

12.

$$a) \quad T(n) = \begin{cases} 1, & n \leq 1 \\ T(n-1) + c, & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso ①

$$T(n-1) + c$$

paso ②

$$[T(n-2) + c] + c$$

$$T(n-2) + 2c$$

paso ③

$$[T(n-3) + c] + 2c =$$

$$= T(n-3) + 3c$$

paso i

$$T(n-i) + ic$$

caso base

$$n-i \leq 1$$

$$n \leq 1 + i$$

$$n-1 \leq i$$

reemplazo i

$$T(n - (n-1)) + (n-1) \cdot C =$$

$$= T(1) + (n-1)C = T(n)$$

$$= 1 + nc - C = T(n)$$

calculo orden

$$1 + nk - k$$

orden candidato $\rightarrow n$

siendo k una
constante

primer termino

$$1 \leq n \cdot C_1 \quad \text{para todo } n \gg n_0$$

con $C_1 = 1$ y $n_0 = 1$ sigue valiendo desigualdad.

segundo termino

$$n \cdot k \leq n \cdot C_2 \quad \text{para todo } n \gg n_0$$

con $C_2 = k$ y $n_0 = 0$

tercer termino

al ser negativo no se tiene en cuenta para la justificación
ya que se puede ocurrir con $C=0$ y $n_0=0$

obtención de C y n_0 para $T(n)$

$$1 + nk - k \leq C_1 \cdot n + C_2 \cdot n$$

$$1 + nk - k \leq (C_1 + C_2) \cdot n$$

$$C = C_1 + C_2 = 1 + k \quad n_0 = 1 \rightarrow \text{mas restrictivo}$$

ENTONCES

$$T(n) \leq O(n) \quad \text{con } C = 1 + k \quad \text{para todo } n \gg n_0 \text{ con}$$

$$n_0 = 1$$

b)

$$T(n) = \begin{cases} 1 & n=1 \\ T(n/2) + c & n \geq 2 \end{cases}$$

Suponemos que $n \geq 2$

paso ①

$$T(n/2) + c$$

paso ②

$$\begin{aligned} & \left[T\left(\frac{n}{2^2}\right) + c \right] + c = \\ & = T\left(\frac{n}{2^2}\right) + 2c \end{aligned}$$

paso ③

$$\begin{aligned} & \left[T\left(\frac{n}{2^3}\right) + c \right] + 2c = \\ & = T\left(\frac{n}{2^3}\right) + 3c \end{aligned}$$

paso ④

$$T\left(\frac{n}{2^i}\right) + ic$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$\begin{aligned} & T\left(\frac{n}{2^{\log_2(n)}}\right) + \log_2(n)C = \\ &= T(1) + \log_2(n)C = \\ &= 1 + \log_2(n)C = T(n) \end{aligned}$$

$O(\log_2(n))$

$$c) \quad T(n) = \begin{cases} 1, & n=1 \\ 2T(n/2) + C, & n \geq 2 \end{cases}$$

suponemos que $n \geq 2$

paso ①

$$2T\left(\frac{n}{2}\right) + C$$

paso ②

$$\begin{aligned} & 2 \left[2T\left(\frac{n}{2^2}\right) + C \right] + C = \\ &= 4T\left(\frac{n}{2^2}\right) + 2C + C = \\ &= 4T\left(\frac{n}{2^2}\right) + 3C \end{aligned}$$

paso ③

$$\begin{aligned} & 4 \left[2T\left(\frac{n}{2^3}\right) + C \right] + 3C = \\ &= 8T\left(\frac{n}{2^3}\right) + 4C \end{aligned}$$

paso i

$$2^i \cdot T\left(\frac{n}{2^i}\right) + (2^i - 1) \cdot C$$

caso base

$$\frac{n}{2^i} = 1$$

$$n = 2^i$$

$$\log_2(n) = i$$

reemplazo i

$$n \cdot T\left(\frac{n}{2^{\log_2(n)}}\right) + (n - 1) \cdot C =$$

$$= n \cdot T(1) + nc - C =$$

$$= n \cdot 1 + nc - C =$$

$$= n + nc - C = T(n)$$

$O(n)$

$$d) T(n) = \begin{cases} 1, & n \leq 5 \\ T(n-5) + C, & n \geq 6 \end{cases}$$

suponemos $n \geq 6$

paso (1)

