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SELF-ORGANIZATION IN THE BATTLE OF THE SEXES

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This paper presents a spatial version of the iterated battle of the sexes game in which every one individual plays with his nearest partners and imitates the optimal strategy of his nearest mate neighbors. It is concluded that the spatial structure enables the emergence of clusters of coincident choices, leading to the mean payoff per encounter to values that are accessible only in the cooperative two-person game scenario, which constitutes a notable case of self-organization.

Keywords: Spatial games; asymmetric; memory.

1. Introduction

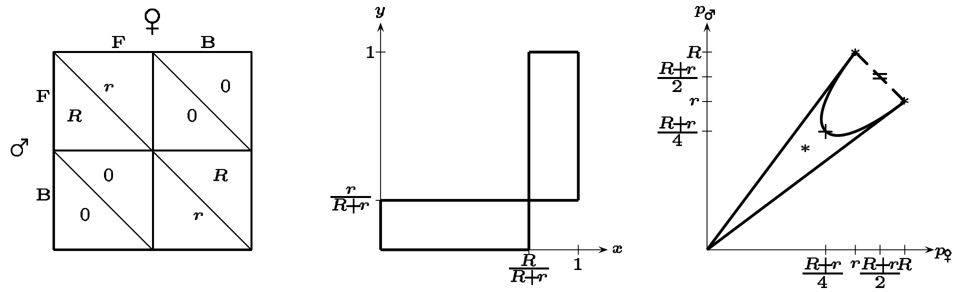
The so-called *battle of the sexes* (BOS for short) is a simple example of a two-person asymmetric game.[8,11] In this game, the preferences of a conventional couple are assumed to fit the traditional stereotypes: the male prefers to attend a Football match, whereas the female prefers to attend a Ballet performance. Both players (which are treated symmetrically), decide in the hope of getting together, so that their payoff matrices are given in the far left panel of Table 1, with rewards $R > r > 0$. Thus, the expected payoffs in the battle of the sexes game, using uncorrelated strategies are:

$$p_{♂}(x;y) = (x, 1-x)\begin{pmatrix} R & 0 \\ 0 & r \end{pmatrix}\begin{pmatrix} y \\ 1-y \end{pmatrix} = ((R+r)y - r)x + r(1-y)$$

$$p_{♀}(y;x) = (x, 1-x)\begin{pmatrix} r & 0 \\ 0 & R \end{pmatrix}\begin{pmatrix} y \\ 1-y \end{pmatrix} = ((R+r)x - R)y + R(1-x)\,.$$

Thus, according to the reaction functions given in Table 1, the pairs of strategies in Nash equilibrium are, the pure (0, 0) and (1, 1), and the mixed $(x^* = R/(R + r), y^* = r/(R+r))$. In the latter, every player assigns to his preferred option (the same) higher probability, i.e. $x^* = 1 - y^* = R/(R+r)$. It is, $p_{♂}(x^*, y^*) = p_{♀}(x^*, y^*) = rR/(R+r) < r < R$, i.e. the geometric mean of R and r.

Table 1. The payoff matrices, reaction functions and payoff region in the *battle of the sexes* game.



Both players get the same payoff, if $y = 1-x$, in which case, $p = (R+r)(1-x)x$. This egalitarian payoff is maximum when $x = y = 1/2$, with $p^+ = (R+r)/4$, the point marked + in the far right panel of Table 1. Thus, the payoff region is closed by the (Pareto efficient) parabola passing by (R, r), (r, R), and (p^+, p^+), as shown in the payoffs panel of Table 1.

In a broader scenario, that of *cooperative* games which allow for correlated strategies, there becomes accessible the payoff region limited by the parabola and the line that joins the (R, r) and (r, R) points. In this scenario, both players may reach a maximum egalitarian payoff $p^= = (R+r)/2$ (the point marked = in the payoff region of Table 1), by agreeing to fully discard the mutually inconvenient *FB* and *BF* combinations and to adopt *FF* and *BB* with 1/2 probability, i.e. $P(F/F) = P(B/B) = 1/2$.

2. The Spatialized Battle of the Sexes

In the spatial version of the BOS we dealt with, each player occupies a site (i, j) in a two-dimensional $N \times N$ lattice. We will consider that *males* and *females* alternate in the site occupation, so that in the chessboard form shown in the far left panel of Table 2, every player is surrounded by four partners (♀-♂, ♂-♀), and four mates (♀-♀, ♂-♂).

