assignment_1

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Question 1: Describing individual variables

Consider the data set in the T1-9.dat file, National track records for women. For 55 different countries we have the national records for 7 variables (100, 200, 400, 800, 1500, 3000m and marathon). Use R to do the following analyses.

a) Describe the 7 variables with mean values, standard deviations e.t.c.

```
Q1_a()
```

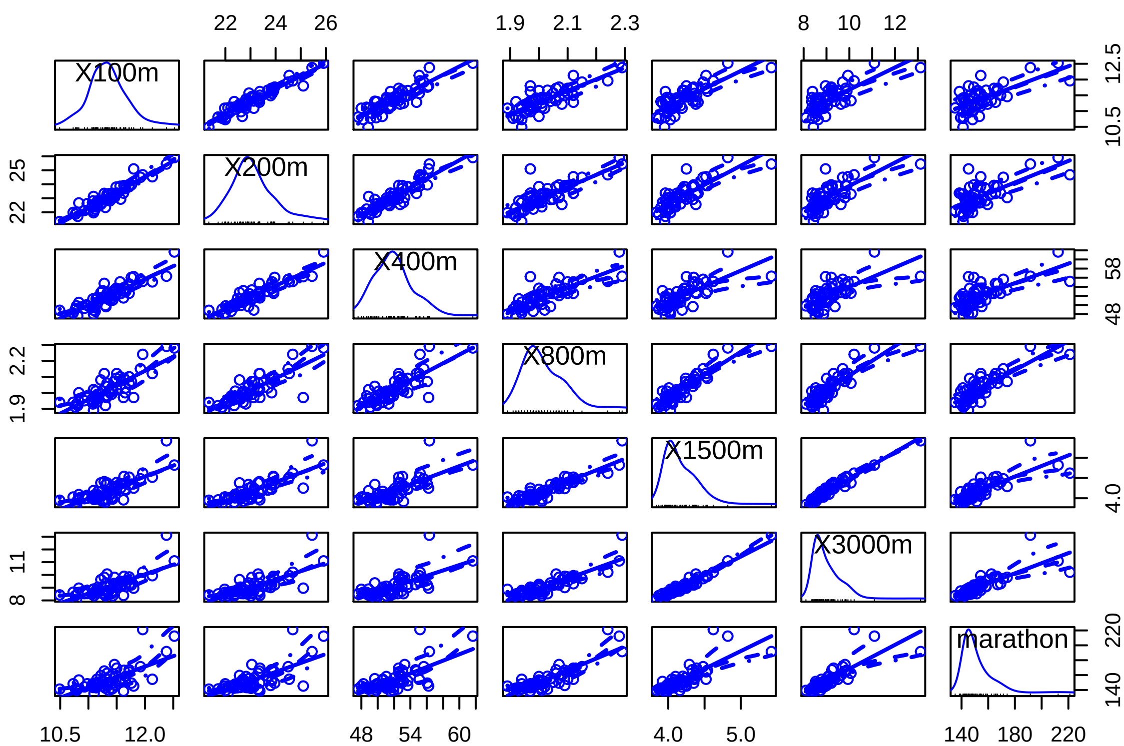
```
## ** Summarizing data **
##
## *data path:  data/T1-9.dat
##
## *Column means:
##       100m       200m       400m       800m      1500m      3000m   marathon
##  11.357778  23.118519  51.989074   2.022407   4.189444   9.080741 153.619259
##
## *Variances:
##                100m       200m       400m        800m       1500m        3000m
## 100m     0.15531572  0.3445608  0.8912960 0.027703564 0.08389119  0.23388281
## 200m     0.34456080  0.8630883  2.1928363 0.066165898 0.20276331  0.55435017
## 400m     0.89129602  2.1928363  6.7454576 0.181807932 0.50917683  1.42681579
## 800m     0.02770356  0.0661659  0.1818079 0.007546925 0.02141457  0.06137932
## 1500m    0.08389119  0.2027633  0.5091768 0.021414570 0.07418270  0.21615514
## 3000m    0.23388281  0.5543502  1.4268158 0.061379315 0.21615514  0.66475793
## marathon 4.33417757 10.3849876 28.9037314 1.219654647 3.53983732 10.70609113
##            marathon
## 100m       4.334178
## 200m      10.384988
## 400m      28.903731
## 800m       1.219655
## 1500m      3.539837
## 3000m     10.706091
## marathon 270.270150
##
## *Correlations:
##               100m      200m      400m      800m     1500m     3000m  marathon
## 100m     1.0000000 0.9410886 0.8707802 0.8091758 0.7815510 0.7278784 0.6689597
## 200m     0.9410886 1.0000000 0.9088096 0.8198258 0.8013282 0.7318546 0.6799537
## 400m     0.8707802 0.9088096 1.0000000 0.8057904 0.7197996 0.6737991 0.6769384
## 800m     0.8091758 0.8198258 0.8057904 1.0000000 0.9050509 0.8665732 0.8539900
## 1500m    0.7815510 0.8013282 0.7197996 0.9050509 1.0000000 0.9733801 0.7905565
## 3000m    0.7278784 0.7318546 0.6737991 0.8665732 0.9733801 1.0000000 0.7987302
## marathon 0.6689597 0.6799537 0.6769384 0.8539900 0.7905565 0.7987302 1.0000000
##
## *Total Sample Variance:
```

