

# 3. National Income

Based on Mankiw, Chapters 3 & 4: *National Income: How It Is Earned & National Income: How It Is Spent*

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# Outline of model

- A closed economy, market-clearing model
- Aggregate supply
  - factor markets (supply, demand, price)
  - determination of output/income
- Aggregate demand
  - determinants of  $C$ ,  $I$ , and  $G$
- Equilibrium
  - goods market
  - loanable funds market

**Aggregate supply**  
Aggregate demand  
Equilibrium

# Factors of production

- $K$ : capital
  - tools, machines, and structures used in production
- $L$ : labor
  - the physical and mental efforts of workers

# The production function: $Y = F(K, L)$

- Shows how much output  $Y$  the economy can produce from:
  - $K$  units of capital
  - $L$  units of labor
- Reflects the economy's level of technology
- Reflects how existing technology transforms capital and labor into output
- Technological improvements result in more output from same level of inputs

# Returns to scale

- We start in period 1 with  $Y_1 = F(K_1, L_1)$
- Scale all inputs by the same factor  $z > 0$ :
  - $K_2 = zK_1$  and  $L_2 = zL_1$
  - E.g., if  $z = 1.2$ , then all inputs are increased by 20%
- What happens to output,  $Y_2 = F(K_2, L_2)$ ?
  - $Y_2 = zY_1$  for any  $z > 0$ : ???
  - $Y_2 > zY_1$  for any  $z > 1$ : ???
  - $Y_2 < zY_1$  for any  $z > 1$ : ???

# Returns to scale

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- Scale all inputs by the same factor  $z > 0$ :
  - $K_2 = zK_1$  and  $L_2 = zL_1$
  - E.g., if  $z = 1.2$ , then all inputs are increased by 20%
- What happens to output,  $Y_2 = F(K_2, L_2)$ ?
  - $Y_2 = zY_1$  for any  $z > 0$ : *constant returns to scale*
  - $Y_2 > zY_1$  for any  $z > 1$ : *increasing returns to scale*
  - $Y_2 < zY_1$  for any  $z > 1$ : *decreasing returns to scale*

# Returns to scale: Example 1

$$F(K, L) = \sqrt{KL}$$

$$F(zK, zL) = ???$$



# Returns to scale: Example 1

$$F(K, L) = \sqrt{KL}$$

$$F(zK, zL) = \sqrt{(zK)(zL)}$$

$$= \sqrt{z^2 KL}$$

$$= z\sqrt{KL}$$

$$= zF(K, L)$$

- This production function has constant returns to scale for any  $z > 0$

# Returns to scale: Example 2

$$F(K, L) = K^2 + L^2$$

$$F(zK, zL) = ???$$

# Returns to scale: Example 2

$$F(K, L) = K^2 + L^2$$

$$F(zK, zL) = (zK)^2 + (zL)^2$$

$$= z^2(K^2 + L^2)$$

$$= z^2 F(K, L)$$

- This production function has increasing returns to scale for any  $z > 1$

## NOW YOU TRY

- Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

a.  $F(K, L) = \frac{K^2}{L}$

b.  $F(K, L) = K + L$

# NOW YOU TRY

a.  $F(K, L) = \frac{K^2}{L}$

$$\begin{aligned} F(zK, zL) &= \frac{(zK)^2}{zL} \\ &= \frac{z^2 K^2}{zL} \\ &= z \frac{K^2}{L} \\ &= zF(K, L) \end{aligned}$$

- This production function has constant returns to scale for any  $z > 0$

# NOW YOU TRY

b.  $F(K, L) = K + L$

$$F(zK, zL) = zK + zL$$

$$= z(K + L)$$

$$= zF(K, L)$$

- This production function has constant returns to scale for any  $z > 0$

# Assumptions

1. Technology  $F(\cdot)$  is fixed over time
2. The economy's supplies of capital and labor are fixed:  $K = \bar{K}$  and  $L = \bar{L}$

# Determining GDP

- Output is determined by the fixed factor supplies and the fixed technology

$$\bar{Y} = F(\bar{K}, \bar{L})$$



# Factor prices

- The per unit prices that firms pay for the factors of production
  - Wage: price of  $L$
  - Rental rate: price of  $K$

# Notation

- $W$ : nominal wage
- $R$ : nominal rental rate
- $P$ : price of output
- $W/P$ : real wage (measured in units of output)
- $R/P$ : real rental rate

# How factor prices are determined

- Quick pause: supply/demand of *factors* vs. *final goods!*
- Factor prices are determined by supply and demand in factor markets
- Recall that the supply of each factor is fixed
- What about demand?

# Demand for labor

- Assume that markets are competitive: each firm takes  $W$ ,  $R$ , and  $P$  as given
- Idea: a firm hires each unit of labor if the cost does not exceed the benefit
  - Cost: ???
  - Benefit: ???

# Demand for labor

- Assume that markets are competitive: each firm takes  $W$ ,  $R$ , and  $P$  as given
- Idea: a firm hires each unit of labor if the cost does not exceed the benefit
  - Cost: real wage
  - Benefit: marginal product of labor ( $MPL$ )

# Marginal product of labor (*MPL*)

- Definition: the extra output firms can produce using an additional unit of labor
- ... holding other inputs fixed

$$MPL = F(K, L + 1) - F(K, L)$$

or

$$MPL = \frac{\partial F(K, L)}{\partial L}$$

# NOW YOU TRY

- a. Determine  $MPL$  at each value of  $L$
- b. Graph the production function
- c. Graph the  $MPL$  curve with  $MPL$  on the vertical axis and  $L$  on the horizontal axis

$L$	$Y$	$MPL$
0	0	n.a.
1	10	
2	19	
3	27	
4	34	
5	40	
6	45	
7	49	
8	52	
9	54	
10	55	

