

Unconstrained Minimization Problems

Your Name

November 14, 2023

Introduction

This document presents the optimization of various unconstrained minimization problems. The optimization algorithms used include [mention the algorithms you are using, e.g., gradient descent, Newton's method, etc.].

Optimization Problems

Helical Valley Function (7)

Objective Function

$$f(x, y, z) = 100 * (z - 10 \operatorname{atan2}(y, x))^2 + (\sqrt{x^2 + y^2} - 1)^2 + z^2$$

This is a function of three variables and have some trigonometric functions.

Optimization Results

To optimize the function we need to find the values of x,y,z z that minimize or maximize the function. Optimization involves finding critical points where the partial derivatives of the function are equal to zero or undefined. These critical points can be minima, maxima, or saddle points.

The partial derivatives of f with respect to x, y, and z are given by:

$$f(x, y, z) = 100 * (z - 10 \operatorname{atan2}(y, x))^2 + (\sqrt{x^2 + y^2} - 1)^2 + z^2$$

$$\frac{\partial f}{\partial x} = \frac{2000y(z - 10 \operatorname{atan2}(y, x))}{x^2 + y^2} + \frac{2(\sqrt{x^2 + y^2} - 1)x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y} = -\frac{2000 \cdot (z - 10 \operatorname{atan2}(y, x))x}{x^2 + y^2} + \frac{2(\sqrt{x^2 + y^2} - 1)y}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial y} = -\frac{2000 \cdot (z - 10 \operatorname{atan2}(y, x))x}{x^2 + y^2} + \frac{2(\sqrt{x^2 + y^2} - 1)y}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial f}{\partial z} = 200(z - 10 \operatorname{atan2}(y, x)) + 2z$$

It looks like you've found the optimal solution for your function [1.00000000e+00,3.25151277e-14,6.60698054e-14], and the corresponding optimal function value is 4.525713535408354e-26 4.525713535408354e-26. x0 was (-1,0,0) This result indicates that at the point (1.00000000e+00,3.25151277e-14,6.60698054e-14) , the function f reaches its minimum value of approximately 4.525713535408354e-26 4.525713535408354e-26. This is a very close-to-zero value, suggesting that the function is essentially at its global minimum at this point. and 2 iterations known minimum is (1,0,0) It's important to note that the quality of the optimization result can be influenced by the optimization algorithm used, as well as the initial values chosen for the variables

Biggs EXP6 Function (18)

Objective Function

To minimize the given function, biggs exp6 function, you can employ optimization techniques to find the values of the variable x that minimize the overall sum of squared differences between the expression $\exp(-t \cdot x)$

$$f(x, t, y) = \sum_{i=1}^n (\exp(-t_i \cdot x_i) - y_i)^2$$

$\exp(-t \cdot x)$ and the target values y . One commonly used approach is to use optimization algorithms, such as gradient descent or other numerical optimization methods.

$$\frac{\partial f}{\partial x_i} = -2 \sum_{i=1}^n t_i \cdot \exp(-t_i \cdot x_i) \cdot (\exp(-t_i \cdot x_i) - y_i)$$

Optimization Results

$$\frac{\partial f}{\partial x_i} = -2 \sum_{i=1}^n t_i \cdot \exp(-t_i \cdot x_i) \cdot (\exp(-t_i \cdot x_i) - y_i)$$

$$H_{ij} = 2 \sum_{k=1}^n t_k^2 \cdot \exp(-t_k \cdot x_k) \cdot (\exp(-t_k \cdot x_k) - y_k) \cdot \delta_{ij} + 2 \sum_{k=1}^n t_k^2 \cdot \exp(-t_k \cdot x_k) \cdot (\exp(-t_k \cdot x_k) - y_k)^2 \cdot \exp(-t_k \cdot x_i - t_k \cdot x_j)$$

x_0 was: [1. 2. 1. 1. 1. 1.] minimum was also [1. 2. 1. 1. 1. 1.] had 0 itera It looks like you have provided the optimal solution and the optimal function value for the given Biggs EXP6 function. The optimal solution is a vector
, and the corresponding optimal function value is approximately 48.74

Gaussian Function (9)

In the context of optimization, it might be used as a component of a larger optimization problem or as a simple test case.

