

Tipos abstractos de datos básicos

Algoritmos y Estructuras de Datos II, DC, UBA.

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1. TAD BOOL

TAD BOOL

géneros `bool`

exporta `bool`, generadores, \neg , \vee , \wedge , \Rightarrow , **if • then • else • fi**

igualdad observacional

$$((\text{true} =_{\text{obs}} \text{true}) \wedge (\text{false} =_{\text{obs}} \text{false}) \wedge \neg(\text{true} =_{\text{obs}} \text{false}) \wedge \neg(\text{false} =_{\text{obs}} \text{true}))$$

generadores

`true` : \longrightarrow `bool`

`false` : \longrightarrow `bool`

otras operaciones

$\neg \bullet$: `bool` \longrightarrow `bool`

$\bullet \vee \bullet$: `bool` \times `bool` \longrightarrow `bool`

$\bullet \wedge \bullet$: `bool` \times `bool` \longrightarrow `bool`

$\bullet \Rightarrow \bullet$: `bool` \times `bool` \longrightarrow `bool`

axiomas $\forall x, y: \text{bool}$

$\neg \text{true}$ \equiv `false`

$\neg \text{false}$ \equiv `true`

`true` $\vee x$ \equiv `true`

`false` $\vee x$ \equiv x

`true` $\wedge x$ \equiv x

`false` $\wedge x$ \equiv `false`

$x \Rightarrow y$ \equiv $\neg x \vee y$

if true then a else b fi \equiv

a

if false then a else b fi \equiv

b

Fin TAD

2. TAD NAT

TAD NAT

géneros `nat`

exporta `nat`, generadores, observadores, $+$, $-$, \times , $<$, \leq , mín, máx

usa `BOOL`

igualdad observacional

$$(\forall n, m : \text{nat}) \left(n =_{\text{obs}} m \iff \left((n = 0? =_{\text{obs}} m = 0?) \wedge (\neg(n = 0?) \Rightarrow (\text{pred}(n) =_{\text{obs}} \text{pred}(m))) \right) \right)$$

observadores básicos

$\bullet = 0?$: `nat` \longrightarrow `bool`

$\text{pred} : \text{nat } n \longrightarrow \text{nat} \quad \{ \neg(n = 0?) \}$

generadores

$0 : \longrightarrow \text{nat}$

$\text{suc} : \text{nat} \longrightarrow \text{nat}$

otras operaciones

$\bullet + \bullet : \text{nat} \times \text{nat} \longrightarrow \text{nat}$

$\bullet - \bullet : \text{nat } n \times \text{nat } m \longrightarrow \text{nat} \quad \{ m \leq_{\text{nat}} n \}$

$\bullet \times \bullet : \text{nat} \times \text{nat} \longrightarrow \text{nat}$

$\bullet < \bullet : \text{nat} \times \text{nat} \longrightarrow \text{bool}$

$\bullet \leq \bullet : \text{nat} \times \text{nat} \longrightarrow \text{bool}$

$\text{mín} : \text{nat} \times \text{nat} \longrightarrow \text{nat}$

$\text{máx} : \text{nat} \times \text{nat} \longrightarrow \text{nat}$

axiomas $\forall n, m: \text{nat}$

$0 = 0? \equiv \text{true}$

$\text{suc}(n) = 0? \equiv \text{false}$

$\text{pred}(\text{suc}(n)) \equiv n$

$n + m \equiv \text{if } m = 0? \text{ then } n \text{ else } \text{suc}(n + \text{pred}(m)) \text{ fi}$

$n - m \equiv \text{if } m = 0? \text{ then } n \text{ else } \text{pred}(n) - \text{pred}(m) \text{ fi}$

$n \times m \equiv \text{if } m = 0? \text{ then } 0 \text{ else } n \times \text{pred}(m) + n \text{ fi}$

$n < m \equiv \neg(m = 0?) \wedge (n = 0? \vee \text{pred}(n) < \text{pred}(m))$

$n \leq m \equiv n < m \vee n = m$

$\text{mín}(n, m) \equiv \text{if } m < n \text{ then } m \text{ else } n \text{ fi}$

$\text{máx}(n, m) \equiv \text{if } m < n \text{ then } n \text{ else } m \text{ fi}$

Fin TAD**3. TAD TUPLA($\alpha_1, \dots, \alpha_n$)****TAD TUPLA($\alpha_1, \dots, \alpha_n$)****igualdad observacional**

