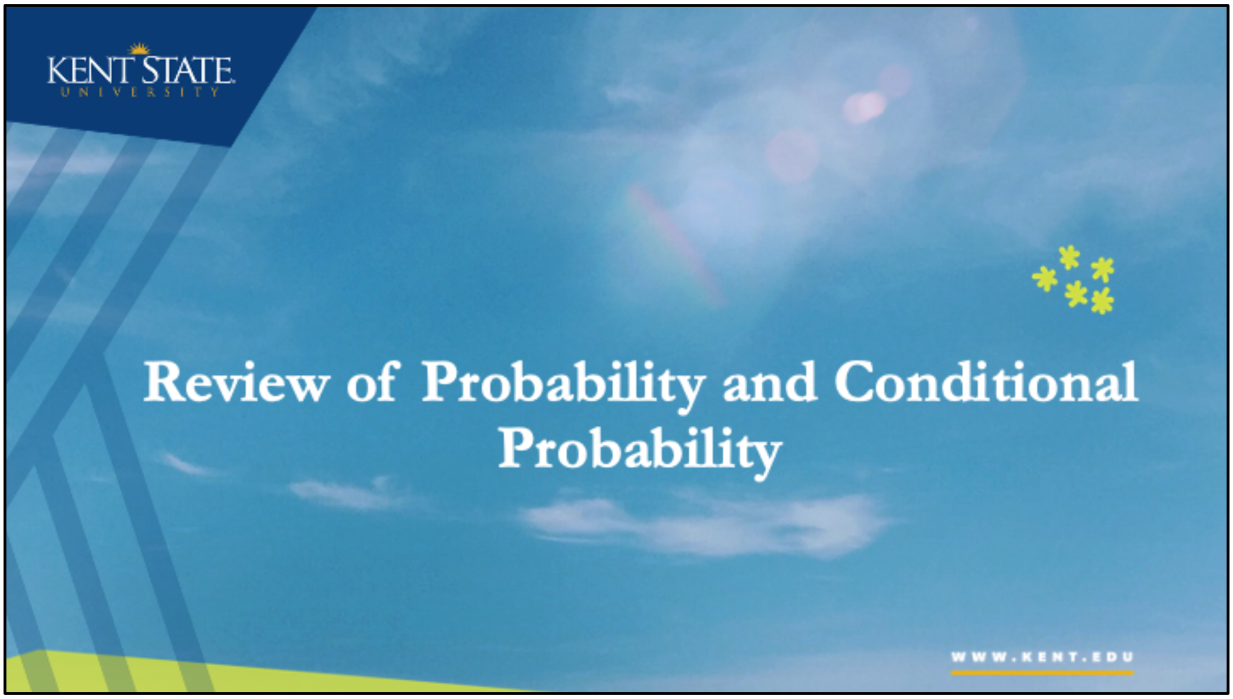


In this module, we briefly review some basic probability concepts

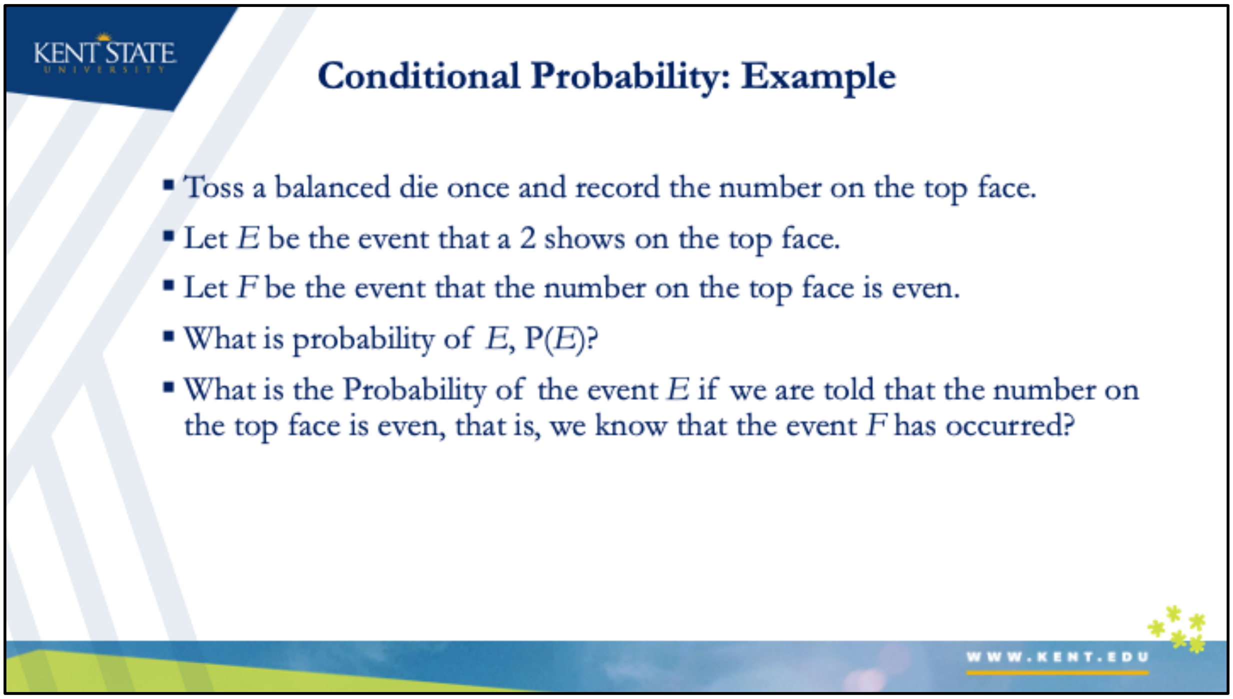
Conditional Probability

- Definition: The conditional probability of E given F is the probability that an event, E, will occur given that another event, F, has occurred

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} \quad \textbf{if} \quad P(F) \neq 0$$

