Edge adaptive schemes and machine learning for high-accuracy finite volume schemes

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Abstract

Edge-adapted methods have been introduced in the context of image processing [1] to reconstruct high-resolution images from coarser cell averages. In particular, when images consist of piecewise smooth functions, the interfaces can be approximated by a prespecified functional class (lines, circle arcs, etc) through optimization (LVIRA) [3] or specific preprocessing (ENO-EA). In this talk, we will first explore some theoretical aspects of these nonlinear approximation spaces that are useful in the context of inverse problems. Secondly we will show an extension of the ENO-EA approach to polynomials of degree higher than 1 and compare this algebraic approach to that introduced in (LVIRA) as well as to learning-based methods [2] in which an artificial neural network (NN) (or in principle any other non linear sufficiently rich function family) is used to attain the same goal.

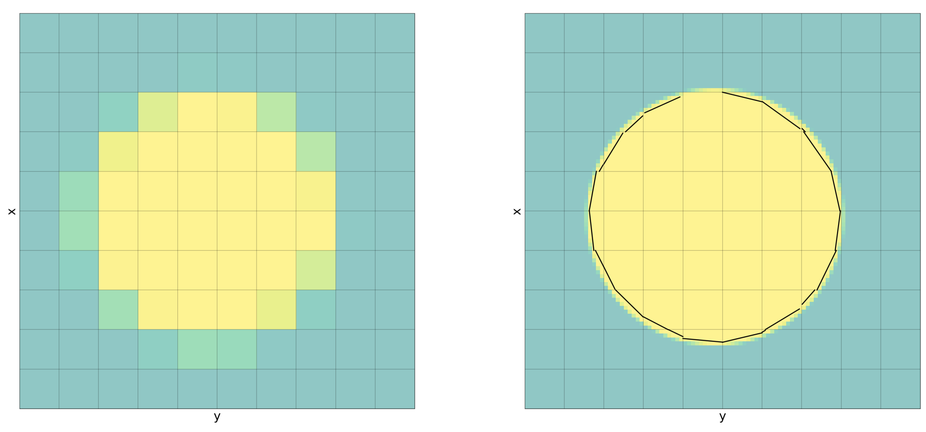
Introduction

In this work we will focus on reconstructing high resolution 2D piece-wise constant images using low-resolution cell-averages information as a first step towards building a finite volume solver. For each cell an approximation of the original image will be inferred locally using the neighboring cells information.

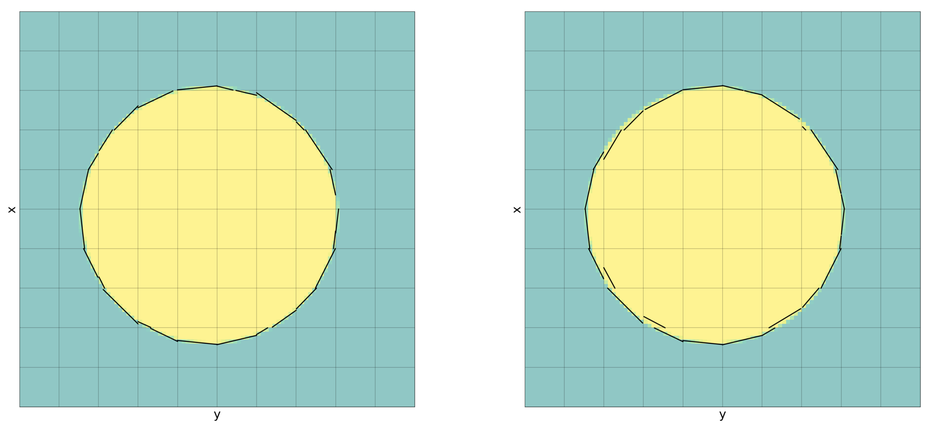
Regular regions: no discontinuity, a 2d-polynomial is fitted using neighboring cell averages.

Edge regions: a discontinuity passing through the cell, a parameterized curve can be fitted through specific inference or optimization taking the reconstruction of neighboring cell averages as the learning loss.

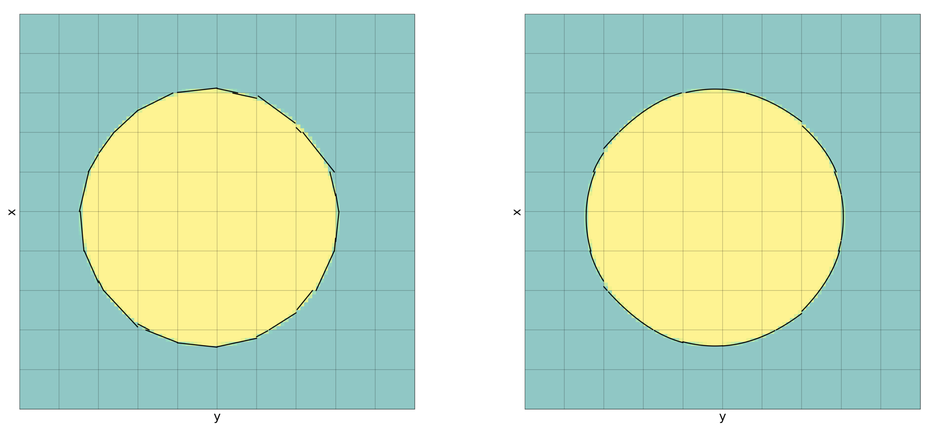
- LinearInterface (LVIRA): a line is fitted to reconstruct the 3x3 centered stencil averages through optimization.
- LinearInterfaceCC: like LinearInterface but adding 100 times more weight on the central cell.
- ELVIRA [3]: the best line over 6 options obtained by calculating the 6 finites differences of the 3x3 centered stencil when it seen as a 1d-function.
- MLLinear: Feed Forward Neural Network to learn the map between 3x3 cell average stencil to line parameters.
- ENO-Quadratic: the quadratic curve that reproduce the vertical averages of the oriented 3xn stencil.