$(\forall t, t' : \text{tupla}(\alpha_1, \dots, \alpha_n)) (t =_{\text{obs}} t' \iff (\Pi_1(t) =_{\text{obs}} \Pi_1(t') \wedge \dots \wedge \Pi_n(t) =_{\text{obs}} \Pi_n(t')))$

parámetros formales

generadores $\alpha_1, \dots, \alpha_n$

géneros $\text{tupla}(\alpha_1, \dots, \alpha_n)$

exporta $\text{tupla}, \text{generadores}, \text{observadores}$

observadores básicos

$\Pi_1 : \text{tupla}(\alpha_1, \dots, \alpha_n) \longrightarrow \alpha_1$

\vdots

$\Pi_n : \text{tupla}(\alpha_1, \dots, \alpha_n) \longrightarrow \alpha_n$

generadores

$$\langle \bullet, \dots, \bullet \rangle : \alpha_1 \times \dots \times \alpha_n \longrightarrow \text{tupla}(\alpha_1, \dots, \alpha_n)$$

axiomas $\forall a_1: \alpha_1 \dots \forall a_n: \alpha_n$

$$\Pi_1(\langle a_1, \dots, a_n \rangle) \equiv a_1$$

$$\vdots \equiv \vdots$$

$$\Pi_n(\langle a_1, \dots, a_n \rangle) \equiv a_n$$

Fin TAD

4. TAD SECUENCIA(α)

TAD SECUENCIA(α)

igualdad observacional

$$(\forall s, s' : \text{secu}(\alpha)) \left(s =_{\text{obs}} s' \iff \left(\text{vac}\dot{\imath}_{\frac{1}{2}}a?(s) =_{\text{obs}} \text{vac}\dot{\imath}_{\frac{1}{2}}a?(s') \wedge \right. \right. \\ \left. \left. (\neg \text{vac}\dot{\imath}_{\frac{1}{2}}a?(s) \Rightarrow (\text{prim}(s) =_{\text{obs}} \text{prim}(s') \wedge \text{fin}(s) =_{\text{obs}} \text{fin}(s'))) \right) \right)$$

parámetros formales

$$\text{g}\ddot{\imath}_{\frac{1}{2}}\text{neros} \quad \alpha$$

géneros $\text{secu}(\alpha)$

exporta $\text{secu}(\alpha)$, generadores, observadores, &, o, ult, com, long, est $\dot{\imath}_{\frac{1}{2}}?$

usa **BOOL**, **NAT**

observadores básicos

$$\text{vac}\dot{\imath}_{\frac{1}{2}}a? : \text{secu}(\alpha) \longrightarrow \text{bool}$$

$$\text{prim} : \text{secu}(\alpha) \longrightarrow \alpha$$

$$\text{fin} : \text{secu}(\alpha) \longrightarrow \text{secu}(\alpha)$$

$$\{\neg \text{vac}\dot{\imath}_{\frac{1}{2}}a?(s)\}$$

$$\{\neg \text{vac}\dot{\imath}_{\frac{1}{2}}a?(s)\}$$

generadores

$$<> : \longrightarrow \text{secu}(\alpha)$$

$$\bullet \bullet \bullet : \alpha \times \text{secu}(\alpha) \longrightarrow \text{secu}(\alpha)$$

otras operaciones

$$\bullet \circ \bullet : \text{secu}(\alpha) \times \alpha \longrightarrow \text{secu}(\alpha)$$

$$\bullet \& \bullet : \text{secu}(\alpha) \times \text{secu}(\alpha) \longrightarrow \text{secu}(\alpha)$$

$$\text{ult} : \text{secu}(\alpha) \longrightarrow \alpha$$

$$\text{com} : \text{secu}(\alpha) \longrightarrow \text{secu}(\alpha)$$

$$\text{long} : \text{secu}(\alpha) \longrightarrow \text{nat}$$

$$\text{est}\dot{\imath}_{\frac{1}{2}}? : \alpha \times \text{secu}(\alpha) \longrightarrow \text{bool}$$

