ONLINE APPENDIX: RANDOM MODELS FOR THE JOINT TREATMENT OF RISK AND TIME PREFERENCES

JOSE APESTEGUIA[†], MIGUEL A. BALLESTER[‡], AND ANGELO GUTIERREZ[§]

1. Datasets

In this section we describe the main features of the experimental datasets used in the paper.

1.1. **Andersen et al. (2018).** This study separately elicited risk and time preferences of a representative sample of 253 subjects from the adult Danish population. In the risk part, four multiple-price lists were implemented, each comprising ten pairs of present lotteries. The four tasks were (i) $([p, 1-p; 3850, 100], 0)$ and $([p, 1-p; 2000, 1600], 0)$, (ii) $([p, 1-p; 4000, 500], 0)$ and $([p, 1-p; 2250, 1500], 0)$, (iii) $([p, 1-p; 4000, 150], 0)$ and $([p, 1-p; 2000, 1750], 0)$, and (iv) $([p, 1-p; 4500, 50], 0)$ and $([p, 1-p; 2500, 1000], 0)$, with $p \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$. All 253 subjects were confronted with the four tasks, with 116 individuals facing all pairs, 67 individuals facing pairs 3, 5, 7, 8, 9, and 10, and the remaining 70 subjects facing pairs 1, 2, 3, 5, 7, and 10, for a total of 7,928 choices in this part. In the time part, six multiple-price lists involving dated degenerate lotteries were implemented. The first degenerate lottery always paid 3.000 DKK after one month. The second degenerate lottery was designed by varying two parameters: (i) the awarding time, which could vary between 2, 5, 7, 13, 19 or 25 months, and (ii) the annual interest obtained by the subject, which increased by multiples of 5, from 5% to 50%.[1] All the subjects faced all 60 temporal binary choices, making a total of 15,180 choices. That is, $\mathcal{O}$ is formed by 23,108 observations with the number of individual

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[†]ICREA, Universitat Pompeu Fabra and Barcelona GSE. E-mail: jose.apesteguia@upf.edu.

[‡]University of Oxford. E-mail: miguel.ballester@economics.ox.ac.uk.

[§]Universitat Pompeu Fabra and Barcelona GSE. E-mail: angelo.gutierrez@upf.edu.

[1]The interest was compounded quarterly to be consistent with general Danish banking practices.

observations varying between 84 and 100. For every pair of options, subjects could either choose one of them or express indifference between the two. In the latter case, they were told that the experimenter would settle indifferences by tossing a fair coin. Only 5% of the choices were indifferences; as in the original paper, we treat them assigning a half choice to each of the two gambles.

1.2. **Coble and Lusk (2010).** This paper reports on the risk and time preferences elicitation of 47 upper-level undergraduate and graduate students from Economics and Business courses. Each subject made a total of 94 choices in 9 different multiple-price lists, for a total of 4,418 observations. There were three parts to the experiment, differing according to the type of choice problem involved. Part (i) consisted of four multiple price list tasks using dated lotteries awarded at the same time period. The pairs were $([p, 1-p; 8, 10], t)$ and $([p, 1-p; 1, 19], t)$, with $p \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$, and $t \in \{0, 1, 13, 37\}$, measured in weeks. Part (ii) involved two tasks, in both of which the earlier degenerate lottery was \$10 to be paid in 1 week, while the later lottery prize-money increased, by multiples of half a dollar, from \$10 to \$15.5, and was paid either in week 13 or 37. Finally, there were three tasks in part (iii) involving the same lotteries as in part (i), with varying periods of times: $([p, 1-p; 8, 10], t)$ and $([p, 1-p; 1, 19], s)$, with $p \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$, and $(t, s) \in \{(1, 13), (1, 37), (13, 37)\}$.

1.3. **Andreoni and Sprenger (2012b).** Andreoni and Sprenger (2012b) introduced risky considerations in the convex budget design of Andreoni and Sprenger (2012a). 80 undergraduate students took part in the experiment, each making 84 choices, for a total of 6,720 observations. The experimental parameters can be described, following our notation, as follows: $([p, 1-p; \alpha x, 0], t)$ and $([q, 1-q; (1-\alpha)y, 0], s)$, with $(p, q) \in \{(1, 1), (0.5, 0.5), (1, 0.8), (0.5, 0.4), (0.8, 0.1), (0.4, 0.5)\}$, $(t, s) \in \{(7, 28), (7, 56)\}$ in days, and $(x, y) \in \{(20, 20), (19, 20), (18, 20), (17, 20), (16, 20), (15, 20), (14, 20)\}$. The decision variable was discrete, with $\alpha \in \{\frac{0}{100}, \frac{1}{100}, \ldots, \frac{100}{100}\}$.

2. Estimating the Tremble Probability ν

As discussed in Section 6 of the main text, we allow for positive choice probabilities in dominated lotteries by introducing a small fixed tremble probability. That is, we assume that with a very large probability $1 - \nu$, the individual chooses according to $\rho_{im\tau}(f)$ and with a very small probability ν, the individual uniformly randomizes. In this section, we check the sensitivity of the main results to this assumption by estimating the parameter ν jointly with the other parameters in the model. Table 1 reports the results of this exercise for each of the three datasets considered in the main text. We can see that the log-likelihood is higher, indicating an improvement in the in-sample fit of the model. This is not surprising, since the estimation of the tremble probability adds an extra degree of freedom to the model which helps in accounting for the observed choices of dominated lotteries. Notice that the estimated parameters of the risk and time preferences do not vary substantially from those estimated under the assumption of a small and fixed ν as used in the main text.

3. Allowing for $r > 1$

In Section 6 of the main text, we have seen that bounding the curvature of the CRRA Bernoulli function at 1 may not accommodate all the risk heterogeneity. In this section we extend the models in order to address this issue.

The CRRA case with $r > 1$ implies negative utilities, which poses no difficulty in the treatment of risk preferences. Unfortunately, this is not the case in the joint treatment of risk and time preferences in DEU-H, due to the exponential discounting part of this representation. Accordingly, we proceed to extend PVCE-H and CEPV-H, which we do by allowing for $r > 1$, while keeping the bound in the intertemporal substitution parameter at 1.[2] We refer to these extended versions as PVCE-H^E and CEPV-H^E. Several comments are in order.

In the setting of dated lotteries, the parameter η is not identifiable and hence we normalize η to 0, and estimate the corrected discount factor $\hat{\delta} = \delta$. Moreover, it immediately follows that PVCE-H^E and CEPV-H^E are equivalent, and obviously different from the baseline DEU-H model. The formulation of PVCE-H^E is basically the one used in the main text for PVCE-H, but simply allowing $r > 1$ and setting $\eta = 0$.

The convex budget experiments involve null experimental payoffs. Since PVCE-H^E and CEPV-H^E are well-defined only for strictly positive monetary payoffs, we incorporate some background consumption $\omega \in \mathbb{R}_{++}$, guaranteeing that all final payoffs, formed by the addition of background consumption and experimental payoffs, are always strictly positive. Hence, we arrive at the following functional forms of PVCE-H^E and CEPV-H^E in convex budgets, with $r \in \mathbb{R} \setminus \{1\}$ and $\eta < 1$:

[2] Given the assumption of continuity over the parameter space, we do not need to specify the representation of preferences when $r = 1$ in our estimation exercises, since this has zero probability mass.

$$
\begin{aligned}
PVCE^{E}_{\delta,\eta,r}(\alpha) = \Bigg[& \delta^{t}\left[p\left(\alpha x+\omega\right)^{1-r}+\left(1-p\right)\omega^{1-r}\right]^{\frac{1-\eta}{1-r}} \\
& + \delta^{s}\left[qu\left(\left(1-\alpha\right)y+\omega\right)^{1-r}+\left(1-q\right)\omega^{1-r}\right]^{\frac{1-\eta}{1-r}}\Bigg]^{\frac{1}{1-\eta}} \\
CEPV^{E}_{\delta,\eta,r}(\alpha) = \Bigg[& pq\left[\delta^{t}\left(\alpha x+\omega\right)^{1-\eta}+\delta^{s}\left(\left(1-\alpha\right)y+\omega\right)^{1-\eta}\right]^{\frac{1-r}{1-\eta}} \\
& + p\left(1-q\right)\left[\delta^{t}\left(\alpha x+\omega\right)^{1-\eta}+\delta^{s}\omega^{1-\eta}\right]^{\frac{1-r}{1-\eta}} \\
& + \left(1-p\right)q\left[\delta^{t}\omega^{1-\eta}+\delta^{s}\left(\left(1-\alpha\right)y+\omega\right)^{1-\eta}\right]^{\frac{1-r}{1-\eta}} \\
& + \left(1-p\right)\left(1-q\right)\left[\delta^{t}\omega^{1-\eta}+\delta^{s}\omega^{1-\eta}\right]^{\frac{1-r}{1-\eta}}\Bigg]^{\frac{1}{1-r}}.
\end{aligned}
$$