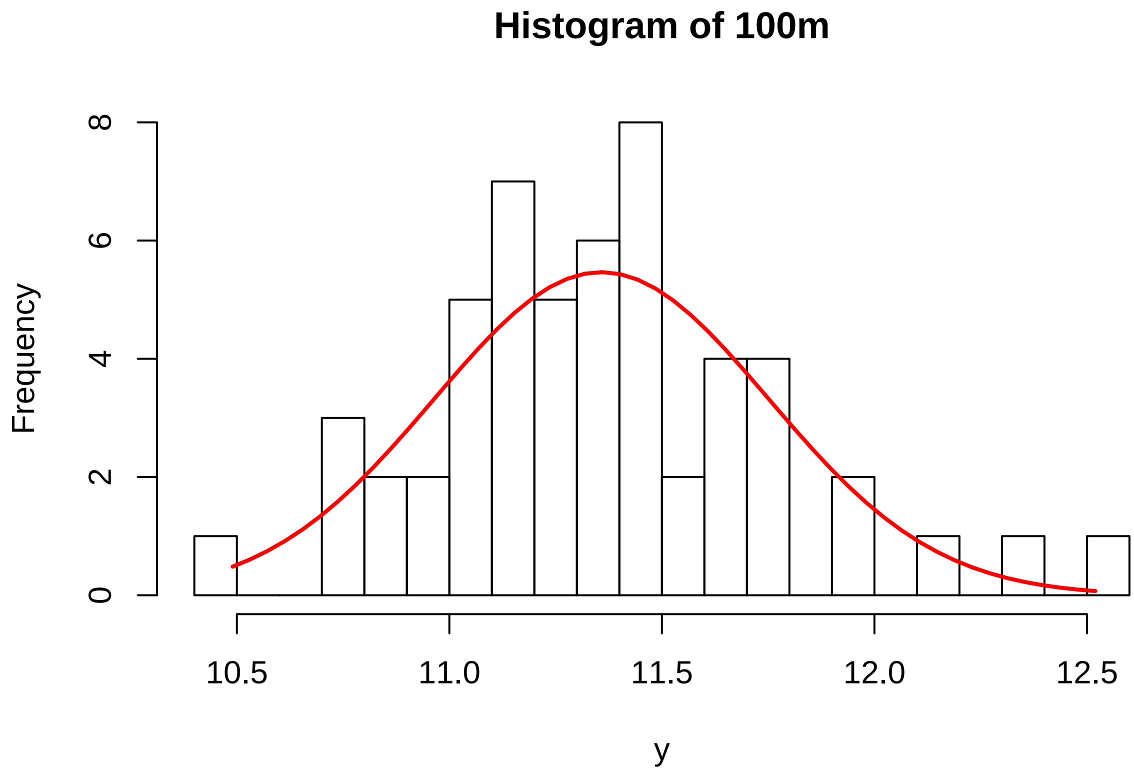
```
## [1] 278.7805
##
## *Generalized Sample Variance:
## [1] 8.195897e-07
```

