Performance of Power Systems Report – Project 2

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EE491 Fall 2024

Project Overview –

In this project, I used **MATLAB** and **MATPOWER** to perform DC State Estimation (SE). The goal was to estimate the voltage angles (X) in a power system using the Weighted Least Squares (WLS) method. I started by generating clean measurements (Z_0) from the DC power flow model using the system's measurement matrix. Then, I added Gaussian noise to these measurements to simulate real-world conditions, creating Z_{noise}. A covariance matrix (R) was calculated to account for the noise variance and ensure numerical stability during estimation. Using the noisy measurements and the WLS algorithm, I successfully estimated the state vector (X_{Cap}). Finally, I analyzed the impact of different noise levels ($\sigma = 0.01, 0.03, 0.05, 0.07$) on the accuracy of the state estimation, demonstrating the robustness of the WLS method.

Requirements –

1) Complete 3 functions

The following three functions were developed to address the requirements:

a) `getZ(X,H)`

Measurements in a power system are modeled as Z = H · X, where:

Z = Measurement vector (power flows, power injections),

H = Measurement matrix derived from network

X = State vector (bus voltage angles).

b) `addNoise(Z0,sig)`

Real-world measurements are subject to noise. Gaussian noise is added to the measurements, modeled as: $Z_{noise}(i) = Z_0(i) + e_i$

$$where, e_i \sim N(0, {\sigma_i}^2)$$

$$and, {\sigma_i}^2 = (\sigma \cdot Z_0(i))^2$$

Covariance matrix R is defined as: $R(i,i) = {\sigma_i}^2$

c) `DC_WLS_SE(H,R,Znoise)`

WLS minimizes the weighted error between measured and estimated values:

$$\hat{X} = (H^T R^{-1} H)^{-1} H^T R^{-1} Z$$

Code Implementations –

```
%% Functions
function Z0 = getZ(X, H)
    % Z0 = H * X as per the DC power system model
    Z0 = H * X;
end
```

This function above, within my code calculates the clean measurement vector using the relationship $Z = H \cdot X$. It is a direct application of the DC power flow measurement model.

```
function [Znoise, R] = addNoise(Z0, sig)
    % Adding Gaussian noise to measurements for construction of covariance matrix
    Znoise = zeros(size(Z0));
    R = zeros(length(Z0));
    for i = 1:length(Z0)
        e = normrnd(0, sig * abs(Z0(i)));  % Generating Gaussian noise
        Znoise(i) = Z0(i) + e;             % Adding noise
        R(i, i) = (sig * abs(Z0(i)))^2;    % Variance for covariance matrix
    end
    % Ensuring numerical stability for near-zero diagonal elements
    R_diag = diag(R);
    idx = find(R_diag < 1e-8);
    R_diag(idx) = 1e-8;
    R = diag(R_diag);
end
```

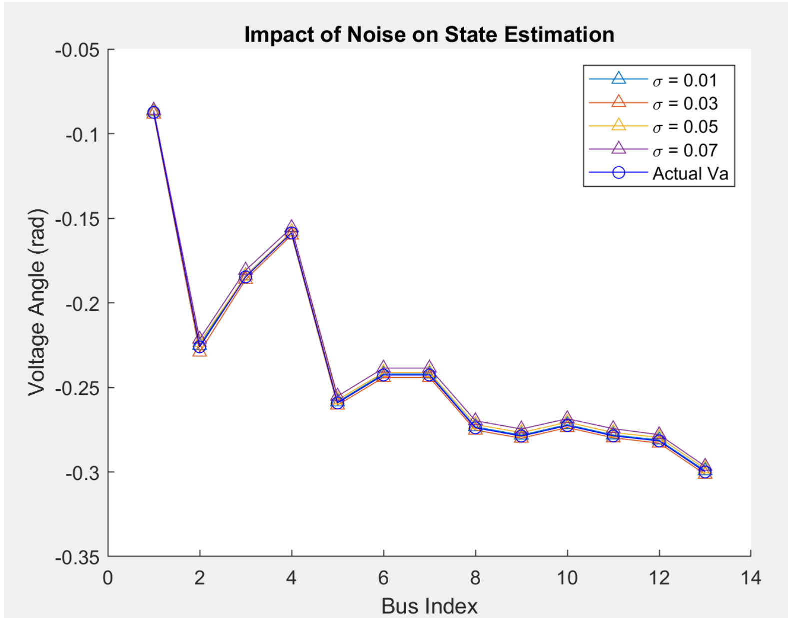
This function above adds Gaussian noise to the clean measurements Z0 to simulate data. The function also constructs the diagonal covariance matrix R with variances proportional to the measurement magnitudes, in addition to Handling edge cases to ensure numerical stability (small diagonal values in R).

```
function Va_se = DC_WLS_SE(H, R, Znoise)
    % Performing Weighted Least Squares (WLS) State Estimation
    R_inv = inv(R);                  % Inverse of covariance matrix
    H_transpose = H';                % Transpose of H
    G = H_transpose * R_inv * H;     % Gain matrix
    Va_se = G \ (H_transpose * R_inv * Znoise); % Solving for state vector
end
```