# NOW YOU TRY

- a. Determine  $MPL$  at each value of  $L$
- b. Graph the production function
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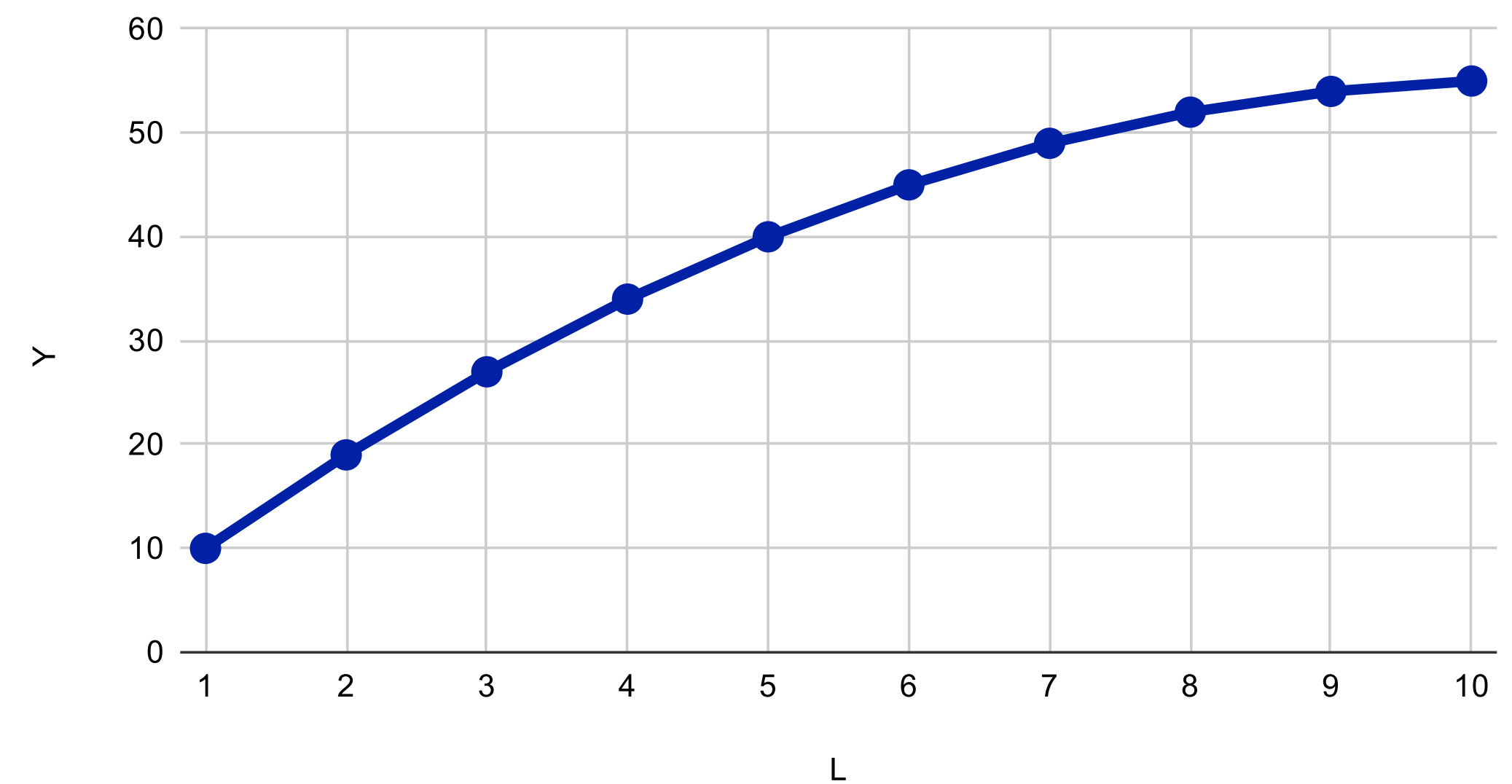
$L$	$Y$	$MPL$
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1



# NOW YOU TRY

- a. Determine  $MPL$  at each value of  $L$
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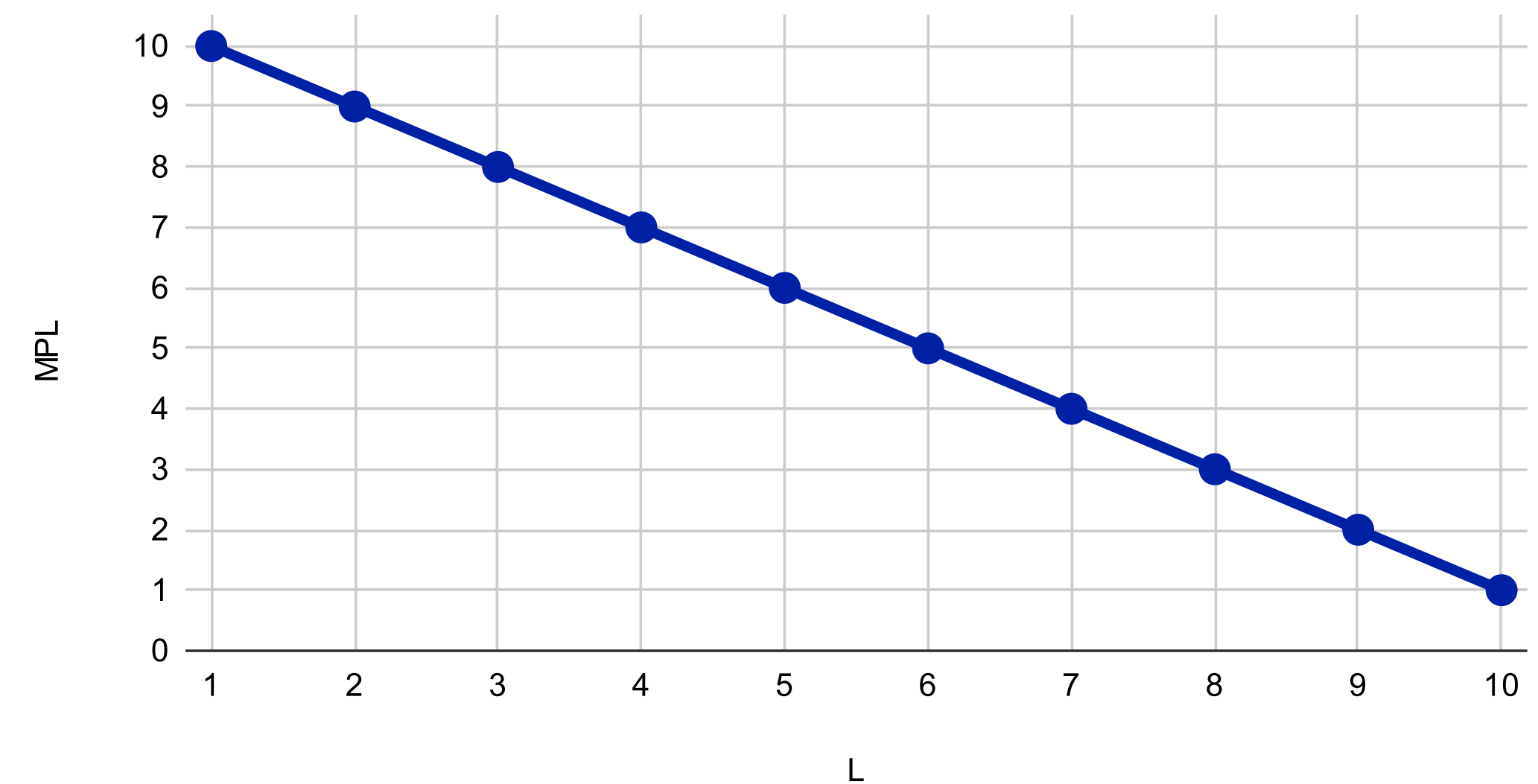
Production function