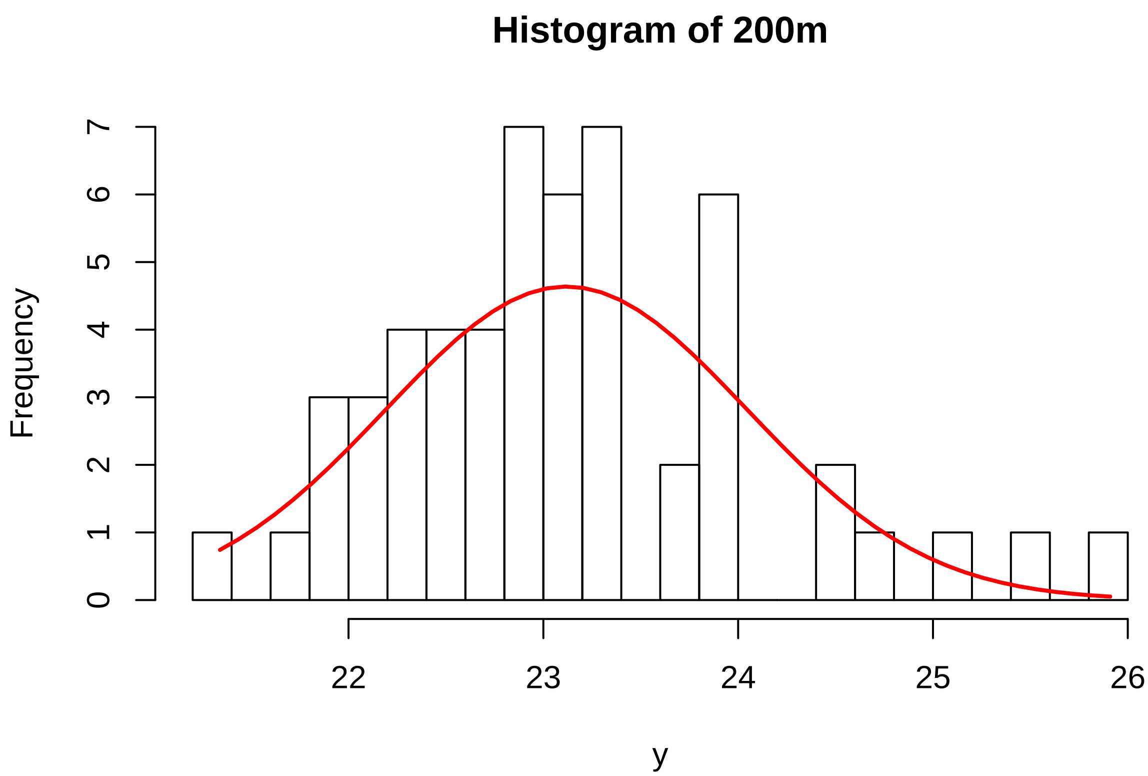
b) Illustrate the variables with different graphs (explore what plotting possibilities R has). Make sure that the graphs look attractive (it is absolutely necessary to look at the labels, font sizes, point types). Are there any apparent extreme values? Do the variables seem normally distributed? Plot the best fitting (match the mean and standard deviation, i.e. method of moments) Gaussian density curve on the data's histogram. For the last part you may be interested in the hist() and density() functions.