$$\{\neg \text{vac}\dot{\imath}_{\frac{1}{2}}a?(s)\}$$

$$\{\neg \text{vac}\dot{\imath}_{\frac{1}{2}}a?(s)\}$$

axiomas $\forall s, t: \text{secu}(\alpha), \forall e: \alpha$

$$\text{vac}\dot{\imath}_{\frac{1}{2}}a?(<>) \equiv$$

true

$$\text{vac}\dot{\imath}_{\frac{1}{2}}a?(e \bullet s) \equiv$$

false

$$\text{prim}(e \bullet s) \equiv e$$

$$\text{fin}(e \bullet s) \equiv s$$

$s \circ e \quad \equiv \text{if } \text{vac}\dot{\iota}_{\frac{1}{2}}a?(s) \text{ then } e \bullet <> \text{ else } \text{prim}(s) \bullet (\text{fin}(s) \circ e) \text{ fi}$
 $s \& t \quad \equiv \text{if } \text{vac}\dot{\iota}_{\frac{1}{2}}a?(s) \text{ then } t \text{ else } \text{prim}(s) \bullet (\text{fin}(s) \& t) \text{ fi}$
 $\text{ult}(s) \quad \equiv \text{if } \text{vac}\dot{\iota}_{\frac{1}{2}}a?(\text{fin}(s)) \text{ then } \text{prim}(s) \text{ else } \text{ult}(\text{fin}(s)) \text{ fi}$
 $\text{com}(s) \quad \equiv \text{if } \text{vac}\dot{\iota}_{\frac{1}{2}}a?(\text{fin}(s)) \text{ then } <> \text{ else } \text{prim}(s) \bullet \text{com}(\text{fin}(s)) \text{ fi}$
 $\text{long}(s) \quad \equiv \text{if } \text{vac}\dot{\iota}_{\frac{1}{2}}a?(s) \text{ then } 0 \text{ else } 1 + \text{long}(\text{fin}(s)) \text{ fi}$
 $\text{est}\dot{\iota}_{\frac{1}{2}}?(e, s) \equiv \neg \text{vac}\dot{\iota}_{\frac{1}{2}}a?(s) \wedge (e = \text{prim}(s) \vee \text{est}\dot{\iota}_{\frac{1}{2}}?(e, \text{fin}(s)))$

Fin TAD

5. TAD CONJUNTO(α)

TAD CONJUNTO(α)

igualdad observacional

$$(\forall c, c' : \text{conj}(\alpha)) (c =_{\text{obs}} c' \iff ((\forall a : \alpha)(a \in c =_{\text{obs}} a \in c'))))$$

parámetros formales

$$\text{g}\ddot{\iota}_{\frac{1}{2}}\text{neros} \quad \alpha$$

géneros $\text{conj}(\alpha)$

exporta $\text{conj}(\alpha)$, generadores, observadores, $\emptyset?$, \cup , \cap , $\#$, $\bullet - \{\bullet\}$, dameUno, sinUno, \subseteq , $\bullet - \bullet$

usa **BOOL**, **NAT**

observadores básicos

$$\bullet \in \bullet \quad : \alpha \times \text{conj}(\alpha) \longrightarrow \text{bool}$$

generadores

$$\emptyset \quad : \longrightarrow \text{conj}(\alpha)$$

$$\text{Ag} \quad : \alpha \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

otras operaciones

$$\emptyset? \quad : \text{conj}(\alpha) \longrightarrow \text{bool}$$

$$\# \quad : \text{conj}(\alpha) \longrightarrow \text{nat}$$

$$\bullet - \{\bullet\} \quad : \text{conj}(\alpha) \times \alpha \longrightarrow \text{conj}(\alpha)$$

$$\bullet \cup \bullet \quad : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

$$\bullet \cap \bullet \quad : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

$$\text{dameUno} : \text{conj}(\alpha) \ c \longrightarrow \alpha$$

$$\{-\emptyset?(c)\}$$

$$\text{sinUno} : \text{conj}(\alpha) \ c \longrightarrow \text{conj}(\alpha)$$

$$\{-\emptyset?(c)\}$$

$$\bullet \subseteq \bullet \quad : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{bool}$$

$$\bullet - \bullet \quad : \text{conj}(\alpha) \times \text{conj}(\alpha) \longrightarrow \text{conj}(\alpha)$$

axiomas $\forall c, d: \text{conj}(\alpha), \forall a, b: \alpha$

$$a \in \emptyset \quad \equiv \text{false}$$

$$a \in \text{Ag}(b, c) \quad \equiv (a = b) \vee (a \in c)$$

$$\emptyset?(\emptyset) \quad \equiv \text{true}$$

$$\emptyset?(\text{Ag}(b, c)) \quad \equiv \text{false}$$

$$\#(\emptyset) \quad \equiv 0$$

$$\begin{array}{ll}
\#(\text{Ag}(a, c)) & \equiv 1 + \#(c - \{a\}) \\
c - \{a\} & \equiv c - \text{Ag}(a, \emptyset) \\
\emptyset \cup c & \equiv c \\
\text{Ag}(a, c) \cup d & \equiv \text{Ag}(a, c \cup d) \\
\emptyset \cap c & \equiv \emptyset \\
\text{Ag}(a, c) \cap d & \equiv \text{if } a \in d \text{ then } \text{Ag}(a, c \cap d) \text{ else } c \cap d \text{ fi} \\
\text{dameUno}(c) \in c & \equiv \text{true} \\
\text{sinUno}(c) & \equiv c - \{\text{dameUno}(c)\} \\
c \subseteq d & \equiv c \cap d = c \\
\emptyset - c & \equiv \emptyset \\
\text{Ag}(a, c) - d & \equiv \text{if } a \in d \text{ then } c - d \text{ else } \text{Ag}(a, c - d) \text{ fi}
\end{array}$$

