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Angular Velocity Estimation in Gyroscope-free Inertial Measurement System Based on Unscented Kalman Filter

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***Abstract* - A gyroscope-free inertial measurement system(GFIMS) uses only accelerometers to measure angular velocity and specific force. It is a low-cost inertial measurement system。 But the value of angular velocity is computed by the outputs of accelerometers, its accuracy is lower than that of a conventional inertial measurement system equipped with gyroscopes. So it is very necessary to research on high accuracy and real time estimation algorithm of angular velocity in gyroscope-free IMS. Unscented kalman filter(UKF) is a novel filter method for the nonlinear gauss system, which can obtain higher accuracy than the conventional. In this paper, the UKF is used to estimate values of angular velocity in gyroscope–free IMS based on analysis of nonlinear gyroscope inertial measurement model. And the extended kalman filter(EKF) is compared with the UKF through a example simulation. Simulation results show that the UKF is superior to the EKF method to estimate the angular velocity in gyroscope-free IMS.**

Index terms - Gyroscope-Free; Angular Velocity, UKF; MEMS accelerometer; State Estimation

I. INTRODUCTION

The gyroscope-free inertial measurement system is a kind of inertial measurement system that uses only accelerometers as sensitive component. Angular velocity and linear accelerations of a rigid body motion are obtained from the accelerometer outputs [1]-[2]. In recent years, The design and fabrication of micro-electro-mechanical-systems(MEMS) inertial sensors are more affordable because of the development of the micro-machining. And also MEMS inertial sensors are several orders of magnitude smaller than the conventional ones and can be fabricated in large quantities by process. But functional low-cost MEMS gyroscope may not be commercialized soon because gyroscopes have more inherent physical complexities than accelerometers. In addition, compared with traditional inertial measurement system equipped with gyroscopes and accelerometers, GFIMS has other advantages of low cost, quick start, easy maintenance and easy batch process. Especially, the GFIMS can bear the impact of high angular velocity and angular acceleration. Therefore, there is great motivation for developing a gyroscope-free inertial measurement system for low cost, medium performance application areas.

At present, domestic and overseas scholars have made extensive and deep research about GFIMS, and obtain some fruitful achievements, such as configuration schemes, theory algorithms and practical analysis. In which, the high accuracy and real time computing algorithm for angular rate is the most important. In this paper, we researched and built angular rate state model with a nine-accelerometer combination. Next, we introduce a new algorithm UKF[3]-[9] to estimate angular velocities instead of conventional method EKF[10][11]. At last, we compare these two methods with each other to test their resolving precision by tracking a reference trajectory.

II. GYROSCOPE-FREE IMS MODEL

Consider the earth-centered inertial frame and body frame in geospace. In accordance with absolute motion and relative motion theory of rigid body, the acceleration expression of a random location P on the rigid body relative to the inertial frame is shown in (1)[12].

$$A=\left(\ddot{R}+\dot{\Omega}*r+\Omega*\Omega*r\right)*\theta \qquad (1)$$

where, A is the acceleration vector output of the location point on the rigid body. $\ddot{R}$ is the line acceleration vector of the origin of body frame. r is the position vector of the location P relative to body frame. θ is sensitive direction vector of accelerometer located in the location P. Ω is a angular velocity information matrix. $\dot{\Omega}$ is derivative of Ω, that is, which is the angular acceleration information matrix. And its expression is shown in (2).

$$\Omega=\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \qquad (2)$$

where, the ω_x, ω_y and ω_z represent the angular velocity along x, y and z axis respectively in body frame.

We expand equation (1) in the body frame and we can get all angular velocity information: ω_x^2, ω_y^2, ω_z^2, $\omega_x\omega_y$, $\omega_y\omega_z$, $\omega_x\omega_z$, $\dot{\omega}_x$, and $\dot{\omega}_y$, $\dot{\omega}_z$.

Then the outputs of accelerometers which contain angular rate information can be used to compute angular velocity values by reasonable accelerometers-combination.

A nine-accelerometer combination designed in reference [13] is shown in Fig. 1.

