

Macroeconomics II

Problem Set 5

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia script that produces the results. The solution must be posted in the student's github repository.

1. Variable z follows an AR(1) process:

$$z' = \rho z + \eta \quad \eta \sim N(0, \sigma_\eta)$$

- (a) Simulate a Markov Chain for z of 10000 draws, starting at $z_0 = 0$.
 - (b) Discretize z using Tauchen and Rouwenhorst's methods, do this for two different grid sizes ($N=5$, $N=15$). Simulate a Markov Chain for each of them.
 - (c) Compute the first four moments of z for each of the simulated Markov Chains, as well as the first 4 autocorrelations. Report your results on a table.
 - (d) Plot the histograms of each of the Markov Chains.
2. Consider the Neo-Classical growth model of problem set 2. There is a representative consumer with period utility:

$$u(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{\ell^{1+\eta}}{1+\eta}$$

Labor hours are constrained to be $\ell \in [0, 1]$. The consumer owns all capital that is rented out to a representative firm operating a Cobb-Douglas technology. Capital depreciates at a rate δ every period. Assume that $\beta = 0.98$, $\alpha = 1/3$, $z = 1$, $\sigma = 2$, $\eta = 1$, $\delta = 0.05$, and set χ such that $\ell_{ss} = 0.4$. Additionally z evolves stochastically according to: $\log z' = \rho \log z + \eta$, where $\eta \sim N(0, \sigma_\eta)$, with $\rho = 0.9$ and $\sigma_\eta = 0.1$.

- (a) Discretize z using Rouwenhorst's method.
- (b) Solve the planner's problem numerically using value function iteration. You must treat all choice variables and capital as continuous. You can choose the method for solving the maximization problem. You are free to use any method to speed up your computation and choose any grid size or curvature. Plot your solution (3D plot and level curves).
- (c) Use the solution to simulate 100 years of data, plot the levels of consumption, capital, labor and output. Report second moments for all variables and their first differences changes.