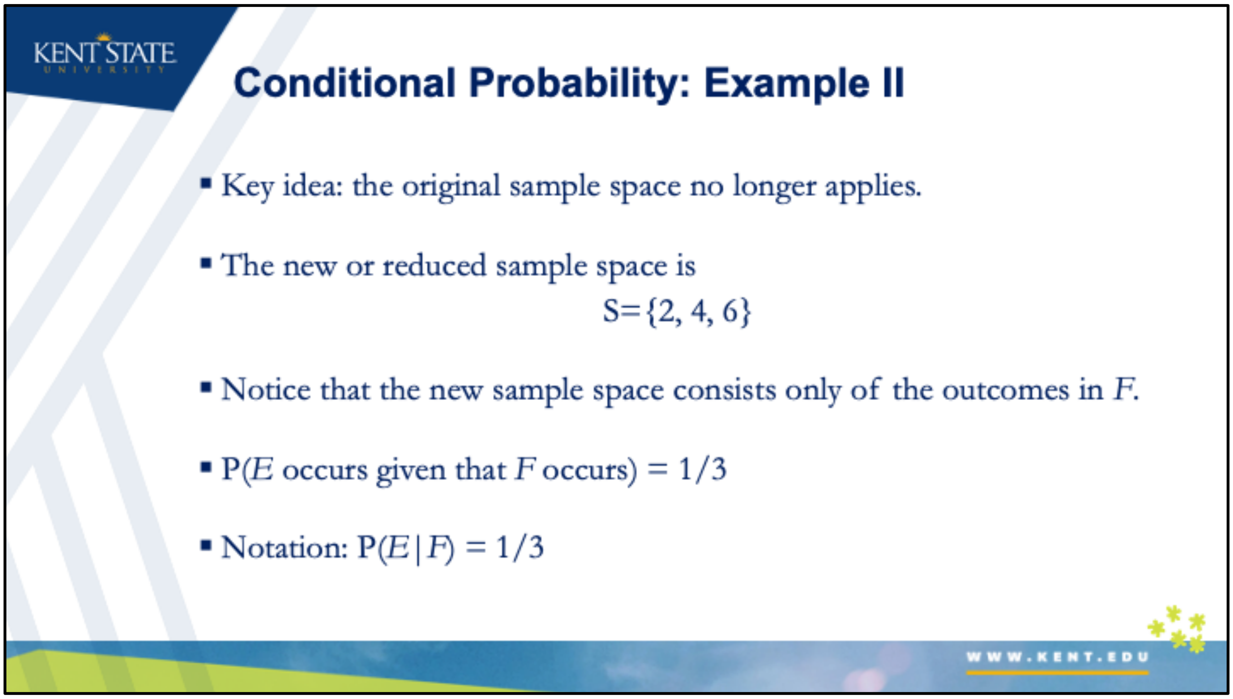
$$P(E \mid F) = \frac{\text{number of outcomes in } E \cap F}{\text{number of outcomes in } F}$$

The conditional probability of E given F is the ratio of the joint probability of E and F happening divided by the marginal probability of F. Or, simply, given that F has happened, what is the probability of E happening? That is the conditional probability. Let us see an example.