We now repeat the estimation of risk and time preferences carried out in the main text under this extension. For the empirical estimations we keep all the distributional assumptions except for the case of r, which we now assume to be normally distributed. We start with the Andersen et al. (2008) dataset, using PVCE-H^E. Table 2 reports the joint estimates, Figure 1 the PDFs of the estimated distributions, and Figure 2 presents a scatterplot of the individual estimates. We immediately see, as expected, that the median risk aversion increases and the corrected discount factor remains unchanged; the heterogeneity in the data, both at the pooled and individual levels is being captured more fully.

Table 3, Figures 3 and 4 report the corresponding results for Coble and Lusk (2010). In this case, median risk aversion decreases slightly, while the variance increases. The results for the distribution of delay aversion are very similar to those of the DEU-H model.

Table 4 reports the results using the models PVCE-H^E and CEPV-H^E in the convex budgets setting of Andreoni and Sprenger (2012b). In the baseline estimations, reported in columns (i) and (v), we assume a fixed, positive but close-to-zero, level of background consumption, $\omega = 10^{-6}$. In the estimations reported in columns (ii) and (vi) we incorporate background consumption as an additional estimated parameter. In columns (iii) and (vii) we report on the estimations at the individual level, providing the median and standard deviation of the medians estimated for each individual. In addition, Figures 5 and 6 plot the estimated PDFs of the preference parameters for the two models and Figures 7 and 8 plot the corresponding CDFs. Figure 9 plots the observed and predicted choice probabilities across the different experimental parameters, and Figure 10 shows scatter-plots with the estimated individual median parameters for each of the 80 subjects and for the two models.

With respect to the results obtained in section 6 of the main text, we note that the consideration of the extended models yields higher levels of risk aversion, very similar results for the corrected discount factor, and slightly lower values of the intertemporal substitution parameter.

4. Observable Characteristics

Some researchers may be interested in studying the effect of individual characteristics, such as age or gender, on risk and time preferences. In this section, we illustrate how our framework can be extended to allow for the dependence of the moments of the distribution of the behavioral traits on observable characteristics.

Consider the case of dated lotteries and the extended model introduced in the previous section, PVCE-H^E. We assume that r follows a normal distribution and allow the mean and variance of this distribution to vary across individuals. Specifically, we assume that the mean and standard deviation of the risk aversion distribution for individual i, denoted by $\mu_{i,r}$ and $\sigma_{i,r}$, are linear functions of observable characteristics. That is, $\mu_{i,r} = x_i'\beta_r$ and $\sigma_{i,r} = x_i'\gamma_r$, where x_i is a $K \times 1$ vector of characteristics which includes a constant term and other observable characteristics such as age, while β_r and γ_r are vectors of coefficients to be estimated.[3] Similarly, we assume that the corrected discount factor follows a beta distribution parameterized as a function of its mean and variance, which also vary across individuals.[4] That is, the mean and standard deviation of $\hat{\delta}$ of individual i, denoted by $\mu_{i,\hat{\delta}}$ and $\sigma_{i,\hat{\delta}}$, are given by $\mu_{i,\hat{\delta}} = x_i'\beta_{\hat{\delta}}$ and $\sigma_{i,\hat{\delta}} = x_i'\gamma_{\hat{\delta}}$.

With this specification we use the methods discussed in the main text to compute the probability of choosing a lottery from a given menu and evaluate the log-likelihood function of the model. We illustrate with the data from Andersen et al. (2008). This dataset includes binary indicators for gender, age below 30, age between 40 and 50,

[3]Notice that this especification nests the original pooled estimates by setting $x_i = 1$.

[4]The beta distribution is usually specified as a function of the shape parameters $\alpha, \beta > 0$. The mean and standard deviation of this distribution are in turn given by $\mu = \frac{\alpha}{\alpha+\beta}$ and $\sigma = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(1+\alpha+\beta)}}$, respectively. It follows that for any value of μ and σ such that $\mu \in (0,1)$ and $\sigma^2 < \mu(1-\mu)$, we can recover the implied shape parameters as $\alpha^* = \left(\frac{\mu}{\sigma^2}(1-\mu) - 1\right)\mu$ and $\beta^* = \alpha^*\left(\frac{1-\mu}{\mu}\right)$.

age over 50, living alone, having children, being a home owner, being retired, being a student, having some post-secondary education, having a substantial level of higher education, having a lower income level in 2002 (below 300,000 DKK), having a higher income level in 2002 (500,000 DKK or more), living in the greater Copenhagen area, and living in a city with 20,000 inhabitants or more. A full description and summary statistics of each variable are given in Table 7.

Tables 5 and 6 show the results of this exercise. The first set of columns in each table show the results when allowing the observed variables to affect only the mean of each distribution. The second set of columns show the results when allowing the observable variables to affect the standard deviation of the distributions only. The third set of columns show the results when allowing the observable variables to affect simultaneously both the mean and the standard deviation of each distribution.

In general, we find no relationship between most of the observable characteristics and the estimated mean and variances of risk and time preferences. The exceptions are gender, with females displaying higher standard deviation of risk aversion across specifications, and individuals with a substantial level of higher education, who display higher mean risk aversion. In terms of delay aversion, older individuals display lower and less volatile corrected discount factors, implying higher degrees of delay aversion. Individuals with substantial levels of higher education also display lower delay aversion, although this result is sensitive to allowing this variable also to affect the variance of delay aversion.

5. Prediction Error

In this section, we illustrate how to evaluate the predictive capacity of the models presented in this paper using data from Andreoni and Sprenger (2012b). That is, we split the data into $K = 10$ roughly equal-sized parts. For every part $k = 1, \dots, 10$, we estimate the DEU-H, PVCE-H and CEPV-H models using the other $K - 1$ parts of the data, and then calculate the prediction error of the fitted model when predicting part k. We use cross-entropy, or log-loss, as the loss function to measure prediction error. We then take the mean and standard deviation of the K estimates of the prediction error.[5]

The results of this exercise are shown in Table 8. Columns (i), (iv) and (vii) show the results of the estimated cross-entropy measures when pooling all observations in

[5]See Hastie (2009) for a detailed exposition of cross-validation for model selection and assessment.

the sample and then splitting the data randomly. We can see that both PVCE-H and CEPV-H significantly outperform DEU-H for out-of-sample prediction. The average prediction error is reduced by almost half for both models. Furthermore, the standard deviation of the prediction error of DEU-H across estimates is four times that of PVCE-H and CEPV-H. Finally, CEPV-H slightly outperforms PVCE-H on average. Columns (ii), (v) and (viii) show the results of the estimated cross-entropy measures when the parts are created by splitting the individuals in the sample into $K = 10$ parts and then assigning the observations for each individual to the corresponding part. Implicitly, this way of splitting the data evaluates the predictive accuracy of the models for the choices of out-of-sample individuals. The average prediction error is similar to that obtained in the baseline exercise but its standard deviation is now higher in all specifications. In relative terms, however, the performance of all three models is very similar to that obtained in the baseline scenario. As an alternative approach, we create the parts by splitting the menus into $K = 10$ parts and then assigning the observations for each menu to the corresponding part. In other words, we evaluate the predictive accuracy of the models for choices in out-of-sample menus. The results of this cross-validation exercise are reported in columns (iii), (vi) and (ix). Once again, the average prediction error obtained for DEU-H is now three times higher, while the volatility of the prediction errors of PVCE-H and CEPV-H is much lower and more stable than that of DEU-H.

