Technical Document
Compensating for Tilt, Hard Iron and Soft Iron Effects

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MEMSENSE TECHNICAL DOCUMENT

COMPENSATING FOR TILT, HARD IRON AND SOFT IRON EFFECTS

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Technical Documentation

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I. Abstract

Heading can be effectively determined through the use of dual-axis magnetometer and triaxial accelerometer sensors available in an inertial measurement unit (IMU) such as the *MEMSense* nIMU[1] or uIMU[2][3]. However, correct implementation of a compass system must compensate for the effect of elevation and bank angle (tilt), as well as attempt to calibrate out hard and soft iron effects as much as possible. This paper discusses the impact of elevation, bank angle and hard and soft iron effects on heading calculations, and methods that may be employed to counter their impact when utilizing magnetometer data from an IMU. It is important to note that although the methods presented address corrections in two dimensions (x-y plane), it is possible, and often necessary, to extend the concepts and functions to three dimensions.

II. Definitions

The following terms are used throughout this document and are defined here to aid in clarity.

- **Accelerometer:** A sensor which measures linear acceleration.
- **Gyro/Gyroscope:** A sensor which measure rotational velocity in degrees per second (deg/s).
- **Inertial Measurement Data:** Any data collected from the **inertial measurement unit**, which consists of information regarding rotational velocity, linear acceleration, and magnetic field.
- **Inertial Measurement Unit (IMU):** A device used to collect **inertial measurement data.**
- **Magnetometer:** A sensor which measures changes in a magnetic field.

III. Frame Conventions

The frame conventions utilized within this document are as shown in Figure 1:

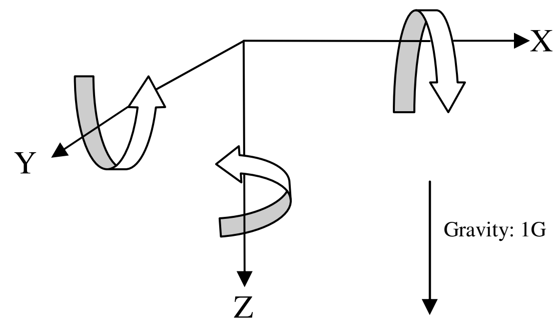

Figure 1: Reference frame conventions.

Referring to Figure 1, positive bank angle will be in the direction of the arrow rotating around the X-axis, positive elevation will be in the direction of the arrow rotating around the Y-axis, and positive heading will be in the direction of the arrow rotating around the Z-axis. More specifically, the 'Right-Hand Rule' is utilized which specifies that positive rotation is in the direction in which the fingers of your right hand curl when the thumb is oriented along the positive axis of rotation (away from the origin). Thus, positive rotation is counter-clockwise when looking along the axis of rotation towards the origin.

It will always be assumed that orientation is relative to a reference coordinate frame where the X-axis points North, Y-axis points East, and Z-axis points down - often referred to as 'NED'. Thus, heading and attitude are always determined relative to this standard, or reference, position.

To differentiate between the local coordinate frame and the reference coordinate frame, the reference frame axes will be in upper- case and/or have the subscript *r,* as in X_r.

IV. Elevation and Bank Angle Effect

Elevation is defined as the angle formed between the x-axis and the horizon/ground, and bank angle is defined as the angle formed between the y-axis and the horizon/ground. Gravity exerts a constant acceleration of 1g, which may be utilized to calculate elevation and bank angle. As shown in Figure 2, use of the arc-tangent function enables calculation of elevation following a negative rotation in the X-Z plane. Note that the Z axis is oriented ‘down’, as specified by the NED convention. Please see [4][5] for further discussion on calculating elevation and bank angle.

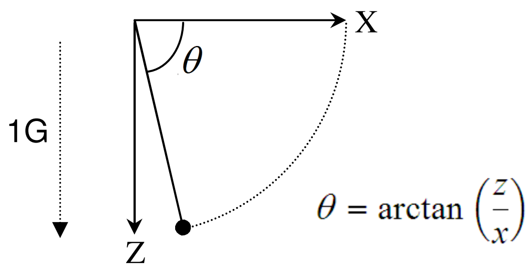

Figure 2: Calculation of theta (elevation). Note that this is a negative rotation which will result in a negative elevation angle.