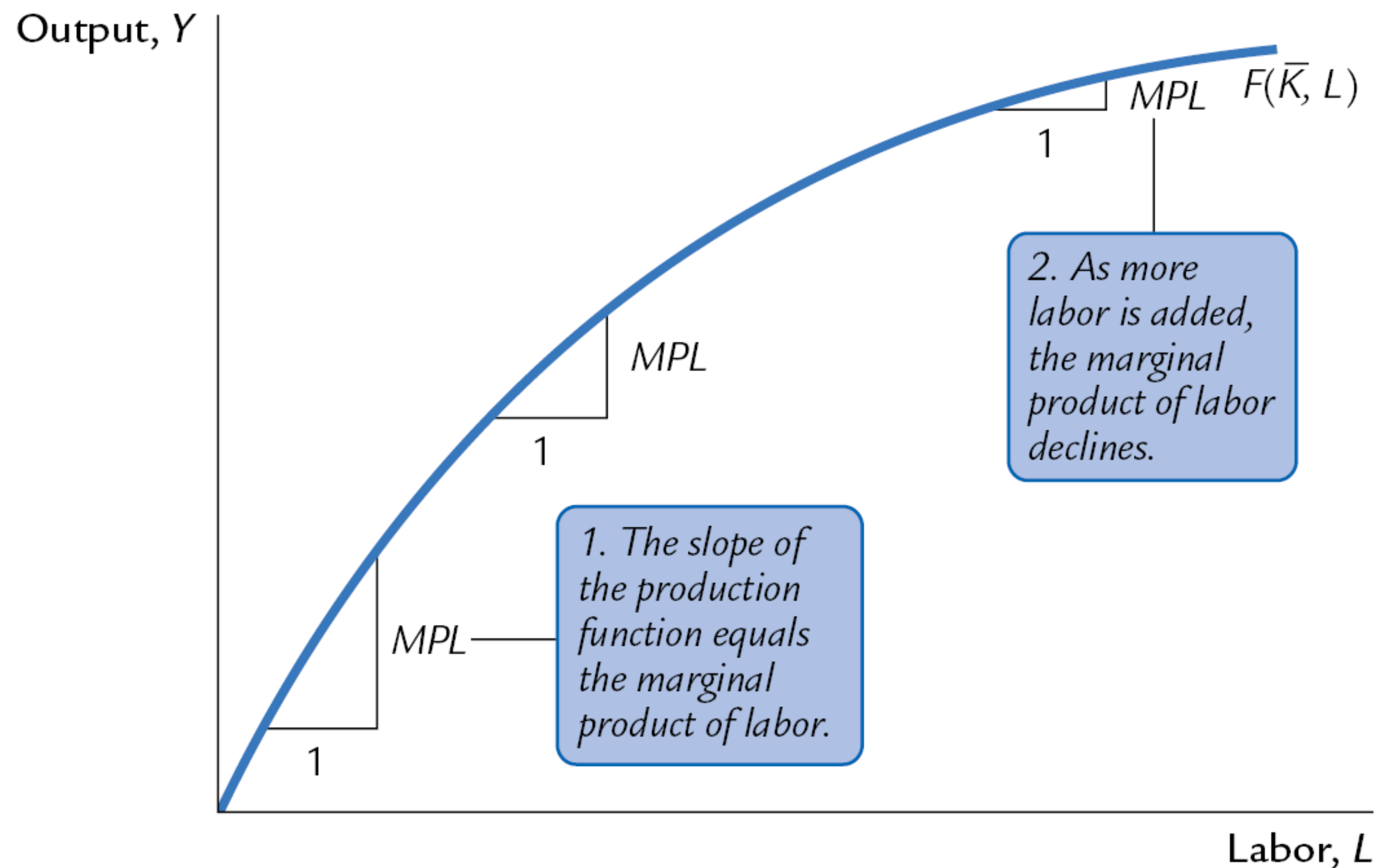
# NOW YOU TRY

- a. Determine  $MPL$  at each value of  $L$
- b. Graph the production function
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Marginal product of labor



# *MPL* and the production function



# Diminishing marginal returns

- As an input is increased (holding other inputs fixed), its marginal product falls
- Intuition:
  - If  $L$  increases while holding  $K$  fixed, machines per worker fall, worker productivity falls

# NOW YOU TRY

- Do these production functions have diminishing marginal returns to labor?
  - a.  $F(K, L) = 2K + 15L$
  - b.  $F(K, L) = \sqrt{KL}$
  - c.  $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

# NOW YOU TRY

- Do these production functions have diminishing marginal returns to labor?

a.  $F(K, L) = 2K + 15L$

$MPL = 15 \implies$  **no**, because it is constant as  $L$  increases

b.  $F(K, L) = \sqrt{KL}$

$$MPL = \frac{\sqrt{K}}{2\sqrt{L}} \implies \text{yes}, \text{ because it is decreasing in } L$$

c.  $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

$$MPL = \frac{15}{2\sqrt{L}} \implies \text{yes}, \text{ because it is decreasing in } L$$

# NOW YOU TRY

- Suppose  $W/P = 6$
- If  $L = 3$ , should the firm hire more or less labor? Why?
- If  $L = 7$ , should the firm hire more or less labor? Why?

