IMPLEMENTATION OF A QUADROTOR UAV

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***Abstract*—This paper describes the design and integration of a control system that allows the autonomous flight of a commercial quadrotor. A mathematical model for the quadrotor is developed, its parameters determined from the characterization of the unit. An external intelligence is integrated to play the rol of the flight controller. A 9 degrees of freedom Inertial Measurement Unit (IMU) equipped with a barometer is calibrated and added to the platform. Data from the IMU is combined with the information provided by a GPS within a Extended Kalman Filter (EKF) to obtain a reliable estimation of the state variables. The control actions are obtained from a proportional-integral controller based on the LQR algorithm. For windless environments, a stable platform is achieved.**

I. INTRODUCTION

A quadrotor platform can be imagined performing countless tasks. Many applications are being developed based on such platforms, but how do they fly? A aerial vehicle is inherently unstable, staying still is not a simple task. This paper focuses on the stabilization of a quadrotor, explaining how to work with an IMU [1] [2] [3], the development of the mathematical model [4] [5] that represents the system, the filtering techniques applied to sensor data [6] [7], and the control system [4] [8] that allows the quadrotor to fly.

The platform is based on a commercial radio controlled quadrotor, the frame and the motor control system were preserved, the IMU and intelligence were replaced by the flight controller that was developed. A BeagleBoard[1] running Linux[2] performs the computations requiered to convert raw data received from the IMU[3] over a UART and combine it using an EKF, overcoming the problems inherent to each sensor and the noise generated by the vibrations of the motors, and providing a reliable estimation of the state vector. Once the current state is known, the LQR algorithm [9] [10] is used to derive the control actions required to bring the system to the desired setpoint.

II. MODEL OF A QUADROTOR

A diagram of the quadrotor is shown in figure (1). Two of the motors rotate clockwise (2 and 4) and the other two (1 and 3) rotate clockwise. This configuration allows the quadrotor to rotate, tilt and gain/lose altitud. Two frames of reference are constantly used throught this work: An intertial frame S_I

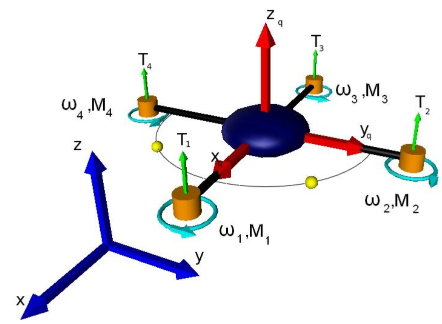
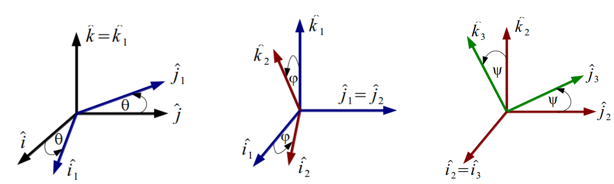


Fig. 1: **Model of the quadrotor -** In blue arrows represent the intertial reference frame $S_I - \{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ $(\{\vec{x}, \vec{y}, \vec{z}\})$, mapped to North, West and Up respectively. The red arrows represent the non-inertial reference frame $S_q - \{\hat{\imath}_q, \hat{\jmath}_q, \hat{k}_q\}$ $(\{\vec{x}_q, \vec{y}_q, \vec{z}_q\})$ relative to the quadrotor. Cyan looped arrows indicate the direction of rotation of each motor (right hand rule). The semicircle and the two yellow spheres indicate the front of the unit.

(relative to the Earth) $\{\hat{\imath}, \hat{\jmath}, \hat{k}\} - \{\text{North}, \text{West}, \text{Up}\}$) and a non-intertial frame S_q relative to the quadrotor. Mapping one frame to the other can be achieved by applying the three rotations show in figure (2), $\{\theta, \varphi, \psi\}$ are know as Euler angles.



(a) Rotation 1: Axis $\hat{k}$ (b) Rotation 2: Axis $\hat{\jmath}$ (c) Rotation 3: Axis $\hat{\imath}$

Fig. 2: Rotations applied on S_I to obtain S_q.

