b. Tiempo de rec2

$$T(n) = (n-1)\,cte2 + cte1$$

Tiempo de rec1

$$T(n) = 2^n \cdot \frac{cte1}{2} + \left(\frac{2^n}{2} - 1\right) \cdot cte2$$

$$T(rec2) < T(rec1)$$

Demostración. $T(rec2) < T(rec1)$

cte1 = 2

cte2 = 3

n = 3

$$(3-1) \cdot 3 + 2 < 2^3 \cdot \frac{2}{2} + \left(\frac{2^3}{2} - 1\right) \cdot 3$$

$$6 + 2 < 8 + (4-1) \cdot 3$$

$$8 < 8 + 9$$

$$\boxed{8 < 17}$$

c.

```
static public int rec3 (int n) {
    int x;
    if (n==0)
        return 0;
    else {
        if (n==1)
            return 1;
        else {  x = rec3(n-2);
                return x * x;
        }
    }
}
```

$$T(n) = \begin{cases} cte_1 & n = 0 \\ T(n2) \begin{cases} cte2 & n = 1 \\ T(n-2) + cte3 & n <> 1 \end{cases} & n <> 0 \end{cases}$$

supongamos que $n <> 0$ y $n <> 1$.

paso 1

$$T(n-2) + cte3.$$

paso 2

$$[T(n-4) + cte3] \; cte3 =$$

$$= T(n-4) + 2cte3.$$

paso 3

$$[T(n-6) + cte3] + 2cte3$$

$$T(n-6) + 3\,cte\,3.$$

paso i

$$T(n-2i) + i.cte3.$$

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caso base

$$n-2i = 1$$

$$n = 1+2i$$

$$n-1 = 2i$$

$$\frac{n-1}{2} = i$$

$n=3$ $cte = 4$

reemplazo i

$$T\left(n-2.\left(\frac{n-1}{2}\right)\right) + \frac{n-1}{2}\,cte3 =$$

$$= T(1) + \left(\frac{n}{2} - \frac{1}{2}\right).cte3 =$$

$$= cte2 + n\,\frac{cte3}{2} - \frac{1}{2}.cte3$$

$O(n)$

$$T(rec3\_2) = cte2 + n\,\frac{cte3}{2} - \frac{1}{2}\,cte3$$

$$T(rec3) = \sqrt{2^n}.\sqrt{\tfrac{1}{2}}.cte2 + \sqrt{2^n}.\sqrt{\tfrac{1}{2}}.cte3 - cte3$$

si $cte3 = 2$ $\rightarrow$ reemplazo

$cte2 = 2$

$n = 2$

$$T(rec3\_2) < T(rec3)$$

$$2 + \cancel{2}.\frac{2}{\cancel{2}} - \frac{1}{\cancel{2}}.\cancel{2} < \sqrt{2^2}.\sqrt{\tfrac{1}{2}}.2 + \sqrt{2^2}.\sqrt{\tfrac{1}{2}}.2 - 2$$

$$2 + 2 - 1 < 4.\sqrt{\tfrac{1}{2}} + 4.\sqrt{\tfrac{1}{2}} - 2$$

$$3 < \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} - 2$$

$$3 < 2\sqrt{2} + 2\sqrt{2} - 2$$

$$3 < 4\sqrt{2} - 2$$

$$\boxed{3 < \approx 3{,}65685}$$