12.

a) $$T(n) = \begin{cases} 1, & n <= 1 \\ T(n-1) + c_1 & n >= 2 \end{cases}$$

suponiendo que $n >= 2$

paso 1

$$T(n-1) + c$$

paso 2

$$\left[T(n-2) + c \right] + c$$

$$T(n-2) + 2c$$

paso 3

$$\left[T(n-3) + c \right] + 2c =$$

$$= T(n-3) + 3c.$$

paso i

$$T(n-i) + ic$$

caso base

$$n-i <= 1$$

$$n <= 1 + i$$

$$n-1 <= i$$

Escaneado con CamScanner

reemplazo i

$$T(n-(n-1)) + (n-1).c =$$

$$= T(1) + (n-1)c =$$

$$= 1 + nc - c = +(n)$$

$\boxed{\text{calculo orden n}}$ candidato $o(n)$.

1er termino

$$1 <= n$$

orden crecimiento

$$c <= n$$

mult. a ambos por 1

$$1 <= n.1$$

$$1 <= n$$

se mantiene desigualdad con $c_1 = 1$, para todos los $n >= 1$

2do termino

$$nc <= n$$

orden crecimiento

$$n <= n$$

multi a ambos por c

$$n.c <= n.c$$

se mantiene desigualdad con $c_2 = c$, para todos los n.

Tercer termino

$$c <= n$$

orden crecimiento

$$c <= n$$

multiplico a ambos por c

$$c <= n \cdot c$$

se mantiene desigualdad con $c_3 = c$ para todos los $n \geq 1$.

calculo c y no para todo $T(n)$.

$$T(n) <= c_1 \cdot n + c_2 \cdot n + c_3 \cdot n$$

$$T(n) <= (c1 + c2 + c3) \cdot n$$

$$T(n) <= (1 + c + c) \cdot n$$

$$T(n) <= (1 + 2c) \cdot n$$

$T(n) <= O(n)$, con $c = 1 + 2c$, para todo $n \geq 1$.

b)

$$T(n) = \begin{cases} 1 & n = 1 \\ T(n/2) + c & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ①

$$T(n/2) + c$$

paso ②

$$\left[T\left(\frac{n}{2^2}\right) + c \right] + c =$$

$$= T\left(\frac{n}{2^2}\right) + 2c$$

paso ③

$$\left[T\left(\frac{n}{2^3}\right) + c \right] + 2c =$$

$$= T\left(\frac{n}{2^3}\right) + 3c$$

paso ⓘ

$$T\left(\frac{n}{2^i}\right) + ic$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$T\left(\frac{n}{2^{\log_2(n)}}\right) + \log_2(n)\,c =$$

$$= T(1) + \log_2(n)\,c =$$

$$= 1 + \log_2(n)\,c = T(n)$$

$O(\log_2(n))$.

c) $T(n) = \begin{cases} 1 & , n = 1 \\ 2T(n/2) + c & , n \geq 2 \end{cases}$

suponemos que $n \geq 2$

paso ①

$$2T\left(\frac{n}{2}\right) + c$$

paso ②

$$2\left[2T\left(\frac{n}{2^2}\right) + c\right] + c =$$

$$= 4T\left(\frac{n}{2^2}\right) + 2c + c =$$

$$= 4T\left(\frac{n}{2^2}\right) + 3c$$

paso ③

$$4\left[2T\left(\frac{n}{2^3}\right) + c\right] + 3c =$$

$$= 8T\left(\frac{n}{2^3}\right) + 7c.$$

paso i

$$2^i \cdot T\left(\frac{n}{2^i}\right) + \left(2^i - 1\right) \cdot c$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$n \cdot T\left(\frac{n}{2^{\log_2(n)}}\right) + (n-1) \cdot c =$$

$$= n \cdot T(1) + nc - c =$$

$$= n \cdot 1 + nc - c =$$

$$= n + nc - c = T(n)$$

$O(n)$.