Objective Function

$$f(x) = \sum_{i=1}^7 \left(x_0 \cdot \exp \left(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2 \right) - y_i \right)^2 + \sum_{i=1}^7 \left(x_0 \cdot \exp \left(-\frac{x_1}{2} \cdot (t_{2,i} - x_2)^2 \right) - y_i \right)^2 + \left(x_0 \cdot \exp \left(-\frac{x_1}{2} \cdot x_2^2 \right) - 0.3989 \right)^2$$

0.1 Optimization results

$$\nabla f = \begin{bmatrix} 2 \sum_{i=1}^7 (x_0 \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) - y_i) \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) \\ 2x_0 \left(\sum_{i=1}^7 (t_{1,i} - x_2)^2 \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) + \sum_{i=1}^7 (t_{2,i} - x_2)^2 \cdot \exp(-\frac{x_1}{2} \cdot (t_{2,i} - x_2)^2) + x_2^2 \cdot \exp(-\frac{x_1}{2} \cdot x_2^2) \right) \\ -2x_0 \cdot x_1 \left(\sum_{i=1}^7 (t_{1,i} - x_2) \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) + \sum_{i=1}^7 (t_{2,i} - x_2) \cdot \exp(-\frac{x_1}{2} \cdot (t_{2,i} - x_2)^2) + x_2 \cdot \exp(-\frac{x_1}{2} \cdot x_2^2) \right) \end{bmatrix}$$

The value of the Gaussian function is: 0.10207049569085339 Optimal x : [0.52388083 0.7058404 0.48088152]
Minimum value of the function: 0.0014489424560909297 x_0 : (0.4, 1, 0) Iterations: 11

$$H = \begin{bmatrix} 2 \sum_{i=1}^7 \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) & 2x_0 \left(\sum_{i=1}^7 (t_{1,i} - x_2)^2 \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) + \sum_{i=1}^7 (t_{2,i} - x_2)^2 \cdot \exp(-\frac{x_1}{2} \cdot (t_{2,i} - x_2)^2) + x_2^2 \cdot \exp(-\frac{x_1}{2} \cdot x_2^2) \right) \\ 2x_0 \left(\sum_{i=1}^7 (t_{1,i} - x_2)^2 \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) + \sum_{i=1}^7 (t_{2,i} - x_2)^2 \cdot \exp(-\frac{x_1}{2} \cdot (t_{2,i} - x_2)^2) + x_2^2 \cdot \exp(-\frac{x_1}{2} \cdot x_2^2) \right) & 2x_0 \cdot x_1 \left(\sum_{i=1}^7 (t_{1,i} - x_2) \cdot \exp(-\frac{x_1}{2} \cdot (t_{1,i} - x_2)^2) + \sum_{i=1}^7 (t_{2,i} - x_2) \cdot \exp(-\frac{x_1}{2} \cdot (t_{2,i} - x_2)^2) + x_2 \cdot \exp(-\frac{x_1}{2} \cdot x_2^2) \right) \end{bmatrix}$$

Three-dimensional box function (12)

Objective Function

$$f(x) = \|\exp(-t \cdot x[0]) - \exp(-t \cdot x[1]) - x[2] \cdot (\exp(-t) - \exp(-10 \cdot t))\|$$

0.1.1 Optimization results

$$\frac{\partial f}{\partial x} = 4x(x^2 + y - 11) + 2(x + y^2 - 7)$$

$$\frac{\partial f}{\partial y} = 2(x^2 + y - 11) + 4y(x + y^2 - 7)$$

$$\frac{\partial f}{\partial z} = 2z$$

Optimal x: [1.00035547e+01 1.00035551e+01 2.03012288e-08] Minimum value of the function: 2.6852718877011665e-08 iterations: 21 x0=(10,10,10) known minimum is x1=x2 and x3=0

$$\nabla f = \begin{bmatrix} -\sum_{i=1}^{10} t_i \cdot \exp(-t_i \cdot x[0]) \\ \sum_{i=1}^{10} t_i \cdot \exp(-t_i \cdot x[1]) \\ -\sum_{i=1}^{10} t_i \cdot (\exp(-t_i) - \exp(-10 \cdot t_i)) \end{bmatrix}$$

0.2 Function of variable dimensions (25)

0.2.1 Objective function

$$f(x) = \|[x_1 - 1, x_2 - 1, \dots, x_n - 1, \sum_{i=1}^n i \cdot (x_i - 1), \left(\sum_{i=1}^n i \cdot (x_i - 1)\right)^2]\|^2$$

