SUPPLEMENTARY MATERIAL: RANDOM MODELS FOR THE JOINT TREATMENT OF RISK AND TIME PREFERENCES

JOSE APESTEGUIA†, MIGUEL A. BALLESTER‡, AND ANGELO GUTIERREZ§

1. Datasets

In this section we describe the main features of the experimental datasets used in the paper.

1.1. **Andersen et al. (2018).** This study separately elicited risk and time preferences of a representative sample of 253 subjects from the adult Danish population. In the risk part, four multiple-price lists were implemented, each comprising ten pairs of present lotteries. The four tasks were (i) $([p, 1-p; 3850, 100], 0)$ and $([p, 1-p; 2000, 1600], 0)$, (ii) $([p, 1-p; 4000, 500], 0)$ and $([p, 1-p; 2250, 1500], 0)$, (iii) $([p, 1-p; 4000, 150], 0)$ and $([p, 1-p; 2000, 1750], 0)$, and (iv) $([p, 1-p; 4500, 50], 0)$ and $([p, 1-p; 2500, 1000], 0)$, with $p \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$. All 253 subjects were confronted with the four tasks, with 116 individuals facing all pairs, 67 individuals facing pairs 3, 5, 7, 8, 9, and 10, and the remaining 70 subjects facing pairs 1, 2, 3, 5, 7, and 10, for a total of 7,928 choices in this part. In the time part, six multiple-price lists involving dated degenerate lotteries were implemented. The first degenerate lottery always paid 3.000 DKK after one month. The second degenerate lottery was designed by varying two parameters: (i) the awarding time, which could vary between 2, 5, 7, 13, 19 or 25 months, and (ii) the annual interest obtained by the subject, which increased by multiples of 5, from 5% to 50%.[1] All the subjects faced all 60 temporal binary choices, making a total of 15,180 choices. That is, $\mathcal{O}$ is formed by 23,108 observations with the number of individual

Date: March, 2020.

* Financial support by Spanish Ministry of Science (PGC2018-098949-B-I00), Balliol College, "la Caixa" Foundation Research Grants on Socioeconomic Well-being (CG-2017-01) and "la Caixa" Severo-Ochoa International Doctoral Fellowship is gratefully acknowledged.

†ICREA, Universitat Pompeu Fabra and Barcelona GSE. E-mail: jose.apesteguia@upf.edu.

‡University of Oxford. E-mail: miguel.ballester@economics.ox.ac.uk.

§Universitat Pompeu Fabra and Barcelona GSE. E-mail: angelo.gutierrez@upf.edu.

[1] The interest was compounded quarterly to be consistent with general Danish banking practices.

observations varying between 84 and 100. For every pair of options, subjects could either choose one of them or express indifference between the two. In the latter case, they were told that the experimenter would settle indifferences by tossing a fair coin. Only 5% of the choices were indifferences; as in the original paper, we treat them assigning a half choice to each of the two gambles.

1.2. **Coble and Lusk (2010).** This paper reports on the risk and time preferences elicitation of 47 upper-level undergraduate and graduate students from Economics and Business courses. Each subject made a total of 94 choices in 9 different multiple-price lists, for a total of 4,418 observations. There were three parts to the experiment, differing according to the type of choice problem involved. Part (i) consisted of four multiple price list tasks using dated lotteries awarded at the same time period. The pairs were $([p, 1-p; 8, 10], t)$ and $([p, 1-p; 1, 19], t)$, with $p \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$, and $t \in \{0, 1, 13, 37\}$, measured in weeks. Part (ii) involved two tasks, in both of which the earlier degenerate lottery was \$10 to be paid in 1 week, while the later lottery prize-money increased, by multiples of half a dollar, from \$10 to \$15.5, and was paid either in week 13 or 37. Finally, there were three tasks in part (iii) involving the same lotteries as in part (i), with varying periods of times: $([p, 1-p; 8, 10], t)$ and $([p, 1-p; 1, 19], s)$, with $p \in \{.1, .2, .3, .4, .5, .6, .7, .8, .9, 1\}$, and $(t, s) \in \{(1, 13), (1, 37), (13, 37)\}$.

1.3. **Andreoni and Sprenger (2012b).** Andreoni and Sprenger (2012b) introduced risky considerations in the convex budget design of Andreoni and Sprenger (2012a). 80 undergraduate students took part in the experiment, each making 84 choices, for a total of 6,720 observations. The experimental parameters can be described, following our notation, as follows: $([p, 1-p; \alpha x, 0], t)$ and $([q, 1-q; (1-\alpha)y, 0], s)$, with $(p, q) \in \{(1, 1), (0.5, 0.5), (1, 0.8), (0.5, 0.4), (0.8, 0.1), (0.4, 0.5)\}$, $(t, s) \in \{(7, 28), (7, 56)\}$ in days, and $(x, y) \in \{(20, 20), (19, 20), (18, 20), (17, 20), (16, 20), (15, 20), (14, 20)\}$. The decision variable was discrete, with $\alpha \in \{\frac{0}{100}, \frac{1}{100}, \dots, \frac{100}{100}\}$.

2. Other Deterministic Utility Models

It is well-known that DEU does not permit a different treatment of money for the risk-evaluation of uncertain payoffs and the time-evaluation of certain payoffs. To visualize this, consider the case of dated lotteries in $\mathcal{L} \times \{0\}$, involving only risk considerations. These are evaluated as $DEU_{\delta,u}(l, 0) = \sum_{n=1}^{N} p_n u(x_n)$, using an expected utility functional based on u. Now, consider the case of dated degenerate lotteries in $\mathcal{D} \times T$, involving only time considerations. These are evaluated as $DEU_{\delta,u}([1;x],t) = \delta^t u(x)$, which is but an exponentially discounted utility using δ and the *same* utility function u. We now introduce two utility representations that allow money to be treated in a different way for risk than for time.[2] Importantly, in so doing, we maintain the standard expected utility treatment of lotteries and exponentially discounted utility of intertemporal payoffs. Below, in Section 2.2, we discuss how to extend these concepts to incorporate behavioral considerations.

The first of these representations, which we call *present value of the certainty equivalent (PVCE)*, starts by eliminating the risk component by means of a certainty equivalent using expected utility. Once the problem has been reduced to the analysis of degenerate certainty equivalents at some future time, the time component is eliminated by transforming that future hypothetical payoff into its equivalent present value, under exponentially discounted utility. Formally,

$$PVCE_{\delta,w,v}(l,t) = w^{-1}[\delta^t w(v^{-1}[\sum_{n=1}^{N} p_n v(x_n)])]$$

$$PVCE_{\delta,w,v}(\alpha) = w^{-1}[\delta^t w(v^{-1}[pv(\alpha x)]) + \delta^s w(v^{-1}[qv((1-\alpha)y)])].$$

Notice that $v^{-1}[\cdot]$ represents a certainty-equivalent mapping, obtained throughout the use of expected utility with Bernoulli utility function $v \in \mathcal{U}$. Similarly, $w^{-1}[\cdot]$ represents a present equivalent value, obtained under exponential discounting with discount parameter δ and monetary utility $w \in \mathcal{U}$.[3] Given the assumptions on $\mathcal{U}$, every lottery has a unique certainty equivalent and every dated payoff has a unique present equivalent value.

---

[2] Kreps and Porteus (1978) and Selden (1978) introduced recursive expected utility, allowing for the separation of risk and time preferences. Later, Epstein and Zin (1989) and Chew and Epstein (1990) further developed the recursive setting, introduced parametric versions and allowed for behavioral considerations. Our models are non-recursive versions of these. Halevy (2008), Baucells and Heukamp (2012), Cheung (2015), Miao and Zhong (2015), Andreoni et al. (2017), Epper and Fehr-Duda (2019), Lanier et al. (2019) and DeJarnette et al. (2019) also study novel extensions separating risk and time preferences in various non-recursive contexts. Remarkably, DeJarnette et al. (2019) characterize, in a different setting involving monetary prizes awarded at uncertain future dates, a generalization of DEU that is essentially equivalent to our second model.

[3] Since $w \in \mathcal{U}$, w^{-1} is strictly increasing and hence, PVCE can be equivalently represented dispensing with w^{-1}.

The second representation, which we call *certainty equivalent of the present values (CEPV)*, reverses the order of analysis. It first eliminates the time component by transforming each of the possible sequences of payoffs into its equivalent present value, using exponentially discounted utility. Once the problem has been reduced to the evaluation of a hypothetical present lottery, the risk component is eliminated by transforming this lottery into its certainty equivalent, throughout expected utility. Formally,[4]

$$CEPV_{\delta,w,v}(l,t) = v^{-1}[\sum_{n=1}^{N} p_n v(w^{-1}[\delta^t w(x_n)])]$$

$$CEPV_{\delta,w,v}(\alpha) = v^{-1}[pqv(w^{-1}[\delta^t w(\alpha x) + \delta^s w((1-\alpha)y)]) + p(1-q)v(w^{-1}[\delta^t w(\alpha x)])+ \\ +(1-p)qv(w^{-1}[\delta^s w((1-\alpha)y)])].$$

The DEU representation is a proper restriction of both the PVCE and the CEPV representations in which the individual uses the same monetary utility to evaluate both intertemporal and risk trade-offs, i.e., $v = w$. Indeed, we can show that, in environments involving dated basic lotteries, the intersection of PVCE and CEPV is exactly DEU.[5] That is, DEU is the only model in which the order of the individual's risk and temporal decision-making processes is inconsequential.

Proposition 1. *Let $\mathcal{B} \times T$. The set of preferences admitting a DEU representation coincides with the set of preferences admitting both a PVCE and a CEPV representation.*

Proof of Proposition 1: Since PVCE and CEPV are extensions of DEU, it is evident that DEU belongs to the intersection of both classes. Now suppose that some behavior over basic lotteries belongs to the intersection of both PVCE and CEPV. Consider the PVCE representation of this behavior, and rewrite it as $\delta^t g(pv(x))$, where $g = w \circ v^{-1}$. We now prove that the function g must be homogeneous.

First, consider any $v_0 \in \mathbb{R}_{++}$ in the range of possible utility values associated to v, i.e., there exists a payoff x_0 such that $v(x_0) = v_0$. Consider $0 < p < 1$ and, whenever it exists, the monetary outcome x_1 such that $v(x_1) = \frac{v_0}{p}$. Then, take the value $t_1 \in T$ such that $w(x_0) = g(v_0) = \delta^{t_1} g(\frac{v_0}{p}) = \delta^{t_1} w(x_1)$. That is, t_1 is the value that makes the dated degenerate lotteries $([1; x_0], 0)$ and $([1; x_1], t_1)$ indifferent. Hence, the

[4] Analogously to PVCE, since v^{-1} is strictly increasing, CEPV can be equivalently represented dispensing with v^{-1}. Note in addition that by writing $\phi = v \circ w^{-1}$, the CEPV representation is basically the generalized DEU model of DeJarnette et al. (2018) in the context of dated lotteries.

[5] Notice that both domains studied in the paper involve the evaluation of dated basic lotteries.

present value of x_1 awarded at t_1 must be x_0. Clearly, the present value of 0 (awarded at t_1) is 0 and since the choice behavior admits a CEPV representation, it must also be that $([p, 1-p; x_0, 0], 0)$ and $([p, 1-p; x_1, 0], t_1)$ provide the same utility. Hence, it must be that $w(v^{-1}[pv(x_0)]) = g(pv_0) = \delta^{t_1} g(p\frac{v_0}{p}) = \delta^{t_1} w(v^{-1}[pv(x_1)])$. This is simply $g(pv_0) = \delta^{t_1} g(v_0)$ or $\delta^{-t_1} g(pv_0) = g(\frac{1}{p}[pv_0])$. By repeated use of this reasoning, we can obtain, for every positive integer ι, $\delta^{-\iota t} g(pv_0) = g(\frac{1}{p^\iota}[pv_0])$, which proves that the function g is homogeneous of degree $\frac{t \log \delta}{\log p}$ on the sequence of utility points $(pv_0, v_0, \frac{v_0}{p}, \dots)$. By taking v_0 as close to zero as desired and p as close to 1 as desired, the continuity of the functions involved guarantees that g must be homogeneous on the positive orthant. Now, the homogeneity of g, with homogeneity of degree ξ, allows us to rewrite the PVCE representation as $\delta^t g(pv(x)) = g((\delta^t)^{\frac{1}{\xi}} pv(x)) = g(\bar{\delta}^t pv(x))$. Hence, preferences can be represented by a monotone transformation of DEU and the claim follows.■

The intuition for Proposition 1 goes as follows. Notice that PVCE can be written as $\delta^t g(pv(x))$, where $g = w \circ v^{-1}$. Consider one present and one future degenerate lottery, $([1; x], 0)$ and $([1; y], t)$, such that the individual is indifferent between the two. If the preferences admit a CEPV representation, the dated basic lotteries $([p, 1-p; x, 0], 0)$ and $([p, 1-p; y, 0], t)$ must also be indifferent. Since this holds for every p, one can select the exact value which, whenever multiplied by the utility of payoff y, reduces utility by as much as the discount δ^t does, thereby showing that g must be homogeneous. Homogeneity of g allows us to write PVCE as $g(\hat{\delta}^t pv(x))$, which is but a monotone transformation of DEU.

We now briefly turn to the study of more risk aversion and more delay aversion in the two utility representations presented here. By reasoning similar to that applied in Proposition 1 in the main text, we can argue that, in these models, risk aversion is connected to the curvature of the utility function v, while delay aversion is naturally connected to the curvature of the normalized utility function formed by the intertemporal substitutability function w and the discount parameter δ. We omit the proof of this immediate corollary.

Corollary 1.