<b><i>L</i></b>	<b><i>Y</i></b>	<b><i>MPL</i></b>
0	0	n.a.
1	10	10
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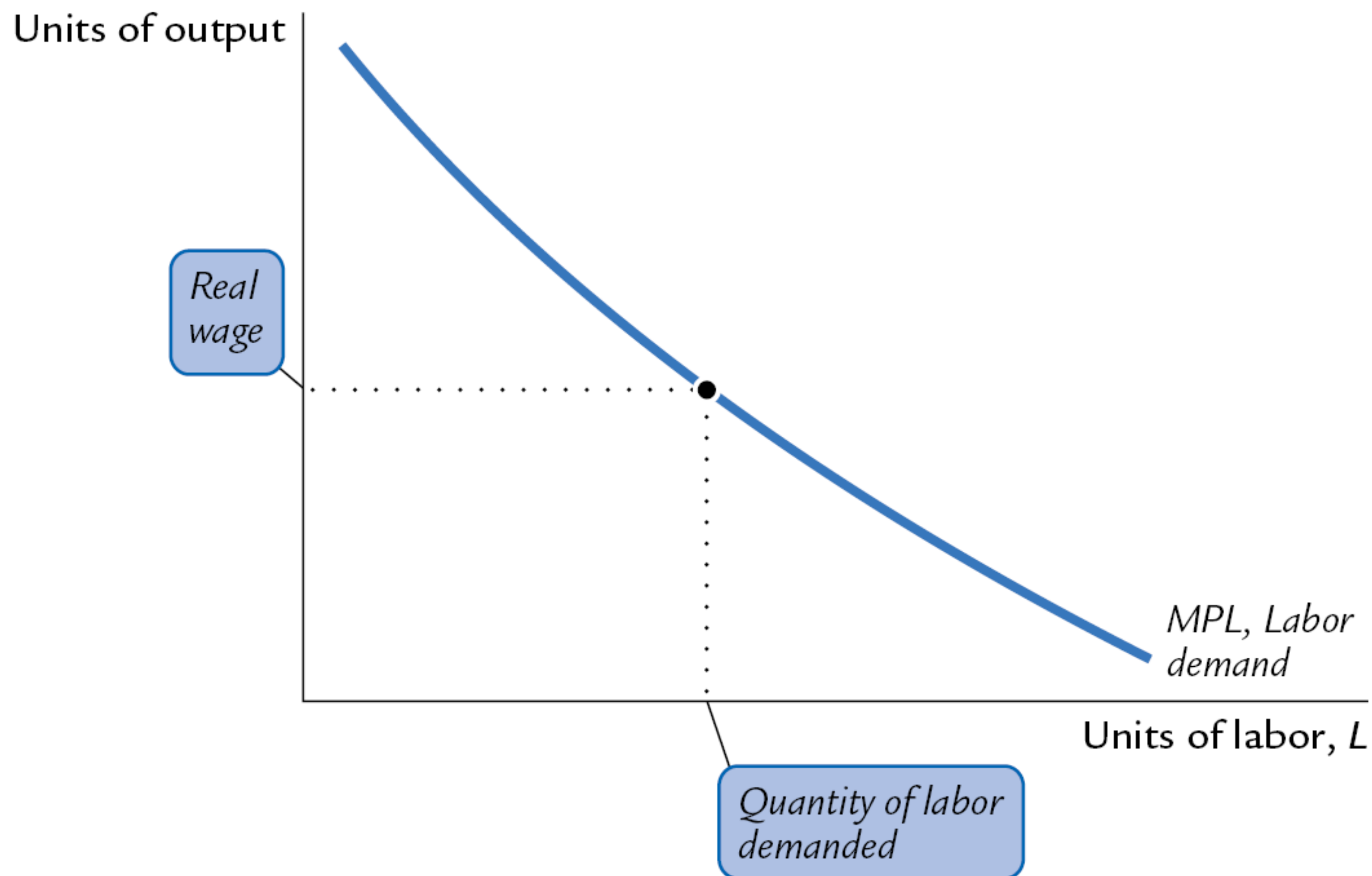
# NOW YOU TRY

- Suppose  $W/P = 6$
- If  $L = 3$ , should the firm hire more or less labor? Why?
  - **Answer:** More, because the benefit of the fourth worker ( $MPL = 7$ ) exceeds its cost ( $W/P = 6$ )
- If  $L = 7$ , should the firm hire more or less labor? Why?
  - **Answer:** Less, because the seventh worker adds  $MPL = 4$  units of output but costs the firm  $W/P = 6$

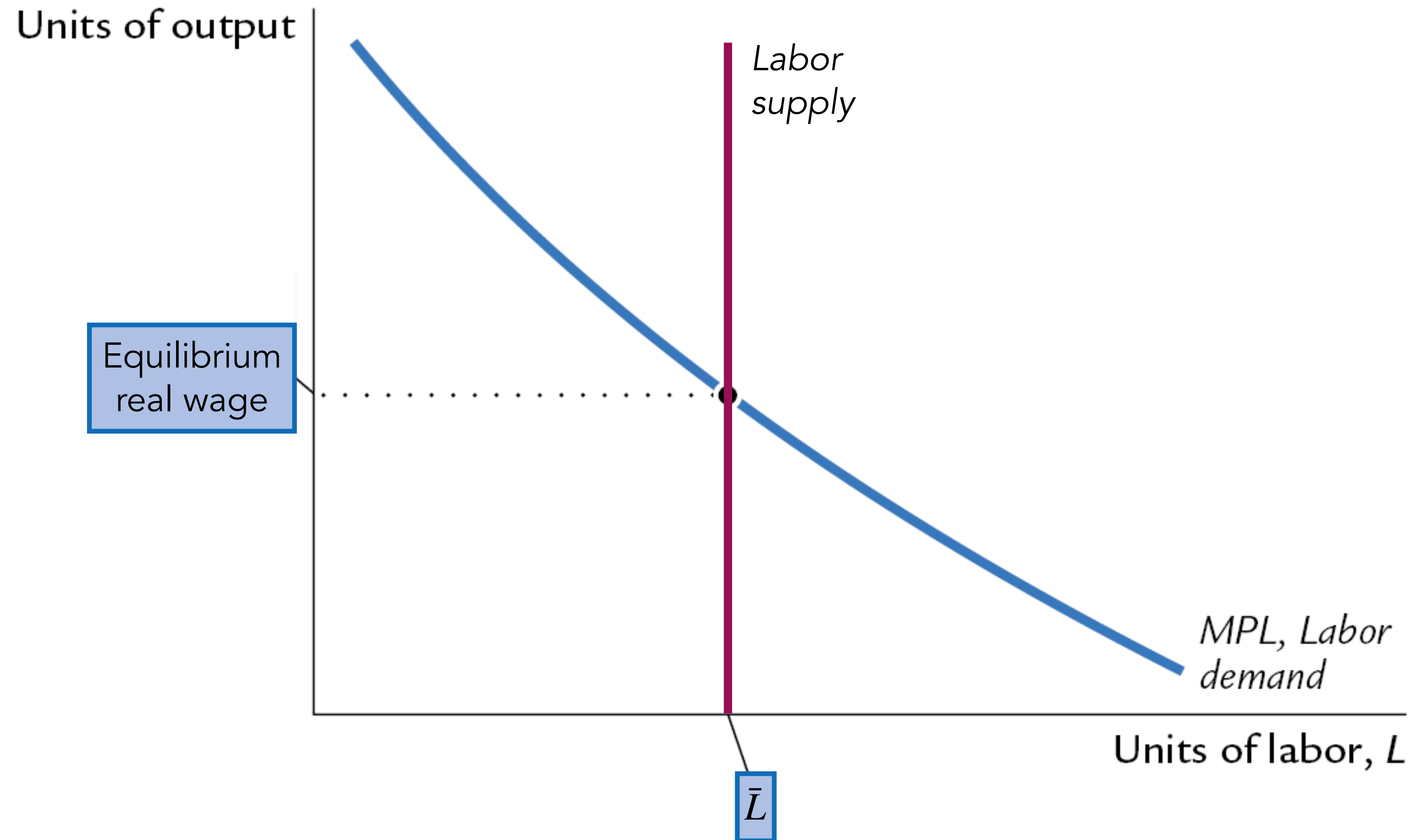
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# *MPL* and the demand for labor