Assume that F is the event of getting an even number when you roll a die, and E is the event of seeing a 2. What is the P(E|F)?

Intuitively, we know that if the number is Even, then the probability it is a 2 is 1/3. Why? There are three even numbers 2, 4, and 6, and the probability of each occurring is the same. Let us now apply the formula to calculate this.



This clearly shows that the $P(E|F) = 1/3$. Note the revised sample space for the calculation. The conditional probability essentially revises the sample space in consideration. We can also use the original formula to calculate this.

Using the Formula

- In the previous example,

$$P(F)=\frac{3}{6}$$

$$P(E\cap F)=\frac{1}{6}$$

- Therefore

$$P(E\mid F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{6}}{\frac{3}{6}}=\frac{1}{3}$$

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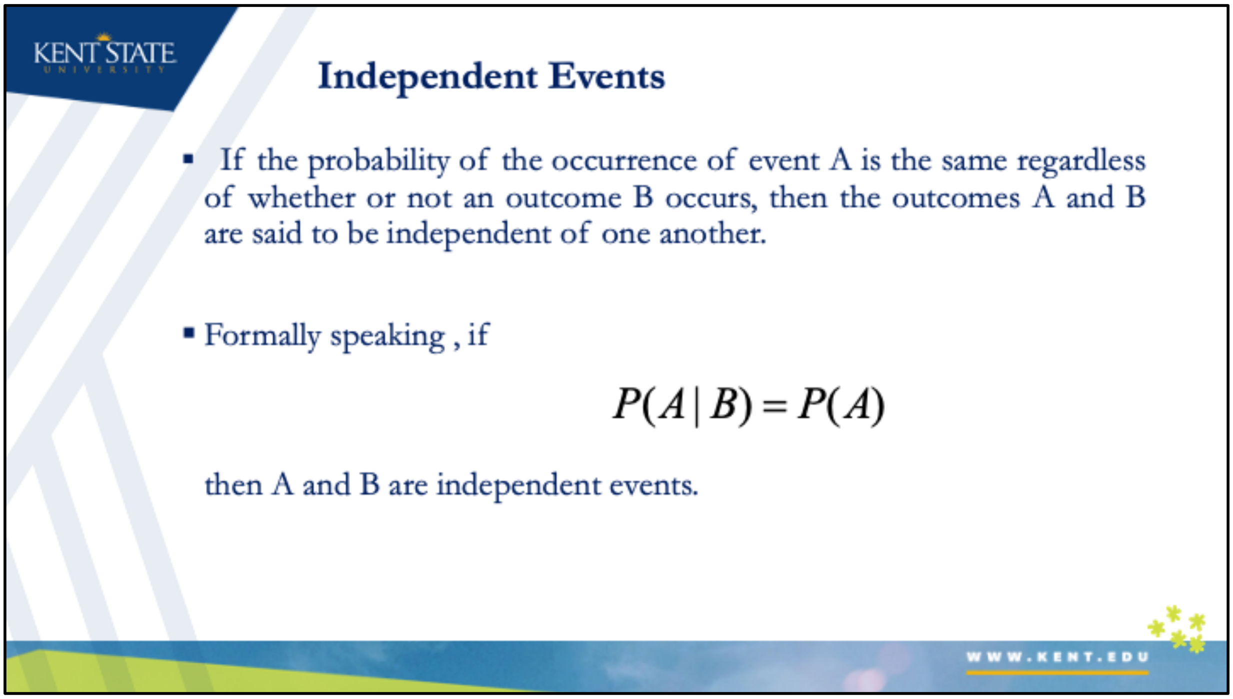
Remember that in calculating the joint probabilities, we are asking what is the probability of getting a 2 and an even number. As there is only one option for that out of 6 choices, the probability of P(E and F) = 1/6. The rest of the calculations are self explanatory.



In other words, if A and B are independent, the conditional probability of A given B is the same as probability of A. That is, the probability of A is not dependent on B. Given this, if A and B are independent, what P(A and B)?