The results of this exercise suggest that the out-of-sample performance of the DEU-H model is substantially improved by accounting for the different curvatures in risk aversion and intertemporal substitution, as in the PVCE-H and CEPV-H models.

References

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[2] Andreoni, J. and C. Sprenger (2012b). "Risk Preferences Are Not Time Preferences." *American Economic Review*, 102(7):3357–3376.

[3] Coble, K.H. and J. Lusk (2010). "At the Nexus of Risk and Time Preferences: An Experimental Investigation." *Journal of Risk and Uncertainty*, 41(1):67–79.

[4] Hastie, T., Tibshirani, R., and Friedman, J. H. (2009). "The elements of statistical learning: data mining, inference, and prediction". 2nd ed. New York: Springer.

Table 1. Estimated Risk and Time Preferences with Tremble Probability

Dataset	**Andersen et al. (2008)**	**Coble and Lusk (2010)**	**Andreoni & Sprenger (2012b)**		
Model	***DEU-H***	***DEU-H***	***DEU-H***	***PVCE-H***	***CEPV-H***
Median h/r	0.710	0.404	−0.060	0.206	0.369
	[0.028]	[0.073]	[0.049]	[0.101]	[0.090]
Std. Dev. h/r	0.329	0.376	0.486	0.720	0.862
	[0.018]	[0.028]	[0.043]	[0.307]	[0.109]
Median $\hat{\delta}$	0.981	0.914	0.967	0.977	0.973
	[0.001]	[0.009]	[0.004]	[0.005]	[0.004]
Std. Dev. $\hat{\delta}$	0.009	0.077	0.058	0.040	0.049
	[0.001]	[0.011]	[0.008]	[0.012]	[0.011]
Median η	–	–	–	0.059	−0.337
				[0.039]	[0.076]
Std. Dev. η	–	–	–	0.448	0.809
				[0.048]	[0.101]
Tremble Prob. ν	0.245	0.063	0.252	0.116	0.110
	[0.016]	[0.021]	[0.029]	[0.025]	[0.022]
Log-Likelihood	−0.513	−0.373	−1.807	−1.714	−1.637

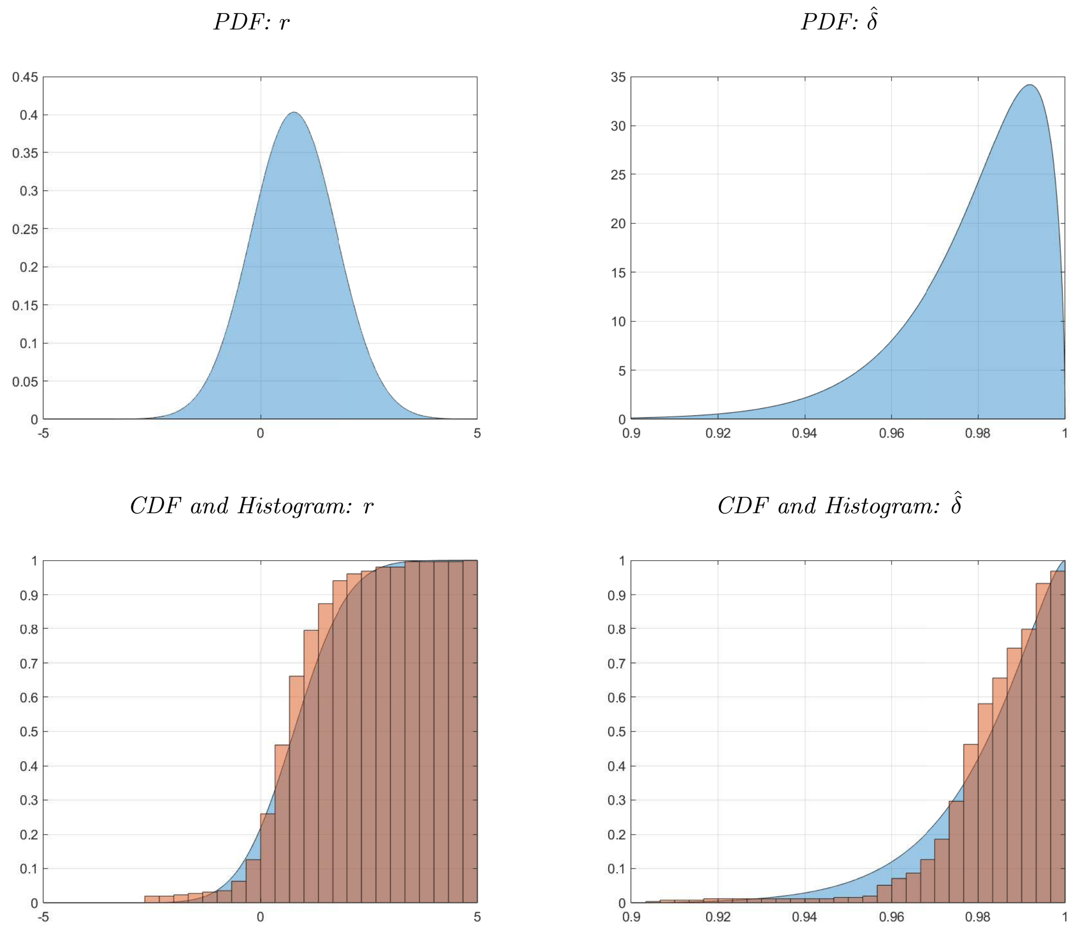
NOTES. The above table reports the maximum-likelihood estimates of the median and standard deviation of the distributions of risk aversion, corrected discount factor and intertemporal substitution under different representations, using data from Andersen et al. (2008), Coble and Lusk (2010) and Andreoni and Sprenger (2012b). The baseline model depicted in Table 1 in the paper is extended by adding a tremble probability ν. The coefficient of risk aversion, denoted h in DEU-H and r in PVCE-H and CEPV-H, follows a normal distribution truncated at 1, the corrected discount factor $\hat{\delta}$ follows a beta distribution and the curvature of intertemporal substitution η follows a normal distribution truncated at 1. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level.

TABLE 2. Estimated Risk and Time Preferences using PVCE-H^E: Andersen et al. (2008)

Dataset	Risk Only	Time Only	Joint	by Individual
Median r	0.767		0.768	0.687
	[0.048]		[0.048]	(0.343)
Std. Dev. r	0.987		0.988	0.888
	[0.058]		[0.058]	(0.514)
Median $\hat{\delta}$		0.983	0.983	0.980
		[0.001]	[0.001]	(0.007)
Std. Dev. $\hat{\delta}$		0.016	0.016	0.084
		[0.001]	[0.001]	(0.051)
# Obs.	7928	15180	23108	23108
Log-Likelihood	−0.679	−0.543	−0.589	−0.358

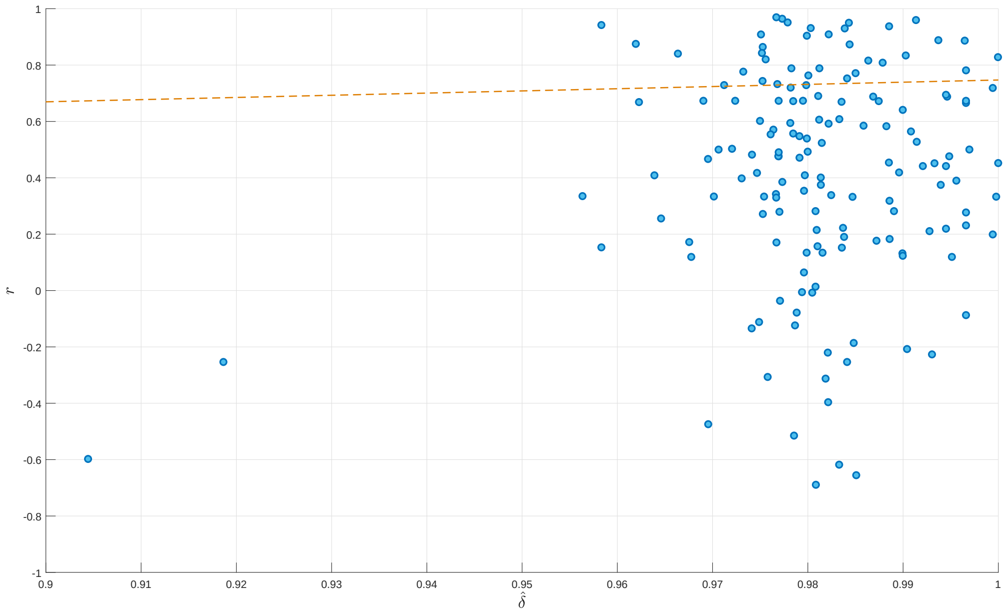
NOTES. The above table reports the maximum-likelihood estimates of the median and standard deviation of the distributions of risk and time preferences under the PVCE-H^E representation with $\eta = 0$ using data from Andersen et al. (2008). The second column shows the results obtained using the menus eliciting risk aversion only, and the third column shows the results obtained using the menus eliciting delay aversion only. The fourth column shows the results of the joint estimation of risk aversion and delay aversion using the pooled sample menus. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The last column shows the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution while the corrected discount factor $\hat{\delta}$ follows a beta distribution.