Fin TAD

6. TAD MULTICONJUNTO(α)

TAD MULTICONJUNTO(α)**igualdad observacional**

$$(\forall c, c' : \text{multiconj}(\alpha)) (c =_{\text{obs}} c' \iff ((\forall a : \alpha)(\#(a, c) =_{\text{obs}} \#(a, c'))))$$

parámetros formales

$$\text{gēneros } \alpha$$

gēneros multiconj(α)**exporta** multiconj(α), generadores, observadores, \in , $\emptyset?$, $\#$, \cup , \cap , \in , $\bullet - \{\bullet\}$, dameUno, sinUno**usa** BOOL, NAT**observadores básicos**

$$\# : \alpha \times \text{multiconj}(\alpha) \longrightarrow \text{nat}$$

generadores

$$\emptyset : \longrightarrow \text{multiconj}(\alpha)$$

$$\text{Ag} : \alpha \times \text{multiconj}(\alpha) \longrightarrow \text{multiconj}(\alpha)$$

otras operaciones

$$\bullet \in \bullet : \alpha \times \text{multiconj}(\alpha) \longrightarrow \text{bool}$$

$$\emptyset? : \text{multiconj}(\alpha) \longrightarrow \text{bool}$$

$$\# : \text{multiconj}(\alpha) \longrightarrow \text{nat}$$

$$\bullet - \{\bullet\} : \text{multiconj}(\alpha) \times \alpha \longrightarrow \text{multiconj}(\alpha)$$

$$\bullet \cup \bullet : \text{multiconj}(\alpha) \times \text{multiconj}(\alpha) \longrightarrow \text{multiconj}(\alpha)$$

$$\bullet \cap \bullet : \text{multiconj}(\alpha) \times \text{multiconj}(\alpha) \longrightarrow \text{multiconj}(\alpha)$$

$$\text{dameUno} : \text{multiconj}(\alpha) \ c \longrightarrow \alpha \quad \{-\emptyset?(c)\}$$

$$\text{sinUno} : \text{multiconj}(\alpha) \ c \longrightarrow \text{multiconj}(\alpha) \quad \{-\emptyset?(c)\}$$

axiomas $\forall c, d : \text{multiconj}(\alpha), \forall a, b : \alpha$

$$\#(a, \emptyset) \equiv 0$$

$\#(a, \text{Ag}(b, c))$	\equiv	if $a = b$ then 1 else 0 fi + $\#(a, c)$
$a \in c$	\equiv	$\#(a, c) > 0$
$\emptyset?(\emptyset)$	\equiv	true
$\emptyset?(\text{Ag}(a, c))$	\equiv	false
$\#(\emptyset)$	\equiv	0
$\#(\text{Ag}(a, c))$	\equiv	1 + $\#(c)$
$\emptyset - \{a\}$	\equiv	\emptyset
$\text{Ag}(a, c) - \{b\}$	\equiv	if $a = b$ then c else $\text{Ag}(a, c - \{b\})$ fi
$\emptyset \cup c$	\equiv	c
$\text{Ag}(a, c) \cup d$	\equiv	$\text{Ag}(a, c \cup d)$
$\emptyset \cap c$	\equiv	\emptyset
$\text{Ag}(a, c) \cap d$	\equiv	if $a \in d$ then $\text{Ag}(a, c \cap (d - \{a\}))$ else $c \cap d$ fi
$\text{dameUno}(c) \in c$	\equiv	true
$\text{sinUno}(c)$	\equiv	$c - \{\text{dameUno}(c)\}$

Fin TAD

7. TAD ARREGLO DIMENSIONABLE(α)

TAD ARREGLO DIMENSIONABLE(α)

igualdad observacional

$$(\forall a, a' : \text{ad}(\alpha)) \left(a =_{\text{obs}} a' \iff \left(\text{tam}(a) =_{\text{obs}} \text{tam}(a') \wedge \left((\forall n : \text{nat}) (\text{definido?}(a, n) =_{\text{obs}} \text{definido?}(a', n) \wedge (\text{definido?}(a, n) \Rightarrow a[n] =_{\text{obs}} a'[n])) \right) \right) \right)$$

parámetros formales

$$\text{g}\ddot{\text{I}}\ddot{\text{L}}_{\frac{1}{2}}\text{neros} \quad \alpha$$

géneros $\text{ad}(\alpha)$

exporta $\text{ad}(\alpha)$, generadores, observadores

usa BOOL, NAT

observadores básicos

$$\text{tam} \quad : \text{ad}(\alpha) \longrightarrow \text{nat}$$

$$\text{definido?} \quad : \text{ad}(\alpha) \times \text{nat} \longrightarrow \text{bool}$$