d) $T(n) = \begin{cases} 1 & , n <= 5 \\ T(n-5) + c & , n \geq 6 \end{cases}$

suponiendo $n \geq 6$

paso (1)

$$T(n-5) + c$$

paso (2)

$$\left[T(n-10) + c\right] + c =$$

$$= T(n-10) + 2c$$

Escaneado con CamScanner

paso 3

$$\left[T(n-15) + c \right] + 2c$$

$$T(n-15) + 3c$$

paso i

$$T(n-5i) + ic$$

caso base

$$n - 5i <= 5$$

$$n <= 5 - 5i$$

$$n - 5 <= 5i$$

$$\frac{n-5}{5} <= i$$

reemplazo i

$$T\left(n - 5\left(\frac{n-5}{5}\right)\right) + \left(\frac{n-5}{5}\right)c =$$

$$= T(5) + \left(\frac{n}{5} - \frac{\cancel{5}^{1}}{\cancel{5}}\right)c =$$

$$= 1 + \left(\frac{n}{5} - 1\right)c =$$

$$= 1 + \frac{n}{5}c - c =$$

$$= 1 + n \cdot \frac{c}{5} - c = T(n)$$

$O(n)$

e)

$$T(n) = \begin{cases} 1 & , n = 1 \\ 2T(n-1) + c & , n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ①

$$2T(n-1) + c$$

paso ②

$$2\left[2T(n-2) + c\right] + c =$$

$$= 4T(n-2) + 3c$$

paso ③

$$4\left[2T(n-3) + c\right] + 3c =$$

$$= 8T(n-3) + 7c$$

paso ⓘ

$$2^i\, T(n-i) + (2^i - 1)c$$

caso base

$$n - i = 1$$

$$n = 1 + i$$

$$\boxed{n - 1 = i}$$

reemplazo i

$$2^{n-1}\, T(n-(n-1)) + (2^{n-1} - 1)c =$$

$$= 2^{n-1}\, T(1) + (2^{n-1} - 1)c =$$

$$= 2^{n-1} \cdot 1 + (2^{n-1} - 1) \cdot c =$$

$$= 2^{n-1} + (2^{n-1} - 1)c$$

$$\frac{2^n}{2} + \left(\frac{2^n}{2} - 1\right)c =$$

$$= 2^n \cdot \frac{1}{2} + \left(2^n \cdot \frac{1}{2} - 1\right) c =$$

$$= 2^n \cdot \frac{1}{2} + 2^n \cdot c - \frac{c}{2} = T(n)$$

$O(2^n)$

F) $T(n) = \begin{cases} 1 & , n <= 7 \\ T\left(\frac{n}{8}\right) + c & , n >= 8 \end{cases}$

suponiendo que $n \geq 8$

paso ①

$$T\left(\frac{n}{8}\right) + c$$

paso ②

$$\left[T\left(\frac{n}{8^2}\right) + c\right] + c =$$

$$= T\left(\frac{n}{8^2}\right) + 2c$$

paso ③

$$\left[T\left(\frac{n}{8^3}\right) + c\right] + 2c =$$

$$= T\left(\frac{n}{8^3}\right) + 3c.$$

paso i

$$T\left(\frac{n}{8^i}\right) + ic$$

caso base

$$\frac{n}{8^i} <= 7$$

$$n <= 7 \cdot 8^i$$

e)

$$\frac{n}{7} <= 8^i$$

$$\log_8\left(\frac{n}{7}\right) <= i$$

reemplazo i

$$T\left(\frac{n}{8^{\log_8\left(\frac{n}{7}\right)}}\right) + \log_8\left(\frac{n}{7}\right) . c =$$

$$= T(7) + \log_8\left(\frac{n}{7}\right) c =$$

$$= 1 + \log_8\left(\frac{n}{7}\right) c =$$

$$= 1 + \left(\log_8(n) - \log_8(7)\right) c =$$

$$= 1 + \log_8(n).c - \log_8(7).c = T(n)$$

$$O\left(\log_8(n)\right).$$