Adaptive Kalman Filtering Algorithms for Integrating GPS and Low Cost INS

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***Abstract*—GPS and Inertial Navigation Systems are used for positioning and attitude determination in a wide range of applications. Over the last few years, a number of low cost inertial sensors have become available. Although they exhibit large errors, GPS measurements can be used correct the INS and sensor errors to provide high accuracy real-time navigation. The integration of GPS and INS measurements is usually achieved using a Kalman filter.**

The measurement and process noise matrices used in the Kalman filter represent the stochastic properties of the GPS and INS systems respectively. Traditionally they are defined *a priori* and remain constant throughout a processing run. In reality, the stochastic properties of the system vary depending on factors such as vehicle dynamics and environmental conditions. This is particularly an issue for low cost inertial sensors where the initial sensor errors can be large, and experience significant temporal variation.

This paper investigates three adaptive Kalman filtering algorithms that can be used to improve the estimation of the stochastic properties of a low cost INS. The algorithms are tested using a low cost Crossbow MEMS IMU integrated with carrier phase GPS for a marine application. The adaptive Kalman filtering algorithms are shown to reduce the dependence on the *a priori* information used in the filter. This results in a reduction in the time required to initialise the sensor errors and align the INS, and results in an improvement in navigation performance.

I. Introduction

GPS and Inertial Navigation Systems (INS) are widely used for position and attitude determination applications. When combined together, GPS and INS provide many complimentary characteristics that overcome the limitations experienced when using each sensor individually. The primary restriction in the proliferation of such technology into a broader range of applications is the high cost of the inertial sensors. Low cost Inertial Motion Units (IMU) that can be integrated with GPS are now available for approximately $5000 or less. However, they suffer from large sensor errors such as biases and scale factor errors. Another problem experienced with low cost sensors is that the error sources are not stable and have to be constantly calibrated using GPS updates.

GPS and INS sensors are typically combined using a Kalman filter. The Kalman filter requires a dynamic model to describe the way in which the errors develop over time, and a stochastic model to describe the noise characteristics of each sensor. The standard inertial navigation system error model (for example as given in [1] and [2]) is generally considered to be sufficient to model the inertial system [3]. However, the stochastic model is recognised to be limited when *a priori* statistics are used to model errors that have time varying characteristics [3] [4] [5] [6]. Therefore, potential performance improvements can be obtained if the stochastic information can be improved to better represent both the GPS and INS errors.

The modification of the stochastic information used within the conventional Kalman filter (CKF) has generally become known as *adaptive Kalman filtering*. This paper uses the term adaptive Kalman filtering to describe the various modifications that can be made to the CKF algorithm that allow the filter to modify the stochastic information on-line. Consequently, these algorithms can also be used in real-time. The algorithms that are identified and tested in this paper are termed process noise scaling, the Adaptive Kalman Filter (AKF) and Multiple Model Adaptive Estimation (MMAE). Therefore a distinction has to be made between the generic term adaptive Kalman filtering, and the AKF (which is one particular algorithm).

There are many characteristics that are desirable from an adaptive Kalman filter. An 'ideal' adaptive Kalman filter would be able to identify the correct stochastic properties without the need for any *a priori* statistics (this includes the *a priori* statistics that are required to initialise the covariance in the Kalman filter). Similarly, we want the filter to rapidly respond to new observations during operation and model any correlation between observations or states. This would have the effect of making the filter more robust, and would maximise the performance that could be obtained from the filter. In addition to this, we require that the algorithms are efficient and can be used in real-time. This paper analyses the adaptive algorithms in view of these desired properties.

Several papers have examined the use adaptive Kalman filtering in navigation applications. The AKF has been used in [6] for integrating navigation grade INS measurements with GPS. The AKF was shown to provide a 20% RMS improvement in attitude accuracy. Reference [3] used the AKF to estimate the GPS measurement noise, again using a tactical grade INS. The AKF has also been investigated in [7] and [8]. In [9] the covariance scaling algorithm is used to improve the dynamic modelling for pseudorange GPS.