Figure 1. PDFs and CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using PVCE-H^E: Andersen et al. (2008)



NOTES.- PDFs and CDFs of the pooled estimates, and histograms of the empirical distributions of the individual estimates using PVCE-H^E with $\eta = 0$.

Figure 2. Individual Estimates using PVCE-H^E: Andersen et al. (2008)



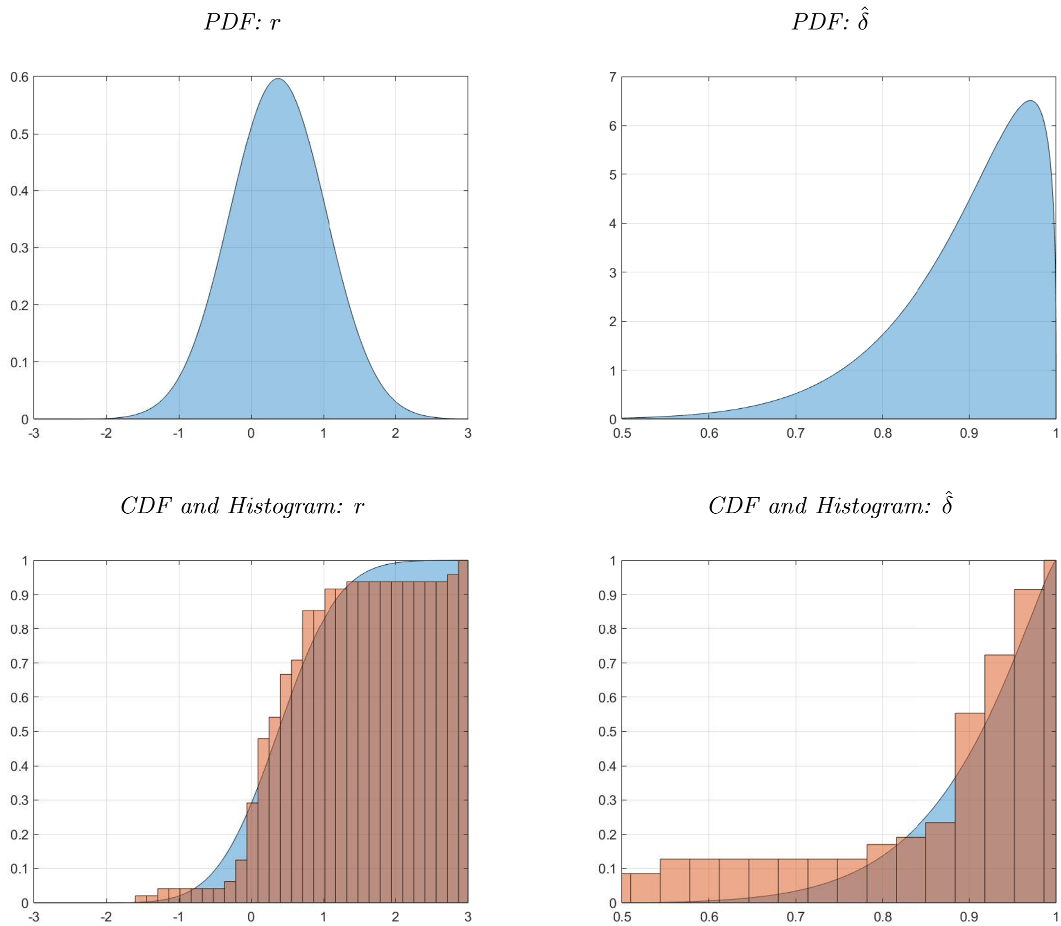
NOTES. Each point represents the median of the estimated distributions of the coefficient of risk aversion h and the corrected discount factor $\hat{\delta}$ for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 2.

Table 3. Estimated Risk and Time Preferences using PVCE-H^E: Coble and Lusk (2010)

Dataset	**Using Risk Tasks Only**	**Using Discount Tasks Only**	**Using Joint Tasks Only**	**All Tasks**	**All Tasks - Correlated Preferences**	**Pooled Individual Estimates**
Median r	0.414 [0.083]	–	0.497 [0.102]	0.371 [0.087]	0.410 [0.082]	0.289 (0.023)
Std. Dev. r	0.669 [0.084]	–	0.476 [0.072]	0.669 [0.089]	0.654 [0.083]	0.744 (0.320)
Median $\hat{\delta}$	–	0.903 [0.012]	0.947 [0.017]	0.916 [0.010]	0.912 [0.010]	0.907 (0.053)
Std. Dev. $\hat{\delta}$	–	0.089 [0.017]	0.133 [0.036]	0.085 [0.013]	0.081 [0.012]	0.240 (0.033)
Corr(r,$\hat{\delta}$)	–	–	–	–	-0.524 [0.260]	-0.214
# Obs.	1880	1128	1410	4418	4418	47
Log-Likelihood	-0.429	-0.436	-0.361	-0.416	-0.414	-0.195

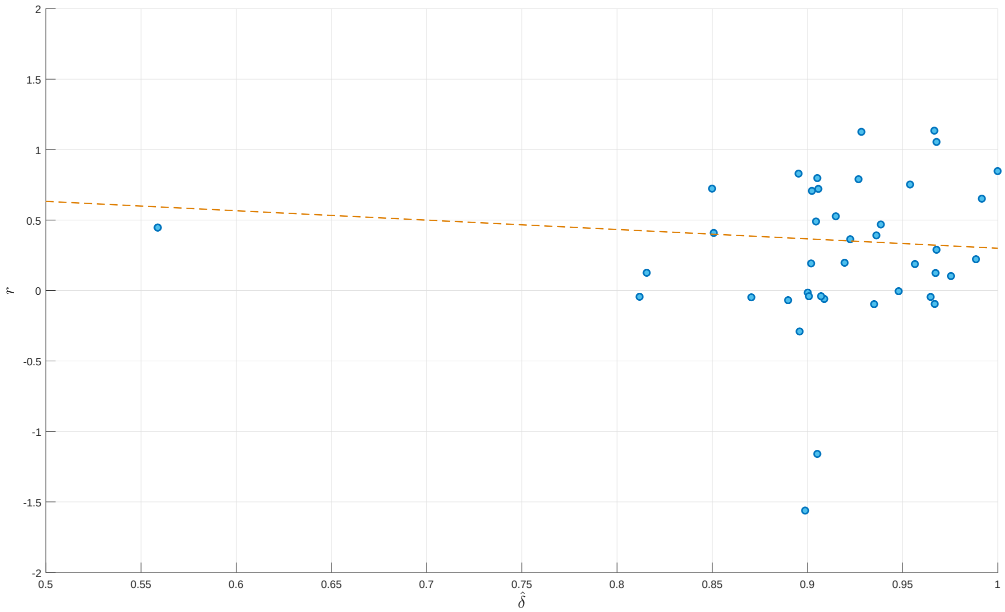
NOTES. The above table reports the maximum-likelihood estimates of the median and standard deviation of the distributions of risk and time preferences under the PVCE-H^E representation with $\eta = 0$ using data from Coble and Lusk (2010). The second column shows the results obtained using the menus eliciting risk aversion only. The third column shows the results obtained using the menus eliciting delay aversion only. The fourth column shows the results using dated lottery menus only. The fifth column shows the results of the joint estimation of risk aversion and delay aversion using the pooled sample menus. The sixth column shows the estimates obtained when allowing correlation between parameters using a Gaussian copula. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The last column shows the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution while the corrected discount factor $\hat{\delta}$ follows a beta distribution.