In a CA-like implementation, in each generation (T) every player plays with his four nearest-neighbor partners, so that the payoff of a given individual ($p_{i,j}^{(T)}$), is the sum over all interactions with his four nearest-neighbor partners. In the next generation, every player will adopt the choice ($d_{i,j}^{(T)}$) of his nearest-neighbor mate (including himself) that received the highest payoff. In case of a tie, the player maintains his choice.

In the initial scenario of Table 2, every player chooses his preferred choice, except a male in the central part of the lattice that chooses *ballet*. As a result, the general income is nil with the only exception arising from the ♂-ballet choice. This reports four units (assuming $r = 1$) to the initial *deviated* male, and fires the change to *ballet* of the four males connected with the initial ♂-ballet as indicated under $T = 2$

Table 2. The battle of the sexes cellular automaton. $R = 5$, $r = 1$.

♂	♀	♂	♀	♂	♀	♂	♀
♀	♂	♀	♂	♀	♂	♀	♂
♂	♀	♂	♀	♂	♀	♂	♀
♀	♂	♀	♂	♀	♂	♀	♂
♂	♀	♂	♀	♂	♀	♂	♀
♀	♂	♀	♂	♀	♂	♀	♂
♂	♀	♂	♀	♂	♀	♂	♀
♀	♂	♀	♂	♀	♂	♀	♂

$\boldsymbol{T = 1}$

F	*B*	*F*	*B*	*F*	*B*	*F*	*B*
B	*F*	*B*	*F*	*B*	*F*	*B*	*F*
F	*B*	*F*	*B*	*F*	*B*	*F*	*B*
B	*F*	*B*	*B*	*B*	*F*	*B*	*F*
F	*B*	*F*	*B*	*F*	*B*	*F*	*B*
B	*F*	*B*	*F*	*B*	*F*	*B*	*F*
F	*B*	*F*	*B*	*F*	*B*	*F*	*B*
B	*F*	*B*	*F*	*B*	*F*	*B*	*F*

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	5	0	0	0	0
0	0	5	4	5	0	0	0
0	0	0	5	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$\boldsymbol{T = 2}$

F	*B*	*F*	*B*	*F*	*B*	*F*	*B*
B	*F*	*B*	*F*	*B*	*F*	*B*	*F*
F	*B*	*B*	*B*	*B*	*B*	*F*	*B*
B	*F*	*B*	*B*	*B*	*F*	*B*	*F*
F	*B*	*B*	*B*	*B*	*B*	*F*	*B*
B	*F*	*B*	*F*	*B*	*F*	*B*	*F*
F	*B*	*F*	*B*	*F*	*B*	*F*	*B*
B	*F*	*B*	*F*	*B*	*F*	*B*	*F*

0	0	0	0	0	0	0	0
0	0	5	0	5	0	0	0
0	5	4	15	4	5	0	0
0	0	15	4	15	0	0	0
0	5	4	15	4	5	0	0
0	0	5	0	5	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

in Table 2. The change ♂-football to ♂-ballet advances in this way at every time step, so that in this simple example, every player will choose ballet in the long term.

3. Random Starting

The ballet frequency and mean payoff per encounter in the $(R = 5, r = 1)$, $(R = 2, r = 1)$, and $(R = 10, r = 1)$, battle of the sexes cellular automata starting with the same random initial configuration of choices are shown in the far left panels of Figs. 1–3 for ten different initial configurations. All the simulations in these figures are run in a 100×100 lattice with periodic boundary conditions.

Initially, as a result of the random assignment of choices, the frequencies of ballet choice (and that of football) are 0.5, and the mean payoffs per encounter commence at the maximum egalitarian payoff, i.e. the arithmetic mean of the matrix payoff values $(R + r)/4$. Thus, $p^+ = 1.5$, 0.75 and 2.75 respectively.