The three algorithms that are described in this paper were first examined for GPS and low cost INS integration in [10]. In [10] the algorithms were investigated using the University of Nottingham's Navigation System Simulator (NSS). NSS is a software simulator that simulates raw GPS and inertial measurements. All of the algorithms were shown to provide a potential decrease in the time required to align and initialise

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the INS when used with simulated data.

The aim of this paper is to extend the results given in [10] by applying the algorithms to a real data set collected on a small marine vessel used for hydrographic surveying. This paper highlights potential modifications to the CKF algorithm, examines the characteristics of these algorithms, and assesses their suitability to the task of integrating GPS with low cost INS.

II. Adaptive Kalman Filtering Algorithms

There are many physical implementations that can be used to blend GPS and INS measurements using a Kalman filter, see for example [11] [12] [13]. The simplest method is the decentralised approach where separate Kalman filters are used to process the GPS and INS observations. The position and velocity from a GPS-only Kalman filter is differenced with the position and velocity from the INS to form the observations for an integration Kalman filter. An alternative approach can be used where the INS is used to predict the GPS range measurements, allowing the GPS and INS to be combined in a single centralised filter. For the purposes of analysing the characteristics of the adaptive Kalman filter, the decentralised approach is used for this paper.

All of the AKF algorithms described here are based on the information provided by the innovation or residual sequences that are computed in the CKF algorithm. For more details of the CKF algorithm see for example [14] or [15]. The innovation, $v_k^{(-)}$, is the difference between the predicted state from the Kalman filter and the new measurement. Similarly, the residual, $v_k^{(+)}$, is the difference between the updated state and the new measurement. Therefore, the innovation and residual sequences contain information about the new measurement, *and* information about the system being modelled by the filter. As a result, the adaptive algorithms based on the innovation and residual sequences are likely to only be able to give an indication of the stochastic properties of the GPS *or* the INS.

In the CKF algorithm applied to GPS and INS integration, the measurement noise matrix, R_k, is used to describe the stochastic properties of the GPS position and velocities. Similarly, the process noise matrix, Q_k, is used to describe the INS. For purpose of this paper, when using the decentralised filtering approach, it is assumed that the GPS stochastic model is accurately represented by the covariance matrix from the GPS filter. Therefore, the GPS covariance matrix is used to form the measurement noise matrix for the integration filter. It is likely that any limitations in the measurement noise model will be introduced as error in the estimation of the process noise model. The analysis of the following algorithms is concerned only with adapting the process noise matrix. This is because the stochastic model for the low cost INS is expected to be deficient due to the temporal variation of the errors in the sensors.

A. Covariance scaling/ Process noise scaling

The first adaptive filter that is considered here is the covariance scaling filter. The covariance scaling filter is a very simple and efficient extension to the CKF algorithm which applies a scale factor, $S_k \geq 1$, to the predicted covariance, $P_k^{(-)}$. When S_k is greater than 1, this has the effect of giving more weight to the new measurement. The modified covariance prediction equation in the Kalman filter is given by,

$$P_k^{(-)} = S_k(\Phi P_{k-1}^{(+)} \Phi^T + Q_{k-1}) \tag{1}$$

where Φ is the state transition matrix. This algorithm was used in [9] where it was used to improve the stochastic modelling for differential pseudorange GPS positioning. Reference [9] also gave an algorithm for computing the scale factor based on the relationship between the innovation at the current epoch k, and the previous N epochs using the equation,

$$S_k \geq \frac{v_k^{(-)T} v_k^{(-)}}{\frac{1}{N}\sum_{j=0}^{N-1} v_{k-N+j}^{(-)}{}^T v_{k-N+j}^{(-)}} \tag{2}$$