FIGURE 3. PDFs and CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using PVCE-H^E: Coble and Lusk (2010)



NOTES. PDFs and CDFs of the pooled estimates, and histograms of the empirical distributions of the individual estimates using PVCE-H^E with $\eta = 0$.

FIGURE 4. Individual Estimates using PVCE-H^E: Coble and Lusk (2010)



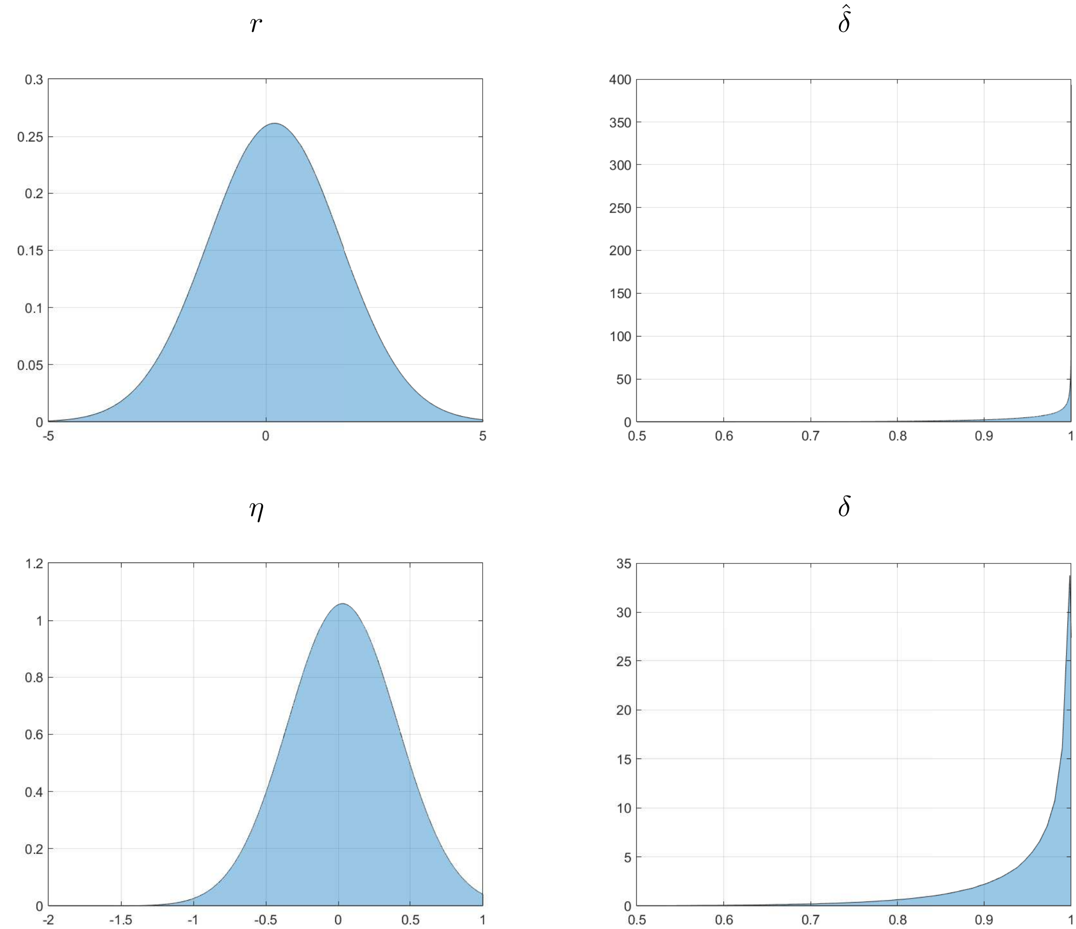
NOTES. Each point represents the median of the estimated distributions of the coefficient of risk aversion h and the corrected discount factor $\hat{\hat{\delta}}$ for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 3.

TABLE 4. Estimated Risk and Time Preferences using PVCE-H^E and CEPV-H^E: Andreoni and Sprenger (2012b)

	$PVCE-H^E$				$CEPV-H^E$			
Dataset	**(i)**	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**
Median h	0.199	0.158	0.248	0.304	0.431	0.391	0.321	0.457
	[0.163]	[0.235]	[0.195]	(0.645)	[0.138]	[0.145]	[0.128]	(0.403)
Std. Dev. h	1.517	1.812	1.564	1.666	1.339	1.351	1.392	1.594
	[0.122]	[0.224]	[0.138]	(0.818)	[0.128]	[0.126]	[0.167]	(0.765)
Median $\hat{\delta}$	0.971	0.972	0.965	0.982	0.968	0.970	0.953	0.978
	[0.004]	[0.004]	[0.005]	(0.054)	[0.005]	[0.004]	[0.008]	(0.057)
Std. Dev. $\hat{\delta}$	0.071	0.070	0.084	0.114	0.083	0.083	0.120	0.189
	[0.010]	[0.010]	[0.013]	(0.068)	[0.012]	[0.012]	[0.022]	(0.065)
Median η	0.029	0.021	-0.038	0.069	-0.486	-0.386	-0.215	-0.421
	[0.041]	[0.040]	[0.040]	(0.260)	[0.229]	[0.187]	[0.081]	(0.438)
Std. Dev. η	0.371	0.358	0.364	0.555	0.823	0.765	0.559	0.671
	[0.037]	[0.040]	[0.036]	(0.267)	[0.107]	[0.106]	[0.069]	(0.578)
Background Consumption $\omega \times 100$	–	0.034 [0.048]	–	–	–	1.017×10^{-4} $[0.594 \times 10^{-4}]$	–	–
Corr(r,$\hat{\delta}$)	–	–	-0.145	-0.076	–	–	-0.297	-0.146
			[0.187]				[0.118]	
Corr(r,s)	–	–	-0.465	-0.019	–	–	-0.333	0.001
			[0.172]				[0.097]	
Corr(η,$\hat{\delta}$)	–	–	-0.295	-0.268	–	–	-0.390	-0.116
			[0.090]				[0.110]	
Log-Likelihood	-1.685	-1.676	-1.671	-1.144	-1.680	-1.678	-1.658	-1.126