(1) *More risk aversion: v_1 is a concave transformation of v_2 if, and only if, for every $l \in \mathcal{L}$ and every $x \in X$, $PVCE_{\delta_2,w_2,v_2}([1; x], 0) \geq PVCE_{\delta_2,w_2,v_2}(l, 0)$ (resp. $CEPV_{\delta_2,w_2,v_2}([1; x], 0) \geq CEPV_{\delta_2,w_2,v_2}(l, 0)$) implies $PVCE_{\delta_1,w_1,v_1}([1; x], 0) \geq PVCE_{\delta_1,w_1,v_1}(l, 0)$ (resp. $CEPV_{\delta_1,w_1,v_1}([1; x], 0) \geq CEPV_{\delta_1,w_1,v_1}(l, 0)$).*

(2) *More delay aversion: Fix* $\theta \in (0,1)$*.* $w_1^{\frac{\log\theta}{\log\delta_1}}$ *is a concave transformation of* $w_2^{\frac{\log\theta}{\log\delta_2}}$ *if, and only if, for every* $x, y \in X$ *and every* $s \in T$*,* $PVCE_{\delta_2,w_2,v_2}([1;x],0) \geq PVCE_{\delta_2,w_2,v_2}([1;y],s)$ *(resp.* $CEPV_{\delta_2,w_2,v_2}([1;x],0) \geq CEPV_{\delta_2,w_2,v_2}([1;y],s)$*) implies* $PVCE_{\delta_1,w_1,v_1}([1;x],0) \geq PVCE_{\delta_1,w_1,v_1}([1;y],s)$ *(resp.* $CEPV_{\delta_1,w_1,v_1}([1;x],0) \geq CEPV_{\delta_1,w_1,v_1}([1;y],s)$*).*

Since the comparative statics results obtained under DEU can be analogously extended to the case of PVCE and CEPV, we omit the basic details here. Given their relevance for the empirical section, we briefly comment on some of the implications in the case of convex budgets. Firstly, as in Proposition 3 in the main text, the convexity of the monetary utility functions is related to corner solutions. In the case of PVCE, for instance, it can be shown that corner solutions require the convexity of w.[6] Secondly, since the first-order condition for PVCE can be written as $\frac{v'(\alpha x)g'(pv(\alpha x))}{v'((1-\alpha)y)g'(qv((1-\alpha)y))} = \frac{\delta^s qy}{\delta^t px}$, with $g = w \circ v^{-1}$, more balanced interior solutions are the result of either more risk aversion or more intertemporal substitutability. Importantly, we can see the relative effect of each of these components through the changes in p and q, which affect only g. To illustrate, notice that the first-order condition of the homogeneous version of PVCE, PVCE-H is $(\frac{(1-\alpha)y}{\alpha x})^\eta = \frac{\delta^s y}{\delta^t x}(\frac{q}{p})^{\frac{1-\eta}{1-r}}$ and thus, variation in prize probabilities enables identification of the parameters.[7]

Thirdly, notice also that PVCE-H is in agreement with DEU-H that the solution depends entirely on the ratio $\frac{p}{q}$. Interestingly, the alternative representation CEPV-H is sensitive to the probabilities beyond the ratio, as the first order condition is simply $(\frac{(1-\alpha)y}{\alpha x})^\eta = \frac{\delta^s y}{\delta^t x}\frac{pqA_\alpha + (p(1-q))}{pqA_\alpha + (q(1-p))}$, where $A_\alpha = [\delta^t(\alpha x)^{1-\eta} + \delta^s((1-\alpha)y)^{1-\eta}]^{\frac{\eta-r}{1-\eta}}$. Notice how the right hand side is now dependent on α, except for the case in which $\eta = r$, i.e., for DEU-H.

2.1. **An Equivalence Result with Dated Lotteries.** Under the assumption of homogeneity, it is immediate to see that, in the setting of convex budgets, DEU-H is a strict subset of both PVCE-H and CEPV-H. That is, in convex budget settings, there

[6]The case of CEPV is more complex, as it requires analyzing the convexity of both v and w.

[7]The homogeneous versions PVCE-H and CEPV-H use the monetary functions $v_r(x) = \frac{x^{1-r}}{1-r}$ and $w_\eta(x) = \frac{x^{1-\eta}}{1-\eta}$, with $r, \eta < 1$, to capture risk aversion and intertemporal substitutability, respectively. Note that we denote the curvature of the Bernoulli function in DEU-H by h and in PVCE-H (and CEPV-H) by r, to emphasize that they represent different attitudes. Ultimately, the objective functions can be simplified to $PVCE_{\delta,\eta,r}(\alpha) = \delta^t p^{\frac{1-\eta}{1-r}}(\alpha x)^{1-\eta} + \delta^s q^{\frac{1-\eta}{1-r}}((1-\alpha)y)^{1-\eta}$ and $CEPV_{\delta,\eta,r}(\alpha) = pq[\delta^t(\alpha x)^{1-\eta} + \delta^s((1-\alpha)y)^{1-\eta}]^{\frac{1-r}{1-\eta}} + p(1-q)\delta^{\frac{t(1-r)}{1-\eta}}(\alpha x)^{1-r} + (1-p)q\delta^{\frac{s(1-r)}{1-\eta}}((1-\alpha)y)^{1-r}$.

are preferences that can be represented by PVCE-H (or alternatively by CEPV-H) but cannot be represented by DEU-H. Thus, convex budget settings allow us to evaluate the empirical content of these models. This is not the case in the setting of dated lotteries, however. Here, the assumption of homogeneity implies that PVCE-H is equivalent to CEPV-H, which in turn is equivalent to DEU-H.

Proposition 2. *Let $\mathcal{L} \times T$. The set of preferences admitting a DEU-H representation coincides with the set of preferences admitting a PVCE-H representation, which in turn coincides with the set of preferences admitting a CEPV-H representation.*

Proof of Proposition 2: In a series of claims, the proof characterizes the models described in the main text, ultimately showing the stated result. Consider a preference $\succsim$ over $\mathcal{L} \times T$. Here is a list of possible properties for such a preference.

Regularity (REG). $\succsim$ satisfies the following conditions:

(1) Rationality (RAT). $\succsim$ is complete and transitive.
(2) Continuity (CON). $\succsim$ is continuous.
(3) Risk-Monotonicity (R-MON). If l strictly first-order stochastically dominates (FOSD) l', then $(l, 0) \succ (l', 0)$.
(4) Time-Monotonicity (T-MON). If $t < s$, then $([1; 0], t) \sim ([1; 0], s)$ and for every $l \neq [1; 0]$, $(l, t) \succ (l, s)$.

Separability (SEP). $(l, t) \succsim (l', t)$ if and only if $(l, s) \succsim (l', s)$.

First-Time-Then-Risk (FTTR). If $([1; x_n], t) \sim ([1; y_n], 0)$ for every $n \in \{1, \ldots, N\}$, then for every $\{p_n\}_{n=1}^N$ with $p_n \geq 0$ and $\sum_{n=1}^N p_n = 1$, $([p_1, \ldots, p_N; x_1 \ldots, x_N], t) \sim ([p_1, \ldots, p_N; y_1 \ldots, y_N], 0)$.

Stationarity (STAT). If $([1; x], t) \sim ([1; y], s)$ then, for every γ such that $t+\gamma, s+\gamma \geq 0$, $([1; x], t+\gamma) \sim ([1; y], s+\gamma)$.

Independence (IND). $(l, 0) \succsim (l', 0)$ if and only if $(\lambda l + (1-\lambda)l'', 0) \succsim (\lambda l' + (1-\lambda)l'', 0)$ for every l'', $\lambda \in (0, 1)$.

Payoff-Scale Invariance (PSI). For every $\kappa > 0$, $([1; x], t) \sim ([1; y], s)$ implies that $[1; \kappa x], t) \sim ([1; \kappa y], s)$.

Claim 1. $\succsim$ satisfies REG and SEP if and only if there exists a continuous mapping $CE : \mathcal{L} \to X$, with $CE([1; x]) = x$ and $CE(l) > CE(l')$ whenever l strictly FOSDs l', and a continuous mapping $PV : X \times T \to X$, with $PV(x, 0) = x$, strictly increasing in X and strictly decreasing in T when $x > 0$, such that $\succsim$ is represented by $(l,t) \succsim (l',s) \Leftrightarrow PV(CE(l),t) \geq PV(CE(l'),s)$.

PROOF OF CLAIM 1: Since the proof of necessity is immediate, we only prove sufficiency. We first show that every lottery admits a unique certainty equivalent when evaluated in the present. That is, there exists a mapping $CE : \mathcal{L} \to X$ with the properties stated in the claim, such that $(l,0) \sim ([1;CE(l)],0)$. We set $CE([1;x]) = x$, which defines a certainty equivalent for every degenerate lottery. If l is not degenerate, R-MON guarantees that $(l,0)$ is strictly better (respectively, strictly worse) than the dated lottery giving the worst (respectively, the best) payoff of lottery l, with probability one, in the present. Hence, RAT and CON guarantee the existence of the certainty equivalent $CE(l)$ in the present. Furthermore, RAT, CON and R-MON guarantee that the constructed mapping satisfies all the properties defining a certainty equivalent mapping. Next, we construct a present equivalent mapping $PV : X \times T \to X$ as follows. For a given amount of money x and a given time t, consider the induced dated degenerate lottery $([1;x],t)$. We claim that we can find a degenerate lottery awarded at time 0 that is indifferent to it, and hence, the corresponding payoff is the required present value. That is, $PV(x,t)$ is such that $([1;x],t) \sim ([1;PV(x,t)],0)$. Whenever $t = 0$ or $x = 0$, the claim can be proved by direct application of T-MON. Whenever both x and t are strictly positive, notice that $([1;x],0) \succ ([1;x],t) \succ ([1;0],t) \sim ([1;0],0)$. That is, the degenerate lotteries giving x and 0 in the present are strictly better and strictly worse, respectively, than the dated lottery giving the degenerate payoff x at t. RAT and CON guarantee the existence of a monetary value $PV(x,t)$ awarded at time 0 and indifferent to $([1;x],t)$. Again, it is evident that RAT, CON and T-MON guarantee that the mapping PV satisfies all the properties required for a present value mapping.

Now consider two dated lotteries (l,t) and (l',s). Since $(l,0) \sim ([1;CE(l)],0)$ and $(l',0) \sim ([1;CE(l')],0)$, SEP guarantees that $(l,t) \sim ([1;CE(l)],t)$ and $(l',s) \sim ([1;CE(l')],s)$. Hence, $(l,t) \succsim (l',s)$ if and only if $([1;CE(l)],t) \succsim ([1;CE(l')],s)$ if and only if $([1;PV(CE(l),t)],0) \succsim ([1;PV(CE(l'),s)],0)$ if and only if $PV(CE(l),t) \geq PV(CE(l'),s)$, as desired. $\diamond$

CLAIM 2. $\succsim$ satisfies REG and FTTR if and only if there exists a continuous mapping $CE : \mathcal{L} \to X$, with $CE([1;x]) = x$ and $CE(l) > CE(l')$ whenever l strictly FOSDs l', and a continuous mapping $PV : X \times T \to X$, with $PV(x,0) = x$, strictly increasing in X and strictly decreasing in T when $x > 0$, such that $\succsim$

can be represented by $(l,t) \succsim (l',s) \Leftrightarrow CE([p_1,\ldots,p_N; PV(x_1,t),\ldots,PV(x_N,t)]) \geq CE([q_1,\ldots,q_M; PV(y_1,t),\ldots,PV(y_M,t)])$.

Proof of Claim 2: Since the proof of necessity is immediate, we only prove sufficiency. That REG implies the existence of certainty equivalents and present values has been proved in Claim 1. Assume FTTR. Consider two dated lotteries (l,t) and (l',s), with $l = [p_1,\ldots,p_N; x_1,\ldots x_N]$ and $l' = [q_1,\ldots,q_M; y_1,\ldots y_M]$. Since, for every n we have that $([1;x_n],t) \sim ([1;PV(x_n,t)],0)$, the direct application of FTTR leads to $(l,t) \sim ([p_1,\ldots,p_N; PV(x_1,t),\ldots,PV(x_N,t)],0)$. Similarly, it must also be that $(l',s) \sim ([q_1,\ldots,q_M; PV(y_1,s),\ldots,PV(y_M,s)],0)$. Now, the lottery constructed by bringing payments of l to the present must be indifferent to one awarding its certainty equivalent. That is, we must have $([p_1,\ldots,p_N; PV(x_1,t),\ldots,PV(x_N,t)],0) \sim ([1; CE(p_1,\ldots,p_N; PV(x_1,t),\ldots,PV(x_N,t))],0)$. A similar reasoning can be applied to the dated lottery (l',s), leading to $([q_1,\ldots,q_M; PV(y_1,s),\ldots,PV(y_M,s)],0) \sim ([1; CE(q_1,\ldots,q_M; PV(y_1,s),\ldots,PV(y_M,s))],0)$. Using REG, we can link $(l,t) \succsim (l',s)$ to the comparison of certainty equivalents $CE(p_1,\ldots,p_N; PV(x_1,t),\ldots,PV(x_N,t)) \geq CE(q_1,\ldots,q_M; PV(y_1,s),\ldots,PV(y_M,s))$, as desired. $\diamond$

Claim 3. $\succsim$ satisfies REG, SEP, STAT and IND if and only if $\succsim$ can be represented by PVCE.

Proof of Claim 3: Since the proof of necessity is immediate, we only prove sufficiency. We start with the representation described in Claim 1. Consider the set $\mathcal{L} \times \{0\}$. Here, our axioms imply those used in the standard treatment of expected utility and, hence, it is immediate to see that there exists a continuous and strictly increasing mapping $v : X \to \mathbb{R}_+$ with $v(0) = 0$ such that a certainty equivalent function is constructed from expected utility with Bernoulli utility function v. For the time dimension, we use the results of Fishburn and Rubinstein (1982). Thus, let $\succsim'$ be the preference on $X \times T$ induced by the restriction of $\succsim$ to the set of all dated degenerate lotteries, i.e. $(x,t) \succsim' (y,s)$ if and only if $([1;x],t) \succsim ([1;y],s)$. Our axioms imply the axioms of Fishburn and Rubinstein's Theorem 2, and hence there exists $\delta \in (0,1)$ and a continuous and strictly increasing mapping $w : X \to \mathbb{R}_+$ with $w(0) = 0$ such that $(x,t) \succsim' (y,s)$ if and only if $\delta^t w(x) \geq \delta^s w(y)$. Since w^{-1} is a strictly monotone transformation of w, it is evident that $(x,t) \succsim' (y,s)$ if and only if $w^{-1}[\delta^t w(x)] \geq w^{-1}[\delta^s w(y)]$. It then follows that $\succsim$ admits a PVCE representation. $\diamond$

Claim 4. $\succsim$ satisfies REG, FTTR, STAT and IND if and only if $\succsim$ can be represented by CEPV.