The covariance scaling algorithm used with (2) to estimate the scale factor, has been shown in [11] to be unstable for the application of GPS and low cost INS integration. The scale factor was estimated to be very large during the INS alignment phase, resulting in the filter becoming divergent. As a result, the scale factor was applied only to the process noise. A short derivation of the algorithm following that given in [9] for the covariance scaling filter is provided in [11]. The modified covariance prediction equation for the process noise scaling filter is written as,

$$P_k^{(-)} = \Phi P_{k-1}^{(+)} \Phi^T + S_k Q_{k-1} \tag{3}$$

B. Multiple Model Adaptive Estimation

Multiple Model Adaptive Estimation (MMAE) was first presented in 1965 in [16]. The MMAE algorithm uses multiple Kalman filters that run simultaneously with, for this purpose, each Kalman filter using a different stochastic model. Different algorithms can be used to identify from the bank of filters which filter is the correct one to use. Alternatively, the estimates can be combined together as below.

The standard MMAE filter, as described in [14], uses the probability density function on the residuals to identify which combination of filters to use. For each filter, the probability density function, $f_n(z_k)$, for the n^{th} Kalman filter is given by,

$$f_n(z_k) = \frac{1}{\sqrt{(2\pi)^m |C_{v_k^{(-)}}|}} e^{-\frac{1}{2} v_k^{(-)} C_{v_k^{(-)}}^{-1} v_k^{(-)T}} \tag{4}$$

where m is the number of measurements. The probability, $p_n(k)$, that the n^{th} model is correct is computed using the recursive formula,

$$p_n(k) = \frac{f_n(z_k).p_n(k-1)}{\sum_{j=1}^{N} f_j(z_k).p_j(k-1)} \tag{5}$$

for N Kalman filters. The optimal state estimate is reconstructed from the individual Kalman filter estimates using,

$$\hat{x}_k^{(+)} = \sum_{j=1}^{N} p_j(k) \hat{x}_{k_j}^{(+)} \tag{6}$$

By using this algorithm, a single correct model is likely to be identified with the probability for this filter converging to unity. This will result in the remaining filters effectively being ignored. For the GPS and INS integration application, it is desired that the filter should continue to respond to the information provided by new measurements. In other words, the filter needs to remain adaptive. This can be achieved by defining a minimum threshold value for the conditional probabilities.

The most significant limitation with the MMAE filter is the significant increase in processing time required. Table I shows the total processing time required per 1 second epoch of data, divided into GPS processing, INS mechanisation (conversion of the raw measurements to estimates of position and attitude) and the INS Kalman filter processing times. The processing times were obtained using a 1.4GHz processor with the KinPos^i integration software described in the following section. It is shown from the table that the INS Kalman filter forms a small part of the overall processing time required for processing the GPS and INS. For the MMAE filter, an extra millisecond of processing will be required for each additional filter. An overall latency of, for example, 20ms for a bank of 7 Kalman filters may be obtained for real-time applications using current processing technology. It should also be noted that the filter also allows parallel processing to be implemented since each filter can be run concurrently.

TABLE I

KinPos^i Processing Time Per 1 Second Epoch

	Time (ms)
GPS	5
INS mechanisation (100Hz)	7
INS Kalman filter	1
Total	13

C. Adaptive Kalman filter

The adaptive Kalman filter is an extension to the CKF that uses extra equations to estimate either the measurement noise or process noise matrices. The principle of the Adaptive Kalman Filter (AKF) is to make the Kalman filter innovations or residuals consistent with their theoretical covariances [17].

An adaptive estimate of the process noise matrix is derived in [6] using the maximum likelihood criterion to give,

$$\hat{Q}_k = \hat{C}_{\Delta x_k} + P_k^{(+)} - \Phi P_{k-1}^{(+)} \Phi^T \tag{7}$$

where Δx_k is the state correction sequence given by $\Delta x_k = x_k^{(-)} - x_k^{(+)}$. An estimate of the state correction covariance is obtained by averaging the last N corrections to give,

$$\hat{C}_{\Delta x_k} \approx \frac{1}{N} \sum_{j=k-N+1}^{k} \Delta x_j \Delta x_j{}^T \tag{8}$$

Using the substitution, $\Delta x_k = K_k v_k^{(-)}$, where K_k is the Kalman gain, an approximation to the state correction sequence can be formed in terms of the innovation covariance using,

$$\hat{C}_{\Delta x_k} \approx K_k \hat{C}_{v_k^{(-)}} K_k^T \tag{9}$$

where the innovation covariance is estimated using,

$$C_{v_k^{(-)}} = \frac{1}{N} \sum_{j=k-N+1}^{k} v_j^{(-)} v_j^{(-)T} \tag{10}$$

Equation (9) is known as the innovation based estimate, and (8) is termed the residual based estimate.