$$\bullet [\bullet] \quad : \text{ad}(\alpha) \ a \times \text{nat} \ n \longrightarrow \alpha \quad \{ \text{definido?}(a, n) \}$$

generadores

$$\text{crearArreglo} : \text{nat} \longrightarrow \text{ad}(\alpha)$$

$$\bullet [\bullet] \leftarrow \bullet \quad : \text{ad}(\alpha) \ a \times \text{nat} \ n \times \alpha \longrightarrow \text{ad}(\alpha) \quad \{ n <_{\text{nat}} \text{tam}(a) \}$$

axiomas $\forall a : \text{ad}(\alpha), \forall e : \alpha, \forall n, m : \text{nat}$

$$\text{tam}(\text{crearArreglo}(n)) \quad \equiv \quad n$$

$$\text{tam}(a [n] \leftarrow e) \quad \equiv \quad \text{tam}(a)$$

$$\text{definido}(\text{crearArreglo}(n), m) \quad \equiv \quad \text{false}$$

$$\text{definido}(a [n] \leftarrow e, m) \quad \equiv \quad n = m \vee \text{definido?}(a, m)$$

$$(a \ [\ n \] \leftarrow e) \ [\ m \] \quad \equiv \quad \text{if } n = m \text{ then } e \text{ else } a \ [\ m \] \text{ fi}$$

Fin TAD

8. TAD PILA(α)

TAD PILA(α)

igualdad observacional

$$(\forall p, p' : \text{pila}(\alpha)) \left(p =_{\text{obs}} p' \iff \left(\text{vac}\ddot{\imath}_{\frac{1}{2}}a?(p) =_{\text{obs}} \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(p') \wedge (\neg \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(p) \Rightarrow \text{tope}(p) =_{\text{obs}} \text{tope}(p') \wedge \text{desapilar}(p) =_{\text{obs}} \text{desapilar}(p')) \right) \right)$$

parámetros formales

$$\text{g}\ddot{\imath}_{\frac{1}{2}}\text{neros} \quad \alpha$$

géneros $\text{pila}(\alpha)$

exporta $\text{pila}(\alpha), \text{generadores}, \text{observadores}, \text{tama}\ddot{\imath}_{\frac{1}{2}}\text{o}$

usa BOOL, NAT

observadores básicos

$$\text{vac}\ddot{\imath}_{\frac{1}{2}}a? : \text{pila}(\alpha) \longrightarrow \text{bool}$$

$$\text{tope} : \text{pila}(\alpha) \longrightarrow \alpha \quad \{ \neg \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(p) \}$$

$$\text{desapilar} : \text{pila}(\alpha) \longrightarrow \text{pila}(\alpha) \quad \{ \neg \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(p) \}$$

generadores

$$\text{vac}\ddot{\imath}_{\frac{1}{2}}a : \longrightarrow \text{pila}(\alpha)$$

$$\text{apilar} : \alpha \times \text{pila}(\alpha) \longrightarrow \text{pila}(\alpha)$$

otras operaciones

$$\text{tama}\ddot{\imath}_{\frac{1}{2}}\text{o} : \text{pila}(\alpha) \longrightarrow \text{nat}$$

axiomas $\forall p : \text{pila}(\alpha), \forall e : \alpha$

$$\text{vac}\ddot{\imath}_{\frac{1}{2}}a?(\text{vac}\ddot{\imath}_{\frac{1}{2}}a) \quad \equiv \quad \text{true}$$

$$\text{vac}\ddot{\imath}_{\frac{1}{2}}a?(\text{apilar}(e, p)) \quad \equiv \quad \text{false}$$

$$\text{tope}(\text{apilar}(e, p)) \quad \equiv \quad e$$

$$\text{desapilar}(\text{apilar}(e, p)) \quad \equiv \quad p$$

$$\text{tama}\ddot{\imath}_{\frac{1}{2}}\text{o}(p) \quad \equiv \quad \text{if } \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(p) \text{ then } 0 \text{ else } 1 + \text{tama}\ddot{\imath}_{\frac{1}{2}}\text{o}(\text{desapilar}(p)) \text{ fi}$$

