



In this module, we briefly review some basic probability concepts

Conditional Probability

- Definition: The conditional probability of E given F is the probability that an event, E , will occur given that another event, F , has occurred

$$P(E | F) = \frac{P(E \cap F)}{P(F)} \quad \text{if} \quad P(F) \neq 0$$

$$P(E | F) = \frac{\text{number of outcomes in } E \cap F}{\text{number of outcomes in } F}$$

The conditional probability of E given F is the ratio of the joint probability of E and F happening divided by the marginal probability of F . Or, simply, given that F has happened, what is the probability of E happening? That is the conditional probability. Let us see an example.

Conditional Probability: Example

- Toss a balanced die once and record the number on the top face.
- Let E be the event that a 2 shows on the top face.
- Let F be the event that the number on the top face is even.
- What is probability of E , $P(E)$?
- What is the Probability of the event E if we are told that the number on the top face is even, that is, we know that the event F has occurred?

Assume that F is the event of getting an even number when you roll a die, and E is the event of seeing a 2. What is the $P(E|F)$?

Intuitively, we know that if the number is Even, then the probability it is a 2 is $1/3$. Why? There are three even numbers 2, 4, and 6, and the probability of each occurring is the same. Let us now apply the formula to calculate this.

Conditional Probability: Example II

- Key idea: the original sample space no longer applies.
- The new or reduced sample space is
$$S = \{2, 4, 6\}$$
- Notice that the new sample space consists only of the outcomes in F .
- $P(E \text{ occurs given that } F \text{ occurs}) = 1/3$
- Notation: $P(E | F) = 1/3$

This clearly shows that the $P(E | F) = 1/3$. Note the revised sample space for the calculation. The conditional probability essentially revises the sample space in consideration. We can also use the original formula to calculate this.

Using the Formula

- In the previous example,

$$P(F) = \frac{3}{6}$$

$$P(E \cap F) = \frac{1}{6}$$

- Therefore

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

Remember that in calculating the joint probabilities, we are asking what is the probability of getting a 2 and an even number. As there is only one option for that out of 6 choices, the probability of $P(E \text{ and } F) = 1/6$. The rest of the calculations are self explanatory.

Independent Events

- If the probability of the occurrence of event A is the same regardless of whether or not an outcome B occurs, then the outcomes A and B are said to be independent of one another.
- Formally speaking , if

$$P(A | B) = P(A)$$

then A and B are independent events.

In other words, if A and B are independent, the conditional probability of A given B is the same as probability of A. That is, the probability of A is not dependent on B. Given this, if A and B are independent, what $P(A \text{ and } B)$?

Independent Events II

- As we have seen, the joint probability of event A and B is given by

$$P(A \cap B) = P(A | B)P(B)$$

- Now, if A and B are independent then

$$P(A | B) = P(A)$$

- Which means that

$$P(A \cap B) = P(A)P(B)$$

So, if A and B are independent, $P(A \text{ and } B) = P(A) P(B)$

Example- Independent Events

- A coin is tossed and a single 6-sided die is rolled. Find the probability of getting a head on the coin and a 3 on the die.
- Probabilities:
$$P(\text{coin is head}) = 1/2$$
$$P(\text{die is 3}) = 1/6$$
- The two events are independent and therefore
$$P(\text{coin is head and die is 3}) = 1/2 * 1/6 = 1/12$$

Obviously, tossing a coin and rolling a die are independent events. As such, the probability of getting a head is not affected by the probability of getting a 3.

Independent Events: Generalization

- If events X_1, X_2, \dots, X_n are independent, then

$$P(X_1 \cap X_2 \cap \dots \cap X_n) = P(X_1) \times P(X_2) \times \dots \times P(X_n)$$

- Finally, note the difference between independent and mutually exclusive events:
- If A and B are independent, then it means that the probability of A would not be changed whether or not B has happened.
- If A and B are mutually exclusive, it means that the probability of A will be zero if B has occurred and vice versa:

$$P(A \cap B) = 0$$

This provides the generalization to n independent events. Question: If A and B are independent, are they also mutually exclusive?

“The consequences of an act affect the probability of its occurring again. ”

B. F. Skinner, social philosopher

WWW.KENT.EDU

This concludes our module on basic probability needed for the next module.