The Attitude Estimation using IMU/GPS and Adaptive Filtering

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This paper deals with the *GPS* usage for the attitude determination and *IMU/GPS* integration. *GPS* system is widely used as the source of position, velocity, and time information; however there is also other possibility of its usage which is the attitude determination capability. Several possible methods to determine *GPS* based attitude are presented after an introduction. The last presented method is closely described with appropriate hardware equipment. The *GPS* based attitude measurement results are presented later on. The last part describes the integration of *IMU* and *GPS* as a potential future development.

Nomenclature

GPS	Global Positioning System
AHRS	Attitude and Heading Reference System
INS	Inertial Navigation System
IMU	Inertial Measurement Unit
GNSS	Global Navigation Satellite System
SV	Space Vehicle
PDD	Pseudorange Double Differences
CPM	Carrier Phase Measurement

I. Introduction

GPS (Global Positioning System)[1,2] is widely known as a system used for position, velocity and time evaluation on the Earth and in free space close to it. Other *GPS* system application, which is not so widely known, can be in determining the aircraft attitude.

The aircraft attitude estimation is one of the oldest problems which had to be solved at the beginning of aviation. *AHRS (Attitude and Heading Reference System)* unit and *INS (Inertial Navigation System)* unit are the most common navigation systems used on board of aircrafts to evaluate the attitude and position. Both systems are based on *IMU (Inertial Measurement Unit)* consisting of three angular rate sensors *(gyroscopes)* and three accelerometers. In case of low-cost navigation systems the evaluation of the attitude and position is influenced by large errors coming from the sensors. Furthermore, attitude angles and position are given from the time integration of angular rates and velocities, and therefore the evaluation can be deflected by an unbound growing error. Hence those navigation systems are generally aided by other information sources such as altimeter, *GPS*, air speed indicator and so on.

Such integration can reduce significant errors and provide suitable accuracy for the application. However, in case of attitude evaluation there are not so many possibilities for aiding as for the positioning. In this paper one method using *GPS* is presented. The attitude correction or standalone estimation using *GPS* receivers is a complex problem, unlike a position inaccuracy elimination which is a simple system junction.

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A. *GPS* system

The *GPS* system employs at least 24 satellites *SVs (Space Vehicle)* for a proper functionality. However, in the present time 32 *SVs* are active (December 2009). *SVs* are divided into groups at six orbital planes. The orbital planes have inclination of approximately 55° and separation by 60°.

For most places on the Earth the *SVs* constellation set provides at least six SVs within the line of sight at any moment. At least 4 *SVs* have to be within the line of sight for *3D* position calculation, three for position and the fourth one for receiver clock synchronization. Eq. (1) shows a relationship among unknown receiver position $[X, Y, Z]$ (*ECEF* coordinates), receiver clock bias (C_{rr}), and distance ρ *(pseudo range)* from each of n visible SV $i \in (1, 2, 3, \ldots, n)$ with known position $[x, y, z]$ decoded and calculated from ephemerides data sent by each *SV*

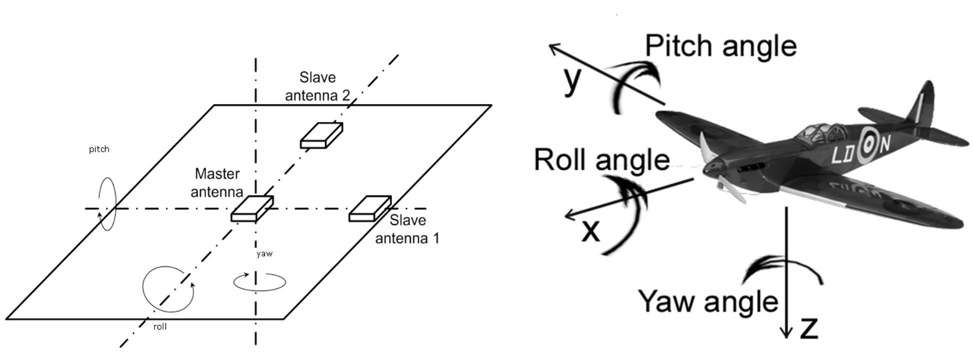
$$\rho_r = \sqrt{(x - X)^2 + (y - Y)^2 + (z - Z)^2} + C_{rr}. \tag{1}$$

Eq. (1) can by linearized by Taylor series and unknown variables estimation solved using a least squares method.