$$T(n-5) + C$$

paso (2)

$$[T(n-10) + C] + C =$$

$$= T(n-10) + 2C$$

paso (3)

$$\left[T(n-15) + c \right] + 2c$$

$$T(n-15) + 3c$$

paso i

$$T(n-5i) + ic$$

caso base

$$n-5i \leq 5$$

$$n \leq 5-5i$$

$$n-5 \leq 5i$$

$$\frac{n-5}{5} \leq i$$

reemplazo i

$$T\left(n-5\left(\frac{n-5}{5}\right)\right) + \left(\frac{n-5}{5}\right)c =$$

$$= T(5) + \left(\frac{n}{5} - \cancel{\frac{5}{5}}\right)c =$$

$$= 1 + \left(\frac{n}{5} - 1\right)c =$$

$$= 1 + \frac{n}{5}c - c =$$

$$= 1 + n \cdot \frac{c}{5} - c = T(n)$$

$O(n)$

e)

$$T(n) = \begin{cases} 1, & n=1 \\ 2T(n-1) + c, & n \geq 2 \end{cases}$$

suponiendo que $n \geq 2$

paso (1)

$$2T(n-1) + c$$

paso (2)

$$2[2T(n-2) + c] + c =$$

$$= 4T(n-2) + 3c$$

paso (3)

$$4[2T(n-3) + c] + 3c =$$

$$= 8T(n-3) + 7c$$

paso (i)

$$2^i T(n-i) + (2^i - 1)c$$

caso base

$$n-i = 1$$

$$n = 1 + i$$

$$\boxed{n-i = 1}$$

reemplazo i

$$2^{n-1} T(n-(n-1)) + (2^{n-1} - 1)c =$$

$$= 2^{n-1} T(1) + (2^{n-1} - 1)c =$$

$$= 2^{n-1} \cdot 1 + (2^{n-1} - 1)c =$$

$$= 2^{n-1} + (2^{n-1} - 1)c$$

$$\begin{aligned}
 & \frac{2^n}{2} + \left(\frac{2^n}{2} - 1 \right) c = \\
 & = 2^n \cdot \frac{1}{2} + \left(2^n \cdot \frac{1}{2} - 1 \right) c = \\
 & = 2^n \cdot \frac{1}{2} + 2^n \cdot c - \frac{c}{2} = T(n)
 \end{aligned}$$

$O(2^n)$

$$f) \quad T(n) = \begin{cases} 1, & n \leq 4 \\ T\left(\frac{n}{8}\right) + c, & n > 8 \end{cases}$$

suponiendo que $n > 8$

paso ①

$$T\left(\frac{n}{8}\right) + c$$

paso ②

$$\left[T\left(\frac{n}{8^2}\right) + c \right] + c =$$

$$= T\left(\frac{n}{8^2}\right) + 2c$$

paso ③

$$\left[T\left(\frac{n}{8^3}\right) + c \right] + 2c =$$

$$= T\left(\frac{n}{8^3}\right) + 3c$$

paso i

$$T\left(\frac{n}{8^i}\right) + ic$$

caso base

$$\frac{n}{8^i} \leq 4$$

$$n \leq 4 \cdot 8^i$$

$$\frac{n}{4} L = 8^i$$

$$\log_8 \left(\frac{n}{4} \right) L = i$$

reemplazo i

$$T \left(\frac{n}{8^{\log_8 \left(\frac{n}{4} \right)}} \right) + \log_8 \left(\frac{n}{4} \right) \cdot C =$$

$$= T \left(\frac{1}{4} \right) + \log_8 \left(\frac{n}{4} \right) C =$$

$$= 1 + \log_8 \left(\frac{n}{4} \right) C =$$

$$= 1 + \left(\log_8 (n) - \log_8 (4) \right) C =$$

$$= 1 + \log_8 (n) \cdot C - \log_8 (4) \cdot C = T(n)$$

$$O \left(\log_8 (n) \right).$$