Piecewise constant, LinearInterface

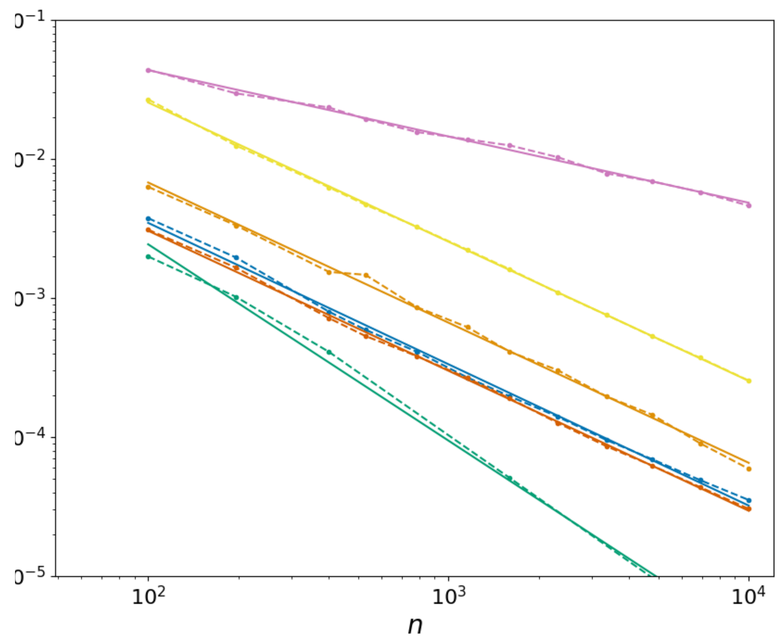


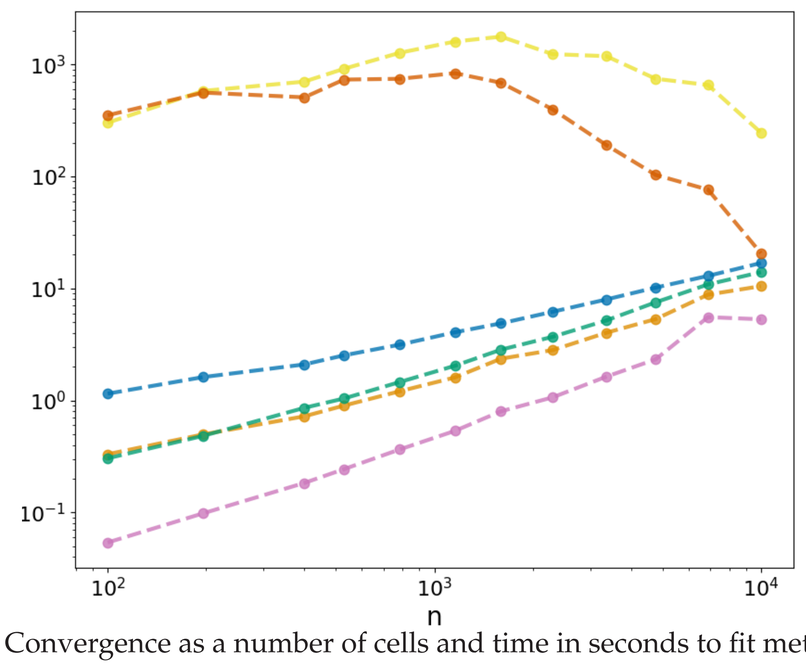
LinearInterfaceCC, ELVIRA



MLLinear, ENO-Quadratic.

Convergence and computing time





Convergence as a number of cells and time in seconds to fit methods.

Vertex regions: the discontinuity passing through the cell forms a vertex.

- OrientedVertex: the angle and orientation allows to build a stencil in which both lines cross the vertical sides. An algebraic calculation can be done to find the parameters of the two lines forming the vertex.
- ExtendedVertex: the gradient of two neighboring curves evaluated at the cell border is used to extend and build the vertex inside the cell.

Arbitrary shape with vertices: using linear, quadratic and vertex methods simultaneously.

Bibliography

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