0.2.2 Optimization results

$$\nabla f = 2 \cdot \begin{bmatrix} x_1 - 1 \\ x_2 - 1 \\ \vdots \\ x_n - 1 \\ \sum_{i=1}^n i \cdot (x_i - 1) \\ 2 \cdot \sum_{i=1}^n i \cdot (x_i - 1) \cdot \sum_{i=1}^n i \cdot (x_i - 1) \end{bmatrix}$$

Optimal x: [1. 1.00000003 1.00000001 1.00000001 0.99999997] Minimum value of the function: 4.4519732194048033e-08 known minimum is (1,1,1....1) Iterations: 34

0.3 Watson function (20)

0.3.1 Objective function

$$f(x) = \|[a_1 - b_1, a_2 - b_2, \dots, a_{29} - b_{29}, x_1, x_2 + x_1^2 - 1]\|$$

0.3.2 Optimization results

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(j - 2 \cdot x_1 \cdot \left(\frac{1}{29}\right)^{j-1}\right) \\ \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(2 \cdot x_2 + 2 \cdot j \cdot x_1 \cdot \left(\frac{1}{29}\right)^{j-1}\right) \\ \vdots \\ \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(2 \cdot x_n + 2 \cdot j \cdot x_{n-1} \cdot \left(\frac{1}{29}\right)^{j-1}\right) \end{bmatrix}$$

$$H = \begin{bmatrix} \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(j \cdot (j-2) - 2 \cdot j \cdot x_1 \cdot \left(\frac{1}{29}\right)^{j-1}\right) & \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(2 \cdot j \cdot x_1 \cdot \left(\frac{1}{29}\right)^{j-1}\right) & \cdots \\ \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(2 \cdot j \cdot x_1 \cdot \left(\frac{1}{29}\right)^{j-1}\right) & \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(2 \cdot j \cdot x_2 + 2 \cdot j \cdot x_1 \cdot \left(\frac{1}{29}\right)^{j-1}\right) & \cdots \\ \vdots & \vdots & \ddots \\ 0 & 0 & \cdots \sum_{j=1}^n j \cdot \left(\frac{1}{29}\right)^{j-1} \cdot \left(2 \cdot j \cdot x_{n-1} \cdot \left(\frac{1}{29}\right)^{j-1}\right) \end{bmatrix}$$

Optimal x: [-1.06312021e-08 9.97966220e-01 1.43529299e-02 3.16885350e-01 -3.19036044e-02 4.94903016e-02]
 Minimum value of the function: 0.006453858327920113
 array([-0.12562862, -0.0959441, -0.01129189, 0.10207429, 0.]) x0=(0,0,0,...,0,0)

0.4 Penalty function I (23)

0.4.1 Objective function

$$f(x) = \|[a \cdot x_1 - 1, a \cdot x_2 - 1, \dots, a \cdot x_n - 1, \|x\|^2 - \frac{1}{4}]\|^2$$

0.4.2 Optimization results

$$\nabla f = \begin{bmatrix} 2 \cdot a \cdot (a \cdot x_1 - 1) \\ 2 \cdot a \cdot (a \cdot x_2 - 1) \\ \vdots \\ 2 \cdot a \cdot (a \cdot x_n - 1) \\ 4 \cdot x_1 \\ 4 \cdot x_2 \\ \vdots \\ 4 \cdot x_n \end{bmatrix}$$

$$\equiv \begin{bmatrix} 2 \cdot a^2 & 0 & \cdots & 0 & 4 & 0 & \cdots & 0 \\ 0 & 2 \cdot a^2 & \cdots & 0 & 0 & 4 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2 \cdot a^2 & 0 & 0 & \cdots & 4 \\ 4 & 0 & \cdots & 0 & 4 & 0 & \cdots & 0 \\ 0 & 4 & \cdots & 0 & 0 & 4 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 4 & 0 & 0 & \cdots & 4 \end{bmatrix}$$

Optimal x: [-1.06312021e-08 9.97966220e-01 1.43529299e-02 3.16885350e-01 -3.19036044e-02 4.94903016e-02]
 Minimum value of the function: 0.006453858327920113 x0=(1,2,3,4,5,6..n)

0.5 Penalty function II (24)

0.5.1 Objective function

$$f(x) = \|[x_1 - 0.2, a^{0.5} \cdot (\exp(x_2/10) + \exp(x_1/10) - \exp((1/10, 2/10, \dots, (n-1)/10)) + a^{0.5} \cdot (\exp(x_3/10) + \exp(x_2/10) - \exp((2/10, 3/10, \dots, (n-1)/10))\|$$

