13) 1.

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + n & n \geq 2 \end{cases}$$

Suponemos que $n \geq 2$

paso ①

$$T(n-1) + n$$

paso ②

$$\left[T(n-2) + (n-1)\right] + n =$$

$$= T(n-2) + (n-1) + n = T(n-2) + 2n - 1$$

paso ③

$$\left[T(n-3) + (n-2)\right] + 2n - 1$$

$$= T(n-3) + (n-2) + 2n - 1$$

$$= T(n-3) + 3n - 3$$

paso ①

$$T(n-i) + i.n - \sum_{j=0}^{i-1} j =$$

$$= T(n-i) + i.n - \frac{(i-1).((i-1)+1)}{2}$$

caso base

$$n-1 = 1$$

$$n = 1+i$$

$$n-1 = i$$

reemplazo i

$$T(n-(n-1)) + (n-1).n - \frac{((n-1)-1).(((n-1)-1)+1)}{2} =$$

$$= T(1) + (n-1).n - \left(\frac{n^2-3n+2}{2}\right)^2 =$$

$$= 2 + n^2 - n - \left(\frac{n^2-3n+2}{2}\right) =$$

$$= 2 + n^2 - n - \left(\frac{n^2-3n+2}{2}\right) =$$

$$= \frac{4 + 2n^2 - 2n - n^2 + 3n - 2}{2} =$$

$$= \frac{2 + n^2 + n}{2} =$$

$$= 1 + \frac{n^2}{2} + \frac{n}{2} =$$

$$= \boxed{n^2 \cdot \frac{1}{2} + n \cdot \frac{1}{2} + 1}$$

$O(n^2)$

Calculo orden

1er termino

$$n^2 \frac{1}{2} \leq n^2$$

orden

$$n^2 \leq n^2$$

mult a ambos por $\frac{1}{2}$

$$n^2 \frac{1}{2} \leq n^2 \frac{1}{2}$$

se mantiene desigualdad con $c_1 = \frac{1}{2}$ para todo $n \geq 0$

2do termino

$$n \frac{1}{2} \leq n^2$$

orden

$$n \leq n^2$$

mult a ambos por $\frac{1}{2}$

$$n \frac{1}{2} \leq n^2 \frac{1}{2}$$

se mantiene desigualdad con $c_2 = \frac{1}{2}$ para todo $n \geq 0$.

3ro

$c = 1 \quad n \geq 1$

calculo c y no para todo $T(n)$

$$T(n) \leq c_1 n^2 + c_2 n^2$$

$$T(n) \leq (c_1 + c_2) n^2$$

$$T(n) \leq \left(\frac{1}{2} + \frac{1}{2} + 1\right) n^2$$

$$T(n) \leq 2 n^2$$

$T(n) \leq O(n^2)$ con $c = 2$ para todo $n \geq 1$.

b)

$$T(n) = \begin{cases} 2 & , n = 1 \\ T(n-1) + \frac{n}{2} & , n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ① $\quad T(n-1) + \frac{n}{2}$

paso ② $\quad \left[T(n-2) + \frac{n-1}{2} \right] + \frac{n}{2} =$

$$= T(n-2) + \frac{n-1}{2} + \frac{n}{2}$$

$$= T(n-2) + \frac{2n-1}{2}$$

paso ③ $\quad \left[T(n-3) + \frac{n-2}{2} \right] + \frac{2n-1}{2} =$

$$= T(n-3) + \frac{3n-3}{2}$$

paso ④ $\quad \left[T(n-4) + \frac{n-3}{2} \right] + \frac{3n-3}{2}$

$$= T(n-4) + \frac{4n-6}{2}$$

paso ① $\quad T(n-i) + \frac{in}{2} - \sum_{j=0}^{i-1} j \cdot \frac{1}{2} =$

$$= T(n-i) + \frac{in}{2} - \frac{1}{2} \cdot \left(\frac{(i-1) \cdot ((i-1)+1)}{2} \right) =$$

$$= T(n-i) + \frac{in}{2} - \left(\frac{i^2 - i}{4} \right) =$$

$$= T(n-i) + \frac{in}{2} + \frac{i - i^2}{4}$$

caso base

$$n - i = 1$$

$$n = 1 - i$$

$$n - 1 = i$$

reemplazo i

$$T\left(n-(n-1)\right) + \frac{(n-1)n}{2} + \frac{(n-1) - (n-1)^2}{4} =$$

$$= T(1) + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= 2 + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= \frac{8 + 2n^2 - 2n + n - 1 - n^2 + 2n - 1}{4}$$

$$\frac{6 + n^2}{4} + n =$$

$$= \frac{3}{2} + n^2 \cdot \frac{1}{4} + n \cdot \frac{1}{4} = T(n)$$

$O(n^2)$

$T(n) <= O(n^2)$ con $c = 2$ para todo $n \geq 1$.