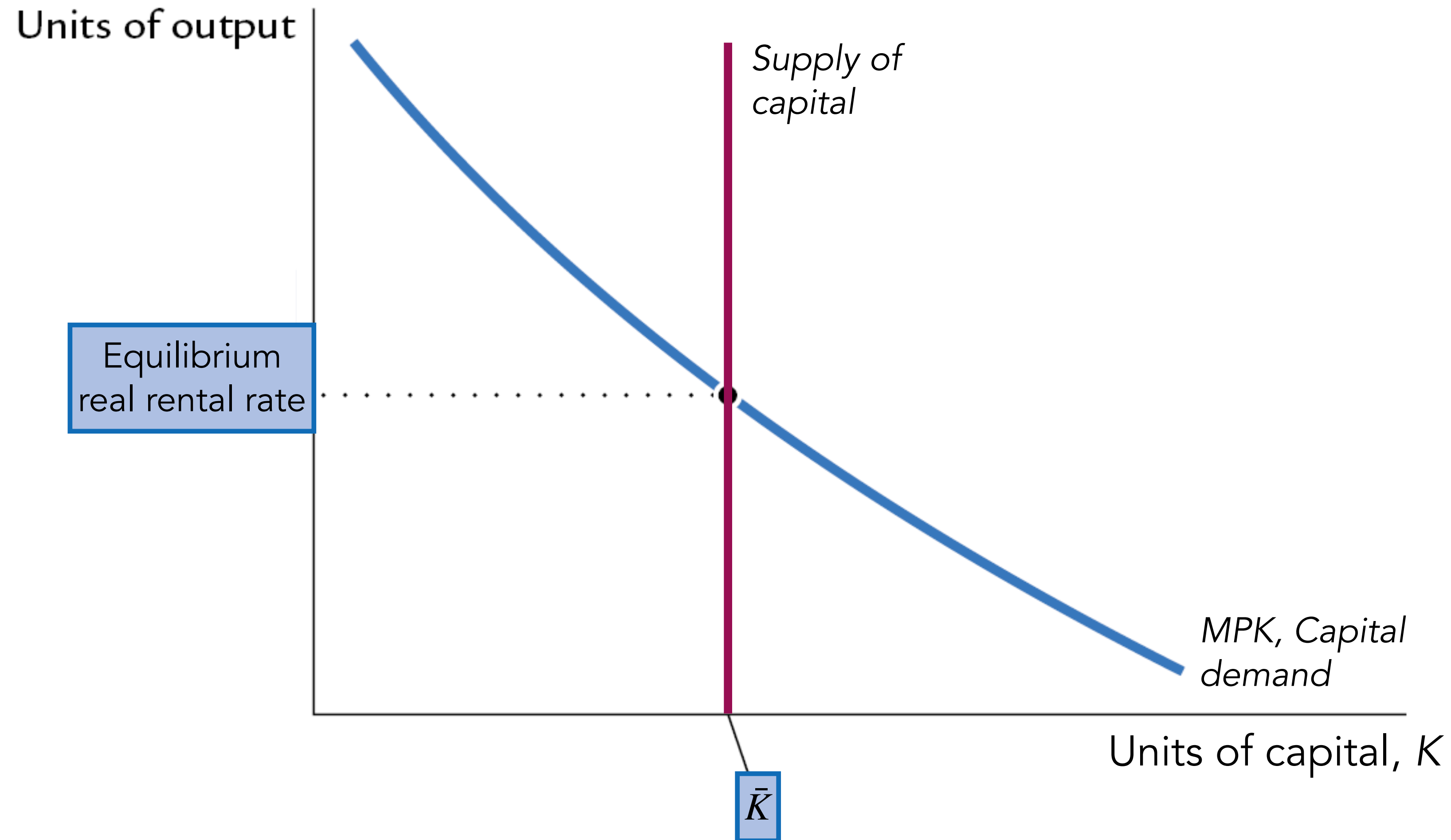
# The equilibrium real wage



# Determining the rental rate

- We have just seen that  $MPL = W/P$
- The same logic shows that  $MPK = R/P$ 
  - Diminishing returns to capital:  $MPK$  falls as  $K$  rises
  - The  $MPK$  curve is the firm's demand curve for renting capital
  - Firms maximize profits by choosing  $K^*$  such that  $MPK = R/P$

# The equilibrium real rental rate



# Neoclassical theory of distribution

- States that each factor input is paid its marginal product
- A good starting point for thinking about income distribution
- Examples where this does not hold? *Class discussion*

# How income is distributed to L and K

- Total labor income:  $\frac{W}{P}\bar{L} = MPL \times \bar{L}$
- Total capital income:  $\frac{R}{P}\bar{K} = MPK \times \bar{K}$
- If the production function has constant returns to scale, then

$$\bar{Y} = \underbrace{MPK \times \bar{K}}_{\text{capital income}} + \underbrace{MPL \times \bar{L}}_{\text{labor income}}$$

# Cobb–Douglas production function

- Cobb–Douglas production function:

$Y = AK^\alpha L^{1-\alpha}$  where  $A$  represents the level of technology

- The Cobb–Douglas production function has constant factor shares
  - $\alpha$ : capital's share of total income. Why?

# Cobb–Douglas production function

- Cobb–Douglas production function:

$Y = AK^\alpha L^{1-\alpha}$  where  $A$  represents the level of technology

- The Cobb–Douglas production function has constant factor shares

- $\alpha$ : capital's share of total income. Why?