Heading is calculated using the same method as that used in elevation and bank angle, but because gravity cannot be used to calculate changes in heading, magnetometer data must instead be used.

A naïve approach to calculating heading would simply apply the arctan approach to raw magnetometer data. However, because magnetometer sensitivity decreases as elevation and bank angles increase[6], errors in the heading will be introduced. Figure 3 shows the change in heading due to the decrease in magnetometer sensitivity as elevation increases. By realigning the local z-axis with the reference frame Z-axis, the heading can be corrected (see Figure 4), with the magnitude of these errors given in Figure 5. It’s also important to note that tilt/sensitivity errors vary with location, thus it is not possible to employ a static correction factor (such as a lookup table).

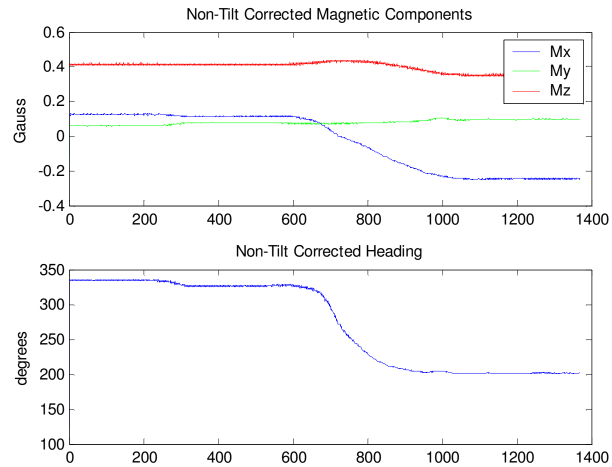

Figure 3: Change in heading as a result of a 50 degree change in elevation (tilt). If properly corrected, a change in heading should not be exhibited.

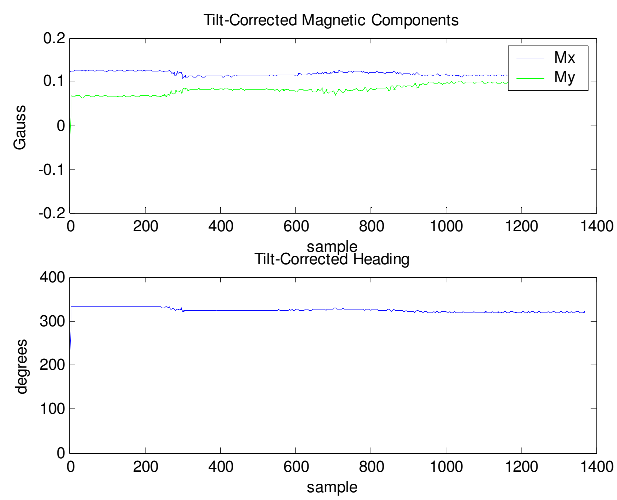

Figure 4: Heading corrected for 50 degree change in elevation (tilt).

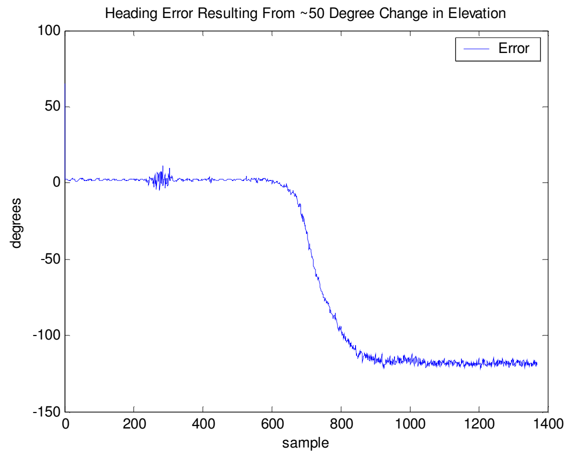

Figure 5: Heading error as a result of a 50 degree change in elevation.