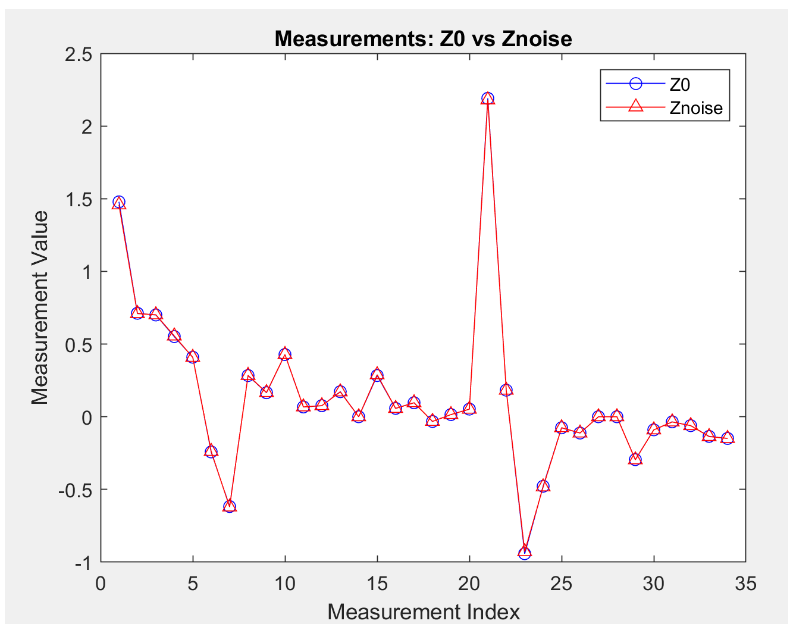
This function above performs Weighted Least Squares (WLS) state estimation. It also minimizes the weighted error between the modeled and measured values. The WLS solution is calculated using:

$$\hat{X} = (H^T R^{-1} H)^{-1} H^T R^{-1} Z$$

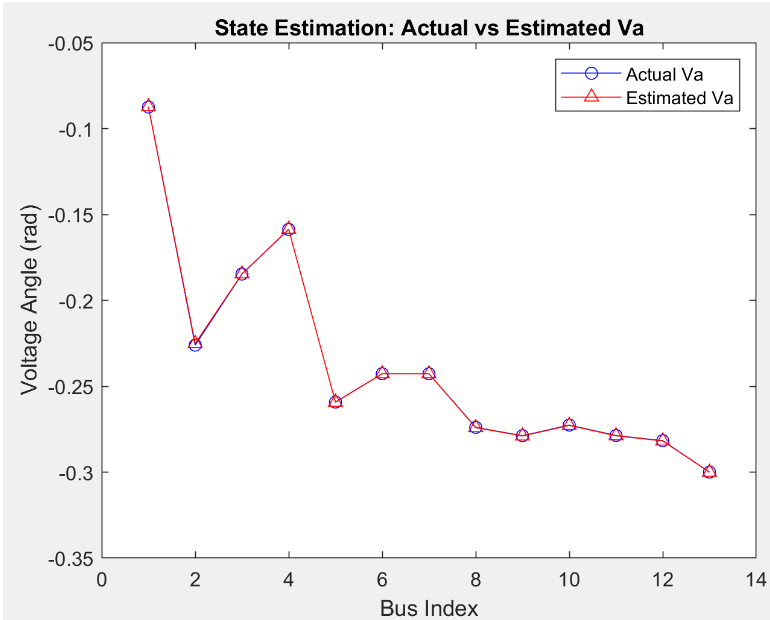
<u>Code Results</u> –



The plot above reveals how increasing noise levels (σ=0.01,0.03,0.05,0.07) affects the accuracy of state estimation. As sigma increases, the deviation of the estimated voltage angles from the actual values grows slightly. However, even at the highest noise level, σ=0.07, the WLS algorithm maintains a high level of accuracy, with the estimated values remaining close to the actual values. This demonstrates that the WLS algorithm is robust and can handle varying levels of noise effectively, making it a reliable method for state estimation in real-world power systems.



The plot above demonstrates the impact of Gaussian noise on the clean measurements. The noisy measurements Znoise closely follow the trend of the clean measurements Z0, with deviations proportional to the applied noise level σ=0.01. This behavior validates the accuracy of the noise modeling function and reflects realistic conditions in power systems, where sensor inaccuracies and environmental factors introduce noise into measurement data.



The plot above confirms the effectiveness of the Weighted Least Squares (WLS) algorithm. The estimated voltage angles (Va hat) align closely with the actual angles Va. showing minimal deviation despite the presence of noise in the measurements. This result highlights the robustness of the WLS method in accurately reconstructing the system's state, even when measurements are imperfect. The minor differences between the actual and estimated values are expected due to the noise in the input data.