From a detailed analysis of the dynamics and kinemtics of the quadrotor, the state vector $\hat{X}$ given by ((1)) is built to describe the system at any given time. Practical considerations lead to choosing the state variables referenced to the quadrotor frame S_q. This choice introduces simplifications introduced in the theoretical development, and simplifies interpretation of the data provided by the IMU, which is mounted on

[1]BeagleBoard development board - http://beagleboard.org/
[2]Angstrom distribution: http://www.angstrom-distribution.org/
[3]*Mongoose* IMU - http://store.ckdevices.com/

the quadrotor and hence provides provides accelerations and angular velocities relative to S_q:

$$\hat{X} = \{x, y, z, \theta, \varphi, \psi, v_{q_z}, v_{q_y}, v_{q_z}, \omega_{q_x}, \omega_{q_y}, \omega_{q_z}\} \quad (1)$$

where $\{x, y, z\}$ are the cartesian coordinates relative to the inertial frame S_I (mapped on $\{\hat{i}, \hat{j}, \hat{k}\}$), $\{\theta, \varphi, \psi\}$ are the Euler angles show in ((2)), $\{v_{q_x}, v_{q_y}, v_{q_z}\}$ are the linear velocities relative to S_I and $\{w_{q_x}, w_{q_y}, w_{q_z}\}$ are the angular velocities relative to S_q (right hand rule applied on $\{\hat{i}_q, \hat{j}_q, \hat{k}_q\}$).

The equations that govern the dynamics of a quadrotor are presented in equation (2). The reference frames used are shown in figure (1).

The model for the quadrotor described by equations (2) is highly non-linear

$$\boxed{\begin{aligned}
\dot{x} &= v_{q_x} \cos\varphi \cos\theta + v_{q_y}(\cos\theta \sin\varphi \sin\psi - \cos\varphi \sin\theta) \\
&\quad + v_{q_z}(\sin\psi \sin\theta + \cos\psi \cos\theta \sin\varphi) \\
\dot{y} &= v_{q_x} \cos\varphi \sin\theta + v_{q_y}(\cos\psi \cos\theta + \sin\theta \sin\varphi \sin\psi) \\
&\quad + v_{q_z}(\cos\psi \sin\theta \sin\varphi - \cos\theta \sin\psi) \\
\dot{z} &= -v_{q_x} \sin\varphi + v_{q_y} \cos\varphi \sin\psi + v_{q_z} \cos\varphi \cos\psi \\
\dot{\psi} &= \omega_{q_x} + \omega_{q_z} \tan\varphi \cos\psi + \omega_{q_y} \tan\varphi \sin\psi \\
\dot{\varphi} &= \omega_{q_y} \cos\psi - \omega_{q_z} \sin\psi \\
\dot{\theta} &= \omega_{q_z} \frac{\cos\psi}{\cos\varphi} + \omega_{q_y} \frac{\sin\psi}{\cos\varphi} \\
\dot{v_{q_x}} &= v_{q_y}\omega_{q_z} - v_{q_z}\omega_{q_y} + g \sin\varphi \\
\dot{v_{q_y}} &= v_{q_z}\omega_{q_x} - v_{q_x}\omega_{q_z} - g \cos\varphi \sin\psi \\
\dot{v_{q_z}} &= v_{q_x}\omega_{q_y} - v_{q_y}\omega_{q_x} - g \cos\varphi \cos\psi + \frac{1}{M} \sum_{i=1}^{4} T_i \\
\dot{\omega_{q_x}} &= \frac{1}{I_{xx}} \omega_{q_y}\omega_{q_z}(I_{yy} - I_{zz}) \\
&\quad + \frac{1}{I_{xx}} \omega_{q_y} I_{zzm}(\omega_1 - \omega_2 + \omega_3 - \omega_4) \\
&\quad - \frac{1}{I_{xx}} dMg \cos\varphi \sin\psi + \frac{1}{I_{xx}} L(T_2 - T_4) \\
\dot{\omega_{q_y}} &= \frac{1}{I_{yy}} \omega_{q_x}\omega_{q_z}(-I_{xx} + I_{zz}) \\
&\quad + \frac{1}{I_{yy}} \omega_{q_x} I_{zzm}(\omega_1 - \omega_2 + \omega_3 - \omega_4) \\
&\quad - \frac{1}{I_{yy}} dMg \sin\varphi + \frac{1}{I_{yy}} L(T_3 - T_1) \\
\dot{\omega_{q_z}} &= \frac{1}{I_{zz}} (-Q_1 + Q_2 - Q_3 + Q_4)
\end{aligned}} \quad (2)$$

para hacerlo rotar para un lado hay q hacer esto, para el otro lo otro

integradores -¿ permiten tomar en consideracion cosas q le cuelges, desequilibrios

