Macroeconomics II

Problem Set 4

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The solution of this problem consists of a PDF with all mathematical derivations and all graphs as well as julia script that produces the results. The solution must be posted in the student's github repository.

Consider the Neo-Classical growth model of problem set 2. There is a representative consumer with period utility:

$$u(c,\ell) = \frac{c^{1-\sigma}}{1-\sigma} - \chi\frac{\ell^{1+\eta}}{1+\eta}$$

Labor hours are constrained to be $\ell \in [0,1]$. The consumer owns all capital that is rented out to a representative firm operating a Cobb-Douglas technology. Capital depreciates at a rate δ every period. Assume that $\alpha = 1/3$, $z = 1$, $\sigma = 2$, $\eta = 1$, $\delta = 0.05$, and set χ such that $\ell_{ss} = 0.4$.

1. Solve the planner' problem numerically using value function iteration. You must treat all choice variables and states as continuous. You must do it using:

   (a) Direct maximization over a pair of variables (c,ℓ) or $\left(k',\ell\right)$.

   (b) Direct maximization over a single variable $\left(k'\right)$ with other variables solved for analytically.

   (c) Solving the first order conditions.

   You are free to use any method to speed up your computation and choose any grid size or curvature.

For each method report the time and number of iterations, and plot the resulting value function, policy functions and Euler equation errors on a fine grid of 1000 equally spaced nodes.

2. Use the solution to the planner's problem to obtain the path of $\{c, \ell, k, r, w, y\}$ starting from the steady state after capital decreases to 80% of its steady state value.

   Make sure to include dotted horizontal lines in all your graphs indicating the initial and final steady state of the variables.

3. Use a shooting algorithm to find the transition. You know that the path variables must take during the transition must satisfy the consumer's FOC at every time. Use this knowledge to code a function that evaluates the residual in the FOC for a transition. The function should have as one of the inputs the length of the transition (a parameter you must find). Use a root finder to solve for the transition path and compare your results with what you obtained using dynamic programming.

4. **Benchmarking Global Optimizers (Arnaud, Guvenen & Kleineberg, 2020)**.

   (a) Read the paper

   (b) Code a version of the TikTak algorithm (use Appendix A)

   (c) Choose at least two of their test functions (Section 4.1).

   (d) Plot the function and its contour lines with $n = 2$ (so they are functions $F : \mathbb{R}^2 \to \mathbb{R}$). Minimize them. Describe your results.

   (e) Minimize the functions for $n = 5$. Describe your results.