Proof of Claim 4: It follows from Claim 2, using the same analysis as in the proof of Claim 3. $\diamond$

Claim 5. $\succsim$ satisfies REG, SEP, FTTR, STAT and IND if and only if $\succsim$ can be represented by DEU.

Proof of Claim 5: The proof of necessity is immediate. For sufficiency, it is enough to see that Proposition 1 extends immediately to $\mathcal{L} \times T$ and hence, the result follows from Claims 3 and 4. $\diamond$

Claim 6. $\succsim$ satisfies REG, SEP, FTTR, STAT, IND and PSI if and only if $\succsim$ can be represented by DEU-H or equivalently, by PVCE-H and by CEPV-H.

Proof of Claim 6: The proof of necessity is immediate. For sufficiency, consider any DEU representation of $\succsim$. We now show that, whenever preferences satisfy PSI, the function u in this representation must be homogeneous, and hence we have in fact DEU-H. First, consider any $x \in X$ and real number $\kappa > 1$. From the strict monotonicity of u, we know that there exists $t_\kappa \in \mathbb{R}_{++}$ such that $u(x) = \delta^{t_\kappa} u(\kappa x)$. PSI allows us to use this argument repeatedly to obtain $([1; \kappa^{\iota-1}x], 0) \sim ([1; \kappa^{\iota}x], t_\kappa)$ for every positive integer $\iota \geq 1$. That is, $u(\kappa^{\iota-1}x) = \delta^{t_\kappa} u(\kappa^{\iota}x)$, or equivalently $u(\kappa^{\iota}x) = \delta^{-t_\kappa} u(\kappa^{\iota-1}x)$ for every $\iota \geq 1$, which means that u is a homogeneous function of degree $\frac{-t_\kappa \log \delta}{\log \kappa}$ on the sequence of points $\{x, \kappa x, \kappa^2 x, \dots\}$. By making x as close to 0 as desired and κ as close to 1 as desired and using continuity, homogeneity must hold for the entire positive orthant, as desired. The equivalence with PVCE-H and CEPV-H follows directly from the defining properties and the fact that PVCE and CEPV contain DEU. $\diamond$

The characterization result in Claim 6 concludes the proof. $\blacksquare$

Our proof of Proposition 2 proceeds in a number of steps axiomatically characterizing the utility representations discussed in the paper, within the framework of preferences over dated lotteries. We believe that these results are of independent interest. The properties used in the characterizations are versions of the classical properties used in the independent treatments of risk and time preferences. The equivalence of all the models when using homogeneous utility functions relies on an additional property, which we call Payoff-Scale Invariance (PSI). PSI is an adaptation of a property of commodity bundles due to Lancaster (1963), which implies that the indifference of two dated degenerate lotteries is preserved when the payoffs are multiplied by the same constant. We show that PSI induces the homogeneity of the monetary utilities involved, and forces the PVCE-H, CEPV-H and DEU-H representations to coincide.

2.2. **Behavioral Models.** Here we outline how to introduce behavioral considerations in the treatment of risk and time preferences, using the dated lottery setting for illustrative purposes. In essence, Claims 1 and 2 in the proof of Proposition 2 axiomatically characterize generalized versions of PVCE and CEPV based on generalized time-invariant certainty equivalent mappings, not necessarily based on expected utility, and generalized present value equivalent mappings, not necessarily based on exponential discounting. The comparative statics in these generalized representations are straightforward, with more risk aversion captured directly by lower values of the certainty equivalent function, and more delay aversion captured by lower values of the present value function. Behavioral considerations can thus be incorporated into this framework through the use of non-standard mappings, as we now illustrate with a simple parametric specification.

In order to save on notation, we assume two-payoff lotteries. In the treatment of risk, we adopt the influential disappointment aversion model of Gul (1991) which, in this setting with two-payoff lotteries, is a special case of the rank-dependent utility of Quiggin (1982). For the treatment of time, we adopt the well-known $\beta-\delta$ model of Laibson (1997), exemplifying with the generalized PVCE representation, and adopting a homogeneous monetary function. Let $([p,1-p;x_1,x_2],t)$ denote a dated binary lottery with $x_1\geq x_2$. The certainty equivalent of Gul's model requires us to evaluate the binary lottery as $[\gamma(p)x_1^{1-r}+(1-\gamma(p))x_2^{1-r}]^{\frac{1}{1-r}}$, with weighting function $\gamma(p)=\frac{p}{1+(1-p)\zeta}$, $\zeta\in(-1,\infty)$, where $\zeta=0$ reduces the model to expected utility, $\zeta>0$ reflects disappointment aversion and $\zeta<0$ elation seeking. In the $\beta-\delta$ model, future payoffs are discounted by means of the standard exponential formula multiplied by a parameter $\beta\in(0,1]$, representing present-bias. Hence, the present value equivalent of a monetary payoff x awarded at time $t>0$ is $[\beta\delta^t x^{1-\eta}]^{\frac{1}{1-\eta}}$.[8] Then the behavioral version of the

[8]Formally, since the present value function of the standard $\beta-\delta$ model is discontinuous, one can use a continuous decreasing piece-wise linear function $\beta(t)$ taking value 1 when $t=0$ and value β for any time above a given t_ϵ. The $\beta-\delta$ model is the limit of these models when ϵ goes to zero. Notice also that this assumption is ineffective in practice, since we can always assume that t_ϵ is lower than the time involved in any future lotteries in an experimental dataset. The exponential discounting model simply assumes $\beta=1$.

PVCE model reduces to $U_{\beta,\delta,\eta,\zeta,r}([p,1-p;x_1,x_2],t) = \beta\delta^t[\gamma(p)x_1^{1-r} + (1-\gamma(p))x_2^{1-r}]^{\frac{1-\eta}{1-r}}$ whenever $t > 0$, and $U_{\beta,\delta,\eta,\zeta,r}([p,1-p;x_1,x_2],0) = [\gamma(p)x_1^{1-r} + (1-\gamma(p))x_2^{1-r}]^{\frac{1-\eta}{1-r}}$ otherwise.

The comparative statics in terms of risk follow immediately when noting that the certainty equivalent of a lottery decreases whenever either r or ζ increases. Similarly, the present value of a future payoff decreases whenever either $\hat{\delta}$ or β decreases.

3. PVCE-H and CEPV-H in Andreoni and Sprenger (2012b)

As mentioned above, in convex budgets DEU-H, PVCE-H and CEPV-H are not equivalent in general, and the high variability of menus in Andreoni and Sprenger (2012b) allows us to estimate all three models. For PVCE-H and CEPV-H, and, in analogy with the estimation procedure for DEU-H in the main text, we assume that the joint probability distribution f is characterized by three independent probability distributions $\bar{f}$, $\tilde{f}$ and $\hat{f}$, defined, respectively, on the curvature of the Bernoulli function r, the curvature of the intertemporal utility function η, and the corrected discount factor $\hat{\delta} = \delta^{\frac{1}{1-\eta}}$. The computation of $\rho_{im\tau}(f)$ now involves a triple integral but is conceptually equivalent. We then use truncated normal distributions for $\bar{f}$ and $\tilde{f}$, and a beta distribution for $\hat{f}$, and therefore, in the baseline model we estimate $(\mu_r, \sigma_r^2, \mu_\eta, \sigma_\eta^2, a_{\hat{\delta}}, b_{\hat{\delta}})$.

Table Table 1 and Figure 1 and Figure 3 report the results of PVCE-H-RUM and CEPV-H-RUM, together with those of DEU-H-RUM in order to facilitate the comparison of the models. Figure 2 suggests that PVCE-H-RUM and CEPV-H-RUM, by being more flexible, bring us even closer to the idiosyncratic nature of the data, being this confirmed by the log-likelihood values.[9] In particular, given the separation of the risk and intertemporal substitution parameters, corner and non-corner choices are now less influenced by risk aversion, and the models may potentially provide different estimates of risk aversion. It proves to be the case that the PVCE-H-RUM estimates low levels of risk aversion while CEPV-H-RUM yields higher levels, more in line with the experimental results on dated lotteries.

[9]CEPV-H-RUM outperforms PVCE-H-RUM in this dataset based on different measures of in-sample fit. Below, in Section 7, we show that CEPV-H-RUM also outperforms PVCE-H-RUM, based on a K-fold cross-validation exercise. PVCE-H-RUM in turn markedly outperforms DEU-H-RUM. These results support the use of CEPV-H-RUM over PVCE-H-RUM and DEU-H-RUM based on in-sample fit and out-sample forecasting performance.

Third, as already mentioned, DEU-H and PVCE-H impose the same predictions across tasks with the same ratio of probabilities. However, Figure 2 clearly shows that this is not observed in the data. For instance, people seem to choose corner solutions more often when both lotteries are degenerate than they do when both outcomes are realized with equally low probability. Since these models must predict the same choices in both scenarios, an intermediate prediction is observed. CEPV-H has no such restriction and hence CEPV-H-RUM fits the data better across these tasks.

4. Estimating the Tremble Probability ν

As discussed in Section 6 of the main text, we allow for positive choice probabilities in dominated lotteries by introducing a small fixed tremble probability. That is, we assume that with a very large probability $1-\nu$, the individual chooses according to $\rho_{im\tau}(f)$ and with a very small probability ν, the individual uniformly randomizes. In this section, we check the sensitivity of the main results to this assumption by estimating the parameter ν jointly with the other parameters in the model. Table 2 reports the results of this exercise for each of the three datasets considered in the main text. We can see that the log-likelihood is higher, indicating an improvement in the in-sample fit of the model. This is not surprising, since the estimation of the tremble probability adds an extra degree of freedom to the model which helps in accounting for the observed choices of dominated lotteries. Notice that the estimated parameters of the risk and time preferences do not vary substantially from those estimated under the assumption of a small and fixed ν as used in the main text.

5. Allowing for $r > 1$

In Section 6 of the main text, we have seen that bounding the curvature of the CRRA Bernoulli function at 1 may not accommodate all the risk heterogeneity. In this section we extend the models in order to address this issue.

The CRRA case with a parameter value above 1 implies negative utilities, which poses no difficulty in the treatment of risk preferences. Unfortunately, this is not the case in the joint treatment of risk and time preferences in DEU-H, due to the exponential discounting part of this representation. Accordingly, we proceed to extend PVCE-H and CEPV-H, which we do by allowing for $r > 1$, while keeping the bound in

TABLE 1. Estimated Risk and Time Preferences: Andreoni and Sprenger (2012b)

	DEU-H			PVCE-H			CEPV-H		
Dataset	**(i)**	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**	**(ix)**
Median h/r	0.039	0.048	0.100	−0.064	−0.057	0.220	0.261	0.221	0.270
	[0.035]	[0.035]	(0.238)	[0.137]	[0.140]	(0.649)	[0.095]	[0.096]	(0.967)
Std. Dev. h/r	0.400	0.420	0.443	1.265	1.260	1.238	0.976	1.018	0.964
	[0.032]	[0.020]	(0.215)	[0.126]	[0.193]	(0.787)	[0.101]	[0.096]	(0.577)
Median $\hat{\delta}$	0.953	0.958	0.957	0.968	0.962	0.985	0.966	0.966	0.978
	[0.005]	[0.005]	(0.079)	[0.004]	[0.007]	(0.043)	[0.004]	[0.005]	(0.065)
Std. Dev. $\hat{\delta}$	0.113	0.135	0.055	0.078	0.122	0.094	0.085	0.102	0.142
	[0.008]	[0.012]	(0.060)	[0.009]	[0.019]	(0.083)	[0.010]	[0.014]	(0.091)
Median η	–	–	–	0.094	0.097	0.114	−0.175	−0.070	−0.076
				[0.030]	[0.033]	(0.223)	[0.062]	[0.034]	(0.216)
Std. Dev. η	–	–	–	0.351	0.375	0.315	0.615	0.431	0.796
				[0.025]	[0.035]	(0.191)	[0.059]	[0.026]	(0.453)
Corr$(r,\hat{\delta})$	–	−0.601	−0.345	–	−0.333	−0.182	–	0.183	−0.177
		[0.046]			[0.143]			[0.160]	
Corr(r,s)	–	–	–	–	0.134	−0.006	–	−0.488	0.137
					[0.272]			[0.070]	
Corr$(\eta,\hat{\delta})$	–	–	–	–	−0.539	−0.201	–	−0.574	−0.160
					[0.103]			[0.202]	
Log-Likelihood	−3.650	−3.602	−3.283	−1.858	−1.839	−1.382	−1.774	−1.751	−1.277

NOTES.- The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distributions of risk aversion, corrected discount factor, and intertemporal substitution under different representations, using data from Andreoni and Sprenger (2012b). For each model, the first column shows the results from estimating the model using the pooled sample of observations. The second column shows the estimates obtained when allowing correlation between parameters using a Gaussian copula. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The third column shows the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion, denoted h in DEU-H and r in PVCE-H and CEPV-H, is assumed to follow a normal distribution truncated at 1, the corrected discount factor $\hat{\delta}$ to follow a beta distribution, and the curvature of intertemporal substitution η to follow a normal distribution truncated at 1.

the intertemporal substitution parameter at 1.[10] We refer to these extended versions as PVCE-H^E and CEPV-H^E. Several comments are in order.

In the setting of dated lotteries, the parameter η is not identifiable and hence we normalize η to 0, and estimate the corrected discount factor $\hat{\hat{\delta}} = \delta$. Moreover, it immediately follows that PVCE-H^E and CEPV-H^E are equivalent, and obviously different from the baseline DEU-H model. The formulation of PVCE-H^E is basically the one used in the main text for PVCE-H, but simply allowing $r > 1$ and setting $\eta = 0$.

