MEMSENSE

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Technical Document
Calculating Heading, Elevation and Bank Angle

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MEMSENSE TECHNICAL DOCUMENT

CALCULATING HEADING, ELEVATION and BANK ANGLE

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Introduction

This article discusses how to calculate heading, elevation and bank angles from inertial data collected via a MEMSense inertial measurement unit (IMU), with an emphasis on MATLAB and Excel implementations. Additional material may be found in the References section of this document.

Definitions

The following terms are used throughout this document and are defined here to aid in clarity.

- **Accelerometer:** A sensor which measures linear acceleration. Specified in G-forces (G's).
- **Gyro/Gyroscope:** A sensor which measure rotational velocity in degrees per second (deg/s).
- **Inertial Measurement Data:** Any data collected from the **inertial measurement unit**.
- **Inertial Measurement Unit (IMU):** A device used to collect **inertial measurement data**, which consists of rotational velocity, linear acceleration, and magnetic field data.
- **Magnetometer:** A sensor which measures changes in a magnetic field in Gauss.

Frame Conventions

The frame conventions utilized within this document will be as shown in Figure 1:

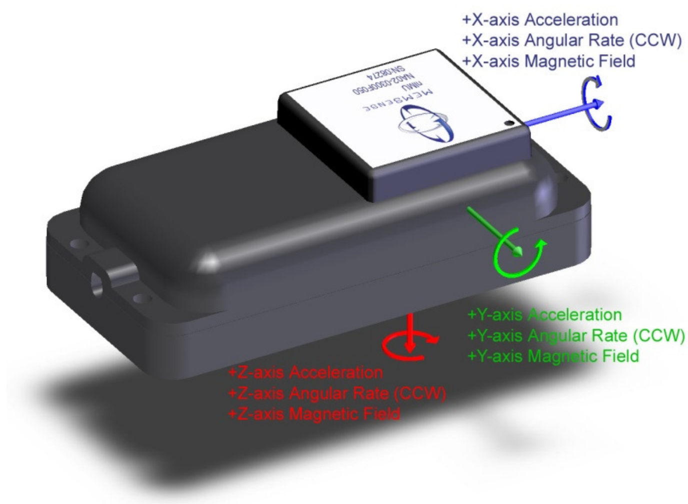


Figure 1: IMU axis conventions.

Referring to Figure 1, positive bank angle will be in the direction of the arrow rotating around the x-axis (blue), positive elevation will be in the direction of the green arrow rotating around the y-axis, and positive heading will be in the direction of the red arrow rotating around the z-axis. More specifically, the 'Right-Hand Rule' is utilized which specifies that positive rotation is in the direction in which the fingers of your right hand curl when the thumb is oriented along the positive axis of rotation (away from the origin). Thus, positive rotation is counter-clockwise when looking along the axis of rotation towards the origin.

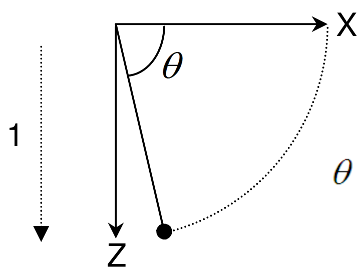
It will always be assumed that orientation is relative to a reference coordinate frame where the X-axis points North, Y-axis points east, and Z-axis points down - often referred to as 'NED'. Thus, heading and attitude are always determined relative to this standard, or reference, position.

To differentiate between the local coordinate frame and the reference coordinate frame, the reference frame axes will be in upper- case and/or have the subscript *r,* as in X_r.

Theory

Elevation and Bank Angle

Elevation is defined as change in the x-axis relative to the horizon/ground, and bank angle is defined as change in the y-axis relative to the horizon/ground. Gravity exerts a constant acceleration of 1g, which may be utilized to calculate attitude and bank angle. As shown in Figure 2, use of the arc-tangent (inverse-tangent) function enables calculation of elevation following a negative rotation in the X-Z plane. Note that the Z axis is oriented 'down', as specified by the NED convention. However, we have defined positive elevation as when the local frame pitches 'up' (a positive rotation of the body frame); thus, we will negate the value in the MATLAB and Excel scripts to achieve this result (covered in greater detail under the Application subsection). This same method is applied to calculate bank angle, the only difference being the rotation takes place in the Y-Z plane. Please see [1] for further discussion.



$$\theta = \arctan\left(\frac{z}{x}\right)$$

Figure 2: Calculation of theta (elevation). Note that this is a negative rotation which will result in a negative elevation angle.

