Problem Set 5

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia script that produces the results. The solution must be posted in the student's github repository.

1. Consider the Neo-Classical growth model of problem set 5. There is a representative consumer with period utility:

$$u\left(c,\ell\right) = \frac{c^{1-\sigma}}{1-\sigma} - \chi\frac{\ell^{1+\eta}}{1+\eta}$$

Labor hours are constrained to be $\ell \in [0,1]$. The consumer owns all capital that is rented out to a representative firm operating a Cobb-Douglas technology. Capital depreciates at a rate δ every period. Assume that $\beta = 0.98$, $\alpha = 1/3$, $z = 1$, $\sigma = 2$, $\eta = 1$, $\delta = 0.05$, and set χ such that $\ell_{ss} = 0.4$. Additionally z evolves stochastically according to: $\log z' = \rho \log z + \eta$, where $\eta \sim N\left(0,\sigma_\eta\right)$, with $\rho = 0.9$ and $\sigma_\eta = 0.01$.

   (a) Discretize z using Rouwenhorst's method.

   (b) Solve the planner' problem numerically using value function iteration. You must treat all choice variables and capital as continuous. Use the endogenous grid method. You are free to use any method to speed up your computation and choose any grid size or curvature. Plot your solution (3D plot and level curves).

(c) Solve the planner' problem numerically using value function iteration. You must treat all choice variables and capital as continuous. Use the envelope condition method. You are free to use any method to speed up your computation and choose any grid size or curvature. Plot your solution (3D plot and level curves).

(d) Use the solution (of one of the methods to simulate 100 years of data, plot the levels of consumption, capital, labor and output. Report second moments for all variables and their first differences changes.

(e) Compare with your previous results from problem set 5.

2. Estimation:

(a) Get data on output (GDP) from any country.

(b) De-trend your data taking out a log-linear trend (so that the growth of the detrended variable is on average 0)

(c) Get the variance of the (detrended) growth rate of GDP

(d) Calibrate your model to match this moment. You should use the parameters of the stochastic process for productivity.