1) i.

$$T(n) \begin{cases} cte_1 & n <= 1 \\ cte_2 + T(n-1) & n > 1 \end{cases}$$

suponiendo que $n > 1$

Paso ①

$$cte_2 + T(n-1)$$

Paso ②

$$cte_2 + [cte_2 + T(n-2)] =$$

$$= 2cte_2 + T(n-2)$$

Paso ③

$$2\,cte_2 + (cte_2 + T(n-3)] =$$

$$= 3cte_2 + T(n-3)$$

Paso i (paso general)

$$T(n) = i.cte_2 + T(n-i)$$

caso base

$$\boxed{n - i <= 1}$$

$$-i <= 1 - n$$

$$i >= n - 1$$

reemplazo i

$$(n-1).cte_2 + T(n-(n-1)) =$$

$$= (n-1)\,cte_2 + cte_1 = T(n)$$

$$O(n)$$

ii.

$$T(n)\begin{cases} cte_1 & , n <= 1 \\ 2 * T(n-1) + cte_2 & , n > 1 \end{cases}$$

suponiendo que $n > 1$

Paso 1

$$2 * T(n-1) + cte_2$$

Paso 2

$$2 * \left[2 * T(n-2) + cte_2\right] + cte_2 =$$

$$= 4\, T(n-2) + 3\, cte_2$$

Paso 3

$$4\left[2 * T(n-3) + cte_2\right] + 3\,cte_2 =$$

$$= 8\, T(n-3) + 7\, cte_2$$

Paso 4

$$8\left[2 * T(n-4) + cte_2\right] + 7\, cte_2 =$$

$$= 16\, T(n-4) + 15\, cte_2$$

Paso i (paso general)

$$2^i \cdot T(n-i) + (2^i - 1) \cdot cte_2$$

Caso base

$$n - i <= 1$$

$$-i <= 1 - n$$

$$i >= n - 1$$

reemplazo i

$$2^{n-1} \cdot T(n - (n-1)) + (2^{n-1} - 1)\, cte_2 =$$

Escaneado con CamScanner

$$= \left(2^{n-1}\right) \cdot cte_1 + \left(2^{n-1} - 1\right) cte_2 = T(n)$$

$$\frac{2^n}{2} \cdot cte_1 + \left(\frac{2^n}{2} - 1\right) \cdot cte_2 = 2^n \cdot \frac{cte_1}{2} + \left(\frac{2^n}{2} - 1\right) \cdot cte_2$$

$$\downarrow$$

$$O\left(2^n\right) ?$$

iii.

$$T(n) = \begin{cases} cte_1 & n=0 \\ T(n)_2 = \begin{cases} cte_2 & n=1 \\ 2 \cdot T(n-2) + cte_3 & n <> 1 \end{cases} & , n <> 0 \end{cases}$$

suponiendo que $n <> 1$ saco $T_2(n)$

paso 1

$$2 \cdot T(n-2) + cte_3$$

paso 2

$$2 \cdot \left[2 \cdot T(n-4) + cte_3 \right] + cte_3 =$$

$$= 4 \cdot T(n-4) + 2\, cte_3 + cte_3 =$$

$$= 4\, T(n-4) + 3\, cte_3 .$$

paso 3

$$4\left[2 \cdot T(n-6) + cte_3\right] + 3\,cte_3$$

$$= 8 \cdot T(n-6) + 4\,cte_3 + 3\,cte_3 =$$

$$= 8\,T(n-6) + 7\,cte_3.$$

paso i

$$2^i\,T(n-2i) + (2^i - 1) \cdot cte_3$$

caso base

$$n - 2i = 1$$

$$n = 1 + 2i$$

$$n - 1 = 2i$$

$$\boxed{\frac{n-1}{2} = i}$$

reemplazo i

$$2^{\frac{n-1}{2}}\,T\left(n - 2\left(\frac{n-1}{2}\right)\right) + \left(2^{\frac{n-1}{2}} - 1\right) \cdot cte_3$$

$$= 2^{\frac{n-1}{2}}\,T(1) + \left(2^{\frac{n-1}{2}} - 1\right) \cdot cte_3 =$$

$$= 2^{\frac{n-1}{2}} \cdot cte_2 + \left(2^{\frac{n-1}{2}} - 1\right) \cdot cte_3 =$$

$$= \sqrt{2^{n-1}} \cdot cte_2 + \left(\sqrt{2^{n-1}} - 1\right) \cdot cte_3 =$$

$$= \sqrt{\frac{2^n}{2}} \cdot cte_2 + \left(\sqrt{\frac{2^n}{2}} - 1\right) \cdot cte_3 =$$

$$= \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_2 + \left(\sqrt{2^n} \cdot \sqrt{\frac{1}{2}} - 1\right) \cdot cte_3 =$$

$$= \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_2 + \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_3 - cte_3$$

$$T(n) = \begin{cases} cte_1 & n = 0 \\ \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_2 + \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_3 - cte_3 & n <> 0 \end{cases}$$

$cte <= \sqrt{2^n}$.

como se trata de un if, el tiempo es el peor de los casos, que es que $n <> 0$ ya que $\sqrt{2^n}$ crece mas rapido que cte, asi que

$$T(n) = \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_2 + \sqrt{2^n} \cdot \sqrt{\frac{1}{2}} \cdot cte_3 - cte_3$$

$\boxed{\text{orden } \sqrt{2^n}}$