1) a) $cte_1 + \sum_{i=1}^{n-1} \left[\sum_{j=i+1}^{n} \left[\sum_{k=1}^{j} cte_2 \right] \right] =$

la saco afuera

$= cte_1 + \sum_{i=1}^{n-1} \left[\sum_{j=i+1}^{n} j \cdot cte_2 \right] =$

$= cte_1 + \sum_{i=1}^{n-1} \left[cte_2 \cdot \sum_{j=i+1}^{n} j \right] =$

$= cte_1 + \sum_{i=1}^{n-1} \left[cte_2 \cdot \left(\sum_{j=1}^{n} j - \sum_{j=1}^{i} j \right) \right] =$

$= cte_1 + \sum_{i=1}^{n-1} \left[cte_2 \cdot \left(\frac{n(n+1)}{2} - \frac{i(i+1)}{2} \right) \right] =$

saco afuera y distribuyo sumatoria

$= cte_1 + \sum_{i=1}^{n-1} cte_2 \cdot \left(\frac{n(n+1)}{2} - i \left(\frac{i+1}{2} \right) \right) =$

$= \frac{i^2 + i}{2}$

$= cte_1 + cte_2 \cdot \left(\sum_{i=1}^{n-1} \frac{n(n+1)}{2} - \sum_{i=1}^{n-1} \frac{i(i+1)}{2} \right) =$

$= cte_1 + cte_2 \cdot \left(\left((n-1) \cdot \left(\frac{n(n+1)}{2} \right) \right) - \left(\sum_{i=1}^{n-1} \frac{i^2 + i}{2} \right) \right) =$

distribuyo

saco afuera

$= cte_1 + cte_2 \cdot \left(\left((n-1) \cdot \left(\frac{n(n+1)}{2} \right) \right) - \left(\sum_{i=1}^{n-1} (i^2 + i) \cdot \frac{1}{2} \right) \right) =$

$= cte_1 + cte_2 \cdot \left(\left((n-1) \cdot \left(\frac{n(n+1)}{2} \right) \right) - \left(\frac{1}{2} \cdot \left(\sum_{i=1}^{n-1} i^2 + \sum_{i=1}^{n-1} i \right) \right) \right) =$

$= cte_1 + cte_2 \cdot \left(\left((n-1) \left(\frac{n(n+1)}{2} \right) \right) - \left(\frac{1}{2} \cdot \left(\frac{(n-1)(n) \cdot (2n-2+1)}{6} \right. \right. \right.$

$$+\left(\frac{(n-1)*n}{2}\right)\right)=$$

$$=ae_1+ae_2\cdot\left(\left((n-1)\cdot\left(\frac{n(n+1)}{2}\right)\right)-\left(\frac{1}{2}\cdot\left(\frac{(n-1)n\cdot(2n-1)}{6}+\left(\frac{(n-1)*n}{2}\right)\right)\right)\right)=$$

$$=ae_1+ae_2\cdot\left(\left((n-1)\cdot\frac{n^2+n}{2}\right)-\left(\frac{1}{2}\cdot\left(\frac{(n^2-n)\cdot(2n-1)}{6}+\frac{n^2-n}{2}\right)\right)\right)$$

$$=ae_1+ae_2\cdot\left(\left((n-1)\cdot\frac{n^2+n}{2}\right)-\left(\frac{1}{2}\left(\frac{2n^3-3n^2+n}{6}+\frac{n^2-n}{2}\right)\right)\right)=$$

$$=ae_1+ae_2\left(\left(\frac{n^3-n}{2}\right)-\left(\frac{n^3-n}{6}\right)\right)=$$

$$=ae_1+ae_2\cdot\left(\frac{n^3-n}{3}\right)$$

$$=ae_1+\left(ae_2\cdot\frac{n^3}{3}-ae_2\frac{n}{3}\right)=$$

$$=ae_1+ae_2\frac{n^3}{3}-ae_2\frac{n}{3}=$$

$$\boxed{=ae_1+n^3\frac{ae_2}{3}-n\frac{ae_2}{3}}=T(n)$$

Escaneado con CamScanner

b) $cte1 + \frac{n^3}{3} \cdot cte2 - \frac{n}{3} cte2$

orden candidato $\rightarrow n^3$

PRIMER TERMINO

$cte1 <= n^3 . C1$ para todo $n >= n0$,

$C1 = cte1 \qquad n0 = 1$

SEGUNDO TERMINO

$\frac{n^3}{3} \cdot cte2 <= n^3 . C2$ para todo $n >= n0_2$

$C2 = \frac{cte2}{3} \qquad n0 = 0$

TERCER TERMINO $-\frac{n}{3} \cdot cte2 <= n^3$

al ser un termino negativo no es necesario tenerlo en cuenta en la justificación ya que se puede acotar con 0.

$c_3 = 0 \qquad no_3 = 0$

DEFINIMOS c y no para T(n)

$$cte1 + \frac{n^3}{3} \cdot cte2 - \frac{n}{3} \cdot cte2 <= c1 \cdot n^3 + c2 \cdot n^3$$

$$T(n) <= (c1 + c2)\, n^3$$

$$c = cte1 + \frac{cte2}{3}$$

$no = 1 \rightarrow$ mas restrictivo

$T(n) <= O(n^3)$ con $c = cte1 + \frac{cte2}{3}$ para todo $n >=$ no con no = 1