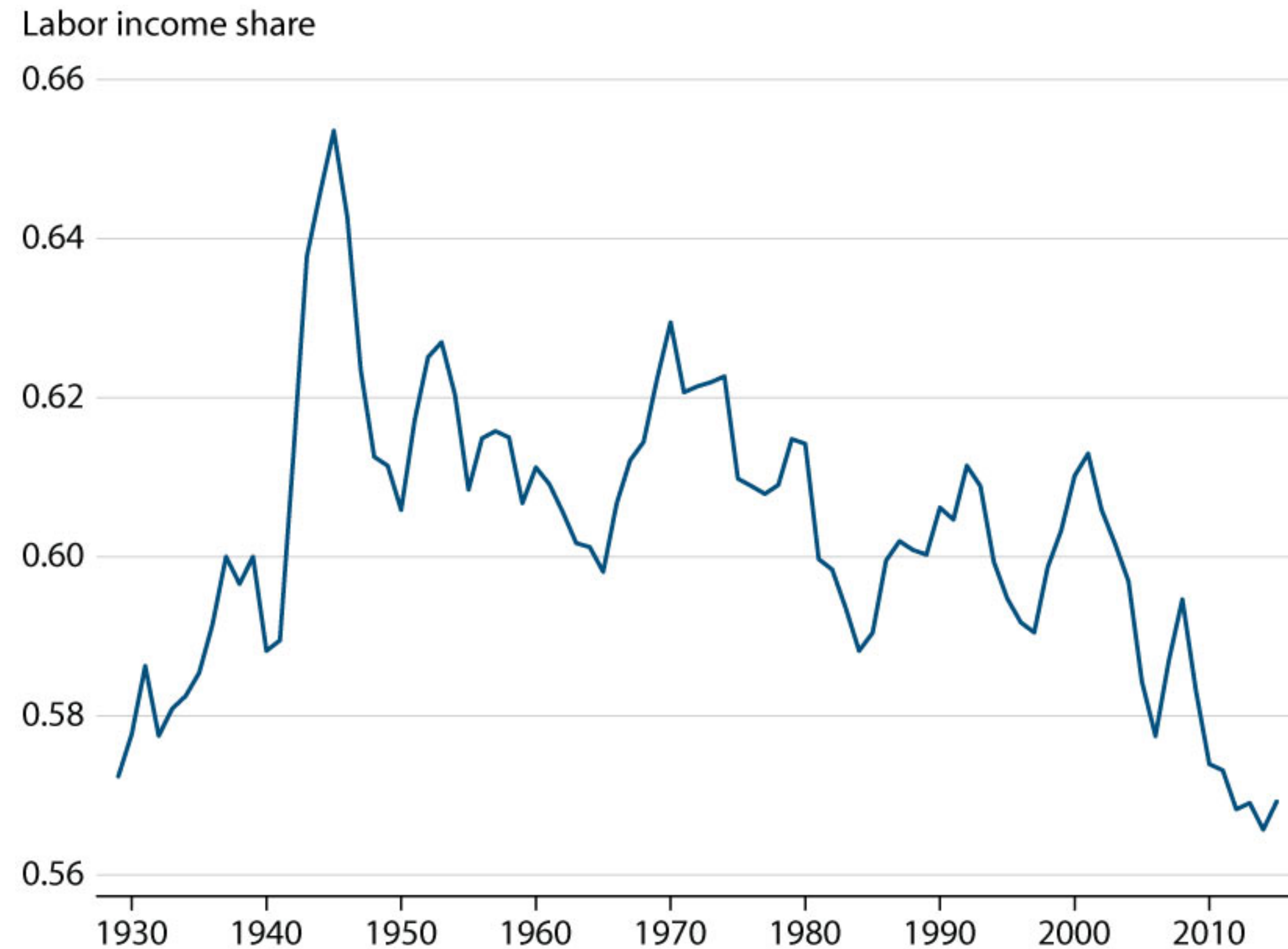
$$MPK = A\alpha K^{\alpha-1}L^{1-\alpha} = \frac{\alpha Y}{K} \text{ and } MPL = AK^\alpha(1-\alpha)L^{-\alpha} = \frac{(1-\alpha)Y}{L}$$

$$\implies \bar{Y} = MPK \times \bar{K} + MPL \times \bar{L} = \alpha \bar{Y} + (1-\alpha)\bar{Y}$$

- Does this hold true in the data?



# Declining labor share in the US



# Labor share and income inequality

- Lower labor share over time exacerbates income disparity
- Why?
  - Technological advances reduce the role of labor in production
  - Market power of firms increases while worker power decreases

# Skill-biased technological change

- Technological progress increased the relative demand for skilled labor
- Since the '70s, technological progress continues but education is stagnant
- Therefore, skill-biased technological change increases wage inequality
  - Growing demand & stagnant supply of skilled workers
  - Skilled workers' wages grow compared to unskilled ones
  - Over time, wage inequality exacerbates income inequality

# Globalization and inequality

- US trade growth also increases the relative demand for skilled labor
- Therefore, inequality grows
- However, trade is still beneficial on aggregate (more on this later)

# Further sources of inequality

- Educational slowdown
- Falling marriage rates among higher-educated workers (assortative matching)

Aggregate supply  
**Aggregate demand**  
Equilibrium

# Demand for goods and services

1 of 2

- Recall the determinants of the four components of GDP
  - $C$ : consumption
  - $I$ : investment
  - $G$ : government expenditures
  - $NX$ : net exports

# Demand for goods and services

2 of 2

- Components of aggregate demand:
  - $C$ : consumer demand for goods and services
  - $I$ : demand for investment goods
  - $G$ : government demand for goods and services
- closed economy: no  $NX$



# Consumption, $C$

1 of 2

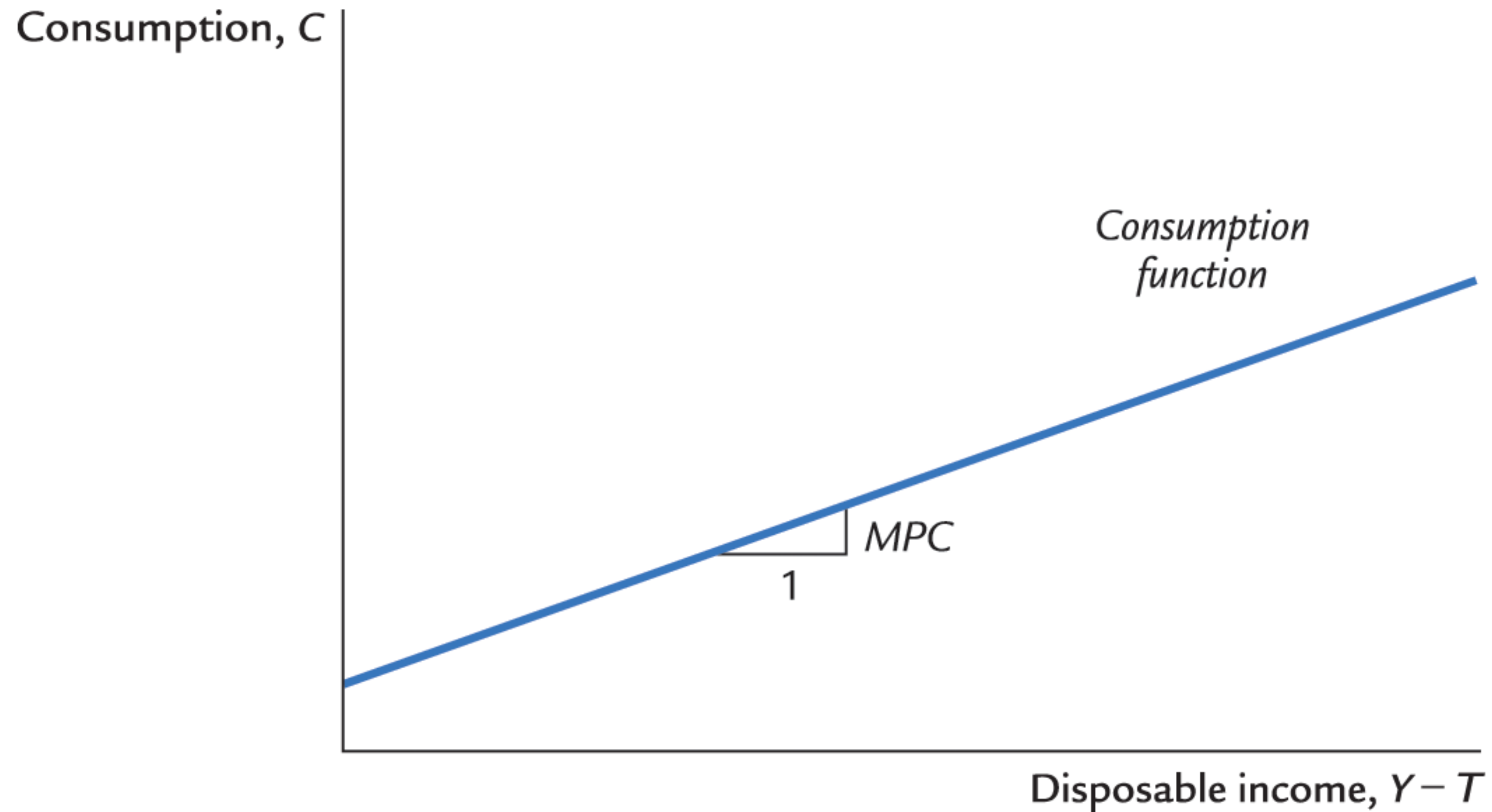
- Households receive income from their labor and from owning capital
- They pay taxes to the government
- They decide how much of their after-tax income to consume and save
- Consumption/savings are the typical decisions in our economic models