Fin TAD

9. TAD COLA(α)

TAD COLA(α)

igualdad observacional

$$(\forall c, c' : \text{cola}(\alpha)) \left(c =_{\text{obs}} c' \iff \left(\text{vac}\ddot{\imath}_{\frac{1}{2}}a?(c) =_{\text{obs}} \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(c') \wedge \left(\neg \text{vac}\ddot{\imath}_{\frac{1}{2}}a?(c) \Rightarrow \left(\text{pr}\ddot{\imath}_{\frac{1}{2}}\text{ximo}(c) =_{\text{obs}} \text{pr}\ddot{\imath}_{\frac{1}{2}}\text{ximo}(c') \wedge \text{desencolar}(c) =_{\text{obs}} \text{desencolar}(c') \right) \right) \right) \right)$$

parámetros formales

gñ_l^{1/2}neros	α		
géneros	$\text{cola}(\alpha)$		
exporta	$\text{cola}(\alpha)$, generadores, observadores, tamañ _l ^{1/2} o		
usa	BOOL, NAT		
observadores básicos			
$\text{vací}_l^{\frac{1}{2}}a?$	$\text{cola}(\alpha)$	$\longrightarrow \text{bool}$	
$\text{prí}_l^{\frac{1}{2}}\text{ximo}$	$\text{cola}(\alpha) \ c$	$\longrightarrow \alpha$	$\{\neg \text{vací}_l^{\frac{1}{2}}a?(c)\}$
desencolar	$\text{cola}(\alpha) \ c$	$\longrightarrow \text{cola}(\alpha)$	$\{\neg \text{vací}_l^{\frac{1}{2}}a?(c)\}$
generadores			
$\text{vací}_l^{\frac{1}{2}}a$		$\longrightarrow \text{cola}(\alpha)$	
encolar	$\alpha \times \text{cola}(\alpha)$	$\longrightarrow \text{cola}(\alpha)$	
otras operaciones			
$\text{tamañ}_l^{\frac{1}{2}}o$	$\text{cola}(\alpha)$	$\longrightarrow \text{nat}$	
axiomas	$\forall c: \text{cola}(\alpha), \forall e: \alpha$		
$\text{vací}_l^{\frac{1}{2}}a?(\text{vací}_l^{\frac{1}{2}}a)$		$\equiv \text{true}$	
$\text{vací}_l^{\frac{1}{2}}a?(\text{encolar}(e,c))$		$\equiv \text{false}$	
$\text{prí}_l^{\frac{1}{2}}\text{ximo}(\text{encolar}(e,c))$		$\equiv \text{if vacia?}(c) \text{ then } e \text{ else } \text{prí}_l^{\frac{1}{2}}\text{ximo}(c) \text{ fi}$	
$\text{desencolar}(\text{encolar}(e,c))$		$\equiv \text{if vací}_l^{\frac{1}{2}}a?(c) \text{ then } \text{vací}_l^{\frac{1}{2}}a \text{ else } \text{encolar}(e, \text{desencolar}(c)) \text{ fi}$	
$\text{tamañ}_l^{\frac{1}{2}}o(c)$		$\equiv \text{if vací}_l^{\frac{1}{2}}a?(c) \text{ then } 0 \text{ else } 1 + \text{tamañ}_l^{\frac{1}{2}}o(\text{desencolar}(c)) \text{ fi}$	

Fin TAD

10. TAD Ĩ_l^{1/2}RBOL BINARIO(α)

TAD Ĩ_l^{1/2}RBOL BINARIO(α)

igualdad observacional

$$(\forall a, a' : \text{ab}(\alpha)) \left(a =_{\text{obs}} a' \iff \left(\text{nil?}(a) =_{\text{obs}} \text{nil?}(a') \wedge (\neg \text{nil?}(a) \Rightarrow (\text{raiz}(a) =_{\text{obs}} \text{raiz}(a'))) \right) \right)$$

parámetros formales

gñ_l^{1/2}neros α géneros $\text{ab}(\alpha)$ exporta $\text{ab}(\alpha)$, generadores, observadores, tamañ_l^{1/2}o, altura, tamañ_l^{1/2}o, inorder, preorder, postorderusa BOOL, NAT, SECUENCIA(α)

observadores básicos

nil?	$\text{ab}(\alpha)$	$\longrightarrow \text{bool}$	
raiz	$\text{ab}(\alpha) \ a$	$\longrightarrow \alpha$	$\{\neg \text{nil?}(a)\}$
izq	$\text{ab}(\alpha) \ a$	$\longrightarrow \text{ab}(\alpha)$	$\{\neg \text{nil?}(a)\}$
der	$\text{ab}(\alpha) \ a$	$\longrightarrow \text{ab}(\alpha)$	$\{\neg \text{nil?}(a)\}$

generadores

$\text{nil} \quad : \quad \longrightarrow \text{ab}(\alpha)$
 $\text{bin} \quad : \text{ab}(\alpha) \times \alpha \times \text{ab}(\alpha) \longrightarrow \text{ab}(\alpha)$