Therefore, we must first apply a rotation that removes the bank angle followed by a second rotation that removes elevation (the reverse sequence is also acceptable). Once this sequence of rotations is completed the local x-y plane will be realigned with the reference X-Y plane, corrections to the magnetometer data will have been made, and we may then proceed with compensating for hard and soft iron effects. Full discussion on determining the necessary rotation matrices, applying the rotations, and calculating the corrected heading can be found in [4].

V. Hard and Soft Iron Distortions

Distortions of the earth's magnetic field are a result of external magnetic influences generally classified as either a hard or soft iron effect. If no distorting effects are present, rotating a magnetometer through a minimum of 360 degrees and plotting the resulting data as y-axis vs. x-axis will result in a circle centered around (0, 0), as shown in Figure 6.

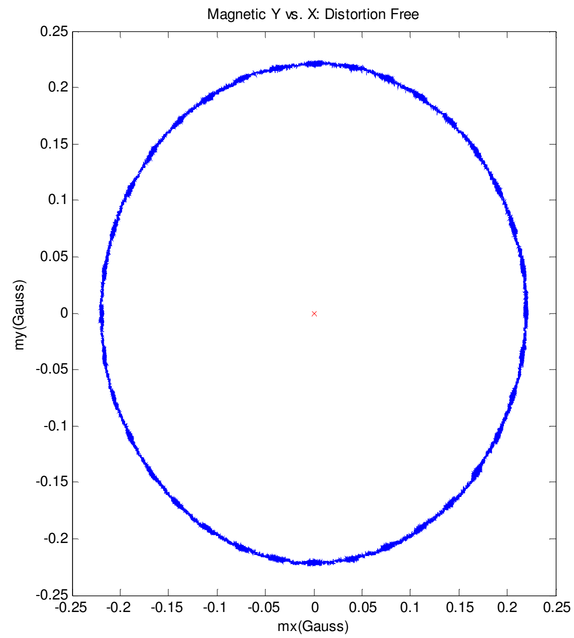

Figure 6: Y vs. X plot of distortion free magnetometer output (Gauss). Note that the output is centered around (0, 0), and circular in shape.

However, the presence of hard and/or soft iron effects may produce a perturbation of the circle as a simple offset from (0, 0) in the case of a hard iron effect, or deform the circle to produce an ellipse in the case of a soft iron effect. It is also possible that both effects will be exhibited simultaneously.

In addition, it is important to recognize that effective compensation of hard and soft iron distortions is dependent upon the distorting material(s) rotating/moving with the sensor. An example would be mounting the sensor in an aircraft; any materials that are part of the aircraft that exhibit a distorting effect would move as the aircraft and mounted sensor move, and it would generally be possible to compensate for the associated hard and soft iron effects. In contrast, it is much more difficult – if not impossible - to compensate for distorting effects exhibited by material external to the aircraft/sensor platform. Thus, it is important to understand not only how compensation may be applied, but also to recognize those conditions under which effective compensation techniques are not possible.

A. Hard Iron Distortion

Hard iron distortion is produced by materials that exhibit a constant, additive, distortion to the earth's magnetic field, thereby generating a constant additive value to the output of each of the magnetometer axes. A hard iron distortion can be visibly identified by an offset of the origin of the ideal circle from (0, 0), as shown in Figure 7.

Compensating for hard iron distortion is straightforward, accomplished by determining the x and y-offsets and then applying these constants directly to the data. It is important to note that tilt compensation must be applied prior to determining hard iron corrections.