0.5.2 Optimization results

$$\nabla f = \begin{bmatrix} 2 \cdot a^{0.5} \cdot \left(\frac{\exp(x_2/10)}{10} + \frac{\exp(x_1/10)}{10} - \frac{\exp((1/10, 2/10, \dots, (n-1)/10))}{10} \right) \\ -2 \cdot a^{0.5} \cdot \frac{\exp((1/10, 2/10, \dots, (n-1)/10))}{10} \\ \vdots \\ -2 \cdot a^{0.5} \cdot \frac{\exp((1/10, 2/10, \dots, (n-1)/10))}{10} \\ 2 \cdot a^{0.5} \cdot \left(\frac{\exp(x_2/10)}{10} \right) \\ 2 \cdot a^{0.5} \cdot \left(\frac{\exp(x_3/10)}{10} \right) \\ \vdots \\ 2 \cdot a^{0.5} \cdot \left(\frac{\exp(x_n/10)}{10} \right) \\ 2 \cdot \sum_{j=1}^n (n-j) \cdot x_j \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 2 \cdot a^{0.5} \cdot \left(\frac{\exp(x_2/10)}{10} + \frac{\exp(x_1/10)}{10} - \frac{\exp((1/10, 2/10, \dots, (n-1)/10))}{10} \right) & -2 \cdot a^{0.5} \cdot \frac{\exp((1/10, 2/10, \dots, (n-1)/10))}{10} & \dots \\ 0 & -2 \cdot a^{0.5} \cdot \frac{\exp((1/10, 2/10, \dots, (n-1)/10))}{10} & 4 \cdot a^{0.5} \cdot \left(\frac{\exp((2/10, 3/10, \dots, (n-1)/10))}{10} \right) & \dots \\ \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \quad \text{Optimal}$$

0.6.1 Objective function

$$f(x) = (x_1 - 1 \times 10^6)^2 + (x_2 - 2 \times 10^{-6})^2 + (x_1 \times x_2 - 2)^2$$

0.6.2 Optimization results

Optimal solution: [1.00000000e+06 1.99254942e-06] Optimal function value: 5.551114377728232e-05 Iterations: 14 x0=(1,1)

$$\nabla f = \begin{bmatrix} 2 \cdot (x_1 - 1 \times 10^6) + 2 \cdot (x_1 \times x_2 - 2) \cdot x_2 \\ 2 \cdot (x_2 - 2 \times 10^{-6}) + 2 \cdot (x_1 \times x_2 - 2) \cdot x_1 \end{bmatrix}$$

gradient is 0 therefore is a critical point and hessian it's negative there fore is a minimum

$$\nabla f = \begin{bmatrix} 2 \cdot (x_1 - 1 \times 10^6) + 2 \cdot (x_1 \times x_2 - 2) \cdot x_2 \\ 2 \cdot (x_2 - 2 \times 10^{-6}) + 2 \cdot (x_1 \times x_2 - 2) \cdot x_1 \end{bmatrix}$$

0.7 Brown and Dennis function (16)

0.7.1 Objective function

$$f(x) = \|[x_1 + t \cdot x_2 - \exp(t), x_3 + x_4 \cdot \sin(t) - \sin(t)]\|^2$$

0.7.2 Optimization results

$$\nabla f = \begin{bmatrix} 2 \cdot \sum_{i=1}^m (x_1 + t_i \cdot x_2 - \exp(t_i)) \\ 2 \cdot \sum_{i=1}^m (x_2 \cdot t_i \cdot (x_1 + t_i \cdot x_2 - \exp(t_i))) \\ 2 \cdot \sum_{i=1}^m (x_3 + x_4 \cdot \sin(t_i) - \sin(t_i)) \\ 2 \cdot \sum_{i=1}^m (x_4 \cdot \sin(t_i) \cdot (x_3 + x_4 \cdot \sin(t_i) - \sin(t_i))) \end{bmatrix}$$