Unfortunately, a real measurement is not an error free. Therefore, it is necessary to include those errors into a position calculation. Mainly those errors are caused by the atmospheric effects (ionosphere and troposphere delay), *SV's* clock errors, ephemeris (*SV* position) errors, and signal multipath.

II. *GPS* based Attitude Estimation

Three angles are needed to know the attitude of an aircraft. The angles in this paper are: *yaw* ψ, *roll* ϕ, and *pitch* θ as shown in Figure 1b. It means that at least three GPS receivers are required and their antennae have to be placed in non-collinear positions, shown in Figure 1a. The antennae are typically placed at the end of wings and on aircraft tail and near aircrafts center of mass. Such a placement guarantees the highest resolution for attitude evaluation with respect of the aircraft configuration.



(a) Antennae placement (b) Attitude angles

Figure 1: Plane Attitude

The first and simplest possible method to estimate the *GPS* based attitude uses three antennae positions calculated individually solving the Eq. (1). Based on the calculated positions three *Euler* angles can be evaluated. Even if the positions of used antennae are taken almost simultaneously there should still be significant inaccuracies caused by different *SVs* used during position calculation.

A. Pseudorange Double Differences *(PDD)*

The second possibility for aircraft attitude estimation is *PDD (Pseudorange Double Difference)*, a method comparable with *DGPS (Differential GPS)* position estimation. A main difference between differential *GPS* and *PDD* is that in *PDD* method there is no external reference station needed, but the method considers one receiver as the reference one.

Pseudorange measurement can be defined as

$$\rho_M = \rho + cdt - cdT + d_{ION} + d_{TROP} + d_{EPH} + d_\rho, \tag{2}$$

where:
- ρ_M is the measured pseudorange between receiver antennae and SV,
- ρ the true range between a receiver and SV,
- c the speed of light,
- dt SV clock error,
- dT a receiver clock error,
- d_{ION} an ionosphere delay error,
- d_{TROP} an troposphere delay error,
- d_{EPH} an ephemerides error,
- d_ρ other errors caused by environment and multipath.

A single difference of *pseudorange* measurement between receivers a and b shows Eq. (3), where the advantage of SV clock error elimination comes along.

$$\begin{aligned} \Delta\rho_{Mab} &= \rho_{Ma} - \rho_{Mb} \\ &= \Delta\rho_{ab} - c\underbrace{(dT_a + dT_b)}_{\Delta T_{ab}} + \Delta d_{ION} + \Delta d_{TROP} + \Delta d_{EPH} + \Delta d_\rho. \end{aligned} \tag{3}$$

Another important point is a significant reduction of values Δd_{ION}, Δd_{TROP} and Δd_{EPH}, because all these variables vary slightly with distance (the difference is negligible for ranges up to $50\,km$).

Double difference is made using two Eq. (3) with pseudoranges between receivers a and b and two satellites i and j. The advantage of *PDD*, double difference Eq. (4) respectively, is the elimination of receivers a and b clock errors dT_{ab} same as satellites i and j clock errors dt in Eq. (3)

$$\begin{aligned} \nabla\Delta\rho^{ij}_{Mab} &= \Delta\rho^i_{Mab} - \Delta\rho^j_{Mab} \\ &= \nabla\Delta\rho^{ij}_{ab} + \nabla\Delta d_{ION} + \nabla\Delta d_{TROP} + \nabla\Delta d_{EPH} + \nabla\Delta d_\rho. \end{aligned} \tag{4}$$

In Eq. (4) terms $\nabla\Delta d_{ION}$, $\nabla\Delta d_{TROP}$ and $\nabla\Delta d_{EPH}$ are reduced; however, last remain error $\nabla\Delta d_\rho$ importance rises.

B. Carrier Phase Measurement *(CPM)*

A special hardware available on market opens new fields of possibilities in the attitude estimation – *CPM (Carrier Phase Measurement)*. The hardware with measured raw data output and *carrier phase* can be used, and thus more precise measurement can be made. A basic *CPM* principle is shown in Figure 2 where Φ is measured signal phase difference, n integer wavelength count λ of receivers' distance difference from SV.