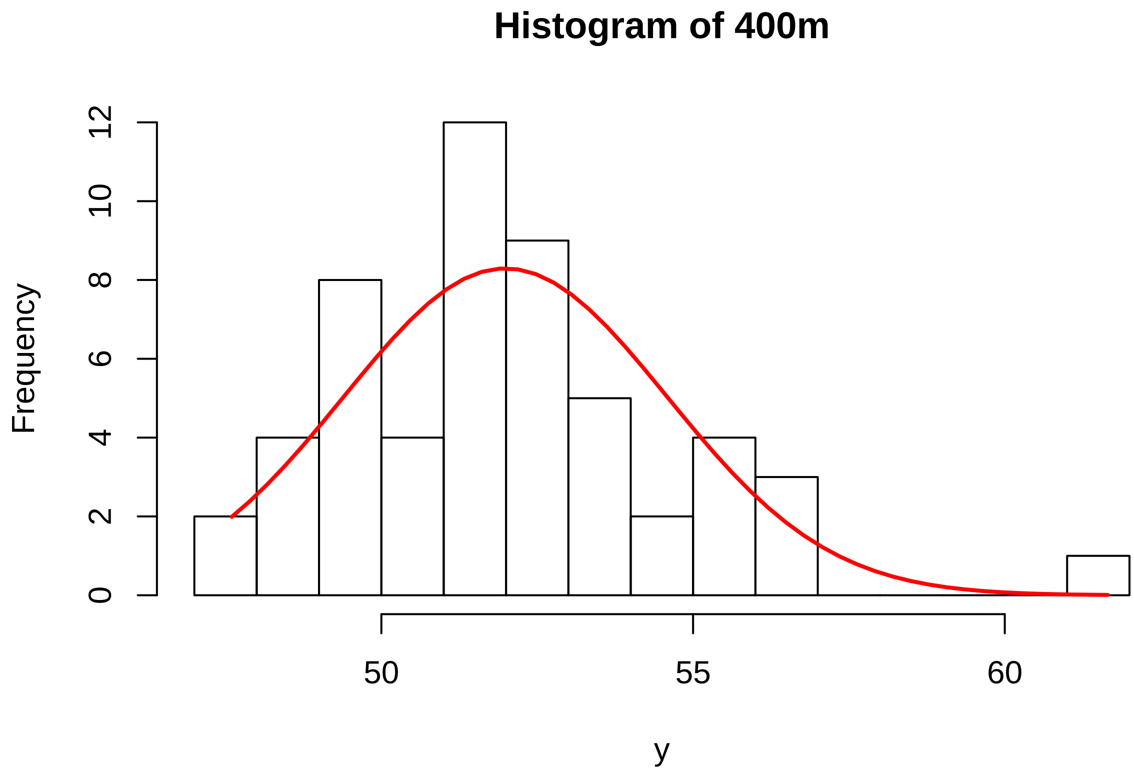
```
Q1_b()
```

```
## Using country as id variables
```