The convex budget experiments involve null experimental payoffs. Since PVCE-H^E and CEPV-H^E are well-defined only for strictly positive monetary payoffs, we incorporate some background consumption $\omega \in \mathbb{R}_{++}$, guaranteeing that all final payoffs, formed by the addition of background consumption and experimental payoffs, are always strictly positive. Hence, we arrive at the following functional forms of PVCE-H^E and CEPV-H^E in convex budgets, with $r \in \mathbb{R} \setminus \{1\}$ and $\eta < 1$:

$$
\begin{aligned}
PVCE^E_{\delta,\eta,r}(\alpha) = & \Bigg[\delta^t \left[p\left(\alpha x+\omega\right)^{1-r} + (1-p)\,\omega^{1-r}\right]^{\frac{1-\eta}{1-r}} \\
& + \delta^s \left[qu\left(\left(1-\alpha\right)y+\omega\right)^{1-r} + (1-q)\,\omega^{1-r}\right]^{\frac{1-\eta}{1-r}}\Bigg]^{\frac{1}{1-\eta}} \\
CEPV^E_{\delta,\eta,r}(\alpha) = & \Bigg[pq\left[\delta^t\left(\alpha x+\omega\right)^{1-\eta} + \delta^s\left(\left(1-\alpha\right)y+\omega\right)^{1-\eta}\right]^{\frac{1-r}{1-\eta}} \\
& + p\,(1-q)\left[\delta^t\left(\alpha x+\omega\right)^{1-\eta} + \delta^s\omega^{1-\eta}\right]^{\frac{1-r}{1-\eta}} \\
& + (1-p)\,q\left[\delta^t\omega^{1-\eta} + \delta^s\left(\left(1-\alpha\right)y+\omega\right)^{1-\eta}\right]^{\frac{1-r}{1-\eta}} \\
& + (1-p)\,(1-q)\left[\delta^t\omega^{1-\eta} + \delta^s\omega^{1-\eta}\right]^{\frac{1-r}{1-\eta}}\Bigg]^{\frac{1}{1-r}}.
\end{aligned}
$$

We now repeat the estimation of risk and time preferences carried out in the main text under this extension. For the empirical estimations we keep all the distributional assumptions except for the case of r, which we now assume to be normally distributed. We start with the Andersen et al. (2008) dataset, using PVCE-H^E. Table 3 reports the joint estimates, Figure 4 the PDFs of the estimated distributions, and Figure 5 presents a scatterplot of the individual estimates. We immediately see, as expected, that the

[10]Given the assumption of continuity over the parameter space, we do not need to specify the representation of preferences when $r = 1$ in our estimation exercises, since this has zero probability mass.

median risk aversion increases and the corrected discount factor remains unchanged; the heterogeneity in the data, both at the pooled and individual levels is being captured more fully.

Table 4, Figures 6 and 7 report the corresponding results for Coble and Lusk (2010). In this case, median risk aversion decreases slightly, while the variance increases. The results for the distribution of delay aversion are very similar to those of DEU-H-RUM.

Table 5 reports the results using PVCE-H^E-RUM and CEPV-H^E-RUM in the convex budgets setting of Andreoni and Sprenger (2012b). In the baseline estimations, reported in columns (i) and (v), we assume a fixed, positive but close-to-zero, level of background consumption, $\omega = 10^{-6}$. In the estimations reported in columns (ii) and (vi) we incorporate background consumption as an additional estimated parameter. In columns (iii) and (vii) we report on the estimations at the individual level, providing the median and standard deviation of the medians estimated for each individual. In addition, Figures 8 and 9 plot the estimated PDFs of the preference parameters for the two models and Figures 10 and 11 plot the corresponding CDFs. Figure 12 plots the observed and predicted choice probabilities across the different experimental parameters, and Figure 13 shows scatter-plots with the estimated individual median parameters for each of the 80 subjects and for the two models.

With respect to the results obtained in section 6 of the main text, we note that the consideration of the extended models yields higher levels of risk aversion, very similar results for the corrected discount factor, and slightly lower values of the intertemporal substitution parameter.

6. Observable Characteristics

Some researchers may be interested in studying the effect of individual characteristics, such as age or gender, on risk and time preferences. In this section, we illustrate how our framework can be extended to allow for the dependence of the moments of the distribution of the behavioral traits on observable characteristics.

Consider the case of dated lotteries and the extended model introduced in the previous section, PVCE-H^E. We assume that r follows a normal distribution and allow the mean and variance of this distribution to vary across individuals. Specifically, we assume that the mean and standard deviation of the risk aversion distribution for individual i, denoted by $\mu_{i,r}$ and $\sigma_{i,r}$, are linear functions of observable characteristics. That is, $\mu_{i,r} = x_i'\beta_r$ and $\sigma_{i,r} = x_i'\gamma_r$, where x_i is a $K \times 1$ vector of characteristics which includes a constant term and other observable characteristics such as age, while β_r and γ_r are vectors of coefficients to be estimated.[11] Similarly, we assume that the corrected discount factor follows a beta distribution parameterized as a function of its mean and variance, which also vary across individuals.[12] That is, the mean and standard deviation of $\hat{\delta}$ of individual i, denoted by $\mu_{i,\hat{\delta}}$ and $\sigma_{i,\hat{\delta}}$, are given by $\mu_{i,\hat{\delta}} = x_i'\beta_{\hat{\delta}}$ and $\sigma_{i,\hat{\delta}} = x_i'\gamma_{\hat{\delta}}$.

With this specification we use the methods discussed in the main text to compute the probability of choosing a lottery from a given menu and evaluate the log-likelihood function of the model. We illustrate with the data from Andersen et al. (2008). This dataset includes binary indicators for gender, age below 30, age between 40 and 50, age over 50, living alone, having children, being a home owner, being retired, being a student, having some post-secondary education, having a substantial level of higher education, having a lower income level in 2002 (below 300,000 DKK), having a higher income level in 2002 (500,000 DKK or more), living in the greater Copenhagen area, and living in a city with 20,000 inhabitants or more. A full description and summary statistics of each variable are given in Table 8.

Tables 6 and 7 show the results of this exercise. The first set of columns in each table show the results when allowing the observed variables to affect only the mean of each distribution. The second set of columns show the results when allowing the observable variables to affect the standard deviation of the distributions only. The third set of columns show the results when allowing the observable variables to affect simultaneously both the mean and the standard deviation of each distribution.

In general, we find no relationship between most of the observable characteristics and the estimated mean and variances of risk and time preferences. The exceptions are gender, with females displaying higher standard deviation of risk aversion across specifications, and individuals with a substantial level of higher education, who display higher mean risk aversion. In terms of delay aversion, older individuals display lower and less volatile corrected discount factors, implying higher degrees of delay aversion. Individuals with substantial levels of higher education also display lower delay aversion,

[11] Notice that this especification nests the original pooled estimates by setting $x_i = 1$.

[12] The beta distribution is usually specified as a function of the shape parameters $\alpha, \beta > 0$. The mean and standard deviation of this distribution are in turn given by $\mu = \frac{\alpha}{\alpha+\beta}$ and $\sigma = \sqrt{\frac{\alpha\beta}{(\alpha+\beta)^2(1+\alpha+\beta)}}$, respectively. It follows that for any value of μ and σ such that $\mu \in (0,1)$ and $\sigma^2 < \mu(1-\mu)$, we can recover the implied shape parameters as $\alpha^* = \left(\frac{\mu}{\sigma^2}(1-\mu) - 1\right)\mu$ and $\beta^* = \alpha^* \left(\frac{1-\mu}{\mu}\right)$.

although this result is sensitive to allowing this variable also to affect the variance of delay aversion.

7. Prediction Error

In this section, we illustrate how to evaluate the predictive capacity of the models presented in this paper using data from Andreoni and Sprenger (2012b). That is, we split the data into $K = 10$ roughly equal-sized parts. For every part $k = 1, \ldots, 10$, we estimate DEU-H-RUM, PVCE-H-RUM and CEPV-H-RUM using the other $K - 1$ parts of the data, and then calculate the prediction error of the fitted model when predicting part k. We use cross-entropy, or log-loss, as the loss function to measure prediction error. We then take the mean and standard deviation of the K estimates of the prediction error.[13]

The results of this exercise are shown in Table 9. Columns (i), (iv) and (vii) show the results of the estimated cross-entropy measures when pooling all observations in the sample and then splitting the data randomly. We can see that both PVCE-H-RUM and CEPV-H-RUM significantly outperform DEU-H-RUM for out-of-sample prediction. The average prediction error is reduced by almost half for both models. Furthermore, the standard deviation of the prediction error of DEU-H-RUM across estimates is four times that of PVCE-H-RUM and CEPV-H-RUM. Finally, CEPV-H-RUM slightly outperforms PVCE-H-RUM on average. Columns (ii), (v) and (viii) show the results of the estimated cross-entropy measures when the parts are created by splitting the individuals in the sample into $K = 10$ parts and then assigning the observations for each individual to the corresponding part. Implicitly, this way of splitting the data evaluates the predictive accuracy of the models for the choices of out-of-sample individuals. The average prediction error is similar to that obtained in the baseline exercise but its standard deviation is now higher in all specifications. In relative terms, however, the performance of all three models is very similar to that obtained in the baseline scenario. As an alternative approach, we create the parts by splitting the menus into $K = 10$ parts and then assigning the observations for each menu to the corresponding part. In other words, we evaluate the predictive accuracy of the models for choices in out-of-sample menus. The results of this cross-validation exercise are reported in columns (iii), (vi) and (ix). Once again, the average prediction error obtained for DEU-H-RUM

[13] See Hastie (2009) for a detailed exposition of cross-validation for model selection and assessment.

is now three times higher, while the volatility of the prediction errors of PVCE-H-RUM and CEPV-H-RUM is much lower and more stable than that of DEU-H.

The results of this exercise suggest that the out-of-sample performance of DEU-H-RUM is substantially improved by accounting for the different curvatures in risk aversion and intertemporal substitution, as in PVCE-H-RUM and CEPV-H-RUM.

References

[1] Andersen, S., G.W. Harrison, M.I. Lau, and E.E. Rutstrom (2008). "Eliciting Risk and Time Preferences." *Econometrica* 76(3):583–618.

[2] Andreoni, J., P. Feldman and C. Sprenger (2017). "A Stream of Prospects or a Prospect of Streams: On the Evaluation of Intertemporal Risks." Mimeo.

[3] Andreoni, J. and C. Sprenger (2012a). "Estimating Time Preferences from Convex Budgets." *American Economic Review*, 102(7):3333–3356.

[4] Andreoni, J. and C. Sprenger (2012b). "Risk Preferences Are Not Time Preferences." *American Economic Review*, 102(7):3357–3376.

[5] Chew, S.H. and L.G. Epstein (1990). "Nonexpected Utility Preferences in a Temporal Framework with an Application to Consumption-Savings Behaviour." *Journal of Economic Theory*, 50(1):54–81.

[6] Coble, K.H. and J. Lusk (2010). "At the Nexus of Risk and Time Preferences: An Experimental Investigation." *Journal of Risk and Uncertainty*, 41(1):67–79.

[7] DeJarnette, P., D. Dillenberger, D. Gottlieb and P. Ortoleva (2019). "Time Lotteries and Stochastic Impatience." *Econometrica*, forthcoming.

[8] Epper, T. and H. Fehr-Duda (2019). "Risk in Time: The Intertwined Nature of Risk Taking and Time Discounting." Mimeo.

[9] Epstein, L.G. and S.E. Zin (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica*, 57(4):937–969.

[10] Gul, F. (1991). "A Theory of Disappointment Aversion." *Econometrica*, 59(3):667–686.

[11] Halevy, Y. (2008). "Strotz Meets Allais: Diminishing Impatience and the Certainty Effect." *American Economic Review*, 98(3):1145–62.

[12] Hastie, T., Tibshirani, R., and Friedman, J. H. (2009). "The elements of statistical learning: data mining, inference, and prediction". 2nd ed. New York: Springer.

[13] Kreps, D.M. and E.L. Porteus (1978). "Temporal Resolution of Uncertainty and Dynamic Choice Theory." *Econometrica*, 46(1):185–200.

[14] Laibson, D. (1997). "Golden Eggs and Hyperbolic Discounting." *Quarterly Journal of Economics*, 112(2):443–477.

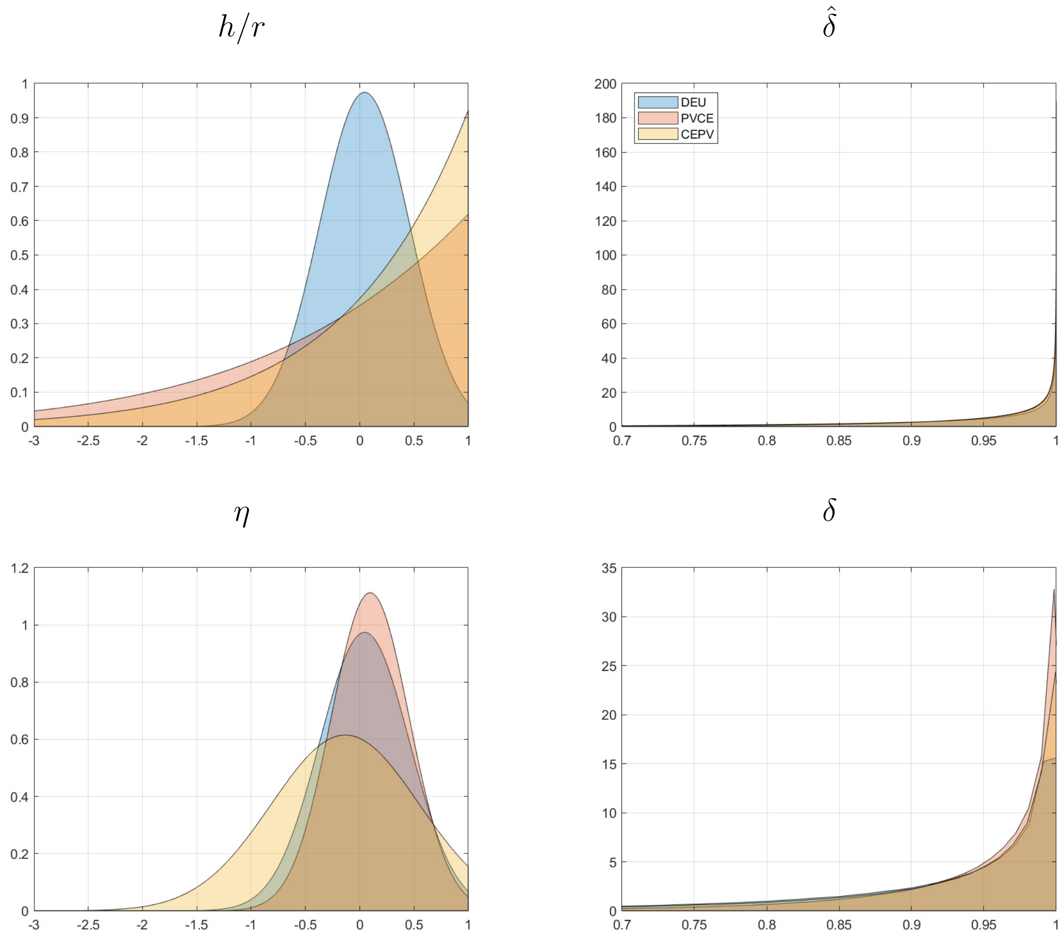
[15] Lancaster, K. (1963). "An Axiomatic Theory of Consumer Time Preference," *International Economic Review*, 4(2):221–231.