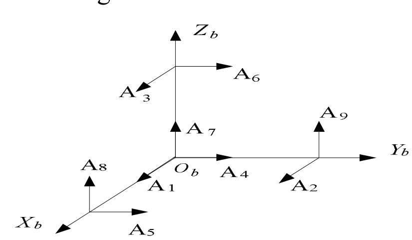

Fig. 1 Nine-Accelerometer Combination

The position vector of Fig. 1 is shown (3)

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$$[r_1 \cdots r_9] = l \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

where, l is distance between accelerometer and the origin of body frame. The sensing direction vector is in (4).

$$[\theta_1 \cdots \theta_9] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{4}$$

Then we can get the expressions of angular velocity information from (1)-(4). Which is shown in (5) and (6).

$$\begin{cases} \dot{\omega}_x = (A9 + A4 - A6 - A7)/2 \cdot l \\ \dot{\omega}_y = (A3 + A7 - A8 - A1)/2 \cdot l \\ \dot{\omega}_z = (A5 + A1 - A2 - A4)/2 \cdot l \end{cases} \tag{5}$$

$$\begin{cases} \omega_x \omega_y = (A5 + A2 - A4 - A1)/2 \cdot l \\ \omega_y \omega_z = (A9 + A6 - A4 - A7)/2 \cdot l \\ \omega_x \omega_z = (A8 + A3 - A1 - A7)/2 \cdot l \end{cases} \tag{6}$$

Consider real situation, output errors of accelerometers' outputs exist. We have to notice these output errors when building system estimation model and these output values can not be used to compute angular rate directly. A system state estimation model built in [14] are shown in (7)-(9).

Define the system state variables as shown in (7).

$$X(t) = \left[\underset{\sim}{\omega}_x(t) \;\; \underset{\sim}{\omega}_y(t) \;\; \underset{\sim}{\omega}_z(t) \;\; \underset{\sim}{\dot{\omega}}_x(t) \;\; \underset{\sim}{\dot{\omega}}_y(t) \;\; \underset{\sim}{\dot{\omega}}_z(t) \right]^T \tag{7}$$

state equation:

$$X(t+1) = \Phi * X(t) + u(t) + \Gamma * w(t) \tag{8}$$

where,

$$\Phi = \begin{bmatrix} 1\,0\,0\,T\,0\,0 \\ 0\,1\,0\,0\,T\,0 \\ 0\,0\,1\,0\,0\,T \\ 0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,1 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} T/2 \\ T/2 \\ T/2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$u(t) = \begin{bmatrix} (T/2) \cdot (\dot{\omega}_x(t+1) - \dot{\omega}_x(t)) \\ (T/2) \cdot (\dot{\omega}_y(t+1) - \dot{\omega}_y(t)) \\ (T/2) \cdot (\dot{\omega}_z(t+1) - \dot{\omega}_z(t)) \\ \dot{\omega}_x(t+1) - \dot{\omega}_x(t) \\ \dot{\omega}_y(t+1) - \dot{\omega}_y(t) \\ \dot{\omega}_z(t+1) - \dot{\omega}_z(t) \end{bmatrix}$$

observation equation:

$$y(t+1) = T \cdot h(X(t+1)) + v(t+1) \tag{9}$$

where,

$$y(t+1) = \begin{bmatrix} \omega_x \omega_y (t+1) - \omega_x \omega_y (t) \\ \omega_y \omega_z (t+1) - \omega_y \omega_z (t) \\ \omega_x \omega_z (t+1) - \omega_x \omega_z (t) \end{bmatrix}$$

$$h(X(t+1)) = \begin{bmatrix} \underset{\sim}{\omega}_x \underset{\sim}{\dot{\omega}}_y (t+1) + \underset{\sim}{\dot{\omega}}_x \underset{\sim}{\omega}_y (t+1) \\ \underset{\sim}{\omega}_y \underset{\sim}{\dot{\omega}}_z (t+1) + \underset{\sim}{\dot{\omega}}_y \underset{\sim}{\omega}_z (t+1) \\ \underset{\sim}{\omega}_x \underset{\sim}{\dot{\omega}}_z (t+1) + \underset{\sim}{\dot{\omega}}_x \underset{\sim}{\omega}_z (t+1) \end{bmatrix}$$