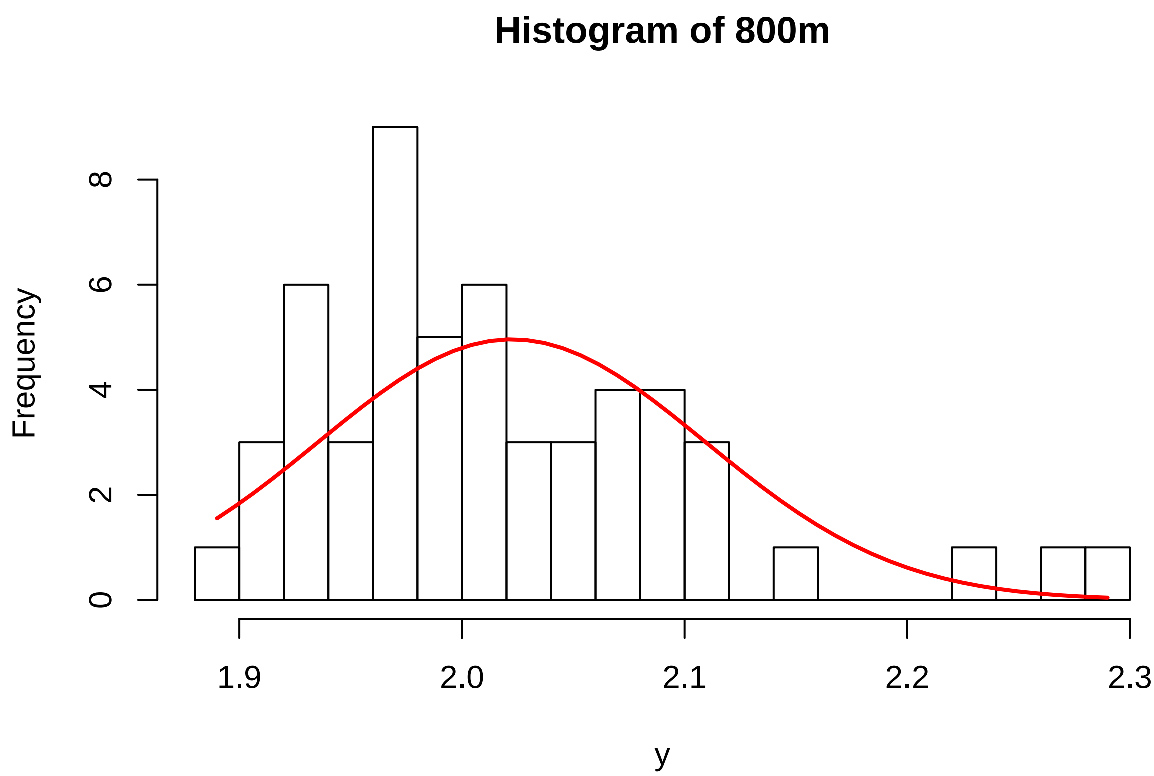



Histogram of 100m
Frequency
0
2
4
6
8
10.5
11.0
11.5
12.0
12.5
y

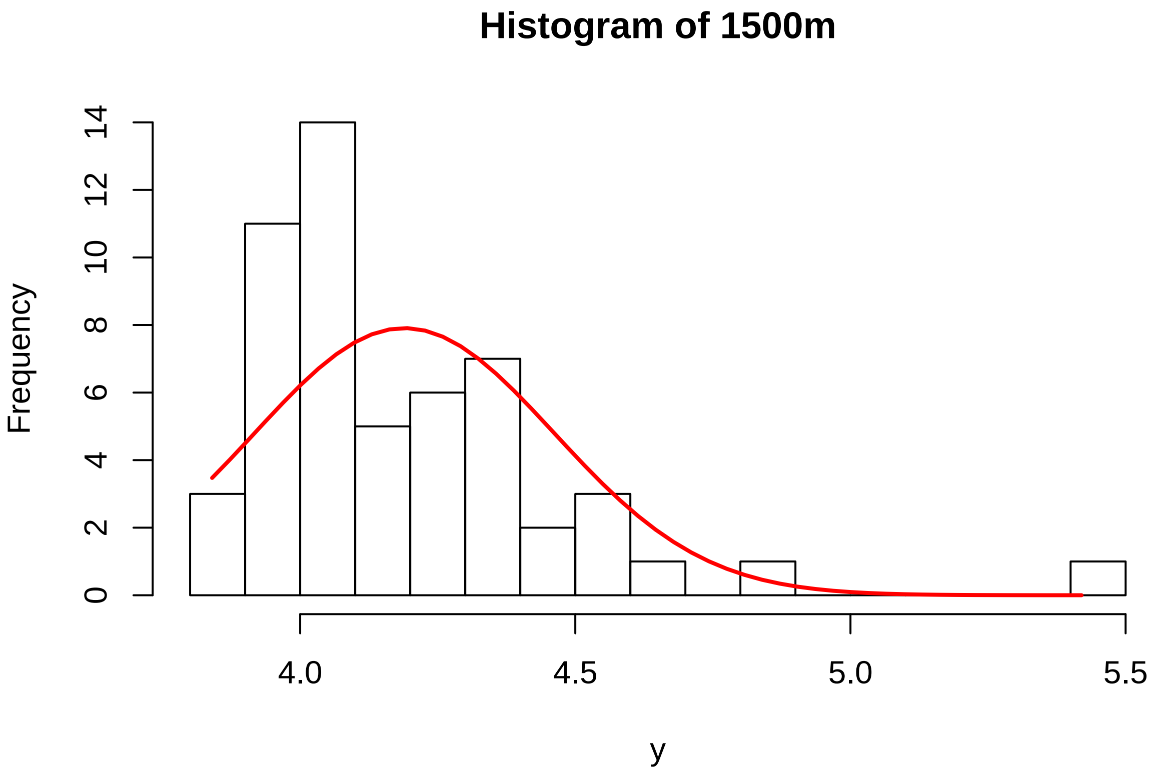

Histogram of 200m
Frequency