A potential limitation of using (7) to estimate the process noise is that the equation is not guaranteed to be positive definite [3]. Consequently, the approximation,

$$\hat{Q}_k \approx \hat{C}_{\Delta x_k} \tag{11}$$

given in [18] is used in this paper. A potential advantage offered by the AKF estimate of the process noise is that a full variance/ covariance matrix is produced. However the filter can become unstable due to the high number of elements that typically need to be estimated. As a result, only the variances from the estimate are used to form the adaptive estimates for this paper.

III. Integration Software

All of the adaptive algorithms described in this paper are implemented in the University of Nottingham's GPS and INS integration software, KinPos^i. KinPos^i provides the ability to process dual frequency differential GPS pseudorange, carrier phase and Doppler observations using one or more reference stations. KinPos^i also provides full processing of raw inertial measurements and integrates the INS with the GPS using decentralised or centralised filtering. Both filters also allow the inertial measurements to be used to check for GPS cycle slips and to aid the integer ambiguity search. The decentralised filtering architecture applied in KinPos^i is given in Fig. 1. The integration filter can be configured to model different sensor errors. For this paper, nine navigation states are modelled with six sensor bias states.

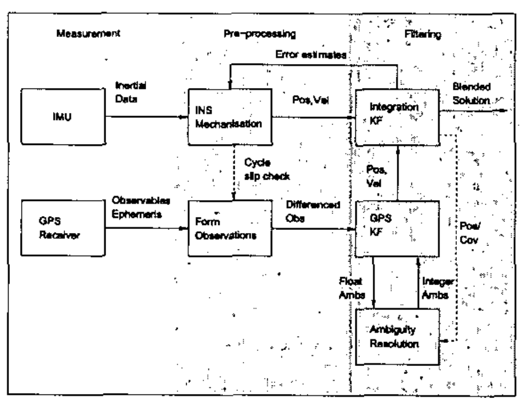


Fig. 1. Decentralised Filter Structure

IV. MARINE TRIAL

A marine trial was conducted on a small surveying vessel at the Royal Hydrographic School in Plymouth, UK in February 2002. The trial was performed in order to assess the performance of the adaptive Kalman filtering algorithms used with a low cost MEMS IMU. The marine application is investigated because the vessel contains a TSS POS/MV system that uses a navigation grade IMU with dual antenna GPS. The POS/MV system is used for georeferencing a multibeam sonar mounted on the front of the vessel. For this trial, the POS/MV was used to provide attitude truth data at the manufacturer's specified accuracy of 0.02°/s.

For the GPS and low cost INS trial, three choke ring antennas connected to Ashtech Z-XII receivers were strapped to a rigid platform on the front of the vessel. This was to allow an independent GPS attitude solution to be computed, however only one roving antenna was used to obtain the following results. A reference GPS receiver was located on the roof of the Royal Hydrographic School, resulting in a maximum baseline separation of approximately 3km. The low cost IMU used was a Crossbow AHRS-DMU-HDX, for which the sensor specification is given in Table II. This was mounted on the cabin floor of the vessel, close to the IMU for the POS/MV. The lever arm separations between the IMU and the GPS were measured using a total station. The integer ambiguities were fixed within two epochs and they remained fixed for the entire dataset. The overall trial configuration is depicted in Fig. 2.