Symbol and variable description:

T is Sample Time. The variables marked with symbol '$\sim$' stand for the truth value and which without symbol '$\sim$' are the outputs of accelerometers. The difference between these two values is output error of accelerometer. For example, the variable $\dot{\omega}_x(t)$ and $\underset{\sim}{\dot{\omega}}_x(t)$, we can get the value of $\dot{\omega}_x(t)$ by measuring output of accelerometer at the moment of t. But we can not get the value of $\underset{\sim}{\dot{\omega}}_x(t)$ with the same method. The relations between these two variables can be expressed in (10).

$$\underset{\sim}{\dot{\omega}}_x(t) = \dot{\omega}_x(t) + e_x(t) \tag{10}$$

where, $e_x(t)$ is output error of accelerometer at the moment of t. And other variables are similar. $w(t)$ and $v(t)$ are irrelevant Gaussian random noise with zero mean. And exist equation (11).

$$\begin{cases} E\left[w(i) w^T(j) \right] = Q\delta_{ij} \\ E\left[v(i) v^T(j) \right] = R\delta_{ij} \end{cases} \tag{11}$$

III. UKF AND EKF STATE ESTIMATION

We could notice that the state equation is linear and observation equation is nonlinear. So the whole model is nonlinear. Compared with the traditional EKF[10][11], which approximate the original nonlinear function through the first-order Taylor linearization, the UKF approximate the original nonlinear function through using a deterministic sampling method to simulate probability density distribution of original nonlinear function. The sampling points(called sigma point) sampled by a certain sampling method are transformed using the nonlinear function. At last the kalman filter algorithm is used to estimate the system state with these transformed sigma points.

A. Unscented Transformation

Unscented transformation is based on priori statistics[3][5]. Consider the nonlinear function: $f = (x)$, where, x is n-dimensional random variable with mean $\bar{x}$ and covariance p_{xx}. So the process for forecasting the mean $\bar{y}$ and covariance p_{yy} of the variable y by unscented transformation is as follow:

1) Sampling: Sampling for the sigma point set $\{\chi_i\}$ of variable x according to one certain sampling method with the mean $\bar{x}$ and the covariance p_{xx}. Where, $i = 1 \cdots L$, and L is the number of sigma points which is determined by the certain sampling method. And also the weight of mean W_i^m and covariance W_i^c are determined by this sampling method.

2) Transformation: Transform sigma points χ_i above into y_i using the nonlinear function $y_i = f(\chi_i)$. And get the function variables point set $\{y_i\}$ where, $i = 1 \cdots L$.

3) Computation: Compute the statistics $\bar{y}$ and p_{yy} using (12).

$$\bar{y} = \sum_{i=1}^{L} W_i^m y_i \ , \quad p_{yy} = \sum_{i=1}^{L} W_i^c (y_i - \bar{y})(y_i - \bar{y})^T \tag{12}$$

At the same time, the accuracy of unscented transformation is proved in [8][9]. Its accuracy is higher than EKF and can arrive the second-order Taylor formula.

B. Sigma Point Sampling

The sampling method of sigma point is the basis of UKF algorithm. There are different sampling point number, weight and estimation accuracy for the different sampling method. At present, the common sampling method are symmetric point sampling[4], minimal simplex sampling[6] and spherical simplex sampling[7] and so on. The symmetric point sampling method is used in this paper to estimate the angular velocity state. This sampling process is shown below and the other sampling methods can be found in related literatures.