0
1
2
3
4
5
6
7
22
23
24
25
26
y

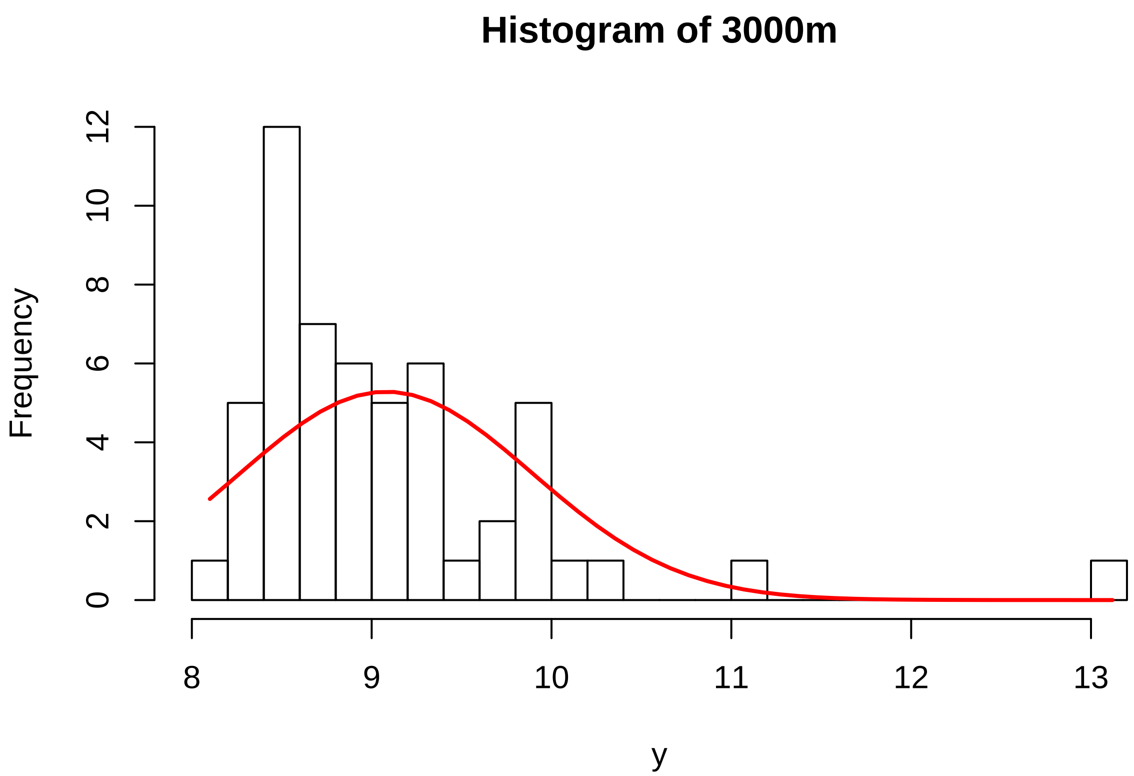


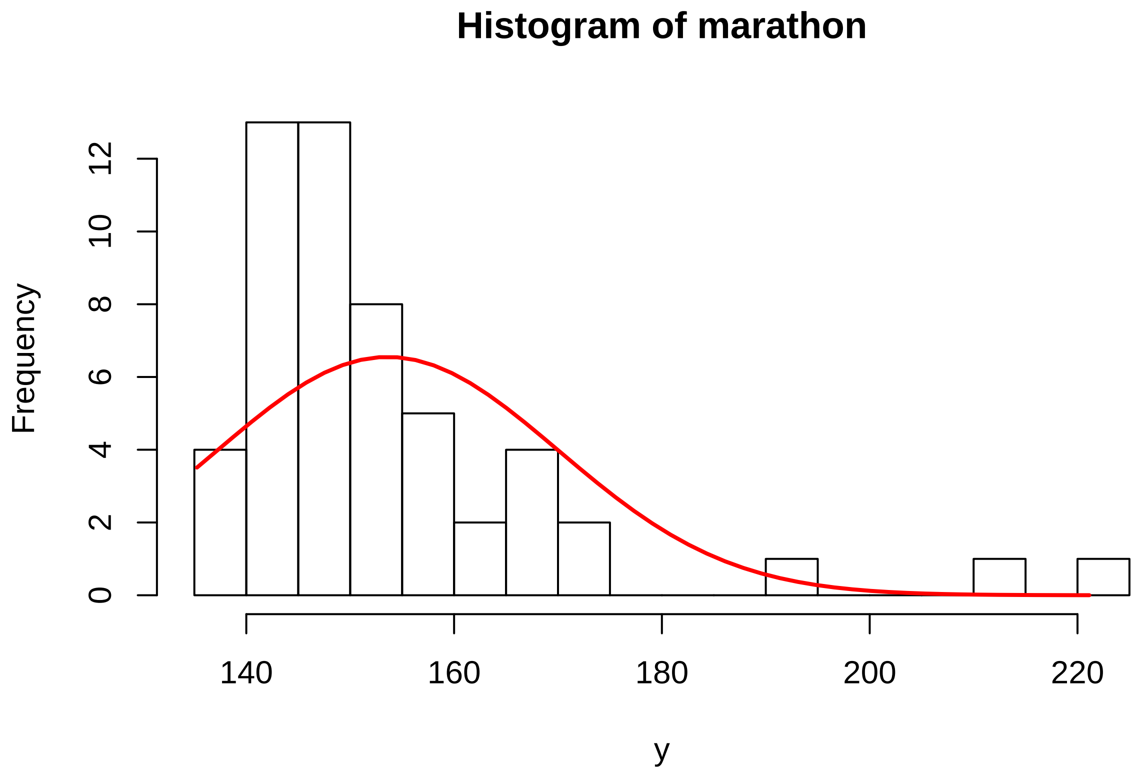
Histogram of 400m
Frequency
0
2
4
6
8
10
12
50
55
60
y