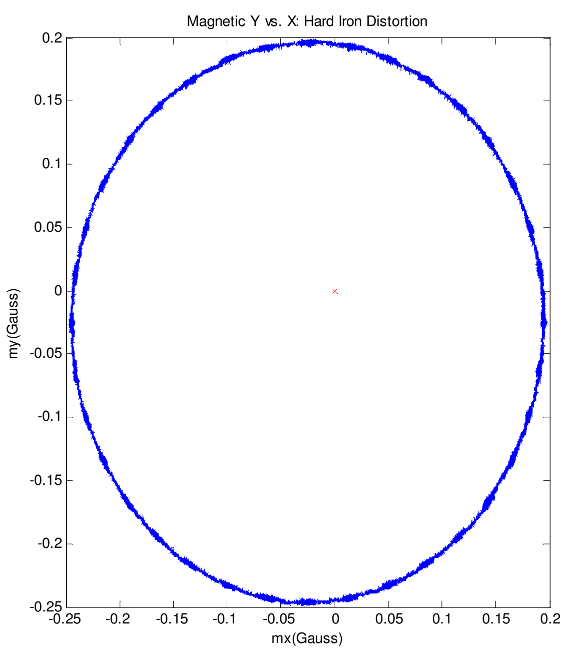

Figure 7: Hard iron distortion exhibited by a constant offset in both X and Y. A red 'x' at (0,0) is the point around which the circle should be centered if no distortion were present.

Hard iron corrections are typically determined by rotating the sensor through a minimum of 360 degrees, then determining the distance from (0, 0) to the center of the circle by identifying the average of the maximum and minimum values for each of the axes, as shown in Equations 1 and 2, respectively.

$$\alpha = \frac{(x_{\max} + x_{\min})}{2} \qquad \text{(Eq. 1)}$$

$$\beta = \frac{(y_{\max} + y_{\min})}{2} \qquad \text{(Eq. 2)}$$

These average values are then subtracted from the raw x and y magnetometer data, thus largely eliminating the hard iron distortion.

It is important to recognize that hard iron effects are constant regardless of orientation or position of the sensing platform. These constant offsets can be stored off once calculated and simply subtracted from the raw magnetometer data.

B. Soft Iron Distortion

Soft iron distortion is similar in nature to hard iron distortion in that it is also additive to the earth's magnetic field. However, the difference is that while hard iron distortion is constant regardless of orientation, the distortion produced by soft iron materials is dependent upon the orientation of the material with the earth's field. Thus, soft iron distortion cannot be compensated with a simple constant; instead, a more complicated procedure is required.

As shown in Figure 8, a soft iron distortion is typically exhibited as a perturbation of the ideal circle into an ellipse. Plotting the magnitudes also shows the characteristic 'two-cycle error'.

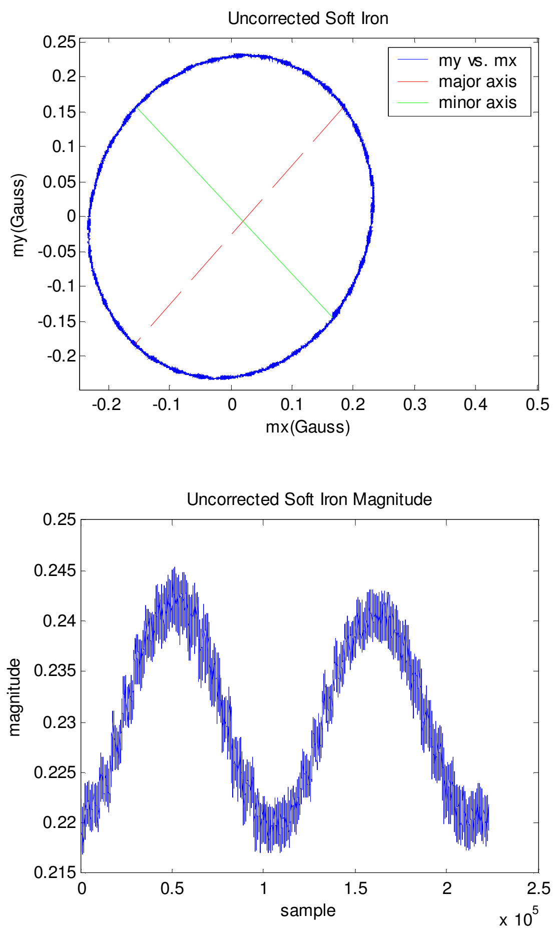

Figure 8: Soft iron effect distorting the ideal circle into an elliptical shape. The corresponding magnitude illustrates the characteristic 'two-cycle error' where the peaks represent the major axis, and valleys the minor-axis.