Heading

Heading is calculated using the same method as that used in elevation and bank angle, but because gravity cannot be used to calculate changes in heading, magnetometer data must instead be used. Also, because magnetometer sensitivity decreases as elevation and bank angles increase [2], if we do not realign the local z-axis with the reference frame Z-axis our results will be incorrect. Therefore, we must first apply a rotation that removes the bank angle followed by a second rotation that removes elevation (the reverse sequence is also acceptable). Once this sequence of rotations is completed the local x-y plane will be realigned with the reference X-Y plane, corrections to the magnetometer data will have been made, and we may then proceed with determining heading.

Rotation Matrices

Rotation such that the local x-y plane and reference X-Y plane are coplanar – which is required for accurate determination of heading - can be achieved either by applying two distinct rotation matrices in sequence, or through the product of the two individual rotation matrices to produce a single rotation matrix. Figure 3 displays the rotation geometrically where elevation and bank angle are removed, thereby enabling the determination of heading.

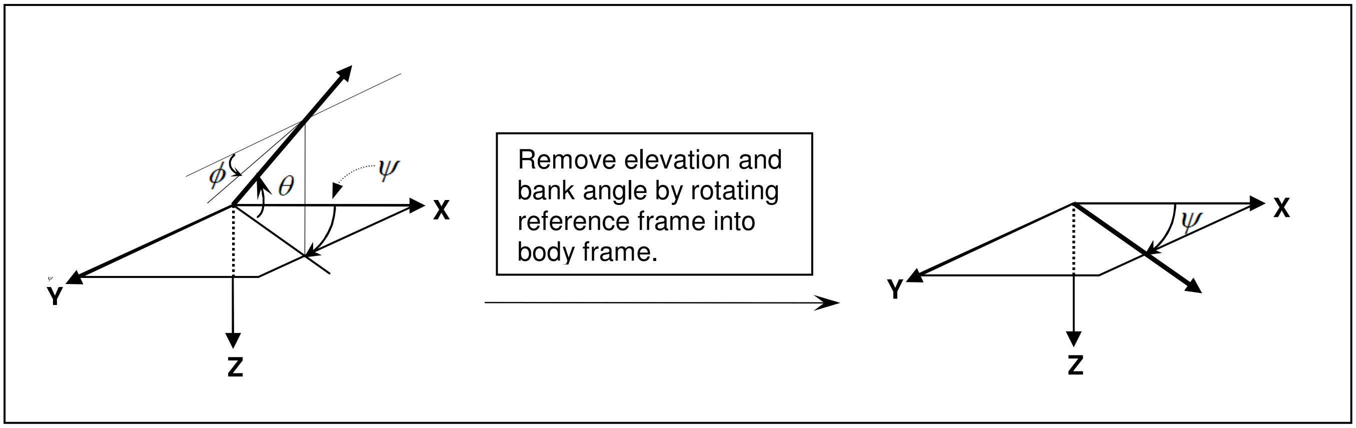


Figure 3: A rotation matrix is applied to remove bank angle and elevation, enabling accurate calculation of heading.

This rotation matrix is then applied to a vector, *v*, as follows

$$v_1 = R\,v$$

where v_1 is the resulting vector in the rotated frame [3]. For this discussion we will combine the two rotations - elevation angle and bank angle - into a single matrix. Additionally, frame rotations will be assumed (points-fixed) where the reference frame is rotated into the body frame.

Rotation about the x-axis (bank angle) can be expressed as follows:

$$R_\phi^x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

where the notation R_ϕ^x means a rotation through an angle ϕ about the x-axis[3]. Rotation about the y-axis (elevation angle) can be expressed as follows:

$$R_\theta^y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

where the notation R_θ^y means a rotation through an angle θ about the y-axis[3]. Each of these rotations can be applied individually; however, a composite rotation can be achieved by taking the product of these two rotations as follows:

$$R = R_\phi^x R_\theta^y$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\phi\sin\theta & \cos\phi & \sin\phi\cos\theta \\ \cos\phi\sin\theta & -\sin\phi & \cos\phi\cos\theta \end{bmatrix}$$

Note that the order of multiplication is important as matrix multiplication is *not* commutative.

For additional discussion on rotation matrices please see [3].

Application

Before applying the rotation sequences, we must first verify that the *sense* of rotations correspond between the reference frame and the data from the sensors, themselves; if the rotations do not correspond then significant errors will be present.

Working in two dimensions we will examine output from the X-Z accelerometers to determine elevation angle. We see the accelerometer data in Figure 4 resulting from performing a 'positive' rotation of the body frame, followed by a 'negative' rotation.