Histogram of 800m
Frequency
0
2
4
6
8
1.9
2.0
2.1
2.2
2.3
y


Histogram of 1500m
Frequency
0
2
4
6
8
10
12
14
4.0
4.5
5.0
5.5
y


Histogram of 3000m
Frequency
0
2
4
6
8
10
12
8
9
10
11
12
13
y



Question 2: Relationships between the variables

a) Compute the covariance and correlation matrices for the 7 variables. Is there any apparent structure in them? Save these matrices for future use.

By analysing the variance covariance matrix, it can be concluded that countries that have a high performance in “shorter” races (100m, 200m and 400m) do not necessarily have high performance in “long distance” races (800m, 1500m, 3000m and marathon). This is coherent to the fact that short and long races require different training. Normally, the athletes are different altogether in these different categories.

b) Generate and study the scatterplots between each pair of variables. Any extreme values?

c) Explore what other plotting possibilities R offers for multivariate data. Present other (at least two) graphs that you find interesting with respect to this data set.

Question 3: Examining for extreme values

a) Look at the plots (esp. scatterplots) generated in the previous question. Which 3–4 countries appear most extreme? Why do you consider them extreme?

b) The most common residual is the Euclidean distance between the observation and sample mean vector, i.e.

$$d(\overrightarrow{x}, \overline{x}) = \sqrt{(\overrightarrow{x} - \overline{x})^T (\overrightarrow{x} - \overline{x})}$$

This distance can be immediately generalized to the L^r, r > 0 distance as

$$d_{L^r}(\overrightarrow{x}, \overline{x}) = \left(\sum_{i=1}^{p} |\overrightarrow{x}_i - \overline{x}_i|^r \right)^{1/r}$$

where p is the dimension of the observation (here p = 7).

Compute the squared Euclidean distance (i.e. r = 2) of the observation from the sample mean for all 55 countries using R's matrix operations. First center the raw data by the means to get x - mean for each country. Then do a calculation with matrices that will result in a matrix that has on its diagonal the requested squared distance for each country. Copy this diagonal to a vector and report on the five most extreme countries. In this questions you MAY NOT use any loops.

```r
#euclidean distance
records <- as.matrix(data[,2:8])

#center the data
#deviation matrix
centered <- scale(x = records, center = TRUE, scale = FALSE)

euclid_dist <- sqrt((centered) %*% t(centered))
```

```
## Warning in sqrt((centered) %*% t(centered)): NaNs produced
```

```r
#diagonal
euclid_diag <- sort(diag(euclid_dist), decreasing = TRUE, index.return = TRUE)
#extreme values (top 5)
euclid_extremes <- head(euclid_diag, n = 5)
#extract extreme countries
ind <- head(euclid_extremes$ix, 5)
euclid_extreme_countries <- data[ind,1]
#Sweden's index
SWE_ind <- which(data[,1] == "SWE")
#Sweden's Position
SWE_rank_euclid <- which(euclid_extremes$ix == SWE_ind)

print("Top 5 distance extreme countries:")
```

```
## [1] "Top 5 distance extreme countries:"
```

```r
euclid_extreme_countries
```

```
## [1] PNG COK SAM BER GBR
## 54 Levels: ARG AUS AUT BEL BER BRA CAN CHI CHN COK COL CRC CZE DEN DOM ... USA
```

```r
paste("Sweden's distance rank: ", SWE_rank_euclid)
```

```
## [1] "Sweden's distance rank:  48"
```