Unlike compensating for hard-iron effects, soft-iron compensation is not a simple constant that can be quickly and easily applied to the raw data. Instead, compensating for soft iron distortion is much more compute-intensive, and it may be more effective from a cost and efficiency perspective – particularly if designing/implementing an embedded system - to simply eliminate the soft-iron material(s) from the proximity of the sensor. However, in many cases this is not an option, and implementation of a soft-iron compensation method is required.

To simplify the following discussion it is assumed that all tilt effects and hard-iron distortions are either not present in the application environment, or tilt and hard-iron compensation methods have previously been applied. Furthermore, if tilt and hard iron effects are present, compensation for these distortions *must* be applied prior to correcting for soft iron distortions. As such, it is safe to assume that the origin of the ellipse is at (0, 0), as shown in Figure 9, and is exhibiting a rotation of θ degrees from the X-axis.

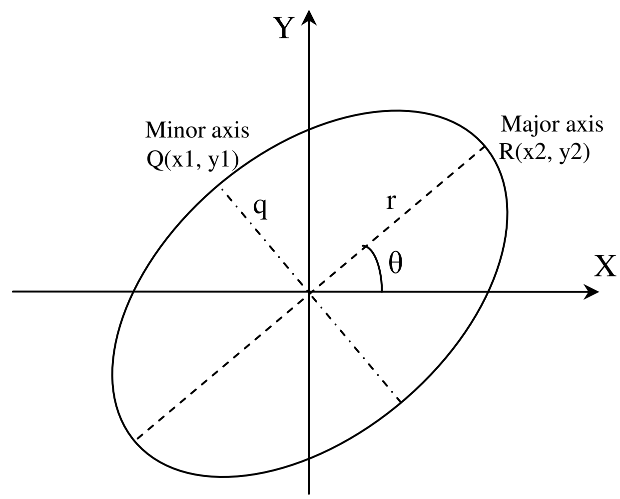

Figure 9: Ellipse generated as a result of soft-iron distortion, centered at (0, 0) with rotation θ.

Identifying θ in Figure 9 is accomplished by using Equation 3 to calculate the magnitude of the line segment r.

$$r = \sqrt{(x_1)^2 + (y_1)^2} \quad \text{(Eq. 3)}$$

From a computing perspective, identifying r is accomplished by calculating the magnitude of each of the points on the ellipse, then identifying the maximum magnitude of all computed values; the coordinates of this value will correspond with major axis.

Once θ has been identified, the rotation matrix given in Equation 4 is applied to the vector of magnetometer x/y values, v, using Equation 5.

For further discussion on rotation matrices and additional material on the derivation of Equations 4 and 5 please see [7].

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \text{(Eq. 4)}$$

$$v_1 = R\,v \quad \text{(Eq. 5)}$$

After the rotation, the major axis of the ellipse will be aligned with the reference frame X-axis and the minor axis will be aligned with the Y-axis, as shown in Figure 10.

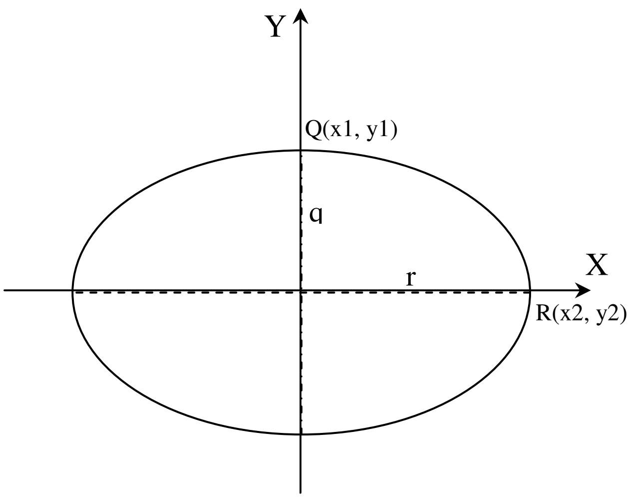

Figure 10: Alignment of the ellipse major and minor axes with the coordinate system x and y-axes, respectively, following the rotation.

