

13) 1.

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + n & n \geq 2 \end{cases}$$

Suponhamos que $n \geq 2$

passo (1)

$$T(n-1) + n$$

passo (2)

$$\left[T(n-2) + (n-1) \right] + n =$$

$$= T(n-2) + (n-1) + n = T(n-2) + 2n - 1$$

passo (3)

$$\left[T(n-3) + (n-2) \right] + 2n - 1$$

$$= T(n-3) + (n-2) + 2n - 1 =$$

$$= T(n-3) + 3n - 3$$

paso ①

$$T(n-i) + i \cdot n - \sum_{j=0}^{i-1} 1 =$$

$$= T(n-i) + i \cdot n - \frac{(i-1) \cdot ((i-1) + 1)}{2}$$

caso base

$$n-1 = 1$$

$$n = 1 + i$$

$$n-1 = i$$

reemplazo i

$$T(n-(n-1)) + (n-1) \cdot n - \frac{((n-1)-1) \cdot ((n-1)-1) + 1}{2} =$$

$$= T(1) + (n-1) \cdot n - \frac{n^2 - 3n + 2^2}{2} =$$

$$= 2 + n^2 - n - \left(\frac{n^2 - 3n + 2^2}{2} \right) =$$

$$= 2 + n^2 - n - \frac{n^2 + 3n - 2^2}{2} =$$

$$= \frac{\cancel{4} + 2n^2 - 2n - n^2 + 3n - \cancel{4}}{2} =$$

$$= \frac{n^2 + n}{2} =$$

$$= \frac{n^2}{2} + \frac{n}{2} =$$

$$= \boxed{n^2 \cdot \frac{1}{2} + n \cdot \frac{1}{2}}$$

$O(n^2)$

calculo orden

1er termino

$$n^2 \cdot \frac{1}{2} \leq n^2$$

orden

$$n^2 \leq n^2$$

mult a ambos por $\frac{1}{2}$

$$n^2 \cdot \frac{1}{2} \leq n^2 \cdot \frac{1}{2}$$

se mantiene desigualdad con $c_1 = \frac{1}{2}$ para todo n .

2do termino

$$n \cdot \frac{1}{2} \leq n^2$$

orden

$$n \leq n^2$$

mult a ambos por $\frac{1}{2}$

$$n \cdot \frac{1}{2} \leq n^2 \cdot \frac{1}{2}$$

se mantiene desigualdad con $c_2 = \frac{1}{2}$ para todo $n \geq 1$.

calculo c y no para todo $T(n)$

$$T(n) \leq c_1 n^2 + c_2 n^2$$

$$T(n) \leq (c_1 + c_2) n^2$$

$$T(n) \leq \left(\frac{1}{2} + \frac{1}{2} \right) n^2$$

$$T(n) \leq 1 n^2$$

$$T(n) \leq O(n^2) \text{ con } c = 1 \text{ para todo } n \geq 1$$

$$b) \quad \tau(n) = \begin{cases} 2 & , n=1 \\ \tau(n-1) + \frac{n}{2} & , n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

pasos ①

$$\tau(n-1) + \frac{n}{2}$$

pasos ②

$$\left[\tau(n-2) + \frac{n-1}{2} \right] + \frac{n}{2} =$$

$$= \tau(n-2) + \frac{n-1}{2} + \frac{n}{2}$$

$$= \tau(n-2) + \frac{2n-1}{2}$$

pasos ③

$$\left[\tau(n-3) + \frac{n-2}{2} \right] + \frac{2n-1}{2} =$$

$$= \tau(n-3) + \frac{3n-3}{2}$$

pasos ④

$$\left[\tau(n-4) + \frac{n-3}{2} \right] + \frac{3n-3}{2} =$$

$$= \tau(n-4) + \frac{4n-6}{2}$$

pasos ⑤

$$\tau(n-i) + \frac{in}{2} = \sum_{j=0}^{i-1} j \cdot \frac{1}{2} =$$

$$= \tau(n-i) + \frac{in}{2} - \frac{1}{2} \cdot \left(\frac{(i-1) \cdot ((i-1)+1)}{2} \right) =$$

$$= \tau(n-i) + \frac{in}{2} - \left(\frac{i^2 - i}{4} \right) =$$

$$= \tau(n-i) + \frac{in}{2} + \frac{i - i^2}{4}$$

caso base

$$n-i = 1$$

$$n = 1-i$$

$$n-1 = i$$

reemplazo i

$$\tau(n-(n-1)) + \frac{(n-1)n}{2} + \frac{(n-1) - (n-1)^2}{4} =$$

$$= \tau(1) + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= 2 + \frac{n^2 - n}{2} + \frac{(n-1) - (n^2 - 2n + 1)}{4} =$$

$$= \cancel{2} + \cancel{2} \frac{n^2}{2} - \cancel{2} n + n - \cancel{1} - \frac{n^2}{4} + \cancel{2} n - \cancel{1}$$

$$= \frac{n^2}{4} + n =$$

$$= \frac{3}{2} + n^2 \cdot \frac{1}{4} + n \cdot \frac{1}{4} = \tau(n)$$

$$O(n^2)$$

$T(n) \leq O(n^2)$ con $C=2$ para todo $n \geq 3$.