c) The different variables have different scales so it is possible that the distances can be dominated by some few variables. To avoid this we can use the squared distance, INSERT LATEX FORMULA HERE where V is a diagonal matrix with variances of the appropriate variables on the diagonal. The effect, is that for each variable the squared distance is divided by its variance and we have a scaled independent distance.

It is simple to compute this measure by standardizing the raw data with both means (cen- tring) and standard deviations (scaling), and then compute the Euclidean distance for the normalized data. Carry out these computations and conclude which countries are the most extreme ones. How do your conclusions compare with the unnormalized ones?

```
#diagonal of variance covariance matrix
covars <- cov(records)
V <- matrix(0, nrow = ncol(records), ncol = ncol(records))
diag(V) <- diag(covars)
#compute distance
dist3c <- (centered %*% solve(V) %*% t(centered))^(1/2)

#diagonal
diag3c <- sort(diag(dist3c), decreasing = TRUE, index.return = TRUE)
#extreme values (top 5)
extremes3c <- head(diag3c, n = 5)
#extract extreme countries
ind3c<- head(extremes3c$ix, 5)
extreme_countries3c <- data[ind3c,1]
#Sweden's Position
SWE_rank_3c <- which(extremes3c$ix == SWE_ind)

print("Top 5 distance extreme countries:")
```

```
## [1] "Top 5 distance extreme countries:"
```

```
extreme_countries3c
```

```
## [1] SAM COK PNG USA SIN
## 54 Levels: ARG AUS AUT BEL BER BRA CAN CHI CHN COK COL CRC CZE DEN DOM ... USA
```

```
paste("Sweden's distance rank: ", SWE_rank_3c)
```

```
## [1] "Sweden's distance rank:  50"
```

d) The most common statistical distance is the Mahalanobis distance, INSERT LATEX FORMULA HERE where C is the sample covariance matrix calculated from the data. With this measure we also use the relationships (covariances) between the variables (and not only the marginal variances as dV(· , ·) does). Compute the Mahalanobis distance, which countries are most extreme now?

```
dist3d <- (centered %*% solve(covars) %*% t(centered))^(1/2)
#diagonal
diag3d <- sort(diag(dist3d), decreasing = TRUE, index.return = TRUE)
#extreme values (top 5)
```

```r
extremes3d <- head(diag3d, n = 5)
#extract extreme countries
ind3d<- head(extremes3d$ix, 5)
extreme_countries3d <- data[ind3d,1]
#Sweden's Position
SWE_rank_3d <- which(extremes3d$ix == SWE_ind)

print("Top 5 distance extreme countries:")
```

```
## [1] "Top 5 distance extreme countries:"
```

```r
extreme_countries3d
```

```
## [1] SAM  PNG  KORN COK  MEX
## 54 Levels: ARG AUS AUT BEL BER BRA CAN CHI CHN COK COL CRC CZE DEN DOM ... USA
```

```r
paste("Sweden's distance rank: ", SWE_rank_3d)
```

```
## [1] "Sweden's distance rank:  54"
```

e) Compare the results in b)–d). Some of the countries are in the upper end with all the measures and perhaps they can be classified as extreme. Discuss this. But also notice the different measures give rather different results (how does Sweden behave?). Summarize this graphically. Produce Czekanowski's diagram using e.g. the RMaCzek package. In case of problems please describe them.

In this case, extreme countries normally have poor performance in races, but that is not always the case. For some distances definitions, high performance countries like USA and Great Britain appear as extremes as well. So "extremism" is not a measure of how "slow" a country is, but rather how far from the overall mean the country is.

By ranking the countries in decreasing order of distance, Sweden's position in the distance rank decreases for the different distances defined in questions b), c), and d) respectively. These distances

```r
df = data.matrix(data)
rownames(df) = data[,1]
df = df[,2:8]
x = czek_matrix(df)
plot(x)
```

Czekanowski's diagram