Following the rotation, it is now possible to properly scale the major axis such that the ellipse is converted to a circle. The scale factor, σ, is determined using Equation 6, which is simply the ratio of the lengths of the major axis to the minor axis. Each magnetometer x-value is then divided by this scale factor to produce the desired circle.

$$\sigma = \frac{q}{r} \quad \text{(Eq. 6)}$$

Once scaling is completed a final rotation must be made to rotate the data back to their original position – thus compensating for the soft iron distortion. This is achieved by applying Equations 4 and 5 as previously performed to align the ellipse and coordinate system axes, but with a negative θ.

VI. Summary

This paper demonstrated that substantial error can be present in heading calculations unless adequate detection and calibration methods are applied. Several methods were presented that may be utilized to correct for tilt, hard iron and soft iron distortion present in data collected via a dual-axis magnetometer, and the proper order of application is critical to realizing improved results in heading calculation. The discussion and corresponding functions are specific to corrections in 2-D/x-y plane, but with the integration of a tri-axial magnetometer it is possible to extend these concepts to 3-D to correct for distortions present in the vertical (Z) axis.

VII. Matlab Script Usage and Instructions

Several Matlab scripts are provided that demonstrate how to compensate for tilt, hard and soft iron effects. A single top-level file, *magCal.m,* is utilized to coordinate calling of the underlying compensation scripts and plot the results (if desired). Three files are used in the post-processing of collected data:

- *magCal.m*: Main function that calls the underlying compensation scripts and plots the results.
- *calcBankElevation.m*: Calculates bank angle and elevation for use in tilt compensation.
- *calcHeading.m*: Utilizes bank angle and elevation to correct for tilt. Produces tilt-compensated heading and magnetometer data, and plots results if requested.

- *idHardIronCorrection.m*: Identifies offsets to be used in hard iron correction.
- *applyHardIronCorrection.m*: Applies the offsets determined in *idHardIronCorrection.m* to tilt compensated magnetometer data.
- *idSoftIronCorrection.m*: Identifies the amount of rotation to be applied to align the ellipse major axes with the coordinate system x and y-axes, respectively, and the scale factor to convert the ellipse to a circle.
- *applySoftIronCorrection.m*: Performs soft-iron correction by applying the rotation and scale factor determined by *idSoftIronCorrection.m* to tilt and hard-iron compensated magnetometer data.

The function signatures are as follows:

ret = magCal(calData, sensData, headingGT180, doPlot)

Identifies and corrects for tilt, soft and hard iron distortions. Returns a three element structure containing the corrected magnetic x and y values specified in Gauss, and heading specified in degrees.

Parameters:

calData: Matrix of accelerometer and magnetometer data to be used as the basis from which calibration values are determined. This should contain data representing a minimum of a 360 degree rotation in the environment for which calibration is to take place. The data shall be in the following order

Column	Sensor/Data
1	Accel X (G's)
2	Ay
3	Az
4	Mag X (Gauss)
5	My
6	Mz

sensData: Data to which calibration is to be applied, and shall be in the following order:

Column	Sensor/Data
1	Accel X (G's)
2	Ay
3	Az
4	Mag X (Gauss)
5	My
6	Mz

headingGT180: True if heading should be plotted in the range (0, 2π), false if (0, +/- π/2).
Default: true

doPlot: True if plotting of data should be performed, else false. Please note that a minimum of 11 plots will generated as part of the calibration process.
Default: false

Returns:

A structure is returned containing the following elements:

mx: Array of corrected magnetometer x-axis data in Gauss.

my: Array of corrected magnetometer y-axis data in Gauss.

heading: Array of heading angles, in radians.

Each element is an array with a size equal to the number of rows in the original data file.