3. $$T(n) = \begin{cases} 1 & n = 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n} & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ① $2T\left(\frac{n}{4}\right) + \sqrt{n}$

paso ② $2\left[2T\left(\frac{n}{4^2}\right) + \sqrt{\frac{n}{4}}\right] + \sqrt{n} =$

$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{\frac{n}{4}} + \sqrt{n} =$

$= 4T\left(\frac{n}{4^2}\right) + \frac{2\sqrt{n}}{2} + \sqrt{n} =$

$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{n}$

paso ③ $4\left[2T\left(\frac{n}{4^3}\right) + \sqrt{\frac{n}{4^2}}\right] + 2\sqrt{n} =$

$= 8T\left(\frac{n}{4^3}\right) + 4\sqrt{\frac{n}{16}} + 2\sqrt{n} =$

$= 8T\left(\frac{n}{4^3}\right) + \frac{4\sqrt{n}}{4} + 2\sqrt{n} =$

$= 8T\left(\frac{n}{4^3}\right) + 3\sqrt{n}$

paso (i)

$$2^i\, T\left(\frac{n}{4^i}\right) + i\sqrt{n}$$

caso base

$$\frac{n}{4^i} = 1$$

$$n = 4^i$$

$$\log_4(n) = i$$

$$n = (z^2)^i$$

$$n = 2^{z*i}$$

$$n = (z^i)^2 \Rightarrow \sqrt{n} = \sqrt{(2^i)^2}$$

$$\sqrt{n} = 2^i$$

reemplazo i

$$\sqrt{n}\; T\left(\frac{n}{4^{\log_4(n)}}\right) + \log_4(n)\sqrt{n} =$$

$$= \sqrt{n}\; T(1) + \log_4(n)\sqrt{n} =$$

$$= \sqrt{n}\; 1 + \log_4(n).\sqrt{n}$$

$$O\left(\log_4(n).\sqrt{n}\right)$$

$T(n) <- O(\log_4(n).\sqrt{n})$ con $c=3$ para todo $n \geq 1$

4.

$$T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{2}\right) + C & n \geq 2 \end{cases}$$

suponemos que $n \geq 2$

paso (i)

$$2T\left(\frac{n}{2}\right) + C$$

paso (2)

$$2\left[2T\left(\frac{n}{2^2}\right)+c\right]+c =$$

$$= 4T\left(\frac{n}{2^2}\right) + 2c + c =$$

$$= 4T\left(\frac{n}{2^2}\right) + 3c$$

paso i

$$2^i T\left(\frac{n}{2^i}\right) + \left(2^i - 1\right).c$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$n.T\left(\frac{n}{2^{\log_2(n)}}\right) + (n-1).c =$$

$$= n.T(1) + nc - c_1 =$$

$$= n + nc - c_1 = T(n)$$

$O(n)$

$T(n) <= O(n)$ con $c = c_1 + 1$ para todo $n >= 1$

5.

$$T(n) = \begin{cases} 1 & n = 1 \\ 4T\left(\frac{n}{2}\right) + n^2 & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ①

$$4T\left(\frac{n}{2}\right) + n^2$$

paso ②

$$4\left[4T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right] + n^2 =$$

$$= 16T\left(\frac{n}{2^2}\right) + 4\frac{n^2}{2^2} + n^2 =$$

$$= 16T\left(\frac{n}{2^2}\right) + 2n^2$$

paso ③

$$16\left[4T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right] + 2n^2 =$$

$$= 64T\left(\frac{n}{2^3}\right) + 16\frac{n^2}{16} + 2n^2 =$$

$$= 64T\left(\frac{n}{2^3}\right) + 3n^2$$

paso (i)

$$4^i T\left(\frac{n}{2^i}\right) + i\,n^2$$

caso base

$\frac{n}{2^i} = 1$

$n = 2^i$

$\log_2(n) = i$

si $n = 2^i$ entonces

$n^2 = (2^i)^2$

$(2^i)^2 = (2^2)^i = 4^i$

reemplazo i

$$n^2 + \left(\frac{n}{2^{\log_2(n)}}\right) + \log_2(n) \cdot n^2 =$$

$$= n^2 \cdot T(1) + \log_2(n) \cdot n^2 =$$

$$= n^2 + \log_2(n) \cdot n^2$$

$O(\log_2(n) \cdot n^2)$

$T(n) <= O(\log_2(n) \cdot n^2)$ con $c = 2$ para todo $n >= 1$