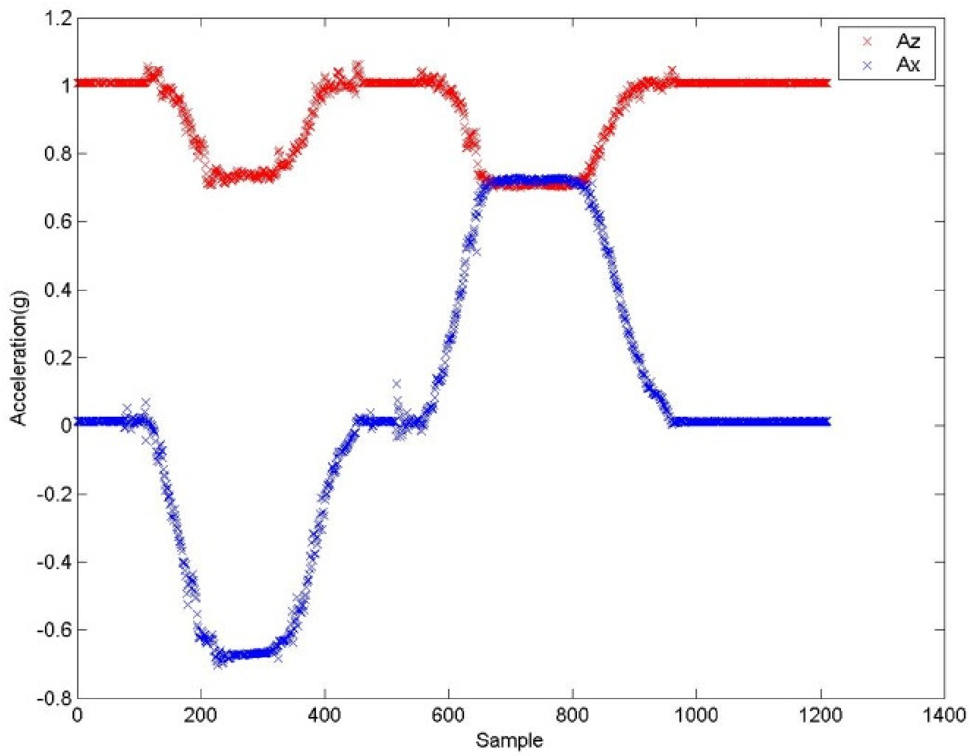


Figure 4: Accelerometer data from a 'positive' rotation, followed by a 'negative' rotation.

We see in Figure 5 that calculating elevation angle from these data results a negative elevation angle, followed by a positive angle, which is the opposite of what is expected.