otras operaciones

$\text{altura} \quad : \text{ab}(\alpha) \longrightarrow \text{nat}$
 $\text{tama\~{i}}_{\frac{1}{2}} \text{o} : \text{ab}(\alpha) \longrightarrow \text{nat}$
 $\text{inorder} \quad : \text{ab}(\alpha) \longrightarrow \text{secu}(\alpha)$
 $\text{preorder} \quad : \text{ab}(\alpha) \longrightarrow \text{secu}(\alpha)$
 $\text{postorder} : \text{ab}(\alpha) \longrightarrow \text{secu}(\alpha)$

axiomas $\forall a, b: \text{ab}(\alpha), \forall e: \alpha$

$\text{nil?}(\text{nil}) \equiv \text{true}$
 $\text{nil?}(\text{bin}(a, e, b)) \equiv \text{false}$
 $\text{raiz}(\text{bin}(a, e, b)) \equiv e$
 $\text{izq}(\text{bin}(a, e, b)) \equiv a$
 $\text{der}(\text{bin}(a, e, b)) \equiv b$
 $\text{altura}(a) \equiv \text{if nil?}(a) \text{ then } 0 \text{ else } 1 + \text{m\~{a}x}(\text{altura}(\text{izq}(a)), \text{altura}(\text{der}(a))) \text{ fi}$
 $\text{tama\~{i}}_{\frac{1}{2}} \text{o}(a) \equiv \text{if nil?}(a) \text{ then } 0 \text{ else } 1 + \text{tama\~{i}}_{\frac{1}{2}} \text{o}(\text{izq}(a)) + \text{tama\~{i}}_{\frac{1}{2}} \text{o}(\text{der}(a)) \text{ fi}$
 $\text{inorder}(a) \equiv \text{if nil?}(a) \text{ then } <> \text{ else } \text{inorder}(\text{izq}(a)) \ \& \ (\text{raiz}(a) \bullet \text{inorder}(\text{der}(a))) \text{ fi}$
 $\text{preorder}(a) \equiv \text{if nil?}(a) \text{ then } <> \text{ else } (\text{raiz}(a) \bullet \text{preorder}(\text{izq}(a))) \ \& \ \text{preorder}(\text{der}(a)) \text{ fi}$
 $\text{postorder}(a) \equiv \text{if nil?}(a) \text{ then } <> \text{ else } \text{postorder}(\text{izq}(a)) \ \& \ (\text{postorder}(\text{der}(a)) \circ \text{raiz}(a)) \text{ fi}$

Fin TAD**11. TAD DICCIONARIO(CLAVE, SIGNIFICADO)****TAD DICCIONARIO(CLAVE, SIGNIFICADO)****igualdad observacional**

$$(\forall d, d' : \text{dicc}(\kappa, \sigma)) \left(d =_{\text{obs}} d' \iff \left((\forall c : \kappa) (\text{def?}(c, d) =_{\text{obs}} \text{def?}(c, d') \wedge (\text{def?}(c, d) \Rightarrow \text{obtener}(c, d) =_{\text{obs}} \text{obtener}(c, d'))) \right) \right)$$

parámetros formales**g\~{i}l_{\frac{1}{2}}neros** clave, significado**géneros** $\text{dicc}(\text{clave}, \text{significado})$ **exporta** $\text{dicc}(\text{clave}, \text{significado}), \text{generadores}, \text{observadores}, \text{borrar}, \text{claves}$ **usa** **BOOL, NAT, CONJUNTO(CLAVE)****observadores básicos**

$\text{def?} \quad : \text{clave} \times \text{dicc}(\text{clave}, \text{significado}) \longrightarrow \text{bool}$
 $\text{obtener} : \text{clave } c \times \text{dicc}(\text{clave}, \text{significado}) \longrightarrow \text{significado} \quad \{\text{def?}(c, d)\}$

generadores

$\text{vac\~{i}}_{\frac{1}{2}} \text{o} : \longrightarrow \text{dicc}(\text{clave}, \text{significado})$
 $\text{definir} : \text{clave} \times \text{significado} \times \text{dicc}(\text{clave}, \text{significado}) \longrightarrow \text{dicc}(\text{clave}, \text{significado})$

otras operaciones

borrar : clave $c \times \text{dicc}(\text{clave}, \text{significado}) \ d \longrightarrow \text{dicc}(\text{clave}, \text{significado}) \quad \{\text{def?}(c, d)\}$
 claves : $\text{dicc}(\text{clave}, \text{significado}) \longrightarrow \text{conj}(\text{clave})$

axiomas $\forall d: \text{dicc}(\text{clave}, \text{significado}), \forall c, k: \text{clave}, \forall s: \text{significado}$

