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Problem 1

Consider the repeated game in which the following stage game is played twice:

	In	Side	Out
Up	$7,7$	$0,0$	$1,8$
Side	$0,0$	$5,4$	$0,0$
Down	$8,1$	$0,0$	$3,2$

Can the payoff $(7,7)$ be achieved in the first period of a SPNE for some $\delta \in (0,1)$? If so, provide strategies that do so. If not, explain why not.

SOLUTION: In the second round, the two players are playing the stage game in the table above, which has Nash Equilibria payoffs of $(5,4)$ and $(3,2)$. In order for $(7,7)$ to be achieved in the first round, we consider the following strategy:

$$s_1 = \begin{cases} U, & t = 1 \\ S, & \text{if } (U,I) \text{ at } t = 1 \\ D, & \text{else} \end{cases}, \qquad s_2 = \begin{cases} I, & t = 1 \\ S, & \text{if } (U,I) \text{ at } t = 1 \\ O, & \text{else} \end{cases}$$

We see that for player 1, the temptation is $8-7$ and the punishment is $(5-3)\delta$, one can check that player 2 also has the same temptations/punishments. Thus, in order for (U,I) to be chosen, we must have that

$$2\delta \geq 1 \iff \delta \geq \frac{1}{2}.$$

Checking the answer with the one deviation property, we have that if (U,I) in the first period, then

$$u_1 = 7 + 5\delta, \quad u_2 = 7 + 4\delta$$

If player 1 deviates to going down then holding player two fixed, we see that

$$u_1' = 8 + 3\delta$$

and similarly:

$$u_2' = 8 + 2\delta.$$

Thus, we must have that

$$u_1 \geq u_1' \iff 7 + 5\delta \geq 8 + 3\delta \iff 2\delta \geq 1 \iff \delta \geq \frac{1}{2}$$

$$u_2 \geq u_2' \iff 7 + 4\delta \geq 8 + 2\delta \iff \delta \geq \frac{1}{2}.$$

$\blacksquare$

Problem 2

Consider an infinitely repeated game with the following stage games:

	Free trade	Protection
Free trade	f, f	$0, f+p$
Protection	$f+p, 0$	p, p

The parameter $f > 0$ represents the value to a country of access to the other country's markets, whereas the parameter $p > 0$ represents the value to a country of protected access to its own markets. Assume that $f > p$.

(a) For what values of δ is there a SPNE in which the players play (Free trade, Free trade) every period?

SOLUTION: Using the one-deviation property and the symmetry of the game, we consider player 1. With the strategy of (F, F), we see that

$$u_i = f + f\delta + \cdots = \frac{f}{1-\delta}$$

Suppose player i deviates to protection, then since player $-i$ will retaliate with playing protection in the next round, we have that

$$u_i' = f + p + p\delta + p\delta^2 \cdots = f + \frac{p}{1-\delta}.$$

Thus, player i will not deviate if

$$u_i \geq u_i' \iff \frac{f}{1-\delta} \geq f + \frac{p}{1-\delta} \iff f \geq f - f\delta + p \iff f\delta \geq p \iff \delta \geq \frac{p}{f}$$

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(b) For δ arbitrarily close to 1, does there exist a SPNE in which the players take turns opening their markets, i.e., in which they play the following infinitely repeated sequence of outcomes?

$$(\text{Free trade, Protection}), (\text{Protection, Free trade})$$

If yes, why? If not, why not?

SOLUTION: We have that if the player's play the above strategy, then

$$u_1 = 0 + (f+p)\delta + 0 + (f+p)\delta^3 + \cdots = (f+p)\delta(1 + \delta^2 + \delta^4 + \dots) = \frac{(f+p)\delta}{1-\delta^2}$$

$$u_2 = (f+p) + 0 + (f+p)\delta^2 + \cdots = \frac{f+p}{1-\delta^2}.$$

Suppose player 1 deviates and plays protection at every turn, then player 2 will retaliate to p and thus

$$u_1' = p + p\delta + p\delta^2 + \cdots = \frac{p}{1-\delta}$$

Similarly, if player 2 deviates to protect at every turn, then

$$u_2' = f + p + p\delta + p\delta^2 + \cdots = f + \frac{p}{1-\delta}.$$

In order for them not to deviate, we must have that

$$u_1 \geq u_1' \iff \delta\frac{f+p}{1-\delta^2} \geq \frac{p}{1-\delta} \iff \delta f + \delta p \geq p(1+\delta) \iff \delta f \geq p \iff \frac{p}{f} \leq \delta$$

and similarly,

$$u_2 \geq u_2' \iff \frac{f+p}{1-\delta^2} \geq f + \frac{p}{1-\delta} \iff f+p \geq f(1-\delta^2)+p(1+\delta) \iff p \geq -f\delta^2+p+p\delta \iff f\delta \geq p,$$

and so

$$\delta \geq \frac{p}{f}.$$

Thus, we have that the players have no incentive to deviate if $\delta \geq \frac{f}{p}$, and so for $\delta \to 1$, we have that the condition is satisfied and it is indeed an equilibrium.

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