After the first round, both types of players drift to their preferred choice, and as a consequence, the mean payoff per encounter plummets to low values at $T = 2$. But immediately the drift to the preferred choice becomes moderated, and the mean payoff per encounter recovers. In the long term the ballet frequencies stabilize at

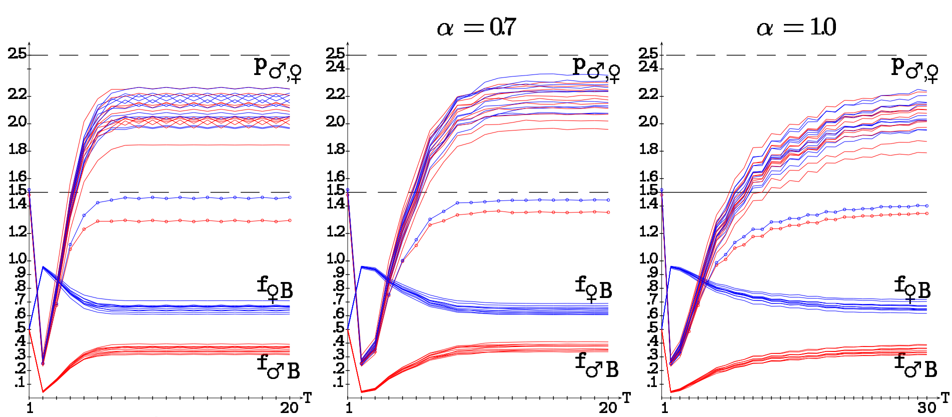


Fig. 1. (Color online) The ballet frequency (f) and mean payoff per encounter (p) in the $(R = 5, r = 1)$ battle of the sexes cellular automaton. Ahistoric (left), $\alpha = 0.7$ (center), and full memory (right) models.

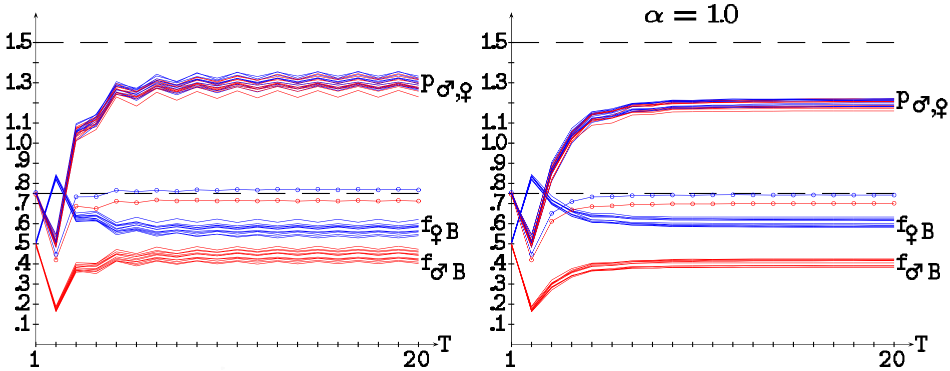


Fig. 2. (Color online) The ballet frequency (f) and mean payoff per encounter (p) in the ($R = 2, r = 1$) battle of the sexes cellular automaton. Ahistoric (left), and full memory (right) models.