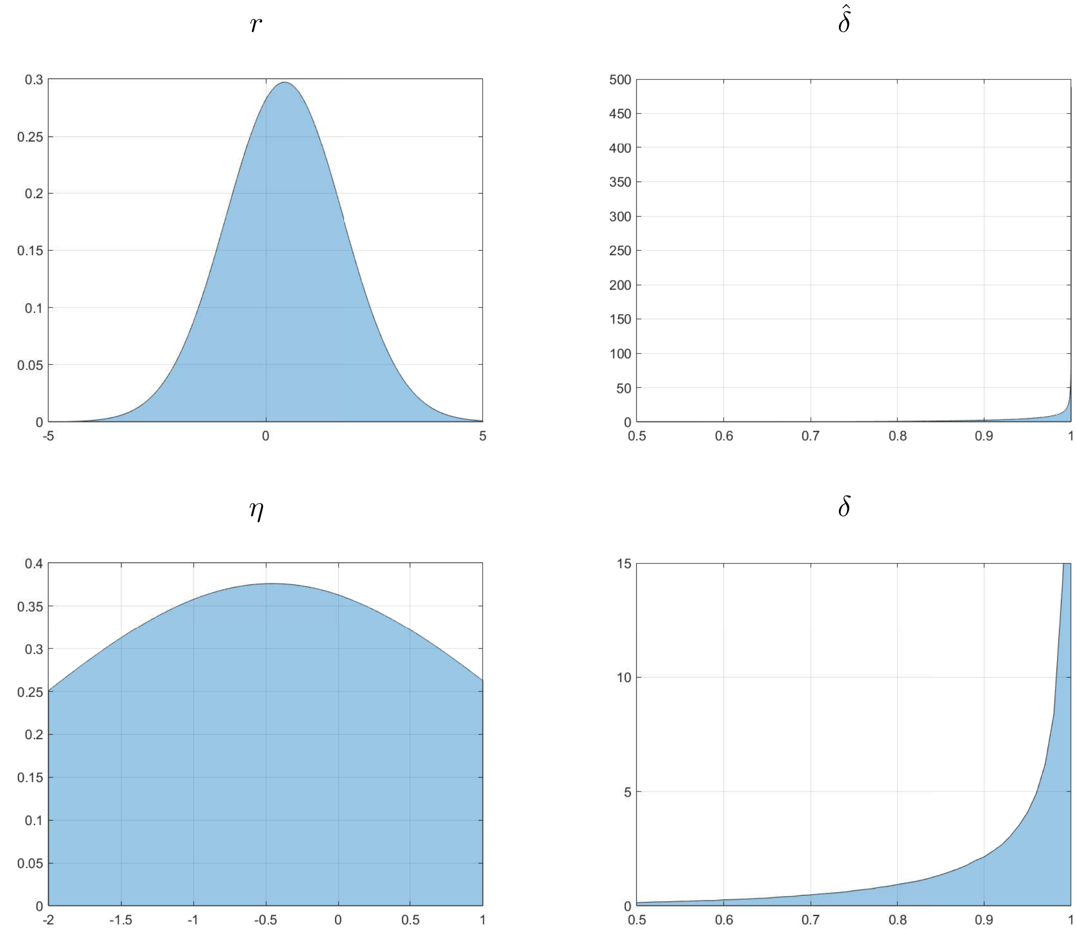
NOTES. The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distributions of risk aversion, corrected discount factor, and intertemporal substitution under PVCE-H^E and CEPV-H^E, using data from Andreoni and Sprenger (2012b). Columns (i) and (v) show the results from estimating each model with the pooled sample observations, assuming a small fixed background consumption level of $\omega = 10^{-6}$. Columns (ii) and (vi) show the results when $\omega > 0$ is estimated together with the other parameters. Columns (iii) and (vii) show the results for the respective models when allowing correlation between parameters using a Gaussian copula. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. Finally, columns (iv) and (viii) report the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution, the corrected discount factor $\hat{\delta}$ to follow a beta distribution and the curvature of intertemporal substitution η to follow a normal distribution truncated at 1.

Figure 5. PDFs of Estimated Risk and Time Preferences using PVCE-H^E: Andreoni and Sprenger (2012b)



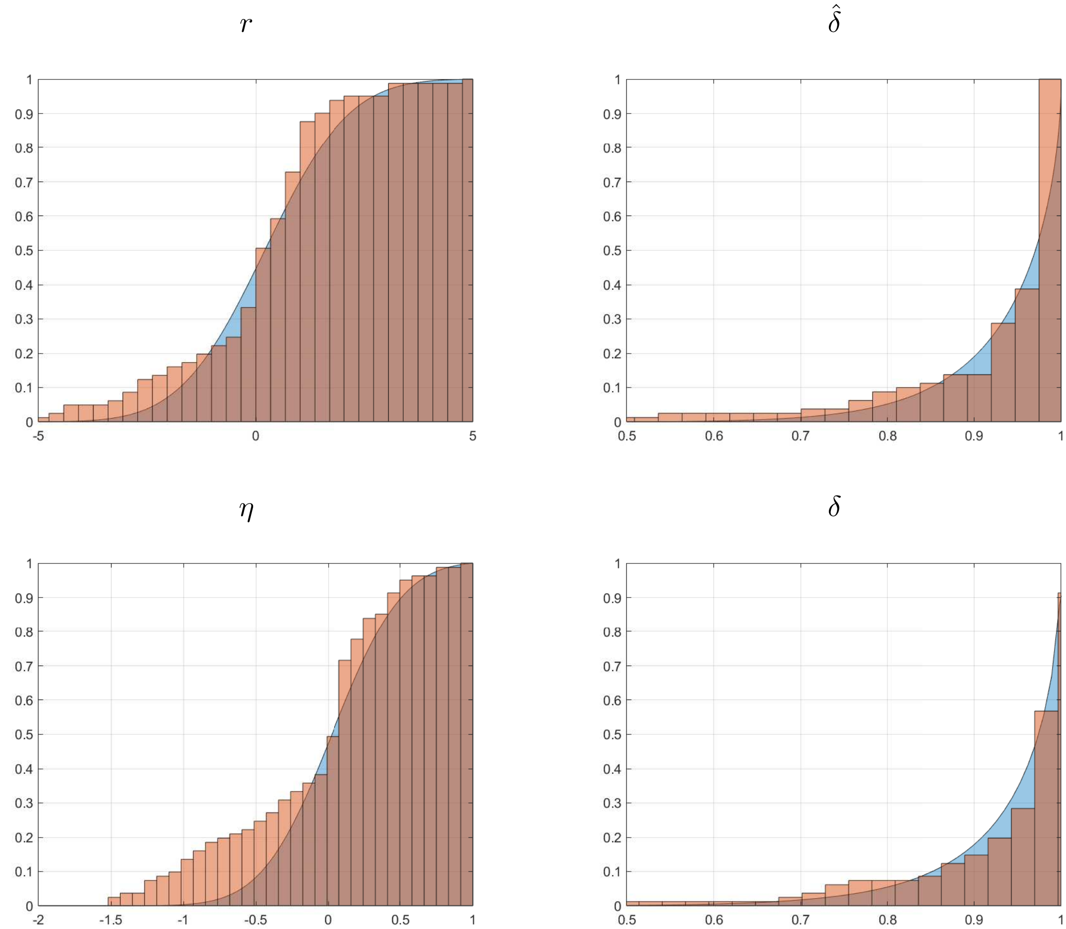
NOTES. PDFs of the estimated distributions reported in Table 4. The PDF of the discount factor $\delta = \hat{\delta}^{1-\eta}$ is estimated non-parametrically from the distributions of intertemporal substitution and the corrected discount factor using a normal kernel.

Figure 6. PDFs of Estimated Risk and Time Preferences using CEPV-H^E: Andreoni and Sprenger (2012b)



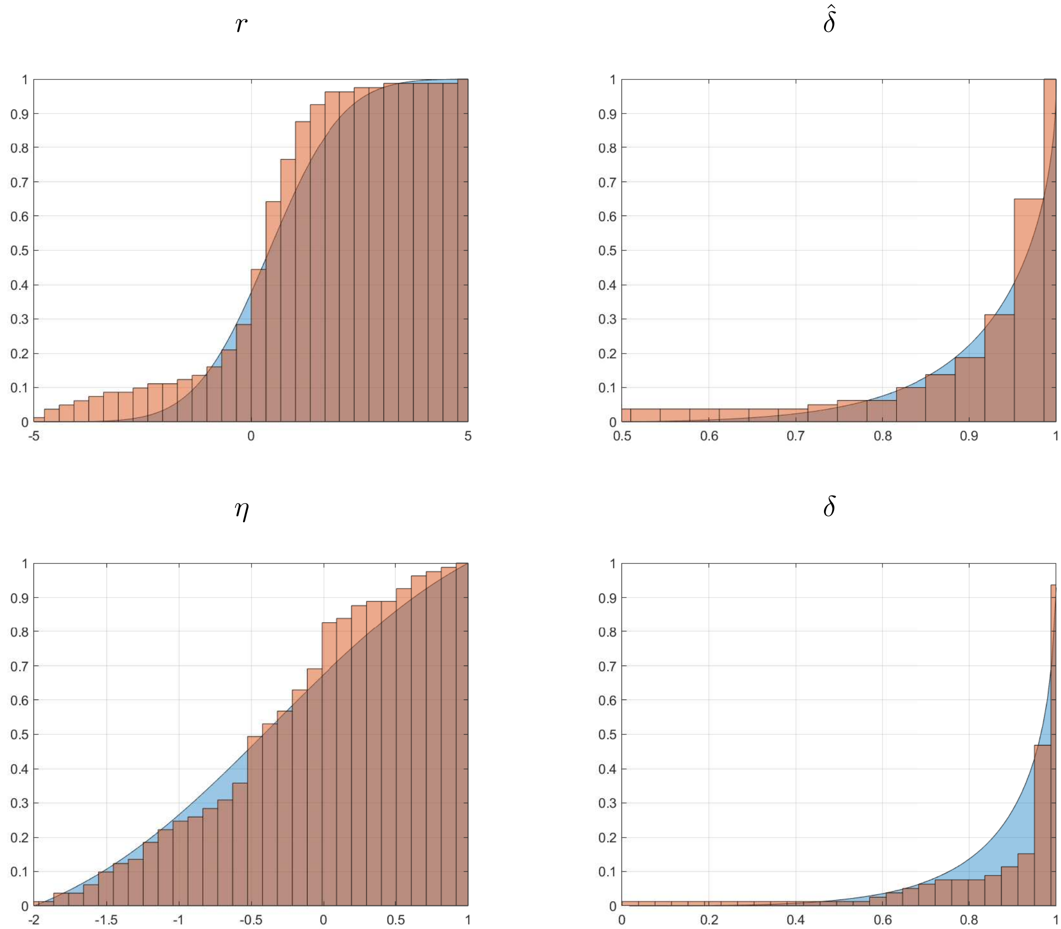
NOTES. PDFs of the estimated distributions reported in Table 4. The PDF of the discount factor $\delta = \hat{\delta}^{1-\eta}$ is estimated non-parametrically from the distributions of intertemporal substitution and the corrected discount factor using a normal kernel.