Consider the nonlinear transformation $y = f(x)$ above, the number of the sigma points and corresponding weight are determined by the symmetric point sampling method and are shown in (13) .

$$L = 2n+1 \ , \quad W_i^m = W_i^c = W_i \tag{13}$$

The specific sampling process are in (14) and (15).

$$\chi_i = \begin{cases} \tilde{x} & i = 0 \\ \tilde{x} + \left(\sqrt{(n+k)p_{xx}}\right)_i & i = 1, \cdots, n \\ \tilde{x} - \left(\sqrt{(n+k)p_{xx}}\right)_{i-n} & i = n+1, \cdots, 2n \end{cases} \tag{14}$$

$$W_i = \begin{cases} k/(n+k), & i = 0 \\ 1/2(n+1), & i \neq 0 \end{cases} \tag{15}$$

where, k is scaling parameter. Which can be used to control the distance between sigma point χ_i and $\tilde{x}$. $\left(\sqrt{(n+k)p_{xx}}\right)_i$ is the line i or row i of square root matrix $\sqrt{(n+k)p_{xx}}$. W_i is weight of sigma point with the number i, and exist relationship (16).

$$\sum_{i=0}^{2L} W_i = 1 \tag{16}$$

C. UKF State Estimation Algorithm

According to the unscented transformation and sigma points sampling method mentioned above, we can get the angular velocity state estimation algorithm. The initial state $x(0|0)$ and covariance $p(0|0)$ are obtained by system initial alignment.

1) Sigma points sampling:

$$\chi_i = \begin{cases} x(t|t) & i = 0 \\ x(t|t) + \left(\sqrt{(n+k)p(t|t)}\right)_i & i = 1, \cdots, n \\ x(t|t) - \left(\sqrt{(n+k)p(t|t)}\right)_{i-n} & i = n+1, \cdots, 2n \end{cases}$$

$$W_i^m = W_i^c = W_i = \begin{cases} k/(n+k), & i = 0 \\ 1/2(n+1), & i \neq 0 \end{cases}$$

2) Unscented transformation of sigma points:

$$\chi_i(t+1|t) = \Phi * \chi_i(t) + u(t)$$
$$y_i(t+1|t) = T \cdot h(\chi_i(t+1|t))$$

3) Prognostic equation:

$$x(t+1|t) = \sum_{i=0}^{L-1} W_i^m \chi_i(t+1|t)$$
$$y(t+1|t) = \sum_{i=0}^{L-1} W_i^m y_i(t+1|t)$$

$$p(t+1|t) = Q + \sum_{i=0}^{L-1} W_i^c \left(\chi_i(t+1|t) - x(t+1|t)\right)\left(\chi_i(t+1|t) - x(t+1|t)\right)^T$$

$$p_{yy}(t+1|t) = R + \sum_{i=0}^{L-1} W_i^c \left(y_i(t+1|t) - y(t+1|t)\right)\left(y_i(t+1|t) - y(t+1|t)\right)^T$$

$$p_{xy}(t+1|t) = \sum_{i=0}^{L-1} W_i^c \left(\chi_i(t+1|t) - x(t+1|t)\right)\left(y_i(t+1|t) - y(t+1|t)\right)^T$$

4) Renewal equation:

$$K(t+1) = p_{xy}(t+1|t)\, p_{yy}^{-1}(t+1|t)$$
$$x(t+1|t+1) = x(t+1|t) + K(t+1)\left(y(t+1) - y(t+1|t)\right)$$
$$p(t+1|t+1) = p(t+1|t) - K(t+1)\, p_{yy}(t+1|t)\, K^T(t+1)$$

D. EKF State Estimation Algorithm

To evaluate the performance of UKF algorithm, which is compared with traditional nonlinear state estimation method EKF by a trajectory simulation. And the EKF model is given in (17) and (18) directly.