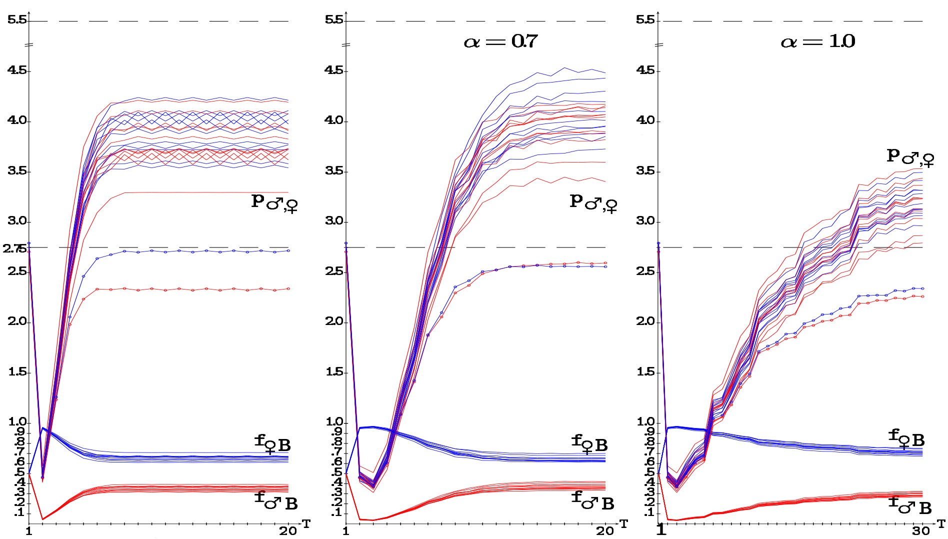


Fig. 3. (Color online) The ballet frequency (f) and mean payoff per encounter (p) in the ($R = 10, r = 1$) battle of the sexes cellular automaton. Ahistoric (left), $\alpha = 0.7$ (center), and full memory (right) models.

values that are not far from that of the values of the ballet probabilities in the mixed equilibrium strategy of a two-person game, $1 - x^* = r/(R+r)$, $1 - y^* = R/(R+r)$, thus, (0.17, 0.83 = 5/6), (0.23, 0.67 = 2/3), and (0.09, 0.91 = 10/11) in Figs. 1–3 respectively.

The dotted curves in Figs. 1–3 show, in one of the simulations, the *theoretical* payoffs of both players in a two-person game with independent strategies using as probabilities the evolving frequencies, namely:

$$p_{\male}^{(T)} = ((R + r)(1 - f_{\female_B}) - r)(1 - f_{\male_B}) + r f_{\female_B}\,,$$

$$p_{\female}^{(T)} = ((R + r)(1 - f_{\male_B}) - R)(1 - f_{\female_B}) + R f_{\male_B}\,.$$

The actual mean payoffs of both kinds of players shown in the figures are over these *expected* values. The divergence starts modestly at $T = 3$, and increases until the virtual stabilization of the p values. In the $R = 2$ scenario, the divergence already becomes apparent at $T = 3$, but in the two other higher R scenarios, the divergence progresses more slowly, although it is already appreciable in the figures at $T = 4$.

The notable increase of the mean payoffs in the proposed cellular automaton is due to the spatial structure, which allows for the emergence of clusters of