# Consumption, $C$

2 of 2

- *Disposable income* is total income minus total taxes:  $Y - T$
- Consumption function:  $C = C(Y - T)$
- *Marginal propensity to consume (MPC)*
  - The change in consumption  $C$  when disposable income  $Y - T$  increases by one dollar

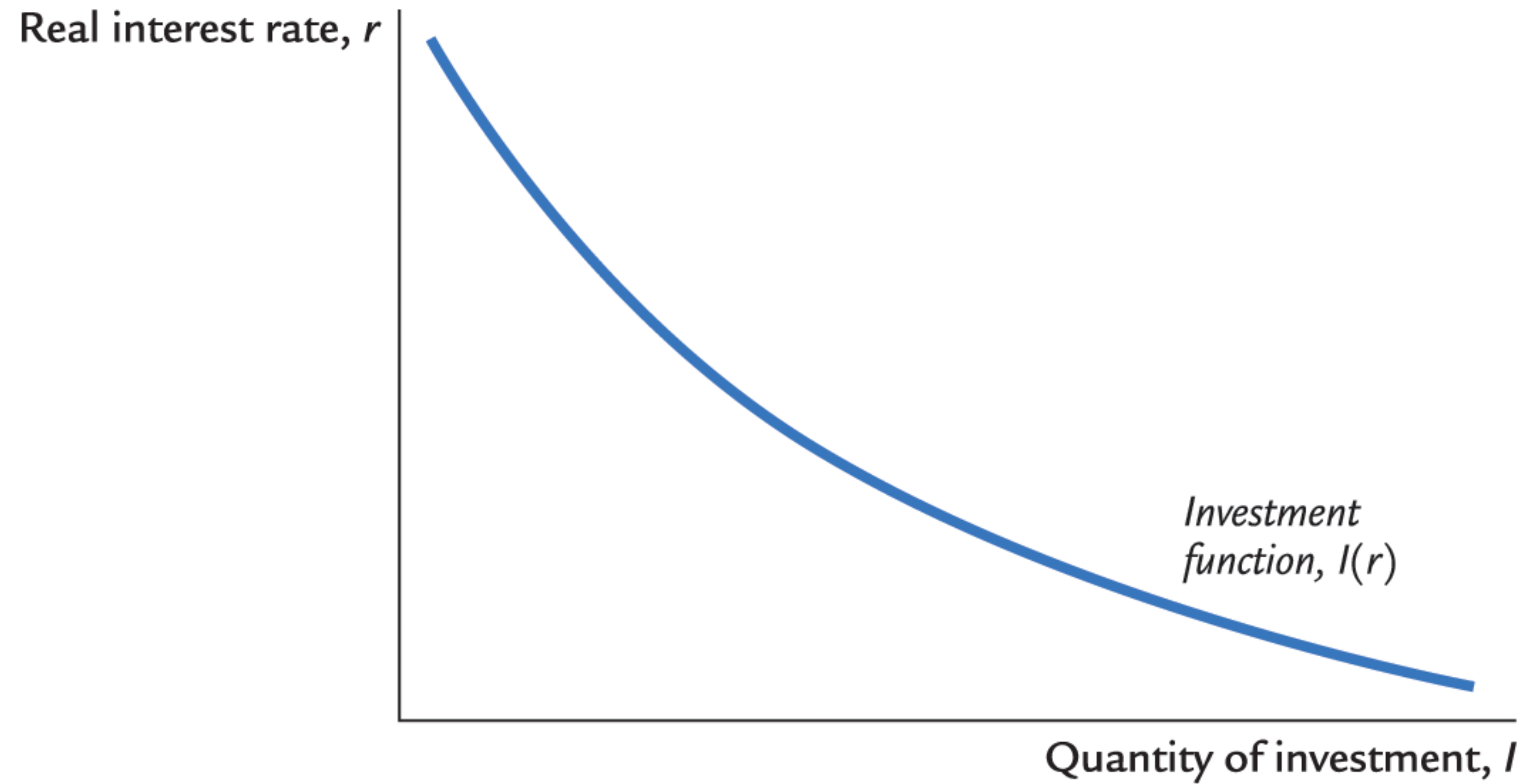
# The consumption function



# Investment, $I$

- Investment function:  $I = I(r)$ 
  - $r$  denotes the real interest rate, the nominal interest rate corrected for inflation
- The real interest rate is
  - ... the cost of borrowing
  - ... the opportunity cost of using one's own funds to finance investment spending
- So,  $I$  depends negatively on  $r$ 
  - Note that  $r$  is the real interest rate, not the rental rate of capital  $R$

# The investment function



# Government spending, $G$

- $G$ : government spending on goods and services
- $G$  excludes transfer payments
  - E.g., Social Security benefits, unemployment insurance benefits
- Assume that government spending and total taxes are exogenous:  
 $G = \bar{G}$  and  $T = \bar{T}$

Aggregate supply  
Aggregate demand  
**Equilibrium**

# The market for goods and services

Aggregate demand:  $C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

Aggregate supply:  $\bar{Y} = F(\bar{K}, \bar{L})$

Equilibrium:  $\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

- The real interest rate adjusts to equate demand with supply



# The loanable funds market

- A simple supply–demand model of the financial system
- One asset: “loanable funds”
  - demand for funds: investment
  - supply of funds: saving
  - “price” of funds: real interest rate