$$X(t+1) = \Phi * X(t) + u(t) + \Gamma * w(t) \tag{17}$$

$$y(t+1) + z(t+1) = H(t+1)X(t+1) + v(t+1) \tag{18}$$

where,

$$H(t+1) = T \cdot \begin{bmatrix} \dot{\underset{\sim}{\omega}}_y & \dot{\underset{\sim}{\omega}}_x & 0 & \underset{\sim}{\omega}_y & \underset{\sim}{\omega}_x & 0 \\ 0 & \dot{\underset{\sim}{\omega}}_z & \dot{\underset{\sim}{\omega}}_y & 0 & \underset{\sim}{\omega}_z & \underset{\sim}{\omega}_y \\ \dot{\underset{\sim}{\omega}}_z & 0 & \dot{\underset{\sim}{\omega}}_x & \underset{\sim}{\omega}_z & 0 & \underset{\sim}{\omega}_x \end{bmatrix}_{(t+1|t)}$$

$$z(t+1) = T \cdot \begin{bmatrix} \underset{\sim}{\omega}_x(t+1|t)\dot{\underset{\sim}{\omega}}_y(t+1|t) + \dot{\underset{\sim}{\omega}}_x(t+1|t)\underset{\sim}{\omega}_y(t+1|t) \\ \underset{\sim}{\omega}_y(t+1|t)\dot{\underset{\sim}{\omega}}_z(t+1|t) + \dot{\underset{\sim}{\omega}}_y(t+1|t)\underset{\sim}{\omega}_z(t+1|t) \\ \underset{\sim}{\omega}_x(t+1|t)\dot{\underset{\sim}{\omega}}_z(t+1|t) + \dot{\underset{\sim}{\omega}}_x(t+1|t)\underset{\sim}{\omega}_z(t+1|t) \end{bmatrix}$$

Then, we linearize the nonlinear observation equation and get the linearization model. The next step is to compute the value of the angular velocity with linear kalman recursive algorithm.

IV. SIMULATION

A computer simulations is performed with matlab to evaluate the performance of the UKF algorithm in this paper. Sampling interval: T=0.01s. Simulation time:100s. The constant biases of the accelerometers is within 100μg and the output random disturbance is white Gaussian noise. An arbitrary trajectory of angular velocity shown in Fig.2 is used for the simulations. The simulation results are presented in Fig.3.

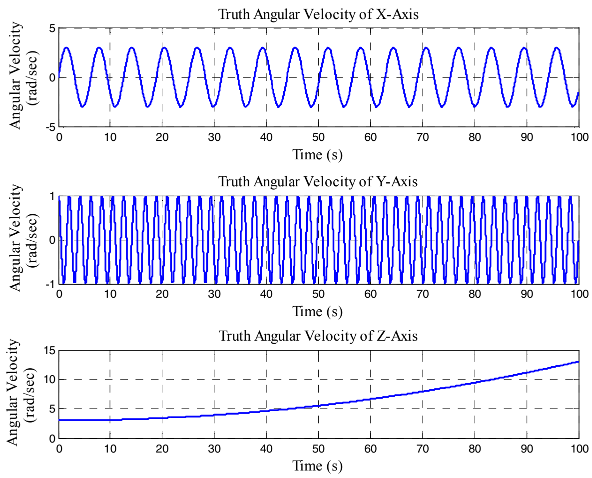

Fig. 2 The reference trajectorys

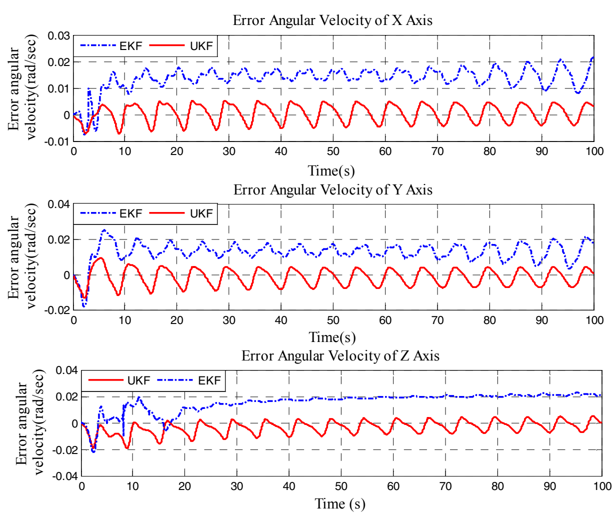

Fig.3 Angular rate estimation errors for three algorithms

The simulation results is presented in Fig.3. In which the angular velocity estimation errors of UKF is compared with the EKF algorithm. It is clear that the performance of the UKF algorithm is superior to the EKF algorithms. It also shows that when the biases of accelerometers' outputs are not compensated, the error of the angular relocity estimated by UKE algorithm remains bounded, but the angular rate estimated by EKF algorithms diverges away from the true trajectory. The simulation results reflects that UKF algorithm is better to approximate the original nonlinear system, and which agrees well with our analytic arguments.