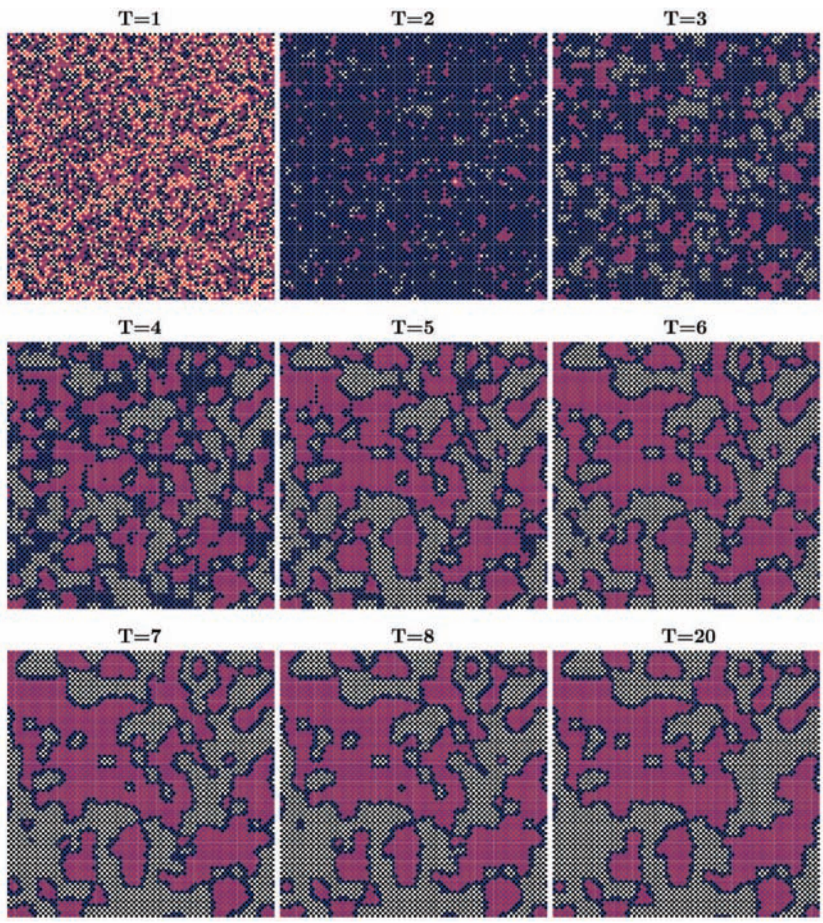


Fig. 4. (Color online) First patterns and pattern at $T = 20$ in the $(R = 5, r = 1)$ battle of the sexes with no memory. 100×100 lattice size. Color code: *red* $\rightarrow \male B$, *blue* $\rightarrow \female B$, *black* $\rightarrow \male F$, *blank* $\rightarrow \female F$. Blue color tends to be masked by red in the agreement clusters.

agreement (or cooperation), shown in Fig. 4 as black-white (FF) and red-blue (BB) regions with interfaces of disagreement among the clusters. This notable case of self-organization of the players explains the high mean payoffs per encounter, clearly in the region accessible only by correlated strategies in the two-person game. It is remarkable that the agreement clusters appear fairly soon, so that in Fig. 4 the pattern at $T = 8$ does not qualitatively differ from that at $T = 20$.

The dynamics in higher size lattices does not qualitatively differ from that presented here in a 100×100 lattice The only difference that can be noticed is a stronger stabilization in the (almost) steady regime, i.e. with a negligible oscillation of the f

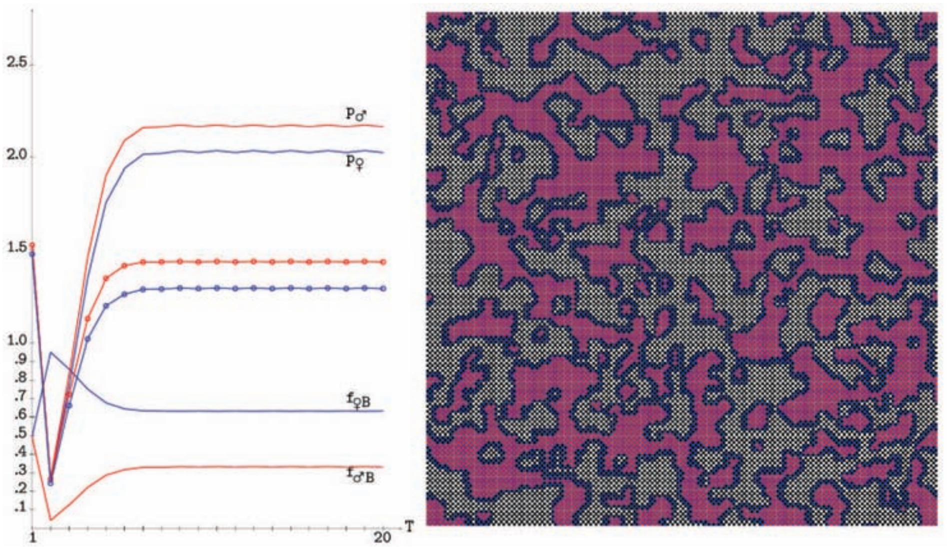


Fig. 5. (Color online) Left: The ballet frequency (f) and mean payoff per encounter (p) in the ahistoric ($R = 5, r = 1$) BOS-CA in a 200×200 lattice. Right: Pattern at $T = 20$.

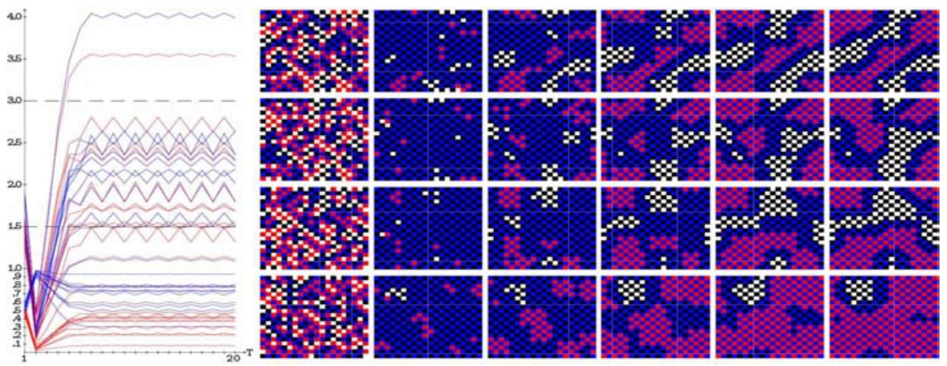


Fig. 6. (Color online) Left: The ballet frequency (f) and mean payoff per encounter (p) in the ($R = 5, r = 1$) ahistoric BOS-CA in a 20×20 lattice size. Right: Initial patterns in four simulations.

and p values. Figure 5 shows an example in the scenario of Fig. 1, but in a 200×200 lattice.