TABLE II

CROSSBOW AHRS SENSOR SPECIFICATION [19]

Sensor	Accel (ms^{-2})	Gyro (°/s)
Bias	$< \pm 0.3$	$< \pm 2.0$
Noise	2.8×10^{-2}	8.5×10^{-2}
Scale Factor	<1.0%	<1.0%
Saturation	$\pm$ 20	$\pm$ 100
Quantisation	1.4×10^{-2}	2.5×10^{-2}

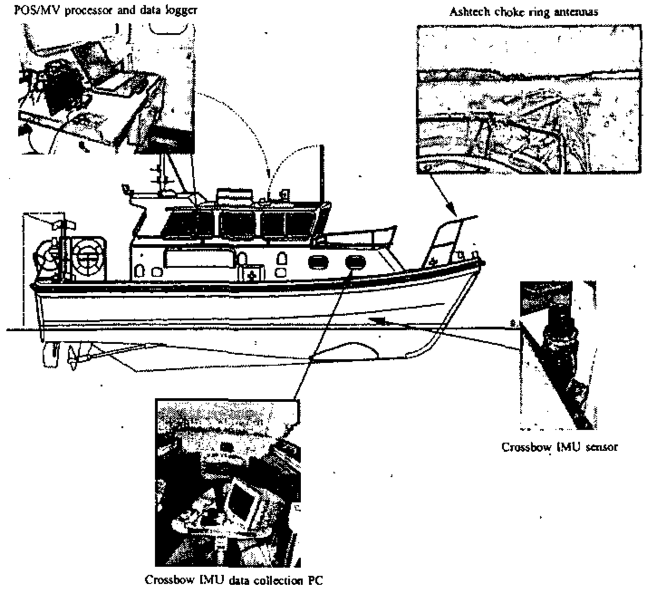


Fig. 2. Trial configuration

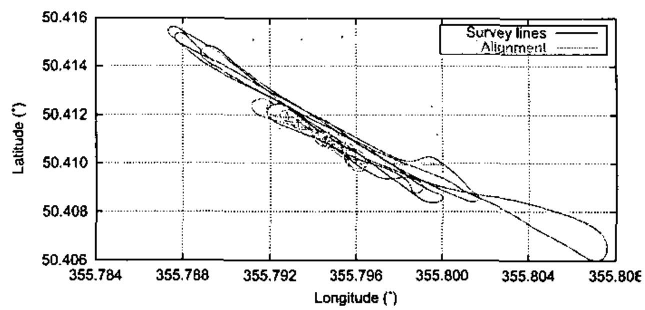


Fig. 3. Marine trial trajectory

The trajectory for the marine trial is shown in Fig. 3. The trajectory shows that an initial alignment manoeuvre was performed where the vessel underwent several figure-of-eight turns. This section of the trajectory lasts for approximately 10 minutes. The alignment manoeuvres are required is order to initialise the INS attitude and estimate the inertial sensor biases. After the alignment section of the trajectory, a number of survey strips are performed, with the strips lasting approximately 5-10 minutes.

V. INNOVATION SEQUENCE

Fig. 4 shows the innovation sequence for the north and east axes for the CKF. This is the difference between the INS predicted position, and the position estimated by the GPS. The figure shows that at the beginning of the dataset the innovation sequence contains differences of over 20cm. This is because the INS is not aligned at the beginning of the dataset. The initial attitude misalignments were 1.6°, 0.6° and 2° for roll, pitch and yaw axes respectively. These values correspond to the initial level error, and a yaw error consistent with that provided by an external aiding source such as a compass.

The innovation sequence is therefore able to provide useful information about the properties of the integrated system. In this case, it is clear that at the beginning of the trajectory, one of the sensors is not working effectively. In this case, it is due to the low cost INS. This information can be used by the adaptive algorithms to adjust the stochastic properties of the filter to give more weight to the measurements during this period. This is because we use the assumption that any error in the innovation sequence is due to the INS. In the following sections, this is shown to improve the time to INS alignment.