[16] Lanier, J., B. Miao, J. Quah and S. Zhong (2019). "Intertemporal Consumption with Risk: A Revealed Preference Analysis." Mimeo.

[17] Quiggin, J. (1982). "A Theory of Anticipated Utility." *Journal of Economic Behavior and Organization*, 3(4):323–343.

[18] Selden, L. (1978). "A New Representation of Preferences over "Certain $\times$ Uncertain" Consumption Pairs: The "Ordinal Certainty Equivalent" Hypothesis." *Econometrica*, 46(5):1045–60.

Figure 1. PDFs of Estimated Risk and Time Preferences: Andreoni and Sprenger (2012b)



NOTES.- PDFs of the estimated distributions reported in Table 1. The PDF of the discount factor $\delta = \hat{\delta}^{1-\eta}$ is estimated non-parametrically from the distributions of intertemporal substitution and the corrected discount factor using a normal kernel.

FIGURE 2. Observed and Predicted Distributions of Choices in Andreoni and Sprenger (2012b)

DEU-H ***PVCE-H*** ***CEPV-H***

Risk Condition 1: $(p, q) = (1, 1)$

Risk Condition 2: $(p, q) = (1, 0.8)$

Risk Condition 3: $(p, q) = (0.8, 1)$

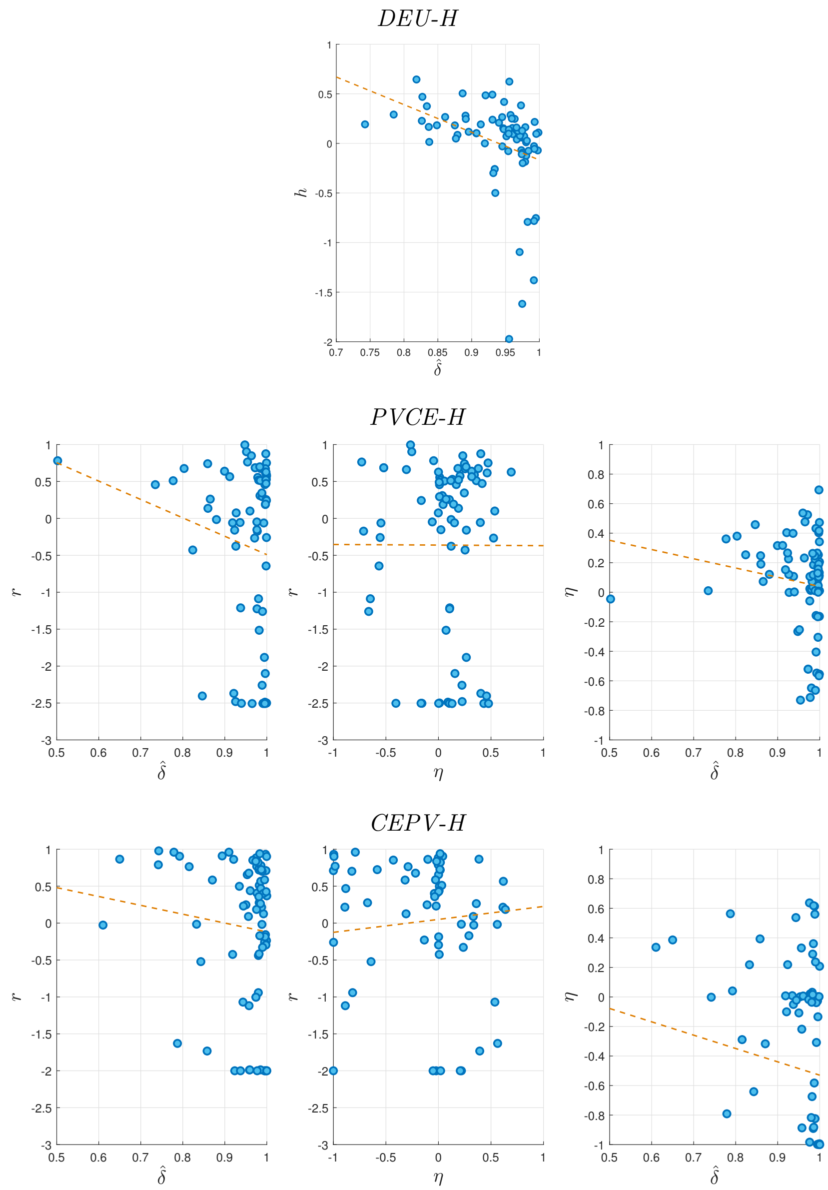
Risk Condition 4: $(p, q) = (0.5, 0.5)$

Risk Condition 5: $(p, q) = (0.5, 0.4)$

Risk Condition 6: $(p, q) = (0.4, 0.5)$

NOTES.- Observed frequencies and predicted probabilities of choosing share α ($\times 100$) under each risk condition considered in Andreoni and Sprenger (2012b). The observed distributions show the relative frequency of each allocation in the data, grouped to the closest multiple of 10. The predicted distributions are computed based on the estimated parameters of the respective representation, as shown in Table 1.

FIGURE 3. Individual Estimates: Andreoni and Sprenger (2012b)



NOTES.- Each point represents a combination of the medians of the estimated distributions of the coefficients of risk aversion h/r, the corrected discount factor $\hat{\delta}$, and the curvature of intertemporal substitution η for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 1.

Table 2. Estimated Risk and Time Preferences with Tremble Probability

Dataset	**Andersen et al. (2008)**	**Coble and Lusk (2010)**	**Andreoni & Sprenger (2012b)**		
Model	***DEU-H***	***DEU-H***	***DEU-H***	***PVCE-H***	***CEPV-H***
Median h/r	0.710	0.404	−0.060	0.206	0.369
	[0.028]	[0.073]	[0.049]	[0.101]	[0.090]
Std. Dev. h/r	0.329	0.376	0.486	0.720	0.862
	[0.018]	[0.028]	[0.043]	[0.307]	[0.109]
Median $\hat{\delta}$	0.981	0.914	0.967	0.977	0.973
	[0.001]	[0.009]	[0.004]	[0.005]	[0.004]
Std. Dev. $\hat{\delta}$	0.009	0.077	0.058	0.040	0.049
	[0.001]	[0.011]	[0.008]	[0.012]	[0.011]
Median η	–	–	–	0.059	−0.337
				[0.039]	[0.076]
Std. Dev. η	–	–	–	0.448	0.809
				[0.048]	[0.101]
Tremble Prob. ν	0.245	0.063	0.252	0.116	0.110
	[0.016]	[0.021]	[0.029]	[0.025]	[0.022]
Log-Likelihood	−0.513	−0.373	−1.807	−1.714	−1.637

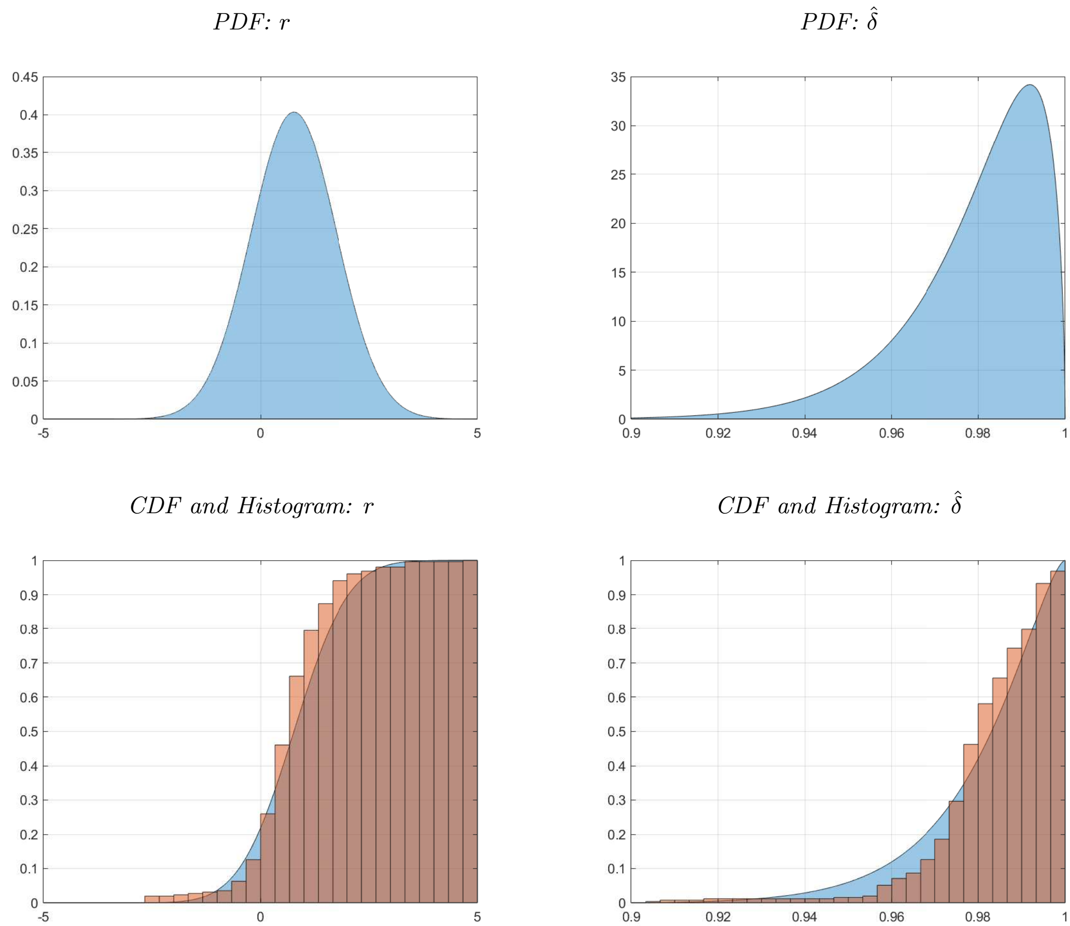
NOTES. The above table reports the maximum-likelihood estimates of the median and standard deviation of the distributions of risk aversion, corrected discount factor and intertemporal substitution under different representations, using data from Andersen et al. (2008), Coble and Lusk (2010) and Andreoni and Sprenger (2012b). The baseline model depicted in Table 1 in the paper is extended by adding a tremble probability ν. The coefficient of risk aversion, denoted h in DEU-H and r in PVCE-H and CEPV-H, follows a normal distribution truncated at 1, the corrected discount factor $\hat{\delta}$ follows a beta distribution and the curvature of intertemporal substitution η follows a normal distribution truncated at 1. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level.

Table 3. Estimated Risk and Time Preferences using PVCE-H^E: Andersen et al. (2008)

Dataset	Risk Only	Time Only	Joint	by Individual
Median r	0.767		0.768	0.687
	[0.048]		[0.048]	(0.343)
Std. Dev. r	0.987		0.988	0.888
	[0.058]		[0.058]	(0.514)
Median $\hat{\delta}$		0.983	0.983	0.980
		[0.001]	[0.001]	(0.007)
Std. Dev. $\hat{\delta}$		0.016	0.016	0.084
		[0.001]	[0.001]	(0.051)
# Obs.	7928	15180	23108	23108
Log-Likelihood	−0.679	−0.543	−0.589	−0.358

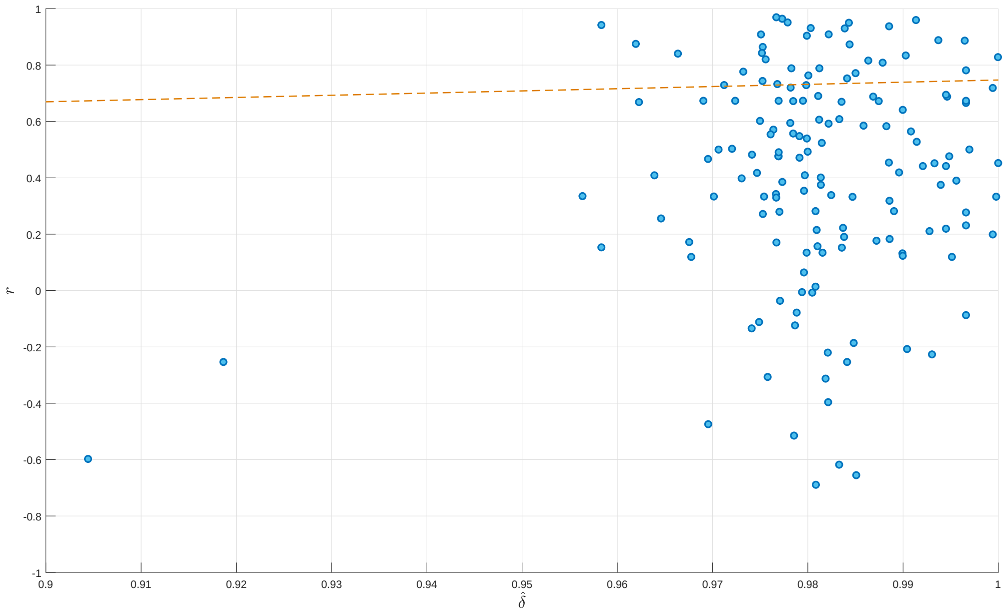
NOTES. The above table reports the maximum-likelihood estimates of the median and standard deviation of the distributions of risk and time preferences under the PVCE-H^E representation with $\eta = 0$ using data from Andersen et al. (2008). The second column shows the results obtained using the menus eliciting risk aversion only, and the third column shows the results obtained using the menus eliciting delay aversion only. The fourth column shows the results of the joint estimation of risk aversion and delay aversion using the pooled sample menus. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The last column shows the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution while the corrected discount factor $\hat{\delta}$ follows a beta distribution.

