# Macroeconomics II

## Problem Set 3

Sergio Ocampo Díaz

---

The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia script that produces the results. The solution must be posted in the student's github repository.

1. This problem asks you to interpolate some utility functions using three different methods. You have to code your own interpolation routine, you cannot use the built-in routines in Julia or MatLab or other high- level programming languages. The goal is for you to understand how each of these methods works. I recommend looking for algorithms in Numerical Recipes, you can copy from there and adapt to your programming language.

   Specifically, write programs for interpolating values of the utility functions given below:

   $$u(c) = \log c \qquad u(c) = \sqrt{c} \qquad u(c) = \frac{c^{1-\sigma}}{1-\sigma} \text{ (for } \sigma = 2, 5, 10)$$

   You must do the following for each function and method:

   (a) Interpolate the function over the domain [0.05, 2] using grids of different size (it is up to you to find the sizes). Plot them all in the same graph.

   (b) Assess the accuracy of the interpolation. Find out the normal ways of benchmarking the accuracy and apply at least one to each of the interpolations (varying by size). Report the results on a matrix that has as rows

the interpolation method and as columns the number of grid points. You have to explain your benchmarking method.

(c) Plot the accuracy of your interpolation as a function of grid size.

(d) Bonus: Vary the curvature of the grid and plot the accuracy of the interpolation against this parameter.

(e) Bonus$^2$: Make a 3D plot and a heatmap (or level curve plot) of the accuracy against grid points and curvature.

2. Code a function that finds the curvature $(\theta)$ that minimizes the accuracy of interpolation. The function should take as inputs the size of the grid $(N)$, the range $(a, b)$, the function to interpolate $(F)$ and the interpolation method $(T)$. You can use bisection to solve for the optimal curvature. Try your function with the CRRA function in the interval [0.05, 2]. As a measure of accuracy use the sup-norm in percentage deviation terms, and set a tolerance of $10^{-5}$.

3. Extrapolate the three functions from problem one until to map the function from $[0.02, 2.5]$. Plot the results for all methods (restrict yourself to one grid size with and without curvature). What method works best?