

Assignment 4 - Canonical Correlation Analysis

GROUP 03

*Agustin Valencia - **aguva779***

*Bayu Brahmantio - **baybr878***

*Joris van Doorn - **jorva845***

*Marcos F Mourao - **marfr825***

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Canonical correlation analysis by utilizing suitable software

Look at the data described in Exercise 10.16 of Johnson, Wichern. You may find it in the file P10-16.DAT. The data for 46 patients are summarized in a covariance matrix, which will be analyzed in R. Read through the description of the different R packages and functions so you may choose the most suitable one for the analysis. Supplement with own code where necessary.

The given matrix is the following:

V1	V2	V3	V4	V5
1106.000	396.700	108.400	0.787	26.230
396.700	2382.000	1143.000	-0.214	-23.960
108.400	1143.000	2136.000	2.189	-20.840
0.787	-0.214	2.189	0.016	0.216
26.230	-23.960	-20.840	0.216	70.560

Thus, separating the variance-covariance matrix it is obtained that

$$\Sigma_{11} =$$

V1	V2	V3
1106.0	396.7	108.4
396.7	2382.0	1143.0
108.4	1143.0	2136.0

$$\Sigma_{22} =$$

V4	V5
0.016	0.216
0.216	70.560

$$\Sigma_{21} =$$

	V1	V2	V3
4	0.787	-0.214	2.189
5	26.230	-23.960	-20.840

$$\Sigma_{12} =$$

V4	V5
0.787	26.23
-0.214	-23.96
2.189	-20.84

It can be computed that

$$S_{11}^{-1/2} S_{12} S_{22}^{-1} S_{21} S_{11}^{-1/2} =$$

45000.99	25864.44	141869.5
25864.44	62898.07	182340.5
141869.53	182340.55	658795.9

Its eigen-decomposition is given by

Eigenvalues :

x
739837.84
26857.14
0.00

Eigenvectors:

-0.2024159	0.7395280	-0.6419705
-0.2619092	-0.6725418	-0.6921640
-0.9436267	0.0280330	0.3298224

Also,

$$S_{22}^{-1/2} S_{21} S_{11}^{-1} S_{12} S_{22}^{-1/2} =$$

0.0009011	0.2611067
0.2611067	81.1397279

Its eigen-decomposition is given by

Eigenvalues :

x
81.1405681
0.0000609

Eigenvectors:

0.0032180	-0.9999948
0.9999948	0.0032180

- a) Test at the 5% level if there is any association between the groups of variables.

```
alpha <- 0.05
#critical value
crit <- qchisq(p = (1-alpha), df = p*q)
#test statistic
```

- b) How many pairs of canonical variates are significant?
- c) Interpret the “significant” squared canonical correlations. Tip: Read section “Canonical Correlations as Generalizations of Other Correlation Coefficients”.
- d) Interpret the canonical variates by using the coefficients and suitable correlations.
- e) Are the “significant” canonical variates good summary measures of the respective data sets? Tip: Read section “Proportions of Explained Sample Variance”.
- f) Give your opinion on the success of this canonical correlation analysis.

Appendix A - Code

```
RNGversion('3.5.1')
knitr::opts_chunk$set(echo = TRUE)
library(expm)
library(knitr)
data <- read.table("./Data/P10-16.DAT")
#number of observations (patients)
n <- 46
#number of primary variables
p <- 3
#number of secondary variables
q <- 2
kable(data)
#separating the variance-covariance matrix
sigma11 <- as.matrix(data[1:3,1:3])
sigma22 <- as.matrix(data[4:5, 4:5])
sigma21 <- as.matrix(data[4:5, 1:3])
sigma12 <- as.matrix(data[1:3, 4:5])

e <- sqrtm(sigma11) %*% sigma12 %*% solve(sigma22) %*% sigma21 %*% sqrtm(sigma11)
f <- sqrtm(sigma22) %*% sigma21 %*% solve(sigma11) %*% sigma12 %*% sqrtm(sigma22)

kable(sigma11)
kable(sigma22)
kable(sigma21)
kable(sigma12)
kable(e)
eEigenDecom <- eigen(e)
eEigVect <- eEigenDecom$vectors
eEigVals <- eEigenDecom$values
kable(eEigVals)
kable(eEigVect)
kable(f)
fEigenDecom <- eigen(f)
fEigVect <- fEigenDecom$vectors
fEigVals <- fEigenDecom$values
kable(fEigVals)
kable(fEigVect)
alpha <- 0.05
#critical value
crit <- qchisq(p = (1-alpha), df = p*q)
#test statistic
```