Figure 4. PDFs and CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using PVCE-H^E: Andersen et al. (2008)



NOTES.- PDFs and CDFs of the pooled estimates, and histograms of the empirical distributions of the individual estimates using PVCE-H^E with $\eta = 0$.

FIGURE 5. Individual Estimates using PVCE-H^E: Andersen et al. (2008)



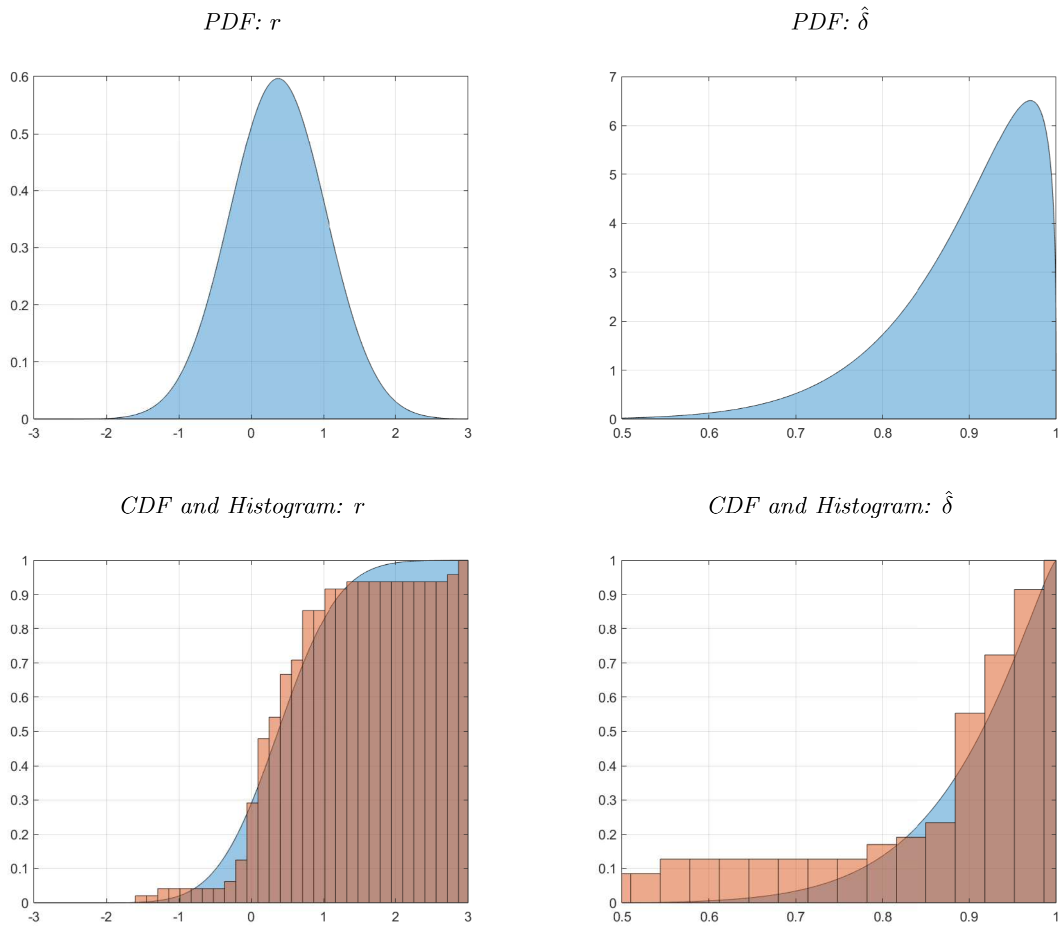
NOTES. Each point represents the median of the estimated distributions of the coefficient of risk aversion h and the corrected discount factor $\hat{\delta}$ for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 3.

Table 4. Estimated Risk and Time Preferences using PVCE-H^E: Coble and Lusk (2010)

Dataset	**Using Risk Tasks Only**	**Using Discount Tasks Only**	**Using Joint Tasks Only**	**All Tasks**	**All Tasks - Correlated Preferences**	**Pooled Individual Estimates**
Median r	0.414 [0.083]	–	0.497 [0.102]	0.371 [0.087]	0.410 [0.082]	0.289 (0.023)
Std. Dev. r	0.669 [0.084]	–	0.476 [0.072]	0.669 [0.089]	0.654 [0.083]	0.744 (0.320)
Median $\hat{\delta}$	–	0.903 [0.012]	0.947 [0.017]	0.916 [0.010]	0.912 [0.010]	0.907 (0.053)
Std. Dev. $\hat{\delta}$	–	0.089 [0.017]	0.133 [0.036]	0.085 [0.013]	0.081 [0.012]	0.240 (0.033)
Corr(r,$\hat{\delta}$)	–	–	–	–	-0.524 [0.260]	-0.214
# Obs.	1880	1128	1410	4418	4418	47
Log-Likelihood	-0.429	-0.436	-0.361	-0.416	-0.414	-0.195

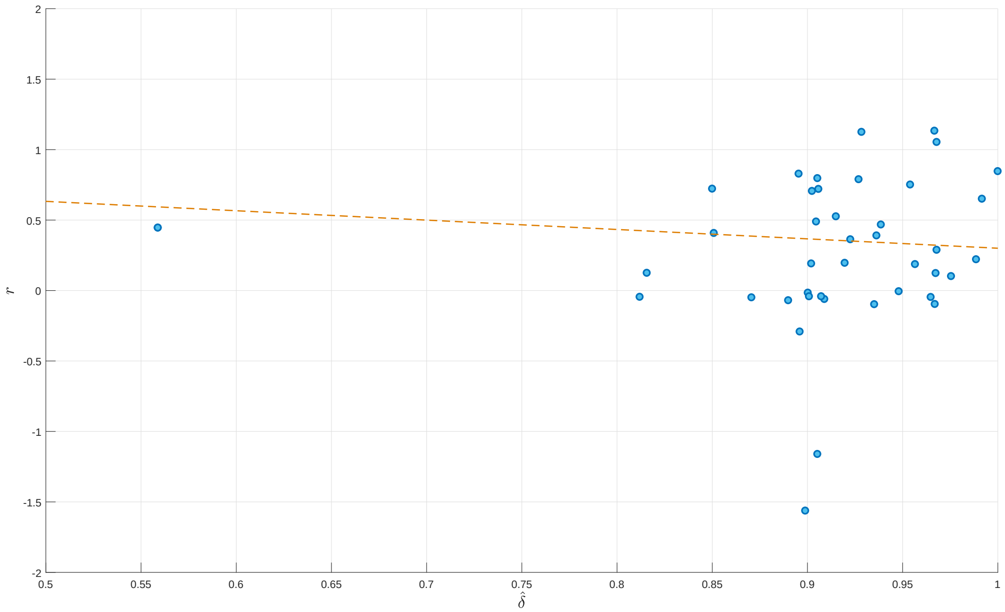
NOTES. The above table reports the maximum-likelihood estimates of the median and standard deviation of the distributions of risk and time preferences under the PVCE-H^E representation with $\eta = 0$ using data from Coble and Lusk (2010). The second column shows the results obtained using the menus eliciting risk aversion only. The third column shows the results obtained using the menus eliciting delay aversion only. The fourth column shows the results using dated lottery menus only. The fifth column shows the results of the joint estimation of risk aversion and delay aversion using the pooled sample menus. The sixth column shows the estimates obtained when allowing correlation between parameters using a Gaussian copula. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The last column shows the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution while the corrected discount factor $\hat{\delta}$ follows a beta distribution.

Figure 6. PDFs and CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using PVCE-H^E: Coble and Lusk (2010)



NOTES. PDFs and CDFs of the pooled estimates, and histograms of the empirical distributions of the individual estimates using PVCE-H^E with $\eta = 0$.

Figure 7. Individual Estimates using PVCE-H^E: Coble and Lusk (2010)



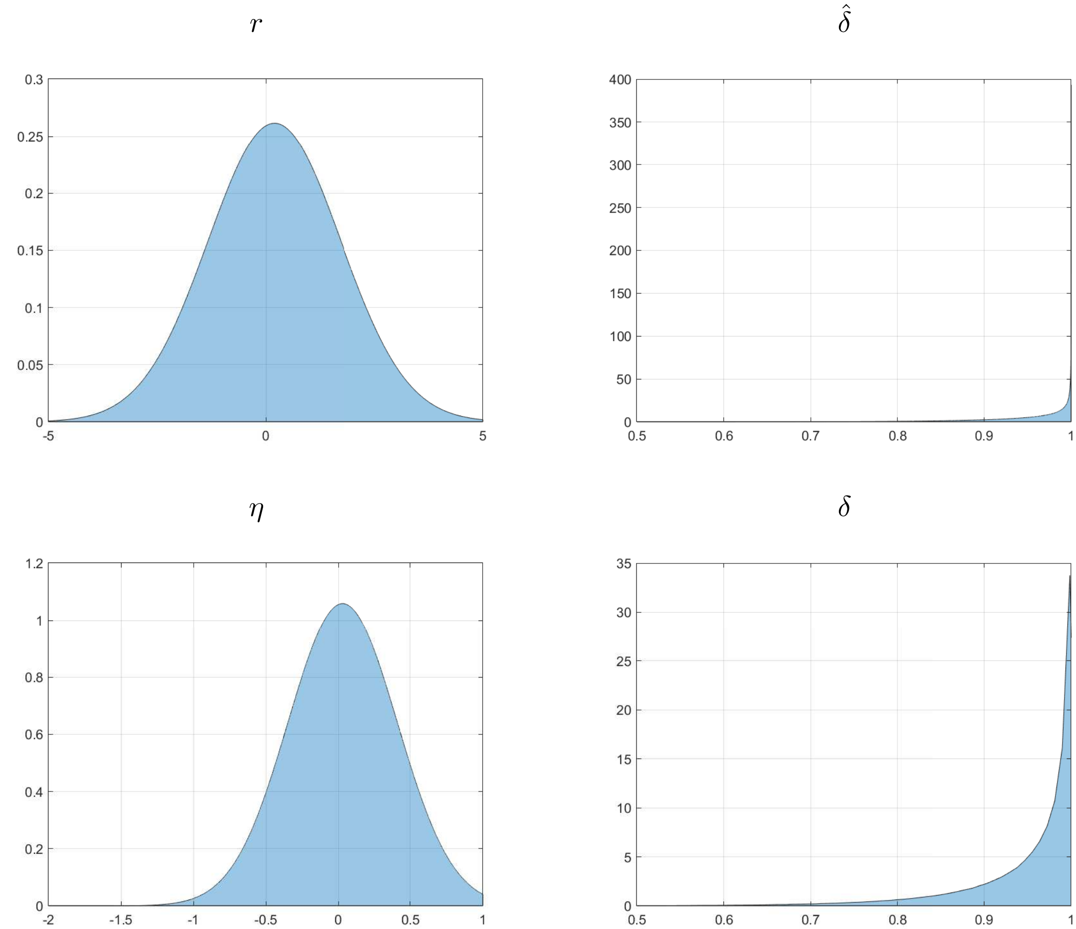
NOTES. Each point represents the median of the estimated distributions of the coefficient of risk aversion h and the corrected discount factor $\hat{\hat{\delta}}$ for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 4.