# Demand for funds: investment

- The demand for loanable funds
  - ... comes from investment
    - Firms borrow to finance spending on plants and equipment, new office buildings, etc.
    - Consumers borrow to buy new houses
  - ...depends negatively on  $r$ 
    - $r$  is the “price” of loanable funds (cost of borrowing)

# Supply of funds: saving

- The supply of loanable funds comes from saving
  - Households use their savings to make bank deposits and purchase assets
  - These funds become available to firms to borrow and finance investment spending
  - The government may also contribute to saving if it does not spend all the tax revenue

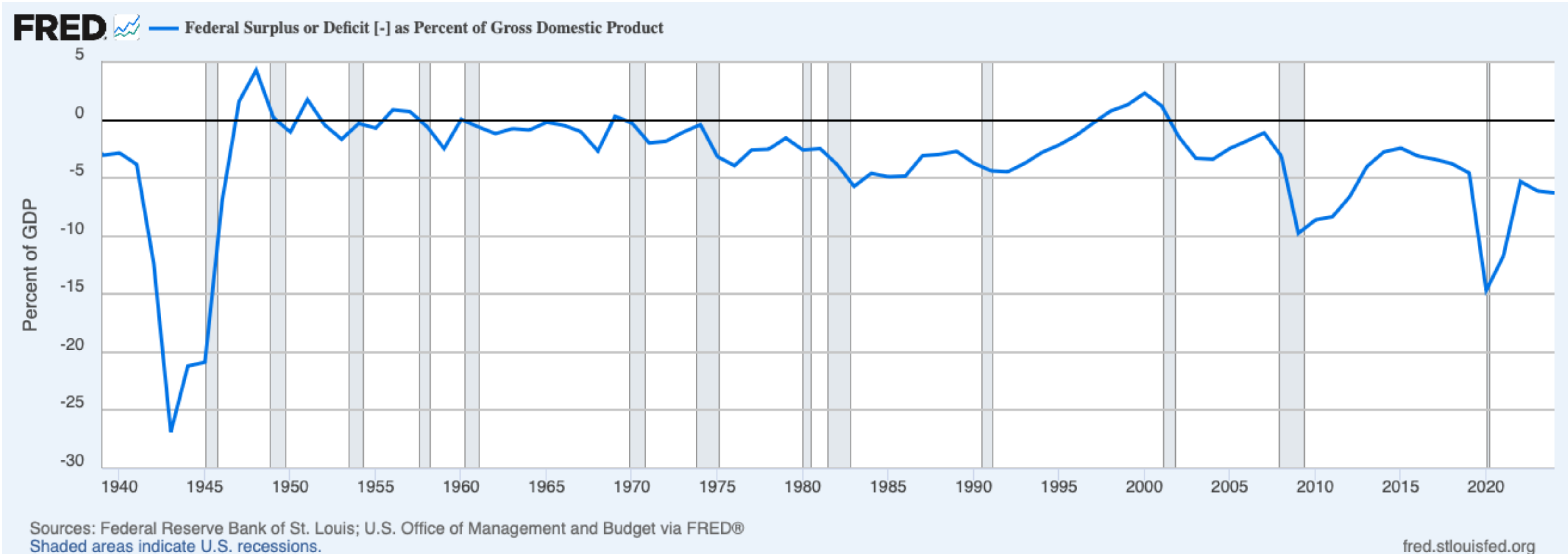
# Types of saving

- *Private saving:*  $(Y - T) - C$
- *Public saving:*  $T - G$
- *National saving:*  $S = \text{private saving} + \text{public saving}$   
 $= (Y - T) - C + T - G$   
 $= Y - C - G$   
 $= I$

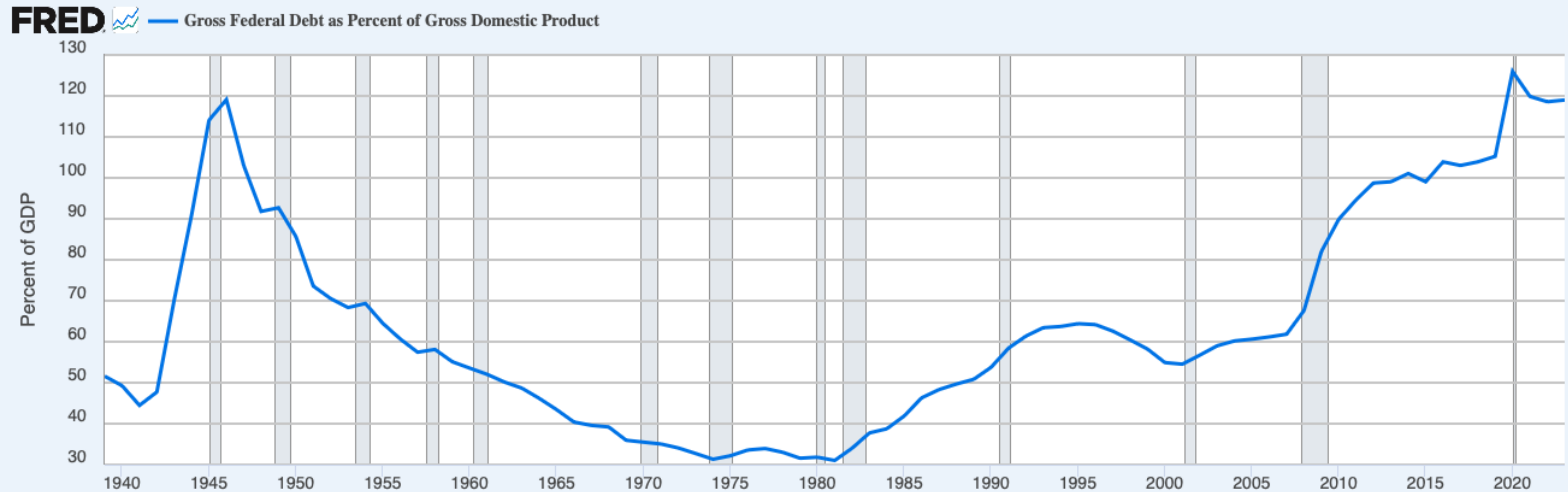
# Budget surpluses and deficits

- *Budget surplus:  $T > G$* 
  - Surplus:  $T - G > 0$
- *Budget deficit:  $T < G$* 
  - Deficit:  $T - G < 0$  (often expressed as  $|T - G|$ )
- *Balanced budget:  $T = G$*
- The U.S. government finances its deficit by issuing Treasury bonds
  - I.e., borrowing from households and firms

# U.S. federal gov't surplus/deficit