Opposite to this, in small lattices the stabilization is much more problematic, and, more importantly, the initial configuration plays an important role in the evolutionary dynamics. According to this, the two types of curves (f and p) in the left panel of Fig. 6 do not amalgamate as they do in Fig. 1, but they cover a broad interval of values. In two of the ten simulations of Fig. 6 with more stabilization, there is a net drift to one of the choices, that leads in one of them to the mean payoff of the female-type to 4, and to the other one to a mean payoff of the male-type to 3.5. The first six patterns of the former, that one with drift to ballet, is shown in the last row of the right-hand side of Fig. 6, where only a small cluster of FF choices (black-white colors) resists the predominant BB mutual election.

4. Memory

As long as only the results from the last round are taken into account and the outcomes of previous rounds are neglected, the model considered up to now may be termed *ahistoric* (memoryless or Markovian). In the *historic* model we consider in this section, after the time step T:

(i) all the payoffs coming from the previous rounds are accumulated, giving $\pi_{i,j}^{(T)}(p_{i,j}^{(1)}, \ldots, p_{i,j}^{(T)}) = p_{i,j}^{(T)} + \sum_{t=1}^{T-1} \alpha^{T-t} p_{i,j}^{(t)}$, and

(ii) players are featured by a summary of past decisions ($\delta_{i,j}^{(T)}$), and not only the last one.

Thus,

$$m_{i,j}^{(T)}(d_{i,j}^{(1)}, \ldots, d_{i,j}^{(T)}) = \frac{d_{i,j}^{(T)} + \sum_{t=1}^{T-1} \alpha^{T-t} d_{i,j}^{(t)}}{1 + \sum_{t=1}^{T-1} \alpha^{T-t}} \equiv \frac{\omega_{i,j}^{(T)}}{\Omega^{(T)}} \Rightarrow \delta_{i,j}^{(T)} = \mathrm{round}(m_{i,j}^{(T)})\,.$$ [a]

The choice of the *memory factor* $0 \leq \alpha \leq 1$ simulates the remnant memory effect: the limit case $\alpha = 1$ corresponds to equally weighted records (*full* memory model), whereas $\alpha \ll 1$ intensifies the contribution of the most recent states and diminishes the contribution of the past ones (*short-term* memory); the choice $\alpha = 0$ leads to the ahistoric model. Due to the rounding, memory of decisions is not operative if $\alpha \leq 0.5$, i.e. $\delta \equiv d$, but memory of payoffs ($\pi \neq p$) affects the dynamics even with low α values.

Again, in each round or generation T, every player plays with each of his/her four partner-neighbors, the decision $\delta^{(T)}$ in the cell of the mate-neighborhood with

[a] This geometric memory mechanism is *accumulative* in its demand for knowledge of past history: the whole $\{p_{i,j}^{(t)}\}$ and $\{d_{i,j}^{(t)}\}$ series need not be known to calculate $\pi_{i,j}^{(T)}$ and the memory charge $\omega_{i,j}^{(T)}$, the numerator of $m_{i,j}^{(T)}$, while it suffices to sequentially compute: $\pi_{i,j}^{(T)} = \alpha\pi_{i,j}^{(T-1)} + p_{i,j}^{(T)}$, and $\omega_{i,j}^{(T)} = \alpha\omega_{i,j}^{(T-1)} + d_{i,j}^{(T)}$.