$$H = \begin{bmatrix} 2 \cdot m & 2 \cdot \sum_{i=1}^m t_i \cdot (x_1 + t_i \cdot x_2 - \exp(t_i)) & 0 & 0 \\ 2 \cdot \sum_{i=1}^m t_i \cdot (x_1 + t_i \cdot x_2 - \exp(t_i)) & 2 \cdot \sum_{i=1}^m t_i^2 \cdot (x_1 + t_i \cdot x_2 - \exp(t_i)) & 0 & 0 \\ 0 & 0 & 2 \cdot m & 2 \cdot \sum_{i=1}^m \sin(t_i) \cdot (x_3 + x_4 \cdot \sin(t_i) - \sin(t_i)) \\ 0 & 0 & 2 \cdot \sum_{i=1}^m \sin(t_i) \cdot (x_3 + x_4 \cdot \sin(t_i) - \sin(t_i)) & 2 \cdot \sum_{i=1}^m \sin(t_i) \cdot (x_3 + x_4 \cdot \sin(t_i) - \sin(t_i)) \end{bmatrix}$$

The value of the Brown and Dennis Function is: 3164.844373227912 Optimal solution: [8.57730812e-01 1.74632931e+00 -1.87750974e-08 1.00000002e+00] Optimal function value: 0.02484595290177809 Iterations: 15 x0:(25,5,-5,-1)

0.8 Gulf Research and Development Function (11)

0.8.1 Objective function

$$f(x) = \left\| \exp \left(-\frac{|y \cdot 100 \cdot t \cdot m \cdot x_1|^{x_2}}{x_0} \right) - t \right\|^2$$

0.8.2 Optimization results

$$\nabla f = 2 \cdot \begin{bmatrix} \sum_{i=1}^m \frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0^2} \cdot \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot \text{sign}(x_1) \cdot x_2 \cdot t_i \\ -2 \cdot \sum_{i=1}^m \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot t_i \\ -2 \cdot \sum_{i=1}^m \frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0^2} \cdot \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot x_2 \cdot \log \left(\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|}{x_0} \right) \end{bmatrix}$$

$$H = 2 \cdot \begin{bmatrix} \sum_{i=1}^m \frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0^2} \cdot \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot \text{sign}(x_1) \cdot x_2 \cdot t_i & -2 \cdot \sum_{i=1}^m \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot t_i \\ -2 \cdot \sum_{i=1}^m \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot t_i & 2 \cdot \sum_{i=1}^m \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \\ -2 \cdot \sum_{i=1}^m \frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0^2} \cdot \exp \left(-\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|^{x_2}}{x_0} \right) \cdot x_2 \cdot \log \left(\frac{|y \cdot 100 \cdot t_i \cdot m \cdot x_1|}{x_0} \right) & 0 \end{bmatrix}$$

Optimal solution: [0.35189333 -0.00093731 0.05808836] Optimal function value: 0.02500939024137528 iterations: 30 x0:(5,2.5,0.15) known minimum : (50,25,1.5)

0.9 Trigonometric function (26)

0.9.1 Objective function

$$f(x) = \left\| n - \sum_{i=1}^n \cos(x_i) + (i+1) \cdot (1 - \cos(x_i)) - \sin(x_i) \right\|^2$$

0.9.2 Optimization results

The value of the Trigonometric Function is: 0.04020653651318907 Optimal solution: [0.0111489 0.01120458 0.01126135 0.01131928 0.01137824 0.01143831 0.01149956 0.01156204 0.01162575 0.01169072 0.01175705 0.01182461 0.01189374 0.12120813 0.01203734 0.01211072 0.01218557 0.01226209 0.01234019 0.01241998 0.01250148 0.01258473 0.01266968 0.01275643 0.01284496 0.01293526 0.01302731 0.01312108 0.0132165 0.01331349 0.01341191 0.01351158 0.01361224 0.01371366 0.0138154 0.01391702 0.01401788 0.0141173 0.0142144 0.01430813

0.01439733 0.01448069 0.01455677 0.01462408 0.01468112 0.01472646 0.0147591 0.01477773 0.01478183 0.01477641]
Optimal function value: 0.00992757965222554 Iterations: 9

$$\nabla f = 2 \cdot \begin{bmatrix} \sum_{i=1}^n \sin(x_i) + \sum_{i=1}^n (i+1) \cdot \sin(x_i) \\ -\sin(x_1) \\ -\sin(x_2) \\ \vdots \\ -\sin(x_n) \end{bmatrix}$$

$$H = 2 \cdot \begin{bmatrix} \sum_{i=1}^n \cos(x_i) + \sum_{i=1}^n (i+1) \cdot \cos(x_i) & \sin(x_1) & \sin(x_2) & \cdots & \sin(x_n) \\ \sin(x_1) & \cos(x_1) + 2 \cdot \sin(x_1) & 0 & \cdots & 0 \\ \sin(x_2) & 0 & \cos(x_2) + 3 \cdot \sin(x_2) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sin(x_n) & 0 & 0 & \cdots & \cos(x_n) + n \cdot \sin(x_n) \end{bmatrix}$$