During the first 200 seconds, the trajectory of the vessel is relatively straight as it navigates to a safe point at which the figure-of-eight turns can be performed. The alignment manoeuvres are then performed for 10 minutes, after which, the vessel begins travelling in straight lines. The 'spikes' in the innovation sequence are caused primarily during turning

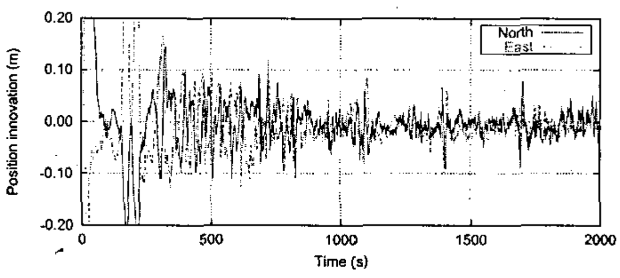
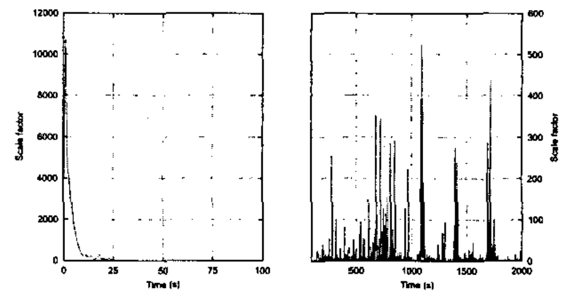


Fig. 4. Position innovation sequence



(a) Scale factor between 1-100 seconds (b) Scale factor between 100-2000 seconds

Fig. 5. Estimated process noise scale factors

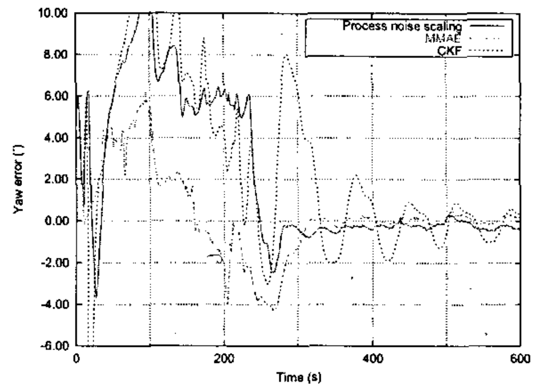


Fig. 6. Comparison of yaw axis alignment

manoeuvres when the yaw error becomes observable. In between the turning manoeuvres, the difference between the GPS and INS position is at the centimetre level. Once again, this information can be used by adaptive Kalman filtering algorithms to adjust the stochastic properties of the filter.

VI. Process Noise Scaling Filter

The process noise scaling filter was used for integrating the GPS and INS dataset. In order to estimate the scale factor, the following formula was used,

$$S_k \geq \frac{v_k^{(-)T} v_k^{(-)}}{E} \tag{12}$$

The value of E is an empirically derived estimate of the expected magnitude of the sum of the squares of the innovations at each epoch. The value is estimated using multiple processing runs to obtain a value expected during normal navigation. The value of E is therefore dependent on the type of GPS solution that is obtained. Since the ambiguities are fixed for the entire dataset, the value is set to a constant. When the scale factor is estimated to be larger than 1, it is used to scale the process noise. The advantage of using (12) is that the adaptive filter can be used from the first epoch (whereas the AKF has to accumulate N epochs using the CKF to form the covariance approximation).

Fig. 5(a) and Fig. 5(b) show the estimated process noise scale factors obtained using (12). The initial scale factors are shown to be large in Fig. 5(a) due to the poor INS alignment. The spikes in the scale factors in Fig. 5 after 800 seconds, correspond directly with the vessel performing the 180° turns at the end of each survey line. This is because the heading error increases during the straight sections of the trajectory, and becomes observable during the turn.

The yaw axis misalignment for the process noise scaling filter is shown in Fig. 6 compared to the CKF and MMAE estimates (the MMAE is examined in the following section). The yaw axis misalignment is shown as it has low observability and is therefore difficult to estimate. The figure shows that the process noise scaling filter resolves the yaw estimate to within 1° after 275 seconds whereas the CKF filter is not aligned to within this value during the entire 10 minute alignment manoeuvre.

