# Vector Math

SC-T-511-TGRA, Tölvugrafík

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Final exam 2014, Problem 3: Window-2-Viewport mapping:

3) Window-2-Viewport mapping

Points are drawn in a 2D world window $(l,r,b,t)=(-10,30,50,80)$.
In which pixels on a 1600x1200 viewport $(l,b,w,h)=(0,0,1600,1200)$
will the following points be rendered?

a) $P1=(-5,70)$

b) $P2=(20,65)$

$$A=\frac{V_r-V_l}{W_r-W_l}=\frac{1600-0}{30-(-10)}=\frac{1600}{30+10}=\frac{1600}{40}=40$$

$$B=\frac{V_t-V_b}{W_t-W_b}=\frac{1200-0}{80-50}=\frac{1200}{30}=40$$

$$C=V_l-A\cdot W_l=0-40\cdot(-10)=400$$

$$D=V_b-B\cdot W_b=0-40\cdot 50=-2000$$

$$S_x=A\cdot x+C \qquad S_y=B\cdot y+D$$

$$\begin{bmatrix}S_x\\S_y\\1\end{bmatrix}=\begin{bmatrix}A&0&C\\0&B&D\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\1\end{bmatrix}$$

a) $$\begin{bmatrix}40&0&400\\0&40&-2000\\0&0&1\end{bmatrix}\begin{bmatrix}-5\\70\\1\end{bmatrix}=\begin{bmatrix}40\cdot(-5)+400\\40\cdot 70-2000\\1\end{bmatrix}=\begin{bmatrix}-200+400\\2800-2000\\1\end{bmatrix}=\begin{bmatrix}200\\800\\1\end{bmatrix}=\underline{\underline{(200,800)}}$$

b) $$\begin{bmatrix}40&0&400\\0&40&-2000\\0&0&1\end{bmatrix}\begin{bmatrix}20\\65\\1\end{bmatrix}=\begin{bmatrix}40\cdot 20+400\\40\cdot 65-2000\\1\end{bmatrix}=\begin{bmatrix}800+400\\2600-2000\\1\end{bmatrix}=\begin{bmatrix}1200\\600\\1\end{bmatrix}=\underline{\underline{(1200,800)}}$$

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7) Vector intersections and reflections

A line has end points (3,8) and (7,6).
A particle starts at (4,2) and travels along in direction (1,3)

a) In which point does the path of the particle cross the line?

$A = (4,2) \qquad B_1 = (3,8) \qquad V = B_2 - B_1 = (7-3, 6-8) = (4,-2)$

$C = (1,3) \qquad B_2 = (7,6) \qquad n = v^{\perp} = (-v_y, v_x) = (2,4)$

$P(t) = A + tC$

$n \circ ((A + t_{hit}C) - B) = 0$

$P(t_{hit}) = A + t_{hit}C$

$n \circ ((A - B) + t_{hit}C) = 0$

$t_{hit} = \frac{n \cdot (B-A)}{n \cdot C}$

$n \circ (A - B) + n \circ t_{hit}C = 0$

$n \circ (\underset{P_{hit}}{\overset{\shortparallel}{R}} - B) = 0$

$t_{hit}(n \circ C) = n \circ (B - A)$

$t_{hit} = \frac{n \circ (B-A)}{n \circ C}$

$B - A = (3-4, 8-2) = (-1,6)$

$$t_{hit} = \frac{(2,4) \circ (-1,6)}{(2,4) \circ (1,3)} = \frac{2 \cdot (-1) + 4 \cdot 6}{2 \cdot 1 + 4 \cdot 3} = \frac{-2+24}{2+14} = \frac{22}{16} = \frac{11}{8}$$

$P_{hit} = A + t_{hit} \cdot C$

$= (4,2) + \frac{11}{8} \cdot (1,3) = (4,2) + (\frac{11}{8}, \frac{33}{8}) = (5\frac{3}{8}, 6\frac{1}{8})$

$3 < 5\frac{3}{8} < 7$ ✓

Svar: Hits the point $(5\frac{3}{8}, 6\frac{1}{8})$

b) If the particle is made to bounce off the line, what will it's new direction vector be?

$$\frac{\cancel{2}^{1} \cdot \cancel{14}^{7}}{\cancel{20}_{\cancel{10}_{5}}} = \frac{7}{5}$$

$$r = (1,3) - 2 \cdot \frac{(1,3) \cdot (2,4)}{(2,4) \cdot (2,4)} \cdot (2,4)$$

$$= (1,3) - 2 \cdot \frac{1 \cdot 2 + 3 \cdot 4}{2 \cdot 2 + 4 \cdot 4} \cdot (2,4) = (1,3) - 2 \cdot \frac{2+12}{4+16} \circ (2,4)$$

$$= (1,3) - 2 \cdot \frac{14}{20} \cdot (2,4) = (1,3) - \frac{7}{5} \cdot (2,4) = (1,3) - (2\tfrac{4}{5}, 5\tfrac{3}{5})$$

$r = a - 2 \frac{(a \circ n)}{(n \circ n)} \cdot n$

Svar: $= (-1\tfrac{4}{5}, -2\tfrac{3}{5})$

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