III. Kalman Filter

In order to perform adequate control actions, a reliable estimation of the state variables must be performed in real time. A Kalman Filter achieves this using the mathematical model for the system to predict what should happen given the current state, and corrects the prediction with the information read from the sensors, taking into consideration how much confidence is placed on the prediction and how much on the measurements. This weighted prediction-correction technique allows a smooth state estimation, without the typical delay introduced by filtering. Even small delays can affect the performance of the system.

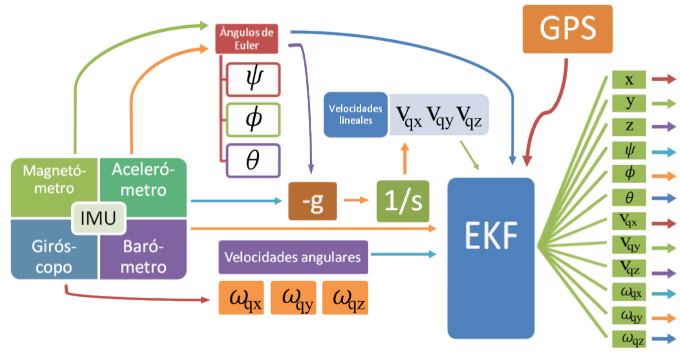


Fig. 3: **EKF -** Outline of how sensor data is combined to estimate the state variables.

Every sensor has its issues: The gyro drifts over time; the accelerometer is very sensible to the vibrations generated by the motors; the magnetometer is distorted by ferromagnetic materials; the GPS has very little precision and a slow update rate. Each sensor by itself is very limited, but they can be combined to compensate for their limitations. The filter takes care of this.

The theory behind a standard Kalman Filter does not hold for a system that is not linear. The model for the quadrotor is highly non-linear, so an EKF is implemented based on [6] and [7]. An EKF is not optimal, and the performance is highly dependant on the linearizacion [11].

Two different types of data will be available, depending on the availability of GPS information. When no GPS data is available there will be no direct feedback from the sensors to correct the position on the horizontal plane, only through integration, so only the prediction step can be performed for these variables. As shown in figure (4), the estimation will drift until GPS data is available.

The EKF proved to play a critical part. For example, the data provided by the accelerometer is unusable without adequate filtering, and a simple low pass filter (LPF) was implemented to reduce noise. The 60ms delay introduced by the LPF severly deteriorated the performance of the system. When the EKF was assigned the task of reducing noise, the perfomance was significatively improved.

IV. Control Design

V. Software

The software runs on several independant boards: An ARM-Cortex-A8 on the flight controller, an ATMega328p on the IMU, and four microprocessors[4] on the ESCs, one for each

[4]C8051F330/1 - Silicon Labs.

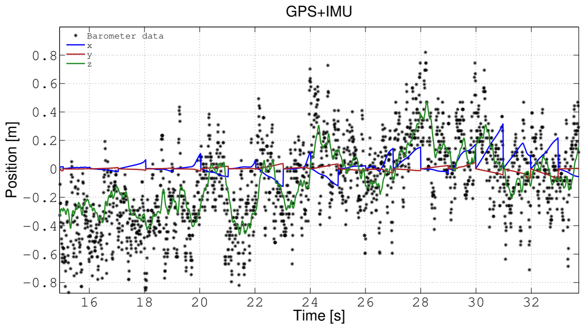


Fig. 4

motor. The code running on the ESCs is not available, but the communication protocol was decoded using a logic analyzer. The firmware on the IMU takes care of reading from all the sensors (using I^2C) and sending a new frame of data every 10ms over a UART. The code running on the flight controller processes the data from the IMU, calculates the necessary control actions, and sets the desired speed for each motor by sending the appropriate commands, via I^2C, to the ESCs.

The code was developed in modules, replacing any part of it should be a straightforward task. The most critical aspect of the flight controller is timing, any delays will severely degrade the performance of the system.

VI. Flight Tests

A. *Subsection Heading Here*

Subsection text here.

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VII. Conclusion

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Acknowledgment

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