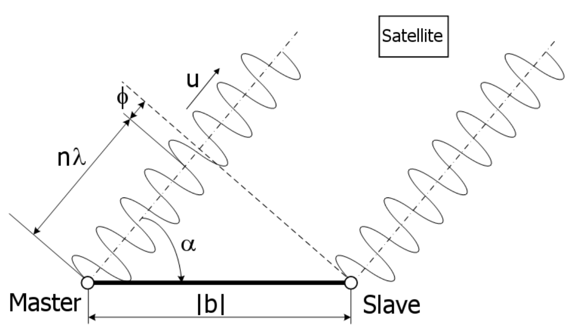


Figure 2: Carrier Phase Measurement

In this paper only *GPS* signal *L1* with carrier frequency $f_1 = 1575.42\,MHz$ is used. The wavelength of L_1 signal is $\lambda_1 = \frac{c}{f_1} \simeq 0.1903\,m$, where c means the speed of light.

CPM attitude estimation is based on Eq.

$$\lambda \cdot \phi_M = \rho + cdt - cdT + \lambda N - d_{ION} + d_{TROP} + d_{EPH} + d_\phi, \tag{5}$$

where new variables, in comparison with Eq. (4), are $\lambda = 0.1903\,m$ that corresponds to a carrier wavelength of the signal *L1*, ϕ_M measured carrier phase and N ambiguity uncertainty, which is an integer number of full wavelengths between the receiver and *SV*.

Clock errors dt and dT in *CPM* are eliminated by a single and double difference as it was in *PDD*. Single difference equation

$$\begin{aligned} \Delta\lambda \cdot \phi_{Mab} &= \lambda \cdot \phi_{Ma} - \lambda \cdot \phi_{Mb} \\ &= \Delta\rho_{ab} - c\Delta dT_{ab} + \lambda \underbrace{(N_a - N_b)}_{\Delta N_{ab}} \\ &\quad -\Delta d_{ION} + \Delta d_{TROP} + \Delta d_{EPH} + \Delta d_\phi, \end{aligned} \tag{6}$$

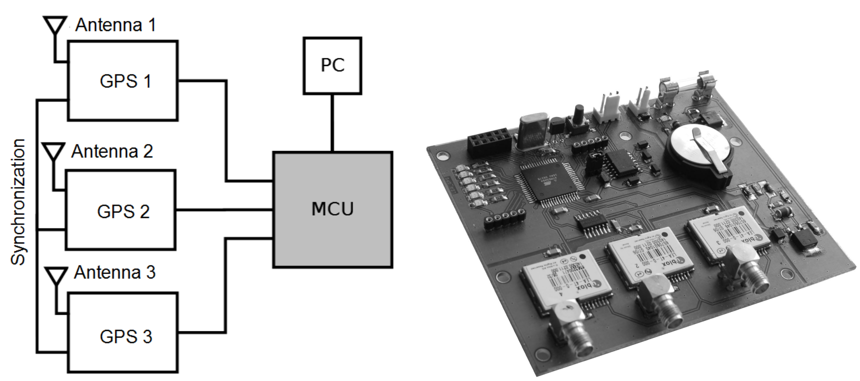
for two receivers a and b shows *SV* clock error elimination. The double difference Eq. (7) can be done between two receivers and two SV's and it eliminates clock errors (SV's and receivers) and atmosphere effects, same as in PDD case. However, errors caused by multipath and receiver noise are amplified through the differencing process, which is a disadvantage of the method. These aspects can be reduced by a shape of antennae and by antennae placements.

$$\begin{aligned} \nabla\Delta\lambda \cdot \phi^{ij}_{Mab} &= \lambda\Delta \cdot \phi^i_{Mab} - \lambda\Delta \cdot \phi^j_{Mab} \\ &= \nabla\Delta\rho^{ij}_{ab} + \lambda \underbrace{(\Delta N^i_{ab} - \Delta N^j_{ab})}_{\nabla\Delta N^{ij}_{ab}} \\ &\quad -\nabla\Delta d_{ION} + \nabla\Delta d_{TROP} + \nabla\Delta d_{EPH} + \nabla\Delta d_\phi, \end{aligned} \tag{7}$$

A latest unknown variable in Eq. (7) is the ambiguity resolution N, ΔN_{ab} or $\nabla\Delta N^{ij}_{ab}$ respectively. This unknown integer number N has to be solved with one of several known method. Most common method is *LAMBDA (Least-squares AMBiguity Decorrelation Adjustment)* from reference,[4,5] but it is not the subject of this paper so the details about this method are not presented.