Output:

None

Usage:

From the MATLAB command line, change to the directory in which the scripts reside:

hdg = magCal(calData, sensData, true, false)

Uses calData to determine tilt, hard and soft iron calibration. The calibration is applied to sensData. *headingGT180* is specified as true, indicating resulting heading values should be in the range 0-2π, and doPlot of false indicates no plotting should be performed. Due to default parameters, the following produces identical results:

hdg = magCal(calData, sensData)

be = calcBankElevation(ax, ay, az)

calc_bankElevation calcualates bank angle and elevation from accelerometer data. Returns a two element structure containing the elements *bank* and *elevation* specified in radians. Each element is an array with a size equal to the number of rows in the original accelerometer data array.

Parameters:
ax: x-axis accelerometer data.

ay: y-axis accelerometer data.

az: z-axis accelerometer data.

Returns:
A structure is returned containing the following elements:

bank: Array of bank angles in radians.

elevation: Array of elevation angles in radians.

Each element is an array with a size equal to the number of rows in the original data file.

Output:
None

ret = calcHeading(bank, elevation, mx, my, mz, headingGT180, doPlot)

Calculates heading using magnetometer values, and applies corrections based on bank and elevation angles. Returns a three element structure containing the corrected magnetic x and y values specified in Gauss, and heading specified in degrees.

Parameters:
bank: Vector of bank angles specified in radians.

elevation: Vector of elevation angles specified in radians.

mx: Vector of magnetometer x-axis values, specified in Gauss.

my: Vector of magnetometer y-axis values, specified in Gauss.

mz: Vector of magnetometer z-axis values, specified in Gauss.

headingGT180: True if heading should be plotted in the range (0, 2π), false if (0, +/-π/2).

doPlot: True if plotting of uncorrected/corrected data should be performed, else false.

Returns:
A structure is returned containing the following elements:

mx: Vector of corrected magnetometer x-axis data in Gauss.

my: Vector of corrected magnetometer y-axis data in Gauss.

heading: Array of heading angles, in radians.

Each element is an array with a size equal to the number of rows in the mx, my and mz vectors.

Output:
If *doPlot* is *true*, numerous plots displaying magnetometer data, uncorrected and corrected heading are displayed.

ret = idHardIronCorrection(mx, my)

Calculates hard iron offsets that may be used to correct for hard iron distortion present in the data. A two element structure is returned containing the x- and y-offset. Not that the x- and y-offset represent the amount of the offset from (0, 0), and should be subtracted from magnetometer x- and y-axis data, respectively, to correct for hard-iron distortion.

Parameters:
mx: Vector of magnetometer x-axis values, specified in Gauss.

my: Vector of magnetometer y-axis values, specified in Gauss.

Returns:
A structure is returned containing the following elements:

xOffset: x-axis offset.

yOffset: y-axis offset.

Output:
none

ret = applyHardIronCorrection(bank, elevation, mx, my, mz, headingGT180, doPlot)

Calculates heading based on magnetometer x/y data, and applies corrections based on bank and elevation angles. Returns a three element structure containing the corrected magnetic x and y values specified in Gauss, and heading specified in radians.

Parameters:
bank: Vector of bank angles specified in radians.

elevation: Vector of elevation angles specified in radians.

mx: Vector of magnetometer x-axis values, specified in Gauss.

my: Vector of magnetometer y-axis values, specified in Gauss.

mz: Vector of magnetometer z-axis values, specified in Gauss.

headingGT180: True if heading should be plotted in the range (0, 2π), false if (0, +/-π/2).

doPlot: True if plotting of uncorrected/corrected data should be performed, else false.

Returns:
A structure is returned containing the following elements:

mx: Vector of corrected magnetometer x-axis data in Gauss.

my: Vector of corrected magnetometer y-axis data in Gauss.

heading: Array of heading angles, in radians.

Each element is an array with a size equal to the number of rows in the mx, my and mz vectors.

Output:
If *doPlot* is *true*, one figure is generated with two plots: the first plots the uncorrected magnetometer x/y data, the second plots the data following application of the hard iron corrections, ideally centered around (0, 0).

ret = idSoftIronCorrection(mx, my, doPlot)

Calculates corrections used to compensate for soft iron distortion. A two element structure is returned containing the angle between the ellipse major-axis and coordinate system X-axis, and the scale factor necessary to scale the ellipse

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to a circular shape. Note that it is expected that both tilt and hard iron corrections have previously been applied to the data.