All of the adaptive Kalman filters in this paper are initialised with the same initial covariance and process noise estimates as used for the CKF. None of the algorithms are able to completely remove the requirement for *a priori* information. The initial covariance is based on the initial position and velocity information from the GPS, the estimated attitude errors, and the estimated sensor biases from the manufacturer's specification. The process noise matrix was obtained using empirical analysis from multiple processing runs.

The attitude errors obtained from the process noise scaling filter after the alignment section of the trajectory are 0.04°, 0.04° and 0.36° for roll, pitch and yaw respectively. The errors are computed by differencing the low cost INS attitude with the attitude from the POS/MV system. These values are standard deviations, with any bias being attributed to the misalignment between the two IMU's. Therefore, multiple runs would be required to validate this assumption. The CKF attitude errors are 0.03°, 0.04° and 0.52° for roll, pitch and yaw. The process noise scaling filter therefore results in approximately a 30% reduction in the yaw error for the marine trial. Comparison of the roll and pitch estimates shows that the values are very similar for both filters.

VII. MMAE

For the MMAE filter, a bank of six Kalman filters was used, with each filter using a different process noise matrix. The process noise in each filter was set up using the *a priori* process noise matrix from the CKF implementation, scaled by a constant. The scale factors used were 0.1 to 10000, increasing in powers of ten. Six filters were used so that there are a wide range of different process noise estimates from which the filter can select. The range is required so that the filter can adapt to find suitable filters during alignment and during normal navigation.

A minimum threshold was applied to the conditional probabilities of 1×10^{-3}. This value was selected so that it forces the filters to remain adaptive instead of converging to a single filter. This is necessary since if this value is not used, the filter converges to one filter during the alignment phase of the trajectory and remains using this filter for the entire dataset. Using the minimum threshold does however mean that a filter cannot be removed from the overall estimate completely. In practice, this is not a problem, since the minimum threshold is chosen to be small enough so that any wrong filters are effectively ignored.

Fig. 7 shows the conditional probability estimates for the six filters during the alignment phase of the trajectory. The graph is shaded according to the probability estimates. It is shown in the figure that the MMAE filter immediately selects the Kalman filters that use the large scale factors at the beginning of the trajectory. This, then effectively gives more weight to the measurements.

The figure shows that after the initial 20 seconds, the MMAE algorithm adapts to use the three smallest filters. This is because the vessel is experiencing low dynamics. When the alignment section of the trajectory begins at 200 seconds, the filter again adapts, this time largely using the filter with the scale factor of 10.

Fig. 8 shows the conditional probabilities for the MMAE filter during the normal navigation part of the trajectory. It is clear from the figure where the vessel is changing heading, resulting in more weight being given to the filter using the larger scale factor of 10.

The MMAE algorithm results in the filter becoming aligned after only 303 seconds. This is a similar performance to the 275 seconds required for the process noise scaling filter. It is again highlighted that the CKF does not maintain alignment to within 1° even after 10 minutes. The attitude errors during normal navigation are 0.03°, 0.03° and 0.45° for roll, pitch and yaw respectively. This yaw alignment is an improvement over the CKF, but is less accurate the process noise scaling filter. This is thought to be because the filter is slower to react to the new measurement.