III. *GPS* based attitude measurement

As shown in Figure 3a for hardware realization three independent GPS receivers were integrated. After studying all possibilities three same u-blox modules LEA-4T were used.[3] A possibility of synchronization between all three modules is shown in simplified schema (in Figure 3a).



(a) Measurement schematics (b) Device realization

Figure 3: GPS attitude measurement

LEA-4T supports not only raw data output with Carrier Phase data, but also synchronization pulses and time mode function with an external synchronization source.

Figure 3a also shows a usage of microcontroller to organize data from all three GPS receivers. The communication is based on UART lines and the controller needs to use software programmed UART's. The Controller sends received data from all GPS receivers to PC for a processing. Hardware realization is shown in Figure 3b.

The design disadvantage is a delay caused by storing and delayed sending data by microcontroller. The delay decreases the system dynamics. The hardware exists as an experimental device with possible testing of all mentioned methods.

A. Experimental Measurement

For experimental measurement *CPM* has been used to evaluate changes in *yaw* ψ and *pitch* θ angles. The baseline between two used antennae was set to $|\vec{b}| = 2\,m$ as shown in Figure 4.

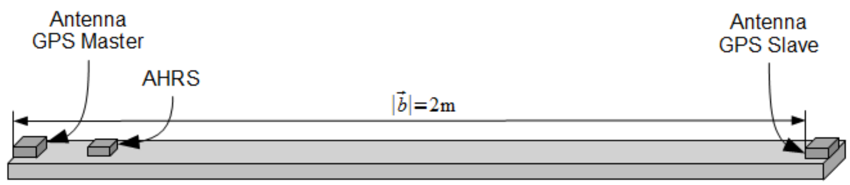
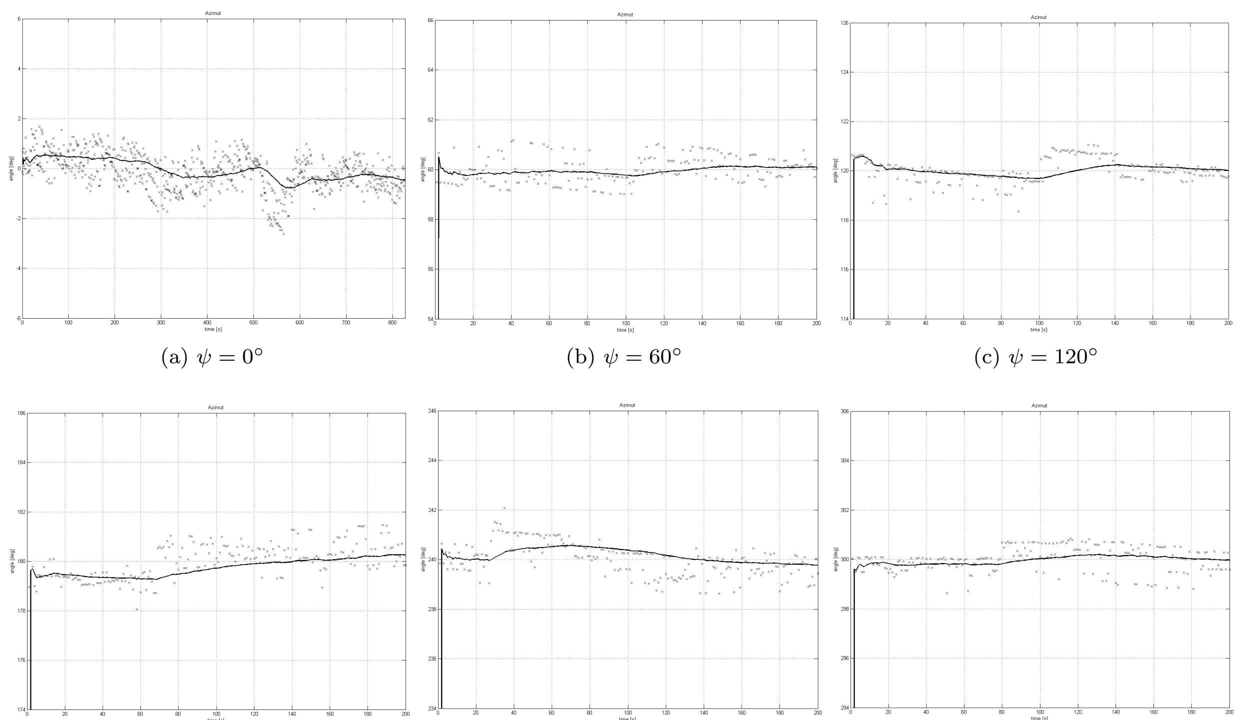