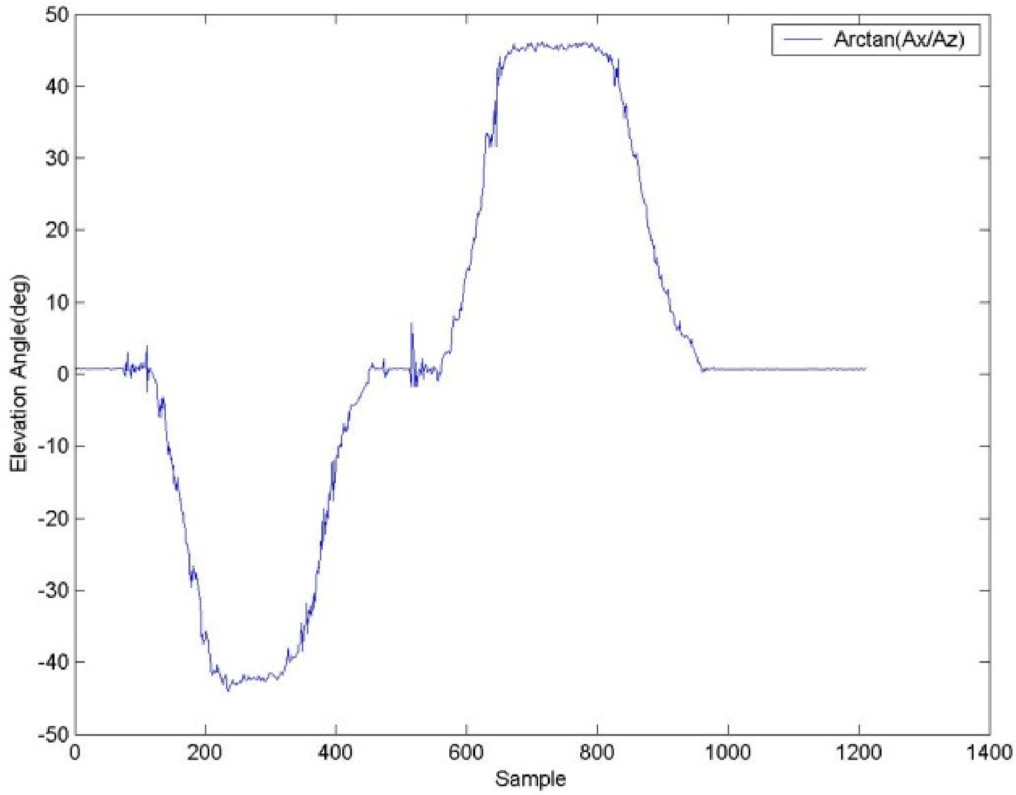


Figure 5: Calculated elevation angle of a 'positive' rotation
followed by a 'negative' rotation, relative to the reference frame.

Therefore, the *sense* of the rotation calculated from the raw accelerometer data differs from that specified for the reference frame rotation, which requires that we negate the elevation angle to maintain consistency with the right-hand rule. However, we must remember that when calculating heading by applying a rotation to remove elevation angle, the elevation angle must again be negated since it is actually a *negative* reference frame rotation due to the inconsistency in *sense*. Figure 6 shows a geometric representation of the rotation relative to both the accelerometer data and the reference frame.

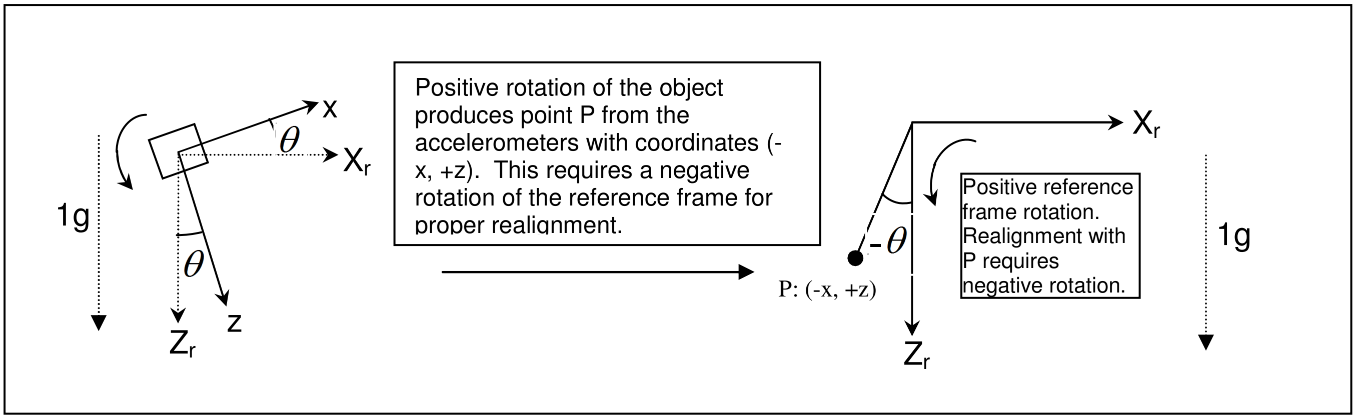


Figure 6: Geometric perspective of removal of elevation angle. A positive body rotation will require a negative reference frame rotation to realign the reference frame with the body frame.

A similar analysis of accelerometer output reveals that a negative reference frame rotation is also required to remove bank angle (assuming a positive body rotation) prior to calculating heading, as well as negation of the calculated heading, again to maintain the correct *sense* relative to the right-hand rule.

Script Usage and Instructions

MATLAB

Three files are used in the post-processing of collected data:

- calc_beh_main.m: Main routine. Plots accelerometer data, bank, elevation and corrected heading.
- calc_bankElevation.m: Calculates bank angle and elevation.
- calc_heading.m: Calculates heading, plots magnetic sensor data and uncorrected heading.

Data is read in from a file specified via the command line, and data is expected to be in the order specified in Table 1.

Column	Sensor/Data
1	Counter data - currently ignored.
2	Gyro X (deg/s)
3	Gy
4	Gz
5	Accel X (G's)
6	Ay
7	Az
8	Mag X (Gauss)
9	My
10	Mz

Table 1: Expected order of sensor data. Note that the counter data may be omitted, but this must also be indicated in the call to *calc_beh_main()*.