0.10 Expanded Rosenbrock function (21)

0.10.1 Objective function

$$f(x) = \sum_{i=1}^{n-1} [100 \cdot (x_{i+1} - x_i^2)^2 + (1 - x_i)^2]$$

0.10.2 Optimization results

$$\nabla f = \begin{bmatrix} -400 \cdot x_1 \cdot (x_2 - x_1^2) - 2 \cdot (1 - x_1) \\ 200 \cdot (x_2 - x_1^2) - 400 \cdot x_2 \cdot (x_3 - x_2^2) - 2 \cdot (1 - x_2) \\ \vdots \\ 200 \cdot (x_{n-1} - x_{n-2}^2) - 400 \cdot x_{n-1} \cdot (x_n - x_{n-1}^2) - 2 \cdot (1 - x_{n-1}) \\ 200 \cdot (x_n - x_{n-1}^2) \end{bmatrix}$$

$$H = \begin{bmatrix} 1200 \cdot x_1^2 - 400 \cdot x_2 + 2 & -400 \cdot x_1 & 0 & \cdots & 0 \\ -400 \cdot x_1 & 1200 \cdot x_2^2 - 400 \cdot x_3 + 2 & -400 \cdot x_2 & \cdots & 0 \\ 0 & -400 \cdot x_2 & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & -400 \cdot x_{n-1} \\ 0 & 0 & \cdots & -400 \cdot x_{n-1} & 200 \end{bmatrix}$$

The value of the Extended Rosenbrock Function is: 0 Optimal solution: [1. 1. 1.] Optimal function value: 0.0 iterations: 1

$$x_0 = (-1.2, 1, -1.2, 1, -1.2, \dots)$$

0.11 Extended Powell singular function (22)

0.11.1 Objective function

$$f(\mathbf{x}) = \sum_{i=0}^{n/4-1} [(x_{4i} + 10x_{4i+1})^2 + 5(x_{4i+2} - x_{4i+3})^2 + (x_{4i+1} - 2x_{4i+2})^4 + 10(x_{4i} - x_{4i+3})^4]$$

0.11.2 Optimization results

$$\frac{\partial f}{\partial x_{4i}} = 2 [(x_{4i} + 10x_{4i+1}) + 40(x_{4i} - x_{4i+3})^3]$$

$$\frac{\partial f}{\partial x_{4i+1}} = 20 [(x_{4i} + 10x_{4i+1}) + 4(x_{4i+1} - 2x_{4i+2})^3]$$

$$\frac{\partial f}{\partial x_{4i+2}} = 10 [5(x_{4i+2} - x_{4i+3}) - 8(x_{4i+1} - 2x_{4i+2})^3]$$

$$\frac{\partial f}{\partial x_{4i+3}} = -10 [5(x_{4i+2} - x_{4i+3}) + 40(x_{4i} - x_{4i+3})^3]$$

$$\frac{\partial^2 f}{\partial x_{4i}^2} = 2 + 120 \cdot (x_{4i} - x_{4i+3})^2$$

$$\frac{\partial^2 f}{\partial x_{4i} \partial x_{4i+1}} = 20$$

$$\frac{\partial^2 f}{\partial x_{4i} \partial x_{4i+2}} = 0$$

$$\frac{\partial^2 f}{\partial x_{4i} \partial x_{4i+3}} = -120 \cdot (x_{4i} - x_{4i+3})^2$$

$$\frac{\partial^2 f}{\partial x_{4i+1}^2} = 200 + 12 \cdot (x_{4i+1} - 2x_{4i+2})^2$$

$$\frac{\partial^2 f}{\partial x_{4i+1} \partial x_{4i+2}} = -24 \cdot (x_{4i+1} - 2x_{4i+2})^2$$

$$\frac{\partial^2 f}{\partial x_{4i+1} \partial x_{4i+3}} = 0$$

$$\frac{\partial^2 f}{\partial x_{4i+2}^2} = 10 - 192 \cdot (x_{4i+1} - 2x_{4i+2})^2$$

$$\frac{\partial^2 f}{\partial x_{4i+2} \partial x_{4i+3}} = -20$$

$$\frac{\partial^2 f}{\partial x_{4i+3}^2} = 120 + 120 \cdot (x_{4i} - x_{4i+3})^2$$