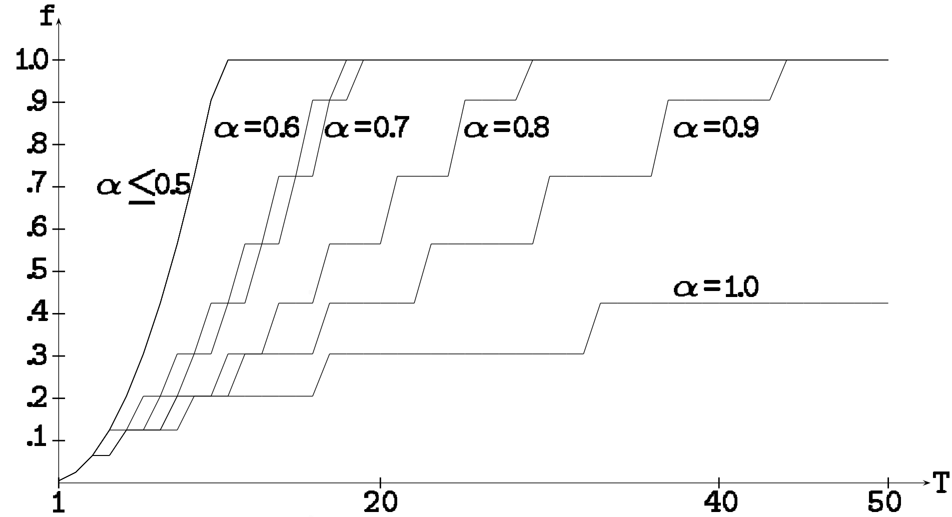


Fig. 7. The frequency of ♂B from a single ♂B in the initial scenario of Table 2 in a 20×20 lattice.

the highest accumulated payoff ($\pi^{(T)}$) being adopted as the choice to play ($d^{(T+1)}$) in the next round.

We have studied the effect of this memory implementation[b] in the spatialized prisoner's dilemma (PD)[1] game in various contexts,[2–6] concluding that memory notably boosts cooperation.

Contrary to the expected after the just-mentioned studies of the effect of memory in the spatialized PD, memory does not boost the ratio of agreement (either FF or BB) in the proposed battle of the sexes cellular automaton. The central and far right panels of Figs. 1–3 indicate that memory tends to decrease the mean payoff, though not at a great extent, by means of a kind of inertial effect that restrains the correction of the initial drift to the preferred choices, that sooner stabilizes the agreement clusters. The evolution with $\alpha = 0.7$ is fairly representative of the effect of memory with the memory factor lying in the [0.5, 1.0) interval, whereas the dynamics with full memory shows a notable intensification of the inertial effect of memory. Low memory charge, namely with the memory factor in the [0.0, 0.5]

[b]The usual approach to considering memory in the spatial formulation of the prisoner's dilemma (PD) consists in designing strategies which determine the next move of a player given the history (genome) of the game. This approach seems at first glance more sophisticated than the somewhat simple-minded adopted here. But it is to be stressed that, at least concerning the memory mechanism activated, this appreciation may be debatable. Thus, for example, in the papers by Lindgren and Nordahl,[9,10] only memory of moves is kept, whereas scores are treated in a Markovian way. We have explored the effect of embedded memory when the players are not spatially structured in Ref. 7. This paper deals with the *Paulov* strategy, a robust strategy that also resulted in the successful probabilistic simulations with genome-type memory in Ref. 12.