Figure 4: Measurement setting

Results are given in Table 1 for different platform position with *yaw* angle $\psi \in (0°, 360°)$ and *pitch* angle $\theta = (0°, 10°, 20°, 30°, 40°, 50°)$, the interesting point here is gained precision $\sigma_\psi < 1°$ for *Yaw* angle and σ_θ for *pitch* angle. The settings comparison is also shown in Figures 5 and 6 where solid line represents applied adaptive Kalman filter.[6]

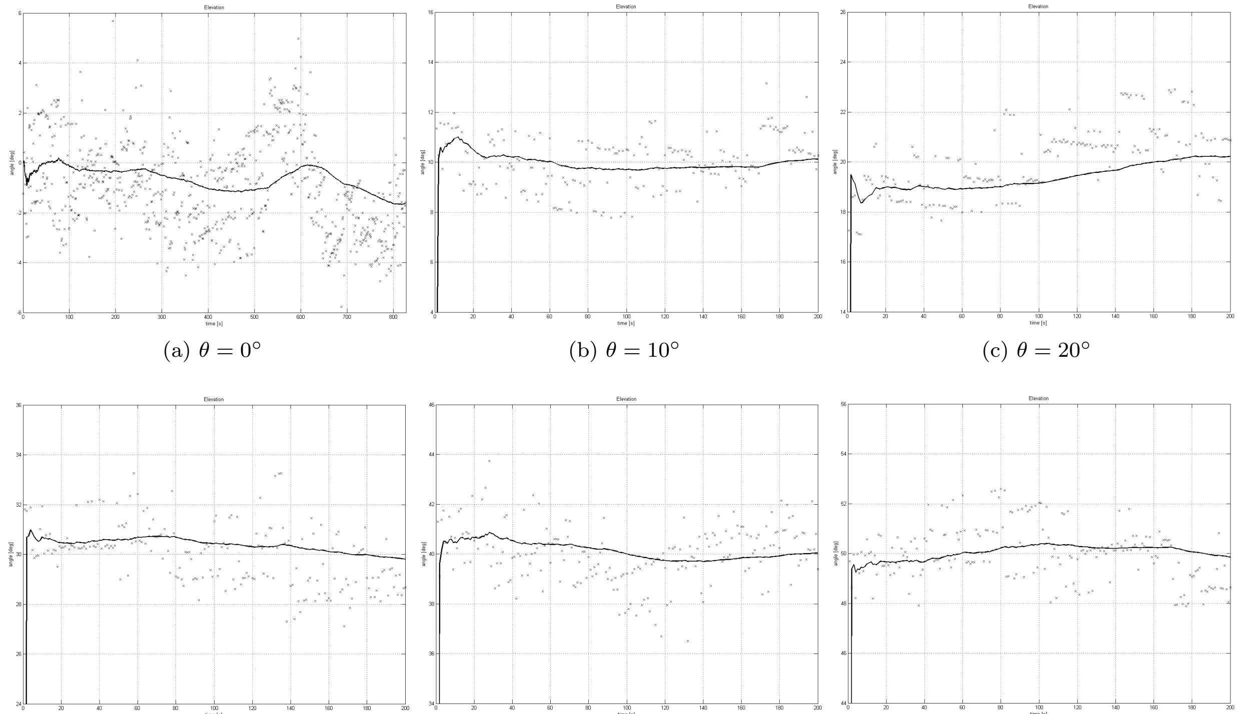


(a) $\psi = 0°$ (b) $\psi = 60°$ (c) $\psi = 120°$

(d) $\psi = 180°$ (e) $\psi = 240°$ (f) $\psi = 300°$

Figure 5: Yaw angle $\psi \in (0°, 360°)$

The *AHRS* unit *MicroStrain Inertia-Link 3DM-GX2* was used as reference and source of attitude, acceleration and angular rate information as shown in Figure 4 during the measurement.