$$3. \quad T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n} & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

pasos ① $2T\left(\frac{n}{4}\right) + \sqrt{n}$

pasos ② $2 \left[2T\left(\frac{n}{4^2}\right) + \sqrt{\frac{n}{4}} \right] + \sqrt{n} =$

$$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{\frac{n}{4}} + \sqrt{n} =$$

$$= 4T\left(\frac{n}{4^2}\right) + \cancel{2} \frac{\sqrt{n}}{\cancel{2}} + \sqrt{n} =$$

$$= 4T\left(\frac{n}{4^2}\right) + 2\sqrt{n}$$

pasos ③ $4 \left[2T\left(\frac{n}{4^3}\right) + \sqrt{\frac{n}{4^2}} \right] + 2\sqrt{n} =$

$$= 8T\left(\frac{n}{4^3}\right) + 4\sqrt{\frac{n}{16}} + 2\sqrt{n} =$$

$$= 8T\left(\frac{n}{4^3}\right) + \cancel{4} \frac{\sqrt{n}}{\cancel{4}} + 2\sqrt{n} =$$

$$= 8T\left(\frac{n}{4^3}\right) + 3\sqrt{n}$$

paso (i)

$$2^i T\left(\frac{n}{4^i}\right) + i\sqrt{n}$$

caso base

$$\frac{n}{4^i} = 1$$

$$n = 4^i$$

$$\log_4(n) = i$$

$$n = (2^2)^i$$

$$n = 2^{2i}$$

$$n = (2^i)^2 \Rightarrow \sqrt{(2^i)^2}$$

$$\sqrt{n} = 2^i$$

reemplazo i

$$\sqrt{n} T\left(\frac{n}{4^{\log_4(n)}}\right) + \log_4(n) \sqrt{n} =$$

$$= \sqrt{n} + T(1) + \log_4(n) \sqrt{n} =$$

$$= \sqrt{n} + 1 + \log_4(n) \cdot \sqrt{n} =$$

$$O(\log_4(n) \cdot \sqrt{n})$$

$$T(n) \leq O(\log_4(n) \cdot \sqrt{n}) \text{ con } c=3 \text{ para todo } n \geq 3$$

4.

$$T(n) = \begin{cases} 1 & n=1 \\ 2T\left(\frac{n}{2}\right) + c & n \geq 2 \end{cases}$$

suponemos que $n \geq 2$

paso (i)

$$2T\left(\frac{n}{2}\right) + c$$

paso (2)

$$2 \left[2T\left(\frac{n}{2^2}\right) + c \right] + c =$$

$$= 4T\left(\frac{n}{2^2}\right) + 2c + c =$$

$$= 4T\left(\frac{n}{2^2}\right) + 3c$$

paso i

$$2^i T\left(\frac{n}{2^i}\right) + (2^i - 1) \cdot c$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$n \cdot T\left(\frac{n}{2^{\log_2(n)}}\right) + (n - 1) \cdot c =$$

$$= n \cdot T(1) + nc - c_1 =$$

$$= n + nc - c_1 = T(n)$$

$O(n)$

$T(n) \leq O(n)$ con $c = 2c_1 + 1$ para todo $n \geq 1$

5.

$$T(n) = \begin{cases} 1 & n=1 \\ 4 + \left(\frac{n}{2}\right) + n^2 & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ①

$$4T\left(\frac{n}{2}\right) + n^2$$

paso ②

$$\begin{aligned} & 4 \left[4T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2 \right] + n^2 = \\ & = 16T\left(\frac{n}{2^2}\right) + \cancel{4} \frac{n^2}{2^2} + n^2 = \\ & = 16T\left(\frac{n}{2^2}\right) + 2n^2 \end{aligned}$$

paso ③

$$\begin{aligned} & 16 \left[4T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2 \right] + 2n^2 = \\ & = 64T\left(\frac{n}{2^3}\right) + \cancel{16} \frac{n^2}{16} + 2n^2 = \\ & = 64T\left(\frac{n}{2^3}\right) + 3n^2 \end{aligned}$$

paso ④

$$4^i T\left(\frac{n}{2^i}\right) + i n^2$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

si $n = 2^i$
entonces

$$n^2 = (2^i)^2$$

$$(2^i)^2 = (2^2)^i = 4^i$$

reemplazo i

$$n^2 + \left(\frac{n}{2^{\log_2(n)}} \right) + \log_2(n) \cdot n^2 =$$

$$= n^2 \cdot T(1) + \log_2(n) \cdot n^2 =$$

$$= n^2 + \log_2(n) \cdot n^2$$

$$O(\log_2(n) \cdot n^2)$$

$$T(n) \leq O(\log_2(n) \cdot n^2) \text{ con } c=2 \text{ para todo } n \geq 2$$