Table 5. Estimated Risk and Time Preferences using PVCE-H^E and CEPV-H^E: Andreoni and Sprenger (2012b)

	$PVCE-H^E$				$CEPV-H^E$			
Dataset	**(i)**	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**
Median h	0.199 [0.163]	0.158 [0.235]	0.248 [0.195]	0.304 (0.645)	0.431 [0.138]	0.391 [0.145]	0.321 [0.128]	0.457 (0.403)
Std. Dev. h	1.517 [0.122]	1.812 [0.224]	1.564 [0.138]	1.666 (0.818)	1.339 [0.128]	1.351 [0.126]	1.392 [0.167]	1.594 (0.765)
Median $\hat{\delta}$	0.971 [0.004]	0.972 [0.004]	0.965 [0.005]	0.982 (0.054)	0.968 [0.005]	0.970 [0.004]	0.953 [0.008]	0.978 (0.057)
Std. Dev. $\hat{\delta}$	0.071 [0.010]	0.070 [0.010]	0.084 [0.013]	0.114 (0.068)	0.083 [0.012]	0.083 [0.012]	0.120 [0.022]	0.189 (0.065)
Median η	0.029 [0.041]	0.021 [0.040]	−0.038 [0.040]	0.069 (0.260)	−0.486 [0.229]	−0.386 [0.187]	−0.215 [0.081]	−0.421 (0.438)
Std. Dev. η	0.371 [0.037]	0.358 [0.040]	0.364 [0.036]	0.555 (0.267)	0.823 [0.107]	0.765 [0.106]	0.559 [0.069]	0.671 (0.578)
Background Consumption $\omega \times 100$	–	0.034 [0.048]	–	–	–	1.017×10^{-4} $[0.594 \times 10^{-4}]$	–	–
Corr(r,$\hat{\delta}$)	–	–	−0.145 [0.187]	−0.076	–	–	−0.297 [0.118]	−0.146
Corr(r,s)	–	–	−0.465 [0.172]	−0.019	–	–	−0.333 [0.097]	0.001
Corr(η,$\hat{\delta}$)	–	–	−0.295 [0.090]	−0.268	–	–	−0.390 [0.110]	−0.116
Log-Likelihood	−1.685	−1.676	−1.671	−1.144	−1.680	−1.678	−1.658	−1.126

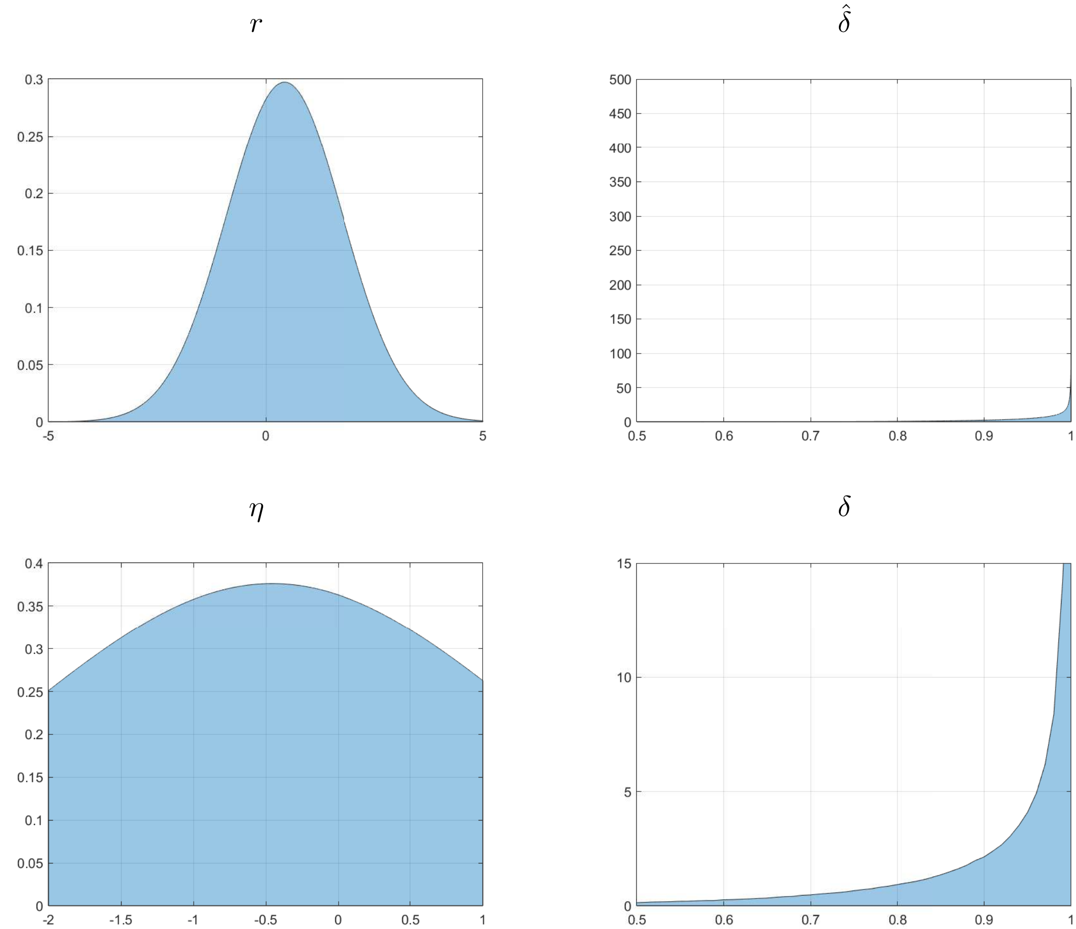
NOTES. The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distributions of risk aversion, corrected discount factor, and intertemporal substitution under PVCE-H^E and CEPV-H^E, using data from Andreoni and Sprenger (2012b). Columns (i) and (v) show the results from estimating each model with the pooled sample observations, assuming a small fixed background consumption level of $\omega = 10^{-6}$. Columns (ii) and (vi) show the results when $\omega > 0$ is estimated together with the other parameters. Columns (iii) and (vii) show the results for the respective models when allowing correlation between parameters using a Gaussian copula. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. Finally, columns (iv) and (viii) report the median and standard deviation (in parentheses) of the distribution of individual estimates of the respective parameter. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution, the corrected discount factor $\hat{\delta}$ to follow a beta distribution and the curvature of intertemporal substitution η to follow a normal distribution truncated at 1.

Figure 8. PDFs of Estimated Risk and Time Preferences using PVCE-H^E: Andreoni and Sprenger (2012b)



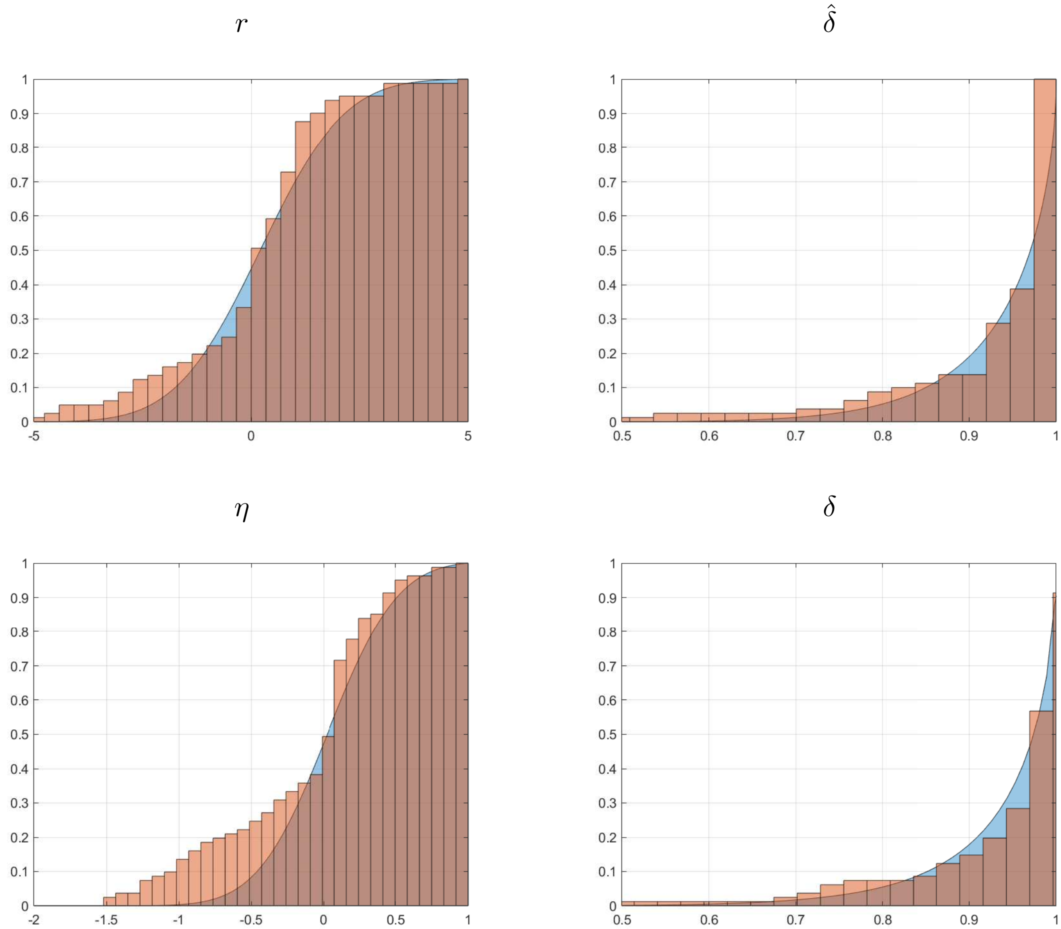
NOTES.— PDFs of the estimated distributions reported in Table 5. The PDF of the discount factor $\delta = \hat{\delta}^{1-\eta}$ is estimated non-parametrically from the distributions of intertemporal substitution and the corrected discount factor using a normal kernel.

Figure 9. PDFs of Estimated Risk and Time Preferences using CEPV-H^E: Andreoni and Sprenger (2012b)



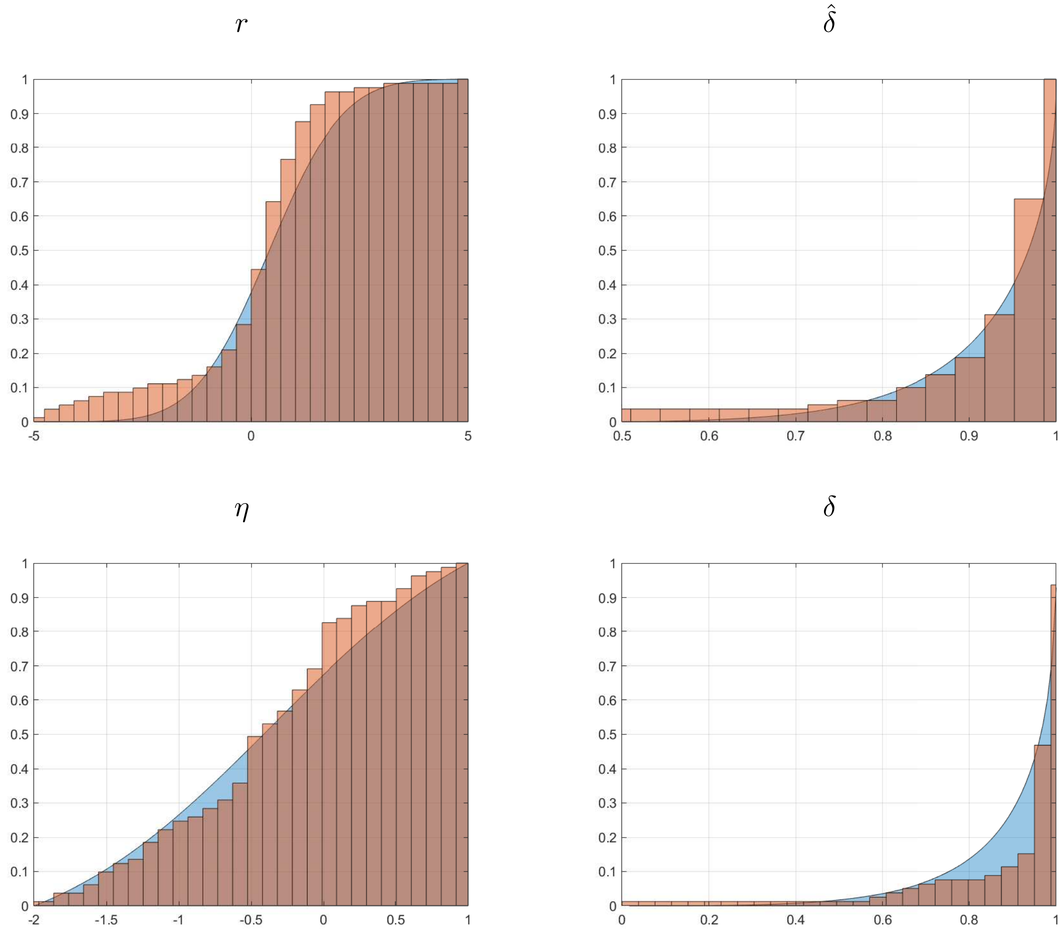
NOTES.— PDFs of the estimated distributions reported in Table 5. The PDF of the discount factor $\delta = \hat{\delta}^{1-\eta}$ is estimated non-parametrically from the distributions of intertemporal substitution and the corrected discount factor using a normal kernel.

FIGURE 10. CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using PVCE-H^E: Andreoni and Sprenger (2012b)



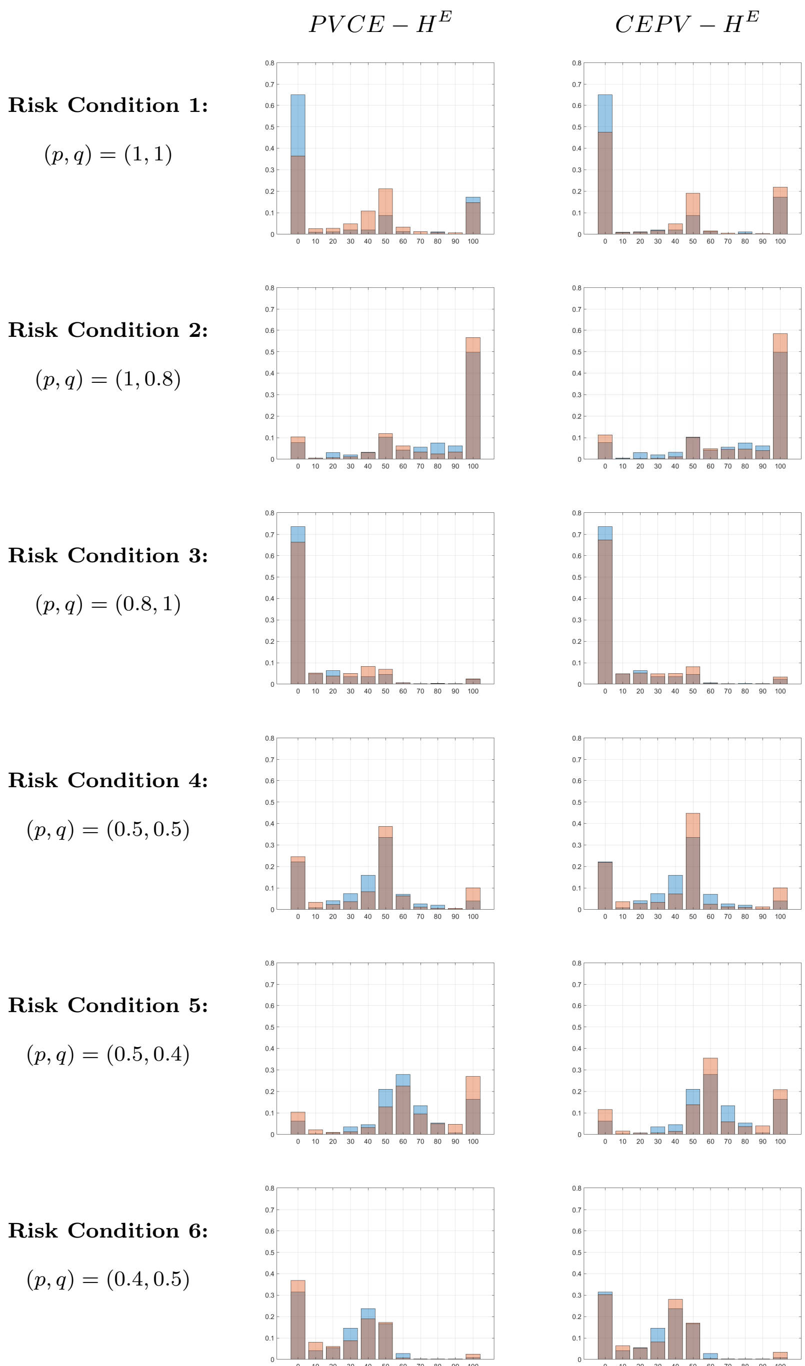
NOTES.— CDFs of the pooled estimation and histograms of the empirical distributions of the individual estimates using PVCE-H^E.