Figure 7. CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using PVCE-H^E: Andreoni and Sprenger (2012b)



NOTES. CDFs of the pooled estimation and histograms of the empirical distributions of the individual estimates using PVCE-H^E.

Figure 8. CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using CEPV-H^E: Andreoni and Sprenger (2012b)



NOTES. CDFs of the pooled estimation and histograms of the empirical distributions of the individual estimates using CEPV-H^E.

FIGURE 9. Observed and Predicted Distributions of Choices using PVCE-H^E and CEPV-H^E: Andreoni and Sprenger (2012b)

$PVCE - H^E$ | $CEPV - H^E$

Risk Condition 1: $(p, q) = (1, 1)$

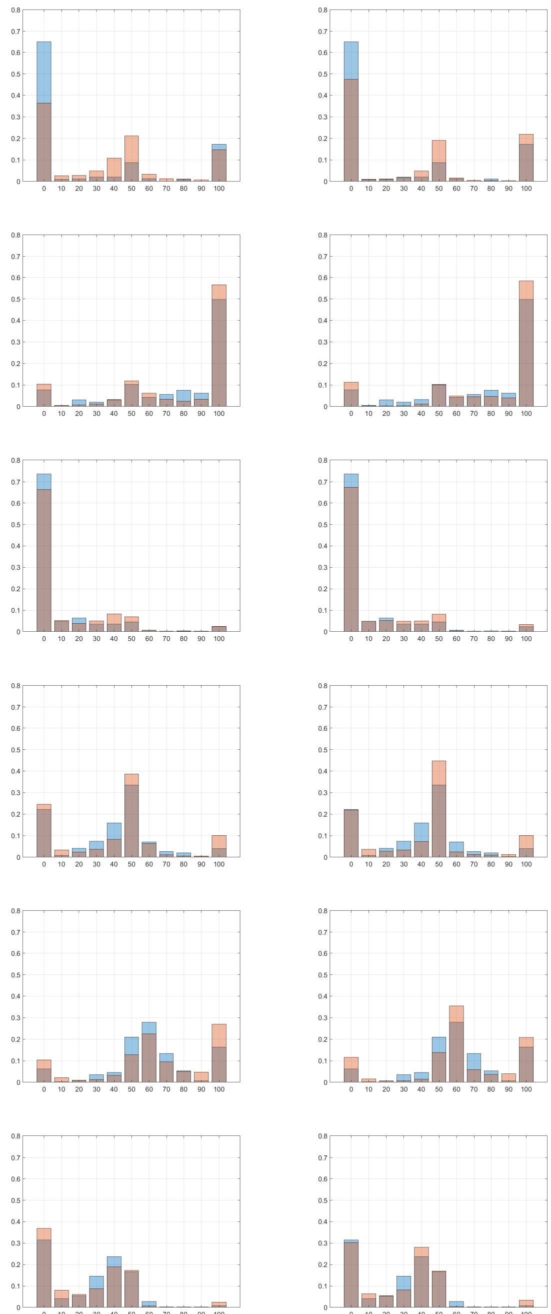
Risk Condition 2: $(p, q) = (1, 0.8)$

Risk Condition 3: $(p, q) = (0.8, 1)$

Risk Condition 4: $(p, q) = (0.5, 0.5)$

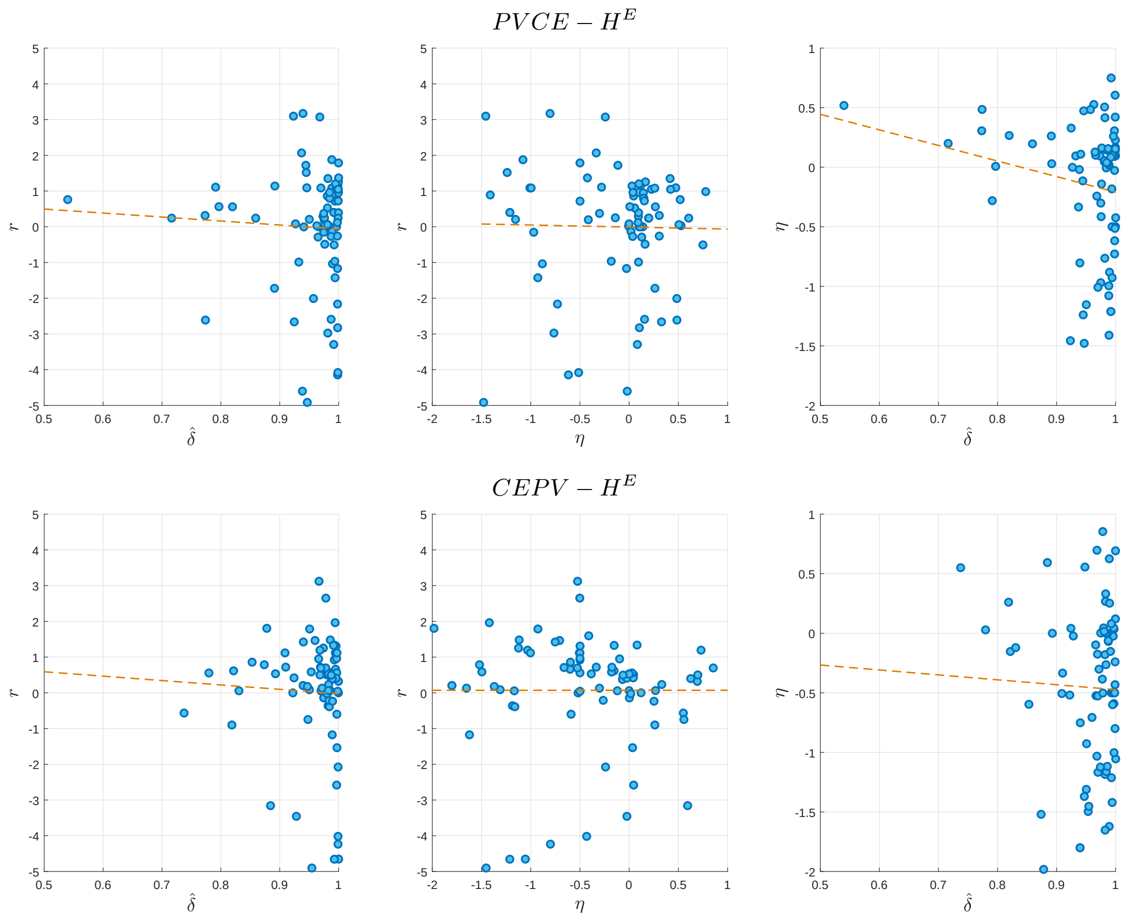
Risk Condition 5: $(p, q) = (0.5, 0.4)$

Risk Condition 6: $(p, q) = (0.4, 0.5)$



NOTES. Observed frequencies and predicted probabilities of choosing share α ($\times 100$) under each risk condition considered in Andreoni and Sprenger (2012b). The observed distributions show the relative frequency of each allocation in the data, grouped to the closest multiple of 10. The predicted distributions are computed based on the estimated parameters of the respective representation, as shown in Table 4.