Optimal solution: [-0.00310417 0.00156839 -0.00015684 -0.0031042] Optimal function value: 6.1078478163066054e-09

0.12 Beale function (5)

0.12.1 Objective function

$$f(x, y) = (1.5 - x + xy)^2 + (2.25 - x + xy^2)^2 + (2.625 - x + xy^3)^2$$

0.12.2 Optimization results

$$\frac{\partial f}{\partial x} = 2 [(1.5 - x + xy)(-1 + y) + (2.25 - x + xy^2)(-1 + y^2) + (2.625 - x + xy^3)(-1 + y^3)]$$

$$\frac{\partial f}{\partial y} = 2 [(1.5 - x + xy)x + 2(2.25 - x + xy^2)xy + 3(2.625 - x + xy^3)xy^2]$$

Optimal solution: [2.99999967 0.49999991] Optimal function value: 1.9911045053339106e-14 gradient is 0 and hessian is positive therefore is minimum

$$\frac{\partial^2 f}{\partial x^2} = 2 [(1 - y) + 2(1 - y^2) + 3(1 - y^3)]$$

$$\frac{\partial^2 f}{\partial y^2} = 2 [x^2 + 2x^2 + 6x^2y + 6x^2y^2]$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 2 [-1 + 2xy + 3xy^2]$$

0.13 Wood function (14)

0.13.1 Objective function

$$f(x) = 100 \cdot (x_2 - x_1^2)^2 + (1 - x_1)^2 + 90 \cdot (x_4 - x_3^2)^2 + (1 - x_3)^2 + 10 \cdot (x_2 + x_4 - 2)^2 + \frac{1}{10} \cdot (x_2 - x_4)^2$$

0.13.2 Optimization results

$$\nabla f(x) = [-400 \cdot x_1 \cdot (x_2 - x_1^2) - 2 \cdot (1 - x_1), 200 \cdot (x_2 - x_1^2) + 20 \cdot (x_2 + x_4 - 2) + 0.2 \cdot (x_2 - x_4), -360 \cdot x_3 \cdot (x_4 - x_3^2) - 2 \cdot (1 - x_3), 20 \cdot (x_2 + x_4 - 2) + 0.2 \cdot (x_2 - x_4)]$$

Optimal solution: [0.99999968 0.99999938 1.00000019 1.00000004] Optimal function value: 7.954383299019438e-13 optimal is x=(1,1,1,1) and gradient is 0 and hessian is positivetherefore is a minimum

0.14 chebyquad function

0.14.1 Objective function

$$f(x) = \|T_{n-1}(x)\|$$

0.14.2 Optimization results

$$\nabla f(x) = \frac{2}{n} \sum_{k=0}^{n-1} [k \cdot U_{k-1}(x) \cdot T_k(x)]$$

$$H_{ij}(x) = \frac{2}{n} \sum_{k=0}^{n-1} [U_{k-1}(x) \cdot U_{k-1}(x) \cdot \delta_{ij} - k \cdot U_{k-2}(x) \cdot U_{k-1}(x)]$$

Optimal solution: [-1.29537067e-08 1.66052438e-09 4.33883736e-01 4.33883738e-01 4.33883739e-01 7.81831475e-01 7.81831480e-01 7.81831479e-01] Optimal function value: 1.3846731514262084e-07 Iterations: 64

Conclusion

Helical Valley Function (7):

Optimal solution: [1. 0. 0.] Optimal function value: 0.0 Iterations: 2

Conclusion: The function is essentially at its global minimum, indicating successful optimization. Biggs EXP6 Function (18):

Optimal solution: [1. 2. 1. 1. 1. 1.] Optimal function value: 4.248354237778568e-18 Iterarions: 0

Conclusion: The optimization has converged but not known minimum value Gaussian Function (9):

The value of the Gaussian function is: 0.10207049569085339 Optimal x: [0.52388083 0.7058404 0.48088152]

Minimum value of the function: 0.0014489424560909297 Iterations: 11 Converged to minimum known value Three-dimensional Box Function (12):

Optimal x: [1.00035547e+01 1.00035551e+01 2.03012288e-08] Minimum value of the function: 2.6852718877011665e-08 iterations: 21 Converged to a minimum value function

Function of Variable Dimensions (25):

Optimal x: [1. 1.00000003 1.00000001 1.00000001 0.99999997] Minimum value of the function: 4.4519732194048033e-08 Iterations: 34 Conclusion: The function has been minimized, and the Hessian matrix confirms it is a minimum.

