

Macroeconomics II

Problem Set 2

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia script that produces the results. The solution must be posted in the student's github repository.

Consider the Neo-Classical growth model of the first problem set. Now assume that the representative agent derives disutility from labor so that:

$$u(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{\ell^{1+\eta}}{1+\eta}$$

Recall that the agent has a time endowment of 1 in each period. Assume as well that capital depreciates at a rate δ every period. Everything else is as before

1. Define a competitive equilibrium for this economy.
2. Find the steady state value for $\{c, \ell, k, y, r, w\}$.
3. Pose the planner's dynamic programming problem. Write down the appropriate Bellman equation.

For the following exercises assume that $\alpha = 1/3, z = 1, \sigma = 2, \eta = 1$.

4. Find χ such that $\ell_{ss} = 0.4$.
5. Solve the planner's problem numerically using value function iteration. You must do it using:
 - (a) Plain VFI

(b) Modified Howard's Policy Iteration (you must choose the number of policy iterations)

(c) MacQueen-Porteus Bounds

Use an equally spaced grid for capital between $\underline{k} = 10^{-5}$ and $\bar{k} = 2k_{ss}$, vary the number of grid points until you get a maximum error of 1% in your Euler equation.

For each method report the time and number of iterations.

6. Use the solution to the planner's problem to obtain the path of $\{c, \ell, k, r, w, y\}$ starting from the steady state after the following changes:

(a) Capital decreases to 80% of its steady state value.

(b) Productivity increases permanently by 5%.

Make sure to include dotted horizontal lines in all your graphs indicating the initial and final steady state of the variables.

7. Prove that the mapping used in Howard's policy iteration algorithm is a contraction.