Two smoothers are available for use within the script: one to work on the sensor data (pre-smoother) prior to calculating B/E/H, and a second that will smooth out the results (post-smoother). The smoothers may be individually enabled from the command line (disabled by default), but the window size must be modified from within the script, itself, via the constants ***PRE_SMOOTH_WINDOW*** and ***POST_SMOOTH_WINDOW*** found in ***calc_beh_main.m***. The larger the values the greater the number of points included in the smoothing window – and the greater amount of smoothing performed. MATLab defaults to a 5 element window, but for very noisy systems it may be necessary to increase this value.

The function signature is as follows:

beh = calc_beh_main(filename, headingGT180, hasCounterCol, preSmooth, postSmooth)

Calculates bank angle, elevation and heading from accelerometer and magnetometer data. Plots accelerometer and magnetometer data, uncorrected heading, and bank and elevation corrected heading. Returns a three element structure containing the elements *bank*, *elevation* and *heading*, specified in degrees.

Parameters:

filename	Path to data file to process. Data is loaded by function
headingGT180	true if heading should be plotted in 0-360 range, false if 0 to +/-180.
hasCounterCol	true if a counter column is at index 1, else false. Default: false.
preSmooth	true if a smoother should be applied to the sensor data. Default: false. Default window size: 5 points.
postSmooth	true if a smoother should be applied to the calculated bank, elevation and heading data. Default: false. Default window size: 5 points.

Returns:

A structure is returned containing the following elements:

bank	Array of bank angles, in degrees.
elevation	Array of elevation angles, in degrees.
heading	Array of heading angles, in degrees.

Each element is an array with a size equal to the number of rows in the original data file.

Output:

Three plots will be generated:

1. Magnetic components in Gauss and uncorrected heading in degrees.
2. Accelerometer values in g's.
3. Bank angle, elevation and corrected heading in degrees.

Usage:

From the MATLAB command line, change to the directory in which the scripts reside:

beh = calc_beh_main('myDataFile', true, false, false, false)

Using data from *myDataFile*, calculates bank, elevation and heading data and plots accordingly, and captures results in the variable *beh*. Plots heading data in the range 0-360, does not contain counter data in column 1, does not apply a smoother to sensor data or to the calculated results. Due to default parameters, the following produces identical results:

calc_beh_main('myDataFile', true)

EXCEL

To utilize the macro perform the following steps:

1. Open the spreadsheet and enable macros if prompted to do so.
2. Paste data in the 'Data' sheet/tab, starting in Column C; the first two columns are expected to contain counter-related data, but currently are completely ignored by the macro. Do not delete the tabs *Results1*and *Results2.* NOTE: If using pre-Microsoft 2007 the data sets are limited to ~65000 rows, but are unlimited if using Microsoft Excel 2007.
3. To run the macro, go to
   - Pre-Excel 2007: Tools->Macro->Macros…
   - Excel 2007: View->Macros->View Macros

Hilight the macro ***calc_BEH_Main***, and select Run.

If charts exist from a previous run of the macro you will be prompted as to whether they should be deleted; indicate that they should be deleted by selecting the 'Delete' option. The macro completely recreates all charts and populates the *Results1* and *Results2* tabs with each new run, so no housekeeping is necessary - simply run the macro and all else is handled automatically. The tab *Results1*contains the calculated bank, elevation and heading data such that heading is in the range 0-360, whereas *Results2* contains identical bank and elevation, but heading is in the range 0 to +/-180.

The macro source may be viewed by returning to the macro dialog (see step 3, above) and selecting *Edit*. This will bring up a VB editor that allows you to make changes as necessary. Besides plotting the B/E/H data, two additional plots for each angle will be created with the extension EMAS or SMAS, indicating a smoothing function has been applied to B/E/H data; SMAS is a Simple Moving Average Smoother, and EMAS is an Exponential Moving Average Smoother. Smoother window sizes may be adjusted via the constants *HEADING_SPAN, ELEVATION_SPAN,* and *BANK_SPAN* at the top of the Visual Basic source file; the larger the values the greater the number of points included in the smoothing window – and the greater amount of smoothing performed.

References

[1] Tilt-Sensing with Kionix MEMS Accelerometers. Kionix Corp., NY. [Online]. Available: http://www.kionix.com/App-Notes/AN005%20Tilt%20Sensing.pdf

[2] Caruso, M.J. "Applications of Magnetic Sensors for Low Cost Compass Systems," *Position Location and Navigation Symposium, IEEE 2000*, 13-16 March 2000, pp. 177 – 184.

[3] Jack B. Kuipers, *Quaternions and Rotation Sequences, a Primer with Applications to Orbits, Aerospace, and Virtual Reality*, Princeton University Press, Princeton, 2002.

Change History:

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1.0	Released	NEW MEMSENSE TECHNICAL DOCUMENT	04-June-08