FIGURE 11. CDFs of Estimated Risk and Time Preferences and Histograms of Individual Estimates using CEPV-H^E: Andreoni and Sprenger (2012b)



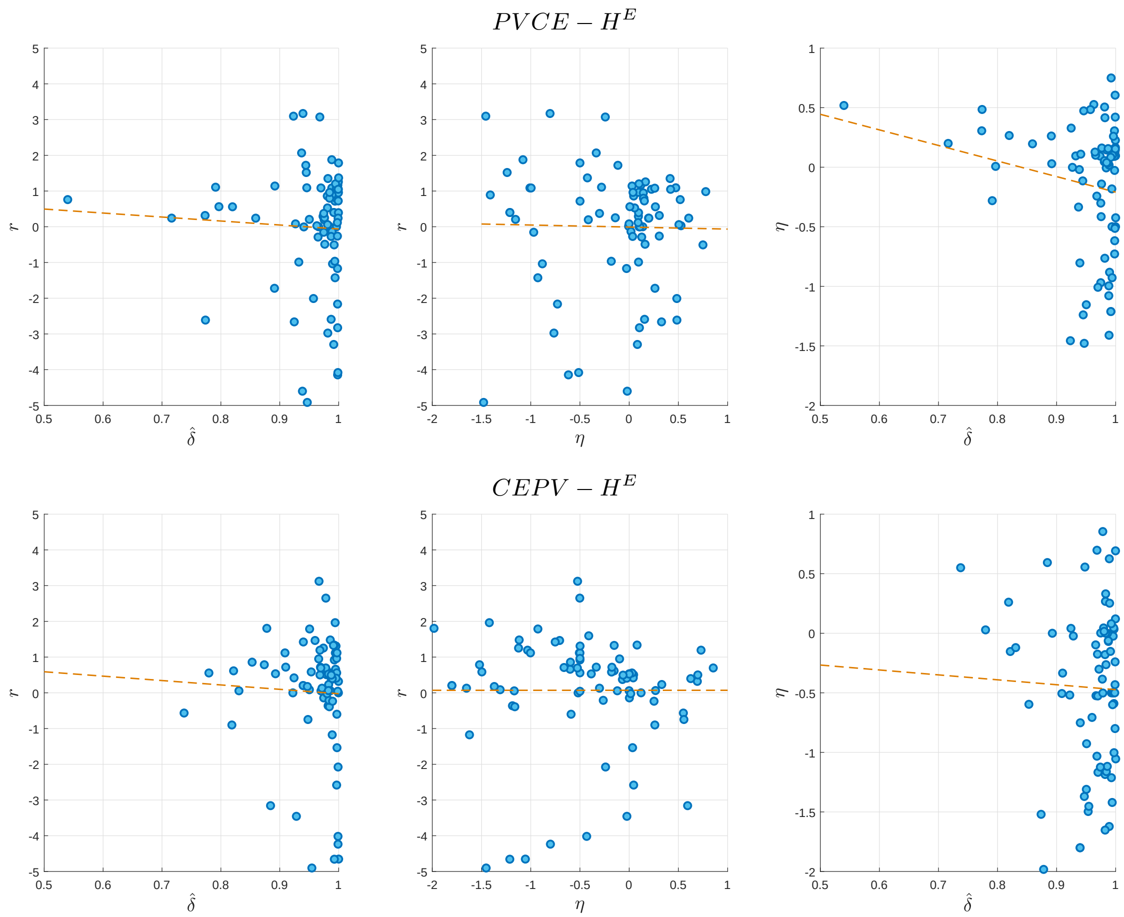
NOTES.— CDFs of the pooled estimation and histograms of the empirical distributions of the individual estimates using CEPV-H^E.

Figure 12. Observed and Predicted Distributions of Choices using PVCE-H^E and CEPV-H^E: Andreoni and Sprenger (2012b)



NOTES. Observed frequencies and predicted probabilities of choosing share α ($\times 100$) under each risk condition considered in Andreoni and Sprenger (2012b). The observed distributions show the relative frequency of each allocation in the data, grouped to the closest multiple of 10. The predicted distributions are computed based on the estimated parameters of the respective representation, as shown in Table 5.

FIGURE 13. Individual Estimates using PVCE-H^E and CEPV-H^E: Andreoni and Sprenger (2012b)



NOTES. Each point represents the median of the estimated distributions of the coefficient of risk aversion h/r, the corrected discount factor $\hat{\delta}$, and the curvature of intertemporal substitution η for the subsample of choice data for a particular individual, following the estimation procedure reported in Table 5.

Table 6. Estimated Risk Preferences and Observable Characteristics

Estimated Coefficient	*(i) Mean Eq. Only*		*(i) XD Eq. Only*		*(i) Both Eqs.*	
	μ_h	σ_h	μ_h	σ_h	μ_h	σ_h
Constant	0.485	0.936	0.805	0.549	0.367	0.227
	[0.185]	[0.052]	[0.051]	[0.156]	[0.100]	[0.068]
female	−0.035			0.266	0.018	0.284
	[0.087]			[0.101]	[0.113]	[0.126]
young	0.260			0.176	0.311	0.289
	[0.179]			[0.135]	[0.200]	[0.176]
middle	−0.256			0.499	−0.167	0.361
	[0.129]			[0.145]	[0.150]	[0.159]
old	−0.157			0.428	−0.060	0.372
	[0.152]			[0.136]	[0.145]	[0.143]
copen	0.102			−0.030	0.085	0.024
	[0.125]			[0.116]	[0.128]	[0.131]
city	−0.004			0.000	0.027	0.078
	[0.105]			[0.107]	[0.108]	[0.131]
owner	0.103			0.245	0.155	0.335
	[0.111]			[0.093]	[0.112]	[0.106]
retired	0.123			−0.118	0.094	−0.038
	[0.138]			[0.177]	[0.139]	[0.139]
student	0.462			−0.172	0.519	0.142
	[0.173]			[0.131]	[0.269]	[0.273]
skilled	0.251			−0.154	0.245	0.057
	[0.105]			[0.116]	[0.101]	[0.127]
longedu	0.307			−0.250	0.242	0.012
	[0.125]			[0.144]	[0.142]	[0.184]
kids	0.109			−0.058	0.087	−0.021
	[0.127]			[0.129]	[0.113]	[0.143]
single	0.207			−0.084	0.197	−0.005
	[0.124]			[0.134]	[0.125]	[0.139]
IncLow	−0.182			0.166	−0.149	0.010
	[0.125]			[0.134]	[0.131]	[0.137]
IncHigh	0.015			−0.054	0.016	−0.082
	[0.119]			[0.141]	[0.123]	[0.139]
Log-Likelihood	−0.658		−0.661		−0.649	

NOTES.The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distribution of the coefficient of risk aversion r under the PVCE-H^E representation with $\eta = 0$, using data from Andersen et al. (2008) and allowing the mean and standard deviation of the distribution to depend linearly on a set of observable characteristics of each individual. Column (i) shows the estimated coefficients when only the mean of the distribution is allowed to depend on the observable variables. Column (ii) shows the estimated coefficients when only the standard deviation of the distribution is allowed to depend on the observable variables. Column (iii) shows the results when both the mean and the standard deviation of the distribution are allowed to depend on the observable variables. In all cases, the coefficient of risk aversion r is assumed to follow a normal distribution. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level.

Table 7. Estimated Time Preferences and Observable Characteristics

Estimated Coefficient	*(i) Mean Eq. Only*		*(i) XD Eq. Only*		*(i) Both Eqs.*	
	$\mu_{\hat{\delta}}$	$\sigma_{\hat{\delta}}$	$\mu_{\hat{\delta}}$	$\sigma_{\hat{\delta}}$	$\mu_{\hat{\delta}}$	$\sigma_{\hat{\delta}}$
Constant	0.981	0.016	0.978	0.021	0.980	0.018
	[0.003]	[0.001]	[0.001]	[0.004]	[0.004]	[0.004]
female	−0.062			0.087	−0.213	0.361
	[0.124]			[0.162]	[0.166]	[0.185]
young	0.196			0.272	0.167	0.050
	[0.245]			[0.464]	[0.317]	[0.476]
middle	−0.195			−0.418	−0.049	−0.414
	[0.192]			[0.273]	[0.274]	[0.319]
old	−0.426			−0.809	−0.082	−0.770
	[0.204]			[0.314]	[0.256]	[0.321]
copen	−0.211			−0.120	−0.353	0.269
	[0.159]			[0.176]	[0.204]	[0.213]
city	−0.206			−0.136	−0.414	0.396
	[0.145]			[0.169]	[0.193]	[0.211]
owner	−0.013			0.086	−0.156	0.237
	[0.166]			[0.153]	[0.188]	[0.196]
retired	0.315			0.449	0.335	0.113
	[0.177]			[0.176]	[0.233]	[0.230]
student	0.099			−0.021	0.288	−0.364
	[0.254]			[0.419]	[0.271]	[0.380]
skilled	0.217			0.131	0.296	−0.158
	[0.148]			[0.179]	[0.166]	[0.171]
longedu	0.397			0.430	0.350	0.118
	[0.166]			[0.231]	[0.231]	[0.285]
kids	−0.277			−0.388	−0.246	−0.108
	[0.165]			[0.262]	[0.199]	[0.212]
single	−0.210			−0.488	0.149	−0.638
	[0.174]			[0.171]	[0.213]	[0.254]
IncLow	−0.218			−0.276	−0.280	0.061
	[0.169]			[0.198]	[0.214]	[0.262]
IncHigh	0.199			0.079	0.397	−0.326
	[0.164]			[0.274]	[0.213]	[0.218]
Log-Likelihood	−0.529		−0.533		−0.525	

NOTES. The above table reports the maximum-likelihood estimates of the median and the standard deviation of the distribution of the corrected discount factor $\hat{\delta}$ under the PVCE-H^E representation with $\eta = 0$, using data from Andersen et al. (2008) and allowing the mean and standard deviation of the distribution to depend linearly on a set of observable characteristics of each individual. Column (i) shows the estimated coefficients when only the mean of the distribution is allowed to depend on the observable variables. Column (ii) shows the estimated coefficients when only the standard deviation of the distribution is allowed to depend on the observable variables. Column (iii) shows the results when both the mean and the standard deviation of the distribution are allowed to depend on the observable variables. In all cases, the corrected discount factor is assumed to follow a beta distribution. Standard errors, shown in brackets, are computed using the delta method and clustered at the individual level. The estimated coefficients and standard errors of all variables, except the constant, are shown multiplied by 100.

Table 8. Description of the Observable Characteristics in Andersen et al. (2008)

Variable	**Description**	**Mean in the Data**	**XD in the Data**
female	Female	0.51	0.50
young	Aged less than 30	0.17	0.38
middle	Aged between 40 and 50	0.28	0.45
old	Aged over 50	0.38	0.49
single	Lives alone	0.27	0.45
kids	Has children	0.39	0.49
owner	Owns home or apartment	0.69	0.46
retired	Being retired	0.16	0.37
student	Being a student	0.09	0.29
skilled	Having some post-secondary education	0.38	0.49
longedu	Having substantial higher education	0.36	0.48
IncLow	Having a lower income level in 2002	0.28	0.45
IncHigh	Having a higher income level in 2002	0.20	0.40
copen	Having in the greater Copenhagen area	0.34	0.48
city	Living in city of 20,000 inhabitants or more	0.34	0.47

NOTES. The above table describes each of the observable characteristics in the data from Andersen et al. (2008), as well as their sample mean and standard deviation. All these characteristics are included in the dataset as indicator variables that take a value of one when the described condition is met by an individual.

Table 9. Model Assessment: DEU-H, PVCE-H and CEPV-H in the Andreoni and Sprenger (2012b) dataset

	DEU-H			**PVCE-H**			**CEPV-H**		
	(i)	**(ii)**	**(iii)**	**(iv)**	**(v)**	**(vi)**	**(vii)**	**(viii)**	**(ix)**
Mean	3.651	3.658	3.653	1.857	1.868	1.856	1.775	1.787	1.771
Std. Dev.	0.416	0.668	1.285	0.092	0.200	0.158	0.099	0.185	0.147

NOTES. The above table reports the mean and standard deviation of the cross-entropy measures obtained in the ten-fold cross-validation of the DEU-H, PVCE-H and CEPV-H representations, using the data in Andreoni and Sprenger (2012b). Columns (i), (iv) and (vii) report the results for the respective models when the full sample is pooled and randomly split into 10 subsamples for cross-validation. Columns (ii), (v) and (viii) report the results when the individuals in the sample are randomly split into 10 groups and their respective observations are used as the 10 subsamples for cross-validation. Columns (iii), (vi) and (ix) report the results when the sample menus are randomly split into 10 groups and their respective observations are used as the 10 subsamples for cross-validation.