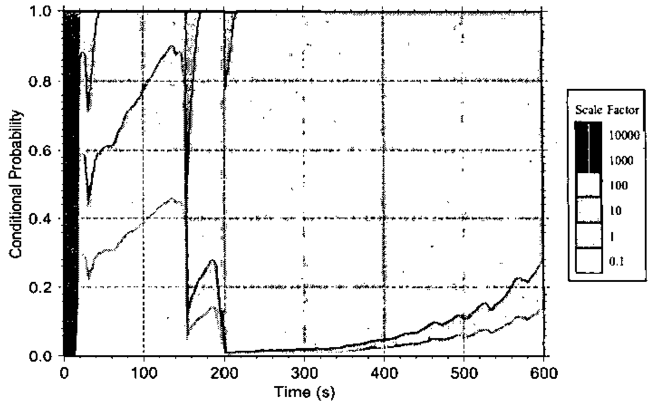


Fig. 7. MMAE conditional probabilities during alignment

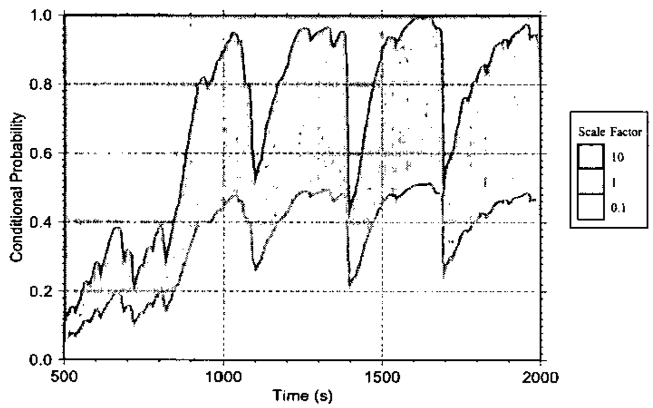


Fig. 8. MMAE conditional probabilities during navigation

VIII. Adaptive Kalman Filter

The yaw error for the AKF is shown in Fig. 9 for the innovation and residual based filters. The figure shows that the AKF results in larger attitude errors than the CKF. The attitude errors are summarised in Table III for the innovation and residual based estimates, along with the attitude errors for the other adaptive Kalman filters. The table shows that the attitude performance is worse in all attitude axes than the CKF.

The adaptive Kalman filter of the process noise matrix has been demonstrated to work successfully in [6] for a higher specification inertial sensor. It is thought that the poor performance of the AKF is due to unmodelled error sources that were introduced in the trial. For example, scale factor errors, axis misalignments, lever arm errors and sensor vibration. The AKF algorithm relies on all the error sources being correctly modelled. However, the filter did remain stable and showed some of the adaptive properties that are required. It is thought that, if the number of elements in the process noise matrix that are being estimated were to be reduced, the AKF may yield a better performance.

IX. Conclusion

This paper has identified and tested three adaptive Kalman filtering algorithms for adapting the stochastic information used in the CKF. The preliminary results obtained using data simulation given in [10] have been reflected in a real marine trial. It has been shown that a significant improvement in time required for initialising and aligning the INS has been

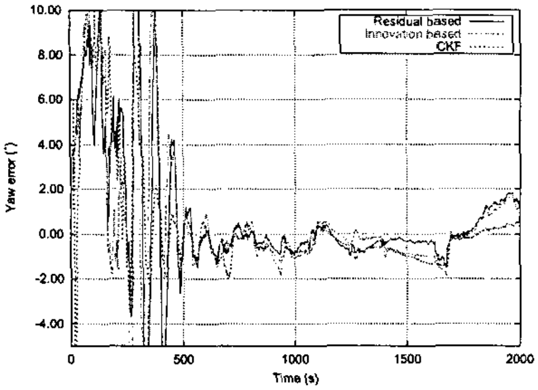


Fig. 9. Adaptive Kalman filter yaw axis misalignment

TABLE III

ATTITUDE PERFORMANCE IN DEGREES

	Roll	Pitch	Yaw
CKF	0.03	0.04	0.52
Process noise scaling	0.04	0.04	0.36
MMAE	0.03	0.03	0.45
Innovation based Adaptive-Q	0.18	0.12	0.83
Residual based Adaptive-Q	0.12	0.10	0.65

obtained using the process noise scaling and MMAE filters. In addition, it has also been demonstrated that these filters provide improved yaw attitude estimation. However, the AKF has been shown to provide less accurate estimates of the attitude parameters than the CKF, and was also unable to align and initialise the INS during a 10 minute alignment procedure.

This paper has shown that there are potential benefits in filter performance to be obtained using adaptive Kalman filtering algorithms. However, none of the algorithms are able to completely remove the need for *a priori* stochastic information and all of the filters still have to be initialised with *a priori* measurement noise, process noise and initial covariances.

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