V. CONCLUSION

The error growth rate of a GFIMS diverges at a rate an order faster than that of a conventional IMS equipped with gyroscopes. To tackle this inherent disadvantage of a GFIMS, we have used a new angular velocity algorithm UKF to bound the error growth rate of a GFIMS. In this algorithm, the sigma points sampling and unscented transformation are adopted before kalman state estimation algorithm. To test its performance, we compared this algorithm to the extended kalman filter algorithm with an example simulation. The simulation results show that the performance of UKF method is superior to the conventional EKF method. Which can bound the angular velocity estimation errors effectively.

REFERENCES

[1] L. D. DiNapoli, "The Measurement of Angular Velocities without the Use of Gyros," *the Moore school of Electrical Engineerring*, University of Pennsylvania, Philadelpia, 1965.
[2] Alfred R. Schuler. "Measuring Rotational Motion with Linear Accelerometers," *IEES Tran. On AES*, 1967,AES-3(3), pp. 465-472.
[3] Julier S J, Uhlmann J K, Durrant-Whyten H F, "A New Approach for Filtreing Nolinear System," *Proc of the American Control Conf*, Washtington:Seattle,1995, pp. 1628-1632.
[4] Julier S J, Uhlmann J K, "A New Extension of the Kalman Filter to Nonlinear Systems," *The Proc of AeroSense: The 11th Int Symposium on Aerospace/Defense Sensing, Simulation and Control*, Orland, 1997, pp. 54-65.
[5] Julier S J, Uhlmann J K. "A Consistent Debiased Method for Converting Between Polar and Cartesian Coordinate Systems," *The Proc of AeroSense: The 11th Int Symposium on Aerospace/Defense Sensing,Simulation and Contro*l, Orland, 1997, pp. 110-121.
[6] Julier S J, Uhlmann J K. "Reduced Sigma Point Filters for the Propagation of Means and Covariances through Nonlinear Transformations," *Proc of American Control Conf*, Jefferson City, 2002, pp. 887-892.
[7] Julier S J. "The spherical simplex unscented transformation," *American Control Conf*, Denver, 2003, pp. 2430-2434.
[8] Julier S J, Uhlman J K, Durrant-Whyten H F. "A General Method for Approximating Nonlinear Transformations of Probabilty Distributions," *http://www.robots.ox.ac.uk/~siju/work/pu-plications/Unscented.zip*, 1997-09-27.
[9] Julier S J, Uhlmann J K, Durrant-Whyten H F, "A New Aprroach for The Nonlinear Transformation of Means and Covariances in Filters and Estimatiors," *IEEE Trans on Autmatiic Control*, 2000, 45(3), pp. 477-482.
[10] Uhlmann J K, "Algorithm for multiple target tracking," *American Science*, 1992, 80(2), pp. 128-141.
[11] Sorenson H W. Kalman filtering, "Theory and Application," *IEEE Press*, New York, 1985.
[12] Wang JinSong, "Research on Theory and Optimized Algorithm of Gyros Free Micro Inertial Measurement Unit," *Harb Institute of Technology*, 2003.
[13] Ding MingLi, Wang Qi, "Design of Non-gyro Inertial Measurement Unit and Research on Rrror Compensation Method of Angular Velocity," *Acta Aeronautica et Astronautica Sinica*, 2006, 27(5), pp. 923-927.
[14] Yang Jie, Shi Zhen, Yue Peng, Cheng ZiJian, "Estimation Algorithm of Angular Velocity in Gyroscope-Free Inertial Measurement System," *Missiles and Guidance Journal*, in press.