FIGURE 10. Individual Estimates using PVCE-H^E and CEPV-H^E: Andreoni and Sprenger (2012b)



NOTES. Each point represents the median of the estimated distributions of the coefficient of risk aversion h/r, the corrected discount factor $\hat{\delta}$, and the curvature of intertemporal substitution η for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 4.

Table 5. Estimated Risk Preferences and Observable Characteristics

Estimated Coefficient	*(i) Mean Eq. Only*		*(i) XD Eq. Only*		*(i) Both Eqs.*	
	μ_h	σ_h	μ_h	σ_h	μ_h	σ_h
Constant	0.485	0.936	0.805	0.549	0.367	0.227
	[0.185]	[0.052]	[0.051]	[0.156]	[0.100]	[0.068]
female	−0.035			0.266	0.018	0.284
	[0.087]			[0.101]	[0.113]	[0.126]
young	0.260			0.176	0.311	0.289
	[0.179]			[0.135]	[0.200]	[0.176]
middle	−0.256			0.499	−0.167	0.361
	[0.129]			[0.145]	[0.150]	[0.159]
old	−0.157			0.428	−0.060	0.372
	[0.152]			[0.136]	[0.145]	[0.143]
copen	0.102			−0.030	0.085	0.024
	[0.125]			[0.116]	[0.128]	[0.131]
city	−0.004			0.000	0.027	0.078
	[0.105]			[0.107]	[0.108]	[0.131]
owner	0.103			0.245	0.155	0.335
	[0.111]			[0.093]	[0.112]	[0.106]
retired	0.123			−0.118	0.094	−0.038
	[0.138]			[0.177]	[0.139]	[0.139]
student	0.462			−0.172	0.519	0.142
	[0.173]			[0.131]	[0.269]	[0.273]
skilled	0.251			−0.154	0.245	0.057
	[0.105]			[0.116]	[0.101]	[0.127]
longedu	0.307			−0.250	0.242	0.012
	[0.125]			[0.144]	[0.142]	[0.184]
kids	0.109			−0.058	0.087	−0.021
	[0.127]			[0.129]	[0.113]	[0.143]
single	0.207			−0.084	0.197	−0.005
	[0.124]			[0.134]	[0.125]	[0.139]
IncLow	−0.182			0.166	−0.149	0.010
	[0.125]			[0.134]	[0.131]	[0.137]
IncHigh	0.015			−0.054	0.016	−0.082
	[0.119]			[0.141]	[0.123]	[0.139]
Log-Likelihood	−0.658		−0.661		−0.649	

NOTES.The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distribution of the coefficient of risk aversion r under the PVCE-H^E representation with $\eta = 0$, using data from Andersen et al. (2008) and allowing the mean and standard deviation of the distribution to depend linearly on a set of observable characteristics of each individual. Column (i) shows the estimated coefficients when only the mean of the distribution is allowed to depend on the observable variables. Column (ii) shows the estimated coefficients when only the standard deviation of the distribution is allowed to depend on the observable variables. Column (iii) shows the results when both the mean and the standard deviation of the distribution are allowed to depend on the observable variables. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level.

Table 6. Estimated Time Preferences and Observable Characteristics

Estimated Coefficient	*(i) Mean Eq. Only*		*(i) XD Eq. Only*		*(i) Both Eqs.*	
	$\mu_{\hat{\delta}}$	$\sigma_{\hat{\delta}}$	$\mu_{\hat{\delta}}$	$\sigma_{\hat{\delta}}$	$\mu_{\hat{\delta}}$	$\sigma_{\hat{\delta}}$
Constant	0.981	0.016	0.978	0.021	0.980	0.018
	[0.003]	[0.001]	[0.001]	[0.004]	[0.004]	[0.004]
female	−0.062			0.087	−0.213	0.361
	[0.124]			[0.162]	[0.166]	[0.185]
young	0.196			0.272	0.167	0.050
	[0.245]			[0.464]	[0.317]	[0.476]
middle	−0.195			−0.418	−0.049	−0.414
	[0.192]			[0.273]	[0.274]	[0.319]
old	−0.426			−0.809	−0.082	−0.770
	[0.204]			[0.314]	[0.256]	[0.321]
copen	−0.211			−0.120	−0.353	0.269
	[0.159]			[0.176]	[0.204]	[0.213]
city	−0.206			−0.136	−0.414	0.396
	[0.145]			[0.169]	[0.193]	[0.211]
owner	−0.013			0.086	−0.156	0.237
	[0.166]			[0.153]	[0.188]	[0.196]
retired	0.315			0.449	0.335	0.113
	[0.177]			[0.176]	[0.233]	[0.230]
student	0.099			−0.021	0.288	−0.364
	[0.254]			[0.419]	[0.271]	[0.380]
skilled	0.217			0.131	0.296	−0.158
	[0.148]			[0.179]	[0.166]	[0.171]
longedu	0.397			0.430	0.350	0.118
	[0.166]			[0.231]	[0.231]	[0.285]
kids	−0.277			−0.388	−0.246	−0.108
	[0.165]			[0.262]	[0.199]	[0.212]
single	−0.210			−0.488	0.149	−0.638
	[0.174]			[0.171]	[0.213]	[0.254]
IncLow	−0.218			−0.276	−0.280	0.061
	[0.169]			[0.198]	[0.214]	[0.262]
IncHigh	0.199			0.079	0.397	−0.326
	[0.164]			[0.274]	[0.213]	[0.218]
Log-Likelihood	−0.529		−0.533		−0.525	

NOTES. The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distribution of the corrected discount factor $\hat{\delta}$ under the PVCE-H^E representation with $\eta = 0$, using data from Andersen et al. (2008) and allowing the mean and standard deviation of the distribution to depend linearly on a set of observable characteristics of each individual. Column (i) shows the estimated coefficients when only the mean of the distribution is allowed to depend on the observable variables. Column (ii) shows the estimated coefficients when only the standard deviation of the distribution is allowed to depend on the observable variables. Column (iii) shows the results when both the mean and the standard deviation of the distribution are allowed to depend on the observable variables. In all cases, the corrected discount factor is assumed to follow a beta distribution. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The estimated coefficients and standard errors of all variables, except the constant, are shown multiplied by 100.

TABLE 7. Description of the Observable Characteristics in Andersen et al. (2008)

Variable	**Description**	**Mean in the Data**	**XD in the Data**
female	Female	0.51	0.50
young	Aged less than 30	0.17	0.38
middle	Aged between 40 and 50	0.28	0.45
old	Aged over 50	0.38	0.49
single	Lives alone	0.27	0.45
kids	Has children	0.39	0.49
owner	Owns home or apartment	0.69	0.46
retired	Being retired	0.16	0.37
student	Being a student	0.09	0.29
skilled	Having some post-secondary education	0.38	0.49
longedu	Having substantial higher education	0.36	0.48
IncLow	Having a lower income level in 2002	0.28	0.45
IncHigh	Having a higher income level in 2002	0.20	0.40
copen	Having in the greater Copenhagen area	0.34	0.48
city	Living in city of 20,000 inhabitants or more	0.34	0.47

NOTES. The above table describes each of the observable characteristics in the data from Andersen et al. (2008), as well as their sample mean and standard deviation. All these characteristics are included in the dataset as indicator variables that take a value of one when the described condition is met by an individual.

TABLE 8. Model Assessment: DEU-H, PVCE-H and CEPV-H in the Andreoni and Sprenger (2012b) dataset

	DEU-H			**PVCE-H**			**CEPV-H**		
	(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**	**(ix)**
Mean	3.651	3.658	3.653	1.857	1.868	1.856	1.775	1.787	1.771
Std. Dev.	0.416	0.668	1.285	0.092	0.200	0.158	0.099	0.185	0.147

NOTES. The above table reports the mean and standard deviation of the cross-entropy measures obtained in the ten-fold cross-validation of the DEU-H, PVCE-H and CEPV-H representations, using the data in Andreoni and Sprenger (2012b). Columns (i), (iv) and (vii) report the results for the respective models when the full sample is pooled and randomly split into 10 subsamples for cross-validation. Columns (ii), (v) and (viii) report the results when the individuals in the sample are randomly split into 10 groups and their respective observations are used as the 10 subsamples for cross-validation. Columns (iii), (vi) and (ix) report the results when the sample menus are randomly split into 10 groups and their respective observations are used as the 10 subsamples for cross-validation.