(a) $\theta = 0°$ (b) $\theta = 10°$ (c) $\theta = 20°$

(d) $\theta = 30°$ (e) $\theta = 40°$ (f) $\theta = 50°$

Figure 6: Pitch angle $\theta \in (0°, 50°)$

Table 1: Yaw ψ and pitch θ measurement

(a) Yaw ψ

Set value ψ	**Mean value** $\bar{\psi}$ [°]	**Variance** σ_ψ [°]
0°	-0.16	0.73
60°	60.54	0.54
120°	119.94	0.51
180°	179.62	0.70
240°	239.98	0.66
300°	299.87	0.50

(b) Pitch θ

Set value θ	**Mean value** $\bar{\theta}$ [°]	**Variance** σ_θ [°]
0°	-0.96	1.84
10°	10.42	1.05
20°	21.01	1.35
30°	28.96	1.23
40°	38.77	1.15
50°	49.54	0.59

In this paper the ambiguity resolution N, ΔN respectively, is solved using known information about initial attitude, estimated position and known *SVs* position or their elevation and azimuth respectively.

IV. IMU/GPS aiding

Low attitude information refresh rate is the most important disadvantage of stand-alone *GPS* based attitude estimation. Thus usage *GPS* attitude estimation together with *IMU* angular rate and acceleration comes up. The *GPS* device will restrict low-cost *IMU* time drift and the *IMU* unit itself will be able to provide attitude information with higher dynamics than a standalone *GPS*.

Data fusion of *GPS* device and *IMU* unit could be made by adaptive filtering, for example Kalman filtering,[7] with appropriate balance of input signals. One of the possible solutions is shown in Figure 7, where *IMU* angular rate and acceleration could be also made inside *Kalman filter* function block.

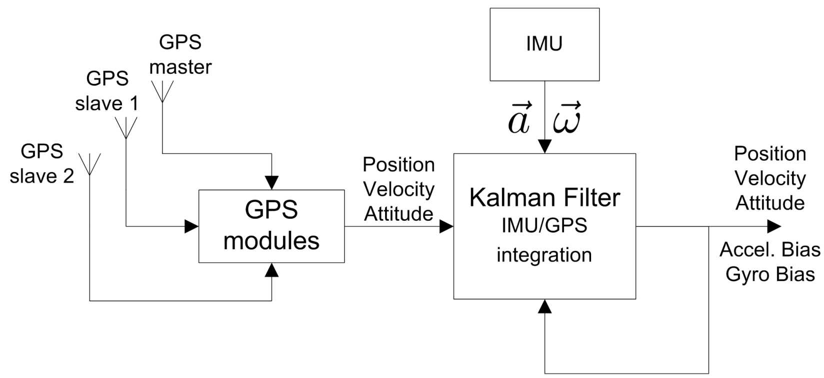


Figure 7: GPS IMU aiding

Above mentioned *GPS* based attitude determination and *IMU* unit will be future field of study and relatively precise *AHRS* could be made combining *GPS* device mentioned above and *IMU* unit with low costs.

V. Conclusion

Several possibilities of GPS usage for the attitude evaluation were discussed. One of them is the most precise, the Carrier Phase Measurement, but special hardware is to be used. Experimental measurement shows possibilities of precise measurement with maximal error from Table 1 $\sigma_\psi < 1°$ and $\sigma_\theta = \sigma_\phi < 2°$.

Achieved results are not as accurate as expected with usage of standalone experimental *GPS* device shown in Figure 3b. The errors are mainly caused by short base line between antennae, $|\vec{b}| = 2\,m$, receivers synchronization and receivers noises.

However, better results can be achieved with larger antennae base line a synchronization improvements. Future work with experimental hardware will lead to achieve more precise results in the attitude evaluation and lead to integration experimental device with existing inertial navigation systems as shown in chapter IV, in Figure 7 respectively.

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