Parameters:

mx: Vector of magnetometer x-axis values, specified in Gauss.

my: Vector of magnetometer y-axis values, specified in Gauss.

doPlot: True if uncorrected and corrected data are to be plotted. Two figures are generated, each containing magnetometer x/y data and their associated magnitude.

Returns:

A structure is returned containing the following elements:

theta: Measured angle in radians between the ellipse major-axis and the X-axis of the coordinate system.

scaleFactor: Scale factor to be applied to the magnetometer x-axis data once rotated such that the ellipse major-axis is aligned with the coordinate system x-axis.

Output:

If *doPlot* is *true,* two figures are generated with two plots each: the first figure plots the uncorrected magnetometer x/y data followed by the magnitude of the corresponding data, while the second figure plots the corrected magnetometer x/y data followed by the corresponding magnitude.

ret = applySoftIronCorrection(mx, my, theta, scaleFactor, doPlot)

Calculates heading based on magnetometer x/y data, and applies previously identified soft iron corrections. Returns a three element structure containing the corrected magnetic x and y values specified in Gauss, and heading specified in radians.

Parameters:

mx: Vector of magnetometer x-axis values, specified in Gauss.

my: Vector of magnetometer y-axis values, specified in Gauss.

theta: The angle, in radians, between the ellipse major-axis and the coordinate system x-axis.

scaleFactor: The scale factor to be applied to the magnetometer x-axis data following alignment of the ellipse major-axis and coordinate system X-axis.

doPlot: True if uncorrected and corrected data are to be plotted. A single figure is generated containing four plots: uncorrected soft iron y vs. x data, uncorrected magnitude, corrected soft iron y vs. x data and the corresponding magnitude.

Returns:

A structure is returned containing the following elements:

mx: Vector of soft iron corrected magnetometer x-axis data in Gauss.

my: Vector of soft iron corrected magnetometer y-axis data in Gauss.

heading: Array of heading angles, in radians.

Each element is an array with a size equal to the number of rows in the mx and my vectors.

Output:

If *doPlot* is *true,* a single figure is generated containing four plots: uncorrected soft iron x vs. y data, uncorrected magnitude, corrected soft ifon x vs. y data and the corresponding magnitude.

VIII. References

[1] Retrieved August 4, 2008, from http://www.memsense.com/products/product/moredetails/display.php?product_id=1

[2] Retrieved August 4, 2008, from http://www.memsense.com/products/product/moredetails/display.php?product_id=3

[3] MEMSense nIMU and uIMU devices ship with tri-axial magnetometers.

[4] Konvalin, C.J., June 28, 2008, "Calculating Bank, Elevation and Heading." Retrieved: August 4, 2008, from http://memsense.com/vishal/MTD-0801_1_0_Calculating_Heading_Elevation_Bank_Angle.pdf

[5] Tilt-Sensing with Kionix MEMS Accelerometers. Kionix Corp., NY. Retrieved August 4, 2008 from http://www.kionix.com/App-Notes/AN005%20Tilt%20Sensing.pdf

[6] Caruso, M.J. "Applications of Magnetic Sensors for Low Cost Compass Systems," *Position Location and Navigation Symposium, IEEE 2000*, 13-16 March 2000, pp. 177 – 184.

[7] Jack B. Kuipers, *Quaternions and Rotation Sequences, a Primer with Applications to Orbits, Aerospace, and Virtual Reality*, Princeton University Press, Princeton, 2002.

IX. Change History:

Rev	Status	Description	Date
1.0	Released	New MEMSense Technical Document	06-August-08
1.1	Update	Updated with suggested modifications from Alan A.: inconsistent use of upper/lower case when referencing coordinate systems, grammar, various inconsistencies in source API.	09-September-08
1.2	Update	An error was identified by Sam F. regarding the rotation matrix for alignment of the ellipse with the x-axis. The columns of row two were swapped – shown as [cos, -sin] when should have been [-sin, cos].	04-December-08