$\text{def?}(c, \text{vací}_{\frac{1}{2}} o) \equiv \text{false}$
 $\text{def?}(c, \text{definir}(k, s, d)) \equiv c = k \vee \text{def?}(c, d)$
 $\text{obtener}(c, \text{definir}(k, s, d)) \equiv \text{if } c = k \text{ then } s \text{ else obtener}(c, d) \text{ fi}$
 $\text{borrar}(c, \text{definir}(k, s, d)) \equiv \text{if } c = k \text{ then}$
 $\quad \text{if } \text{def?}(c, d) \text{ then borrar}(c, d) \text{ else } d \text{ fi}$
 $\quad \text{else}$
 $\quad \text{definir}(k, s, \text{borrar}(c, d))$
 $\quad \text{fi}$
 $\text{claves}(\text{vací}_{\frac{1}{2}} o) \equiv \emptyset$
 $\text{claves}(\text{definir}(c, s, d)) \equiv \text{Ag}(c, \text{claves}(d))$

Fin TAD

12. TAD COLA DE PRIORIDAD(α)

TAD COLA DE PRIORIDAD(α)

igualdad observacional

$$(\forall c, c' : \text{colaPrior}(\alpha)) \left(c =_{\text{obs}} c' \iff \left(\begin{array}{l} \text{vací}_{\frac{1}{2}} a?(c) =_{\text{obs}} \text{vací}_{\frac{1}{2}} a?(c') \wedge \\ (\neg \text{vací}_{\frac{1}{2}} a?(c) \Rightarrow (\text{prí}_{\frac{1}{2}} \text{ximo}(c) =_{\text{obs}} \text{prí}_{\frac{1}{2}} \text{ximo}(c')) \\ \wedge \\ \text{desencolar}(c) =_{\text{obs}} \text{desencolar}(c') \end{array} \right) \right)$$

parámetros formales

$\text{gí}_{\frac{1}{2}} \text{neros } \alpha$

operaciones $\bullet < \bullet : \alpha \times \alpha \longrightarrow \text{bool}$

$\text{Relaci}_{\frac{1}{2}} n$ de orden total estricto¹

géneros $\text{colaPrior}(\alpha)$

exporta $\text{colaPrior}(\alpha), \text{generadores}, \text{observadores}$

usa **BOOL**

observadores básicos

$\text{vací}_{\frac{1}{2}} a? : \text{colaPrior}(\alpha) \longrightarrow \text{bool}$
 $\text{prí}_{\frac{1}{2}} \text{ximo} : \text{colaPrior}(\alpha) \ c \longrightarrow \alpha \quad \{(\neg \text{vací}_{\frac{1}{2}} a?(c))\}$
 $\text{desencolar} : \text{colaPrior}(\alpha) \ c \longrightarrow \text{colaPrior}(\alpha) \quad \{(\neg \text{vací}_{\frac{1}{2}} a?(c))\}$

generadores

$\text{vací}_{\frac{1}{2}} a : \longrightarrow \text{colaPrior}(\alpha)$
 $\text{encolar} : \alpha \times \text{colaPrior}(\alpha) \longrightarrow \text{colaPrior}(\alpha)$

axiomas $\forall c: \text{colaPrior}(\alpha), \forall e: \alpha$

¹Una $\text{relaci}_{\frac{1}{2}} n$ es un orden total estricto cuando se cumple:

Antirreflexividad: $\neg a < a$ para todo $a: \alpha$

Antisimetrí: $\frac{1}{2} a: (a < b \Rightarrow \neg b < a)$ para todo $a, b: \alpha$

Transitividad: $((a < b \wedge b < c) \Rightarrow a < c)$ para todo $a, b, c: \alpha$

Totalidad: $(a < b \vee b < a)$ para todo $a, b: \alpha$

$\text{vac\ddot{i}}_{i\frac{1}{2}}a?(\text{vac\ddot{i}}_{i\frac{1}{2}}a) \equiv \text{true}$

$\text{vac\ddot{i}}_{i\frac{1}{2}}a?(\text{encolar}(e, c)) \equiv \text{false}$

$\text{pri}_{i\frac{1}{2}}\text{ximo}(\text{encolar}(e, c)) \equiv \text{if } \text{vac\ddot{i}}_{i\frac{1}{2}}a?(c) \vee \text{proximo}(c) < e \text{ then } e \text{ else } \text{pri}_{i\frac{1}{2}}\text{ximo}(c) \text{ fi}$

$\text{desencolar}(\text{encolar}(e, c)) \equiv \text{if } \text{vac\ddot{i}}_{i\frac{1}{2}}a?(c) \vee \text{proximo}(c) < e \text{ then } c \text{ else } \text{encolar}(e, \text{desencolar}(c)) \text{ fi}$

Fin TAD