Independent Events II

- As we have seen, the joint probability of event A and B is given by

$$P(A \cap B) = P(A \mid B)P(B)$$

- Now, if A and B are independent then

$$P(A \mid B) = P(A)$$

- Which means that

$$P(A \cap B) = P(A)P(B)$$

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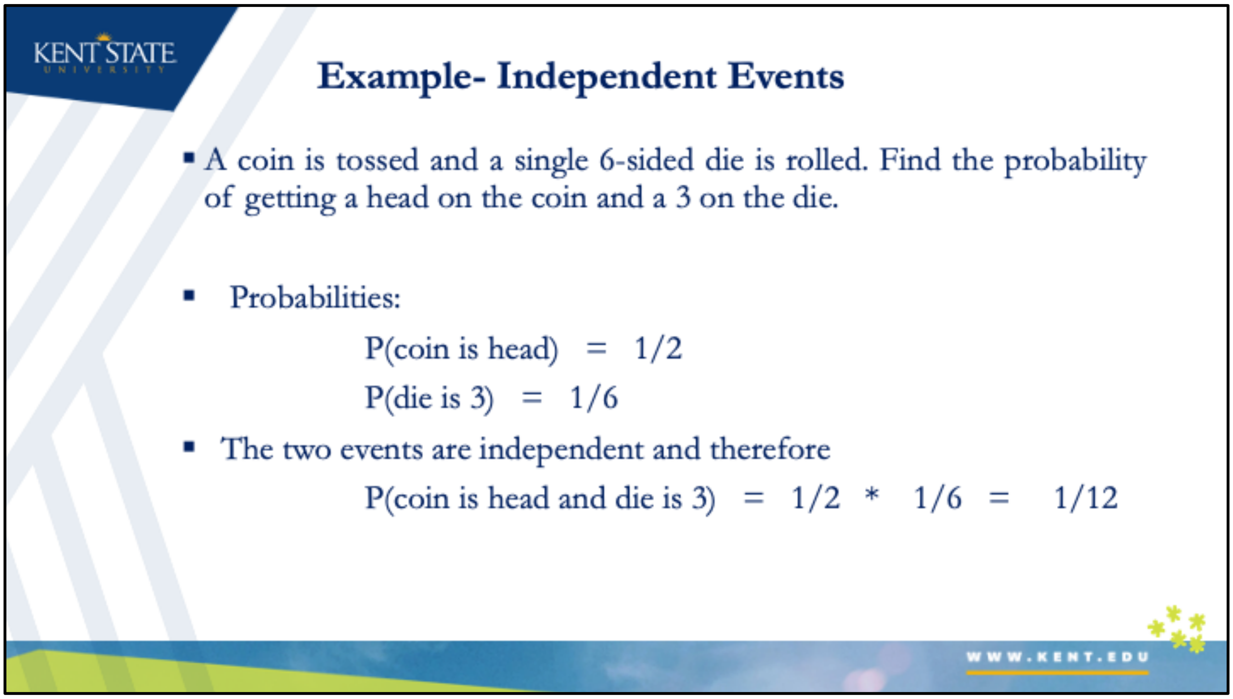
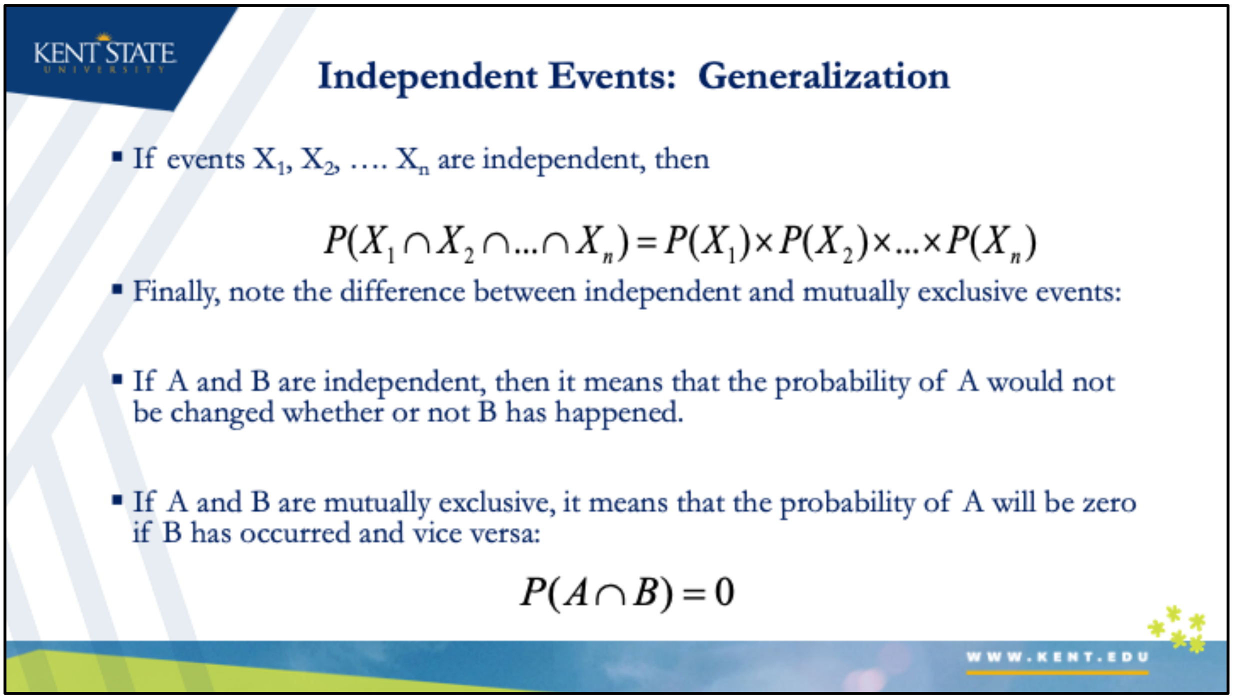
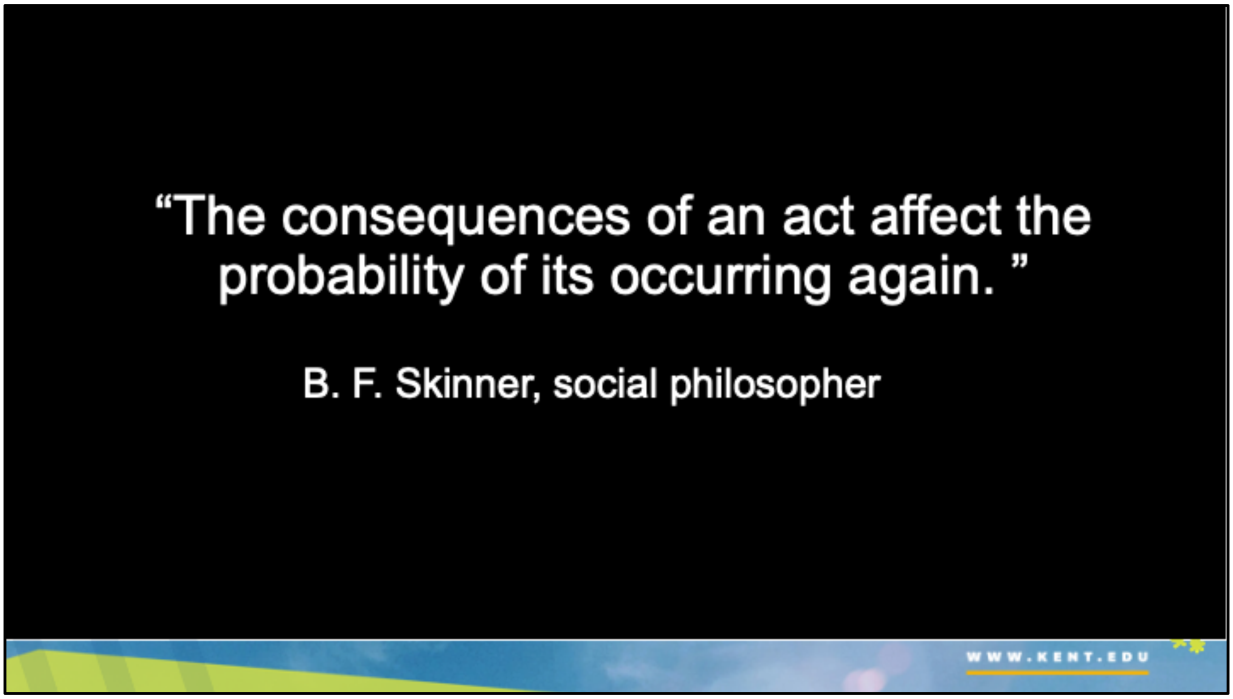
So, if A and B are independent, P(A and B) = P(A) P(B)



Obviously, tossing a coin and rolling a die are independent events. As such, the probability of getting a head is not affected by the probability of getting a 3.



This provides the generalization to n independent events. Question: If A and B are independent, are they also mutually exclusive?



This concludes our module on basic probability needed for the next module.