n \frac{1}{2} <= n^2 \frac{1}{2}$$

se mantiene desigualdad con $c_2 = \frac{1}{2}$ para todo $n \geq 1$

$\boxed{\text{calculo c y no para todo } T(n)}$

$$T(n) <= c_1 n^2 + c_2 n^2$$

$$T(n) <= (c_1 + c_2) n^2$$

$$T(n) <= \left(\frac{1}{2} + \frac{1}{2}\right) n^2$$

$$T(n) <= 1 n^2$$

$T(n) <= O(n^2)$ con $c = 1$ para todo $n \geq 1$

b)

$$T(n) = \begin{cases} 2 & , n = 1 \\ T(n-1) + \frac{n}{2} & , n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso 1

$$T(n-1) + \frac{n}{2}$$

paso 2

$$\left[T(n-2) + \frac{n-1}{2} \right] + \frac{n}{2} =$$

$$= T(n-2) + \frac{n-1}{2} + \frac{n}{2}$$

$$= T(n-2) + \frac{2n-1}{2}$$

paso 3

$$\left[T(n-3) + \frac{n-2}{2} \right] + \frac{2n-1}{2} =$$

$$= T(n-3) + \frac{3n-3}{2}$$

paso 4

$$\left[T(n-4) + \frac{n-3}{2} \right] + \frac{3n-3}{2}$$

$$= T(n-4) + \frac{4n-6}{2}$$

paso i

$$T(n-i) + \frac{in}{2} - \sum_{j=1}^{i-1} j \cdot \frac{1}{2} =$$

Escaneado con CamScanner

$$= T(n-i) + \frac{in}{2} - \frac{1}{2} \cdot \left(\frac{(i-1) \cdot ((i-1)+1)}{2} \right) =$$

$$= T(n-i) + \frac{in}{2} - \left(\frac{i^2 - i}{4} \right) =$$

$$= T(n-i) + \frac{in}{2} + \frac{i - i^2}{4}$$

caso base

$$n - i = 1$$

$$n = 1 - i$$

$$n - 1 = i$$

reemplazo i

$$T(n-(n-1)) + \frac{(n-1)n}{2} + \frac{(n-1) - (n-1)^2}{4} =$$

$$= T(1) + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= 2 + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= \frac{\cancel{8} + \cancel{2}n^2 - \cancel{2}n + n - \cancel{1} - \cancel{n^2} + \cancel{2}n - \cancel{1}}{}$$

$$\frac{6 + n^2 + n}{4} =$$

$$= \frac{3}{2} + n^2 \cdot \frac{1}{4} + n \cdot \frac{1}{4} = T(n)$$

$O(n^2)$

$T(n) <= O(n^2)$ con $c = 2$ para todo $n \geq 3$.

3. $T(n) = \begin{cases} 1 & n = 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n} & n \geq 2 \end{cases}$

suponiendo que $n \geq 2$

paso 1 $\quad 2T\left(\frac{n}{4}\right) + \sqrt{n}$

paso 2 $\quad 2\left[2T\left(\frac{n}{4^2}\right) + \sqrt{\frac{n}{4}}\right] + \sqrt{n} =$

$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{\frac{n}{4}} + \sqrt{n} =$

$= 4T\left(\frac{n}{4^2}\right) + \cancel{2}\frac{\sqrt{n}}{\cancel{2}} + \sqrt{n} =$

$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{n}$

paso 3 $\quad 4\left[2T\left(\frac{n}{4^3}\right) + \sqrt{\frac{n}{4^2}}\right] + 2\sqrt{n} =$

$= 8T\left(\frac{n}{4^3}\right) + 4\sqrt{\frac{n}{16}} + 2\sqrt{n} =$

$= 8T\left(\frac{n}{4^3}\right) + \cancel{4}\frac{\sqrt{n}}{\cancel{4}} + 2\sqrt{n} =$

$= 8T\left(\frac{n}{4^3}\right) + 3\sqrt{n}$

paso i

$$2^i T\left(\frac{n}{4^i}\right) + i\sqrt{n}$$

caso base

$$\frac{n}{4^i} = 1$$

$$n = 4^i$$

$$\log_4(n) = i$$

$$n = (2^2)^i$$

$$n = 2^{2*i}$$

$$n = (2^i)^2 \Rightarrow \cancel{\sqrt[2]{(2^i)^2}}$$

$$\sqrt{n} = 2^i$$

reemplazo i

$$\sqrt{n}\ T\left(\frac{n}{4^{\log_4(n)}}\right) + \log_4(n)\sqrt{n} =$$

$$= \sqrt{n} + T(1) + \log_4(n)\sqrt{n} =$$

$$= \sqrt{n} + 1 + \log_4(n).\sqrt{n}$$

$$O\left(\log_4(n) . \sqrt{n}\right)$$

$T(n) <= O(\log_4(n).\sqrt{n})$ con $c=3$ para todo $n \geq 3$.

4.

$$T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{2}\right) + c & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso 1

$$2T\left(\frac{n}{2}\right) + c$$

Escaneado con CamScanner

paso (2)

$$2\left[2T\left(\frac{n}{2^2}\right)+c\right]+c=$$

$$=4T\left(\frac{n}{2^2}\right)+2c+c=$$

$$=4T\left(\frac{n}{2^2}\right)+3C$$

paso i

$$2^i\,T\left(\frac{n}{2^i}\right)+\left(2^i-1\right).c$$

caso base

$$\frac{n}{2^i}=1$$

$$n=2^i$$

$$\log_2(n)=i$$

reemplazo i

$$n.\,T\left(\frac{n}{2^{\log_2(n)}}\right)+(n-1).c=$$

$$=n.\,T(1)+nc-c_1=$$

$$=n+nc_1-c_1=T(n)$$

$O(n)$

$T(n) <= O(n)$ con $c=2c_1+1$ para todo $n \geq 1$

Escaneado con CamScanner

5.

$$T(n) = \begin{cases} 1 & n=1 \\ 4T\left(\frac{n}{2}\right) + n^2 & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso 1

$$4T\left(\frac{n}{2}\right) + n^2$$

paso 2

$$4\left[4T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right] + n^2 =$$

$$= 16T\left(\frac{n}{2^2}\right) + \cancel{4}\frac{n^2}{\cancel{2^2}} + n^2 =$$

$$= 16T\left(\frac{n}{2^2}\right) + 2n^2$$

paso 3

$$16\left[4T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right] + 2n^2 =$$

$$= 64T\left(\frac{n}{2^3}\right) + \cancel{16}\frac{n^2}{\cancel{16}} + 2n^2 =$$

$$= 64T\left(\frac{n}{2^3}\right) + 3n^2$$

paso i

$$4^i T\left(\frac{n}{2^i}\right) + i n^2$$

caso base.

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

si $n = 2^i$
entonces ¿

$n^2 = (2^i)^2$

$(2^i)^2 = (2^2)^i = 4^i$

reemplazo i

$$n^2 T\left(\frac{n}{2^{\log_2(n)}}\right) + \log_2(n).n^2 =$$

$$= n^2 . T(1) + \log_2(n) . n^2 =$$

$$= n^2 + \log_2(n) . n^2$$

$O(\log_2(n). n^2)$

$T(n) <= O(\log_2(n). n^2)$ con $c = 2$ para todo $n >, 2$