Watson Function (20):

Optimal x: [-1.06312021e-08 9.97966220e-01 1.43529299e-02 3.16885350e-01 -3.19036044e-02 4.94903016e-02] Minimum value of the function: 0.006453858327920113 Conclusion: The optimization result is close to the optimum

Penalty Function I (23):

Optimal x: [0.16000229 0.1580955 0.15681293 0.15673907 0.1580507 0.15754714 0.16227176 0.15465012 0.15931135 0.1585308] Minimum value of the function: 9.999683699006326 Conclusion: The optimization, but the gradient is not zero, suggesting a potential issue.

Penalty Function II (24): Optimal x: [1.29875272e-01 -8.82752006e-05 -1.27193508e-04 -1.10731286e-04] Minimum value of the function: 0.010557098048339484 Iterations: 49 got closer to 0 the optimum could be 0

Poorly Scaled Brown Function:

Optimal Solution: [1.95848211e-06, 1.79366035e-06, 1.45346821e-06, -3.39815933e-07, 2.01403015e-06] Minimum Function Value: 1.8782493991793412e-11 Conclusion: The gradient is zero, indicating a minimum. Brown and Dennis Function (16):

Optimal solution: [1.00000000e+06 1.99254942e-06] Optimal function value: 5.551114377728232e-05 Iterations: 14 Conclusion: The optimization result is totally satisfactory.

Brown and Dennis: The value of the Brown and Dennis Function is: 10016239.906752376 Optimal solution: [8.57717122e-01 1.74634067e+00 -1.11331063e-06 1.00000904e+00] Optimal function value: 0.0006173214141067991 Iterations: 96

Gulf Research and Development Function (11):

Optimal solution: [0.48491083 -0.10046213 0.06349683] Optimal function value: 0.0006379152333885442 iterations: 10 Conclusion: The function has not been successfully minimized. Trigonometric Function (26):

The value of the Trigonometric Function is: 0.0016165655783864058 Optimal solution: [0.01251834 0.01257993 0.01264239 0.01270579 0.01277008 0.01283552 0.01290166 0.01296882 0.01303694 0.013106 0.13091753 0.01324647 0.01331832 0.01339097 0.01346427 0.01353838 0.01361314 0.01368852 0.0137644 0.0138406 0.01391711 0.0139937 0.01407035 0.01414638 0.01422194 0.0142966 0.0143701 0.01444191 0.01451183 0.0145791 0.01464336 0.01470398 0.01476035 0.01481178 0.0148576 0.01489709 0.01492956 0.01495432 0.01497081 0.01497829 0.01497658 0.01496519 0.01494407 0.01491295 0.01487208 0.01482166 0.01476205 0.01469371 0.01461719 0.01453758] Optimal function value: 0.0007811001559582129 Iterations: 6 it seems not the minimum Expanded Rosenbrock Function (21):

Optimal Solution: [1. 1. 1.] Minimum Function Value: 0.0 Conclusion: The Rosenbrock function has been successfully minimized, and the Hessian is positive, confirming it as a minimum. Extended Powell Singular Function (22):

Optimal Solution: [-0.00310417, 0.00156839, -0.00015684, -0.0031042] Minimum Function Value: 6.1078478163066054e-09 Conclusion: The function has been successfully minimized, and the optimal solution has been found. Beale Function (5):

Optimal Solution: [2.99999967, 0.49999991] Minimum Function Value: 1.9911045053339106e-14 Conclusion: The function has been minimized, and the gradient is zero, indicating a minimum. Wood Function (14):

Optimal Solution: [0.99999968, 0.99999938, 1.00000019, 1.00000004] Minimum Function Value: 7.954383299019438e-13 Conclusion: The Wood function has been successfully minimized, and the Hessian is positive, confirming it as a minimum.

Chebyshev Function Optimal solution: [-1.29537067e-08 1.66052438e-09 4.33883736e-01 4.33883738e-01 4.33883739e-01 7.81831475e-01 7.81831480e-01 7.81831479e-01] Optimal function value: 1.3846731514262084e-07 Iterations: 64 the minimum is closer origin