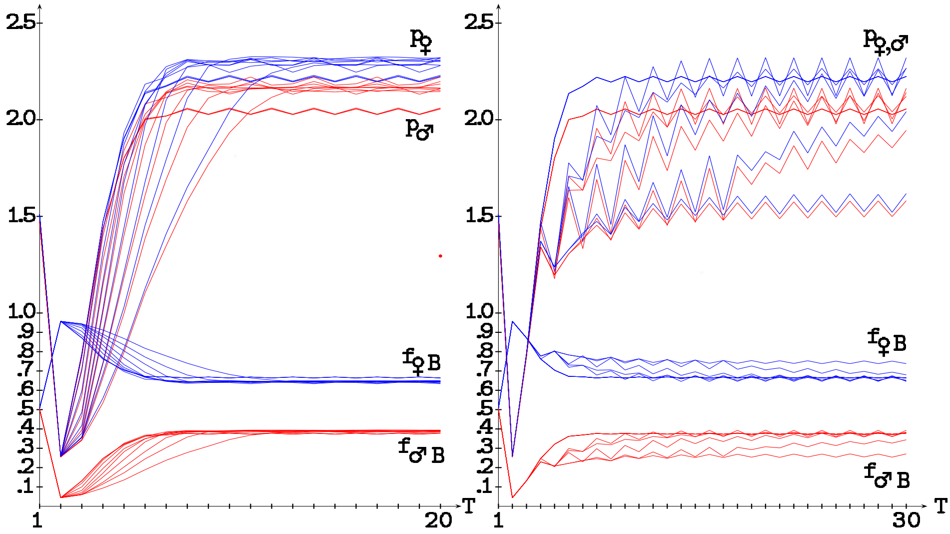


Fig. 8. (Color online) Effect of partial memory in a (5, 1)-BOS-CA. Left: Only memory of payoffs. Right: Only memory of choices.

interval, has little effect in the dynamics, as shown in the left panel of Fig. 8, in which only memory of payoffs is implemented.

As an additional example of the inertial effect of memory in the proposed BOS-CA, Fig. 7 shows the frequency of ♂B from a single ♂B in the initial scenario of Table 2 in a 20×20 lattice. The dynamics are shown up to $T = 50$, in the ahistoric model and with the memory factor varying from 0.0 to 1.0 by 0.1 intervals. In this scenario, the evolution with $\alpha \leq 0.5$ (so with no memory of choices) coincides with the ahistoric one, whereas the velocity of the increase of ♂B frequency falls with $\alpha > 0.5$ memory, by increasing the duration of the stable periods in a *punctuated* equilibrium-like manner. In the full memory context ($\alpha = 1.0$), the stable periods progressively increase their duration in such a way that the full ♂B-occupation is not achieved up to $T = 513$.

Figure 8 refers to the effect of partial memory in the (5, 1)-BOS-CA acting from the same initial configuration, that is shown in Fig. 4. In the left frame of Fig. 8, only memory of payoffs is kept, whereas in the right frame only memory of choices is implemented. Endowing the dynamics with only memory of payoffs does not exert a notable disruptive effect with respect to the ahistoric evolution. Thus, the left panel of Fig. 8 shows a short altered initial transition period, soon stabilized (around $T = 10$) at a fairly similar plateau. In contrast, the implementation of memory of choices with Markovian treatment of payoffs (right panel) generates an oscillatory-like behavior. This oscillation tends to rapidly reach the ahistoric levels (i.e. the less oscillatory lines), except with full memory, in which case the mean payoff oscillation appears fairly stabilized around 1.5.

5. Conclusions and Future Work

The spatial structure enables the emergence of clusters of coincident choices in the iterated battle of the sexes game, leading to the mean payoff per encounter to values that are accessible only in the cooperative two-person game scenario, which constitutes a notable case of self-organization. Memory of past payoffs and choices induces an inertial effect that leads the evolving dynamics to mean payoffs that are slightly under those achieved in the ahistoric scenario.

As a natural extension of the binary model adopted here, the strong 0–1 assumption underlying the model can be relaxed by allowing for *degrees* of choices in a continuous-valued scenario. Denoting by x the degree of F choice of the ♂-player and by y the degree of F choice of the ♀-player, a consistent way to specify the pay-off for values of x and y, other than zero or one, is to simply interpolate between the extreme payoffs of the binary case. Thus, the expected payoffs in the probabilistic strategies binary scenario given in the introduction, become the actual payoff in the continuous-valued context. We have studied such an approach dealing within the spatialized PD game,[3,6] and we plan to implement it in the battle of the sexes game in the near future.

Turning the deterministic mechanism of updating the configurations into a probabilistic one, will allow to relate somehow the results presented here with other *ordering* processes in spatialized social relations. Thus, for example, the "antiferromagnetic" model proposed in Ref. 14.

In real-life situations the preferential choice and refusal of partners plays an important role in the emergence of cooperation.[13] We have studied the effect of memory on a simple, deterministic, structurally dynamic PD game, in which state and link configurations are *both* dynamic and are continually interacting.[4] Further study is due on the structurally dynamic battle of the sexes. Not only in the basic model studied here, but also with probabilistic updating of the configurations, allowing for degrees of cooperation, and endowing memory in the dynamics. The study of the effect of memory in other structurally spatialized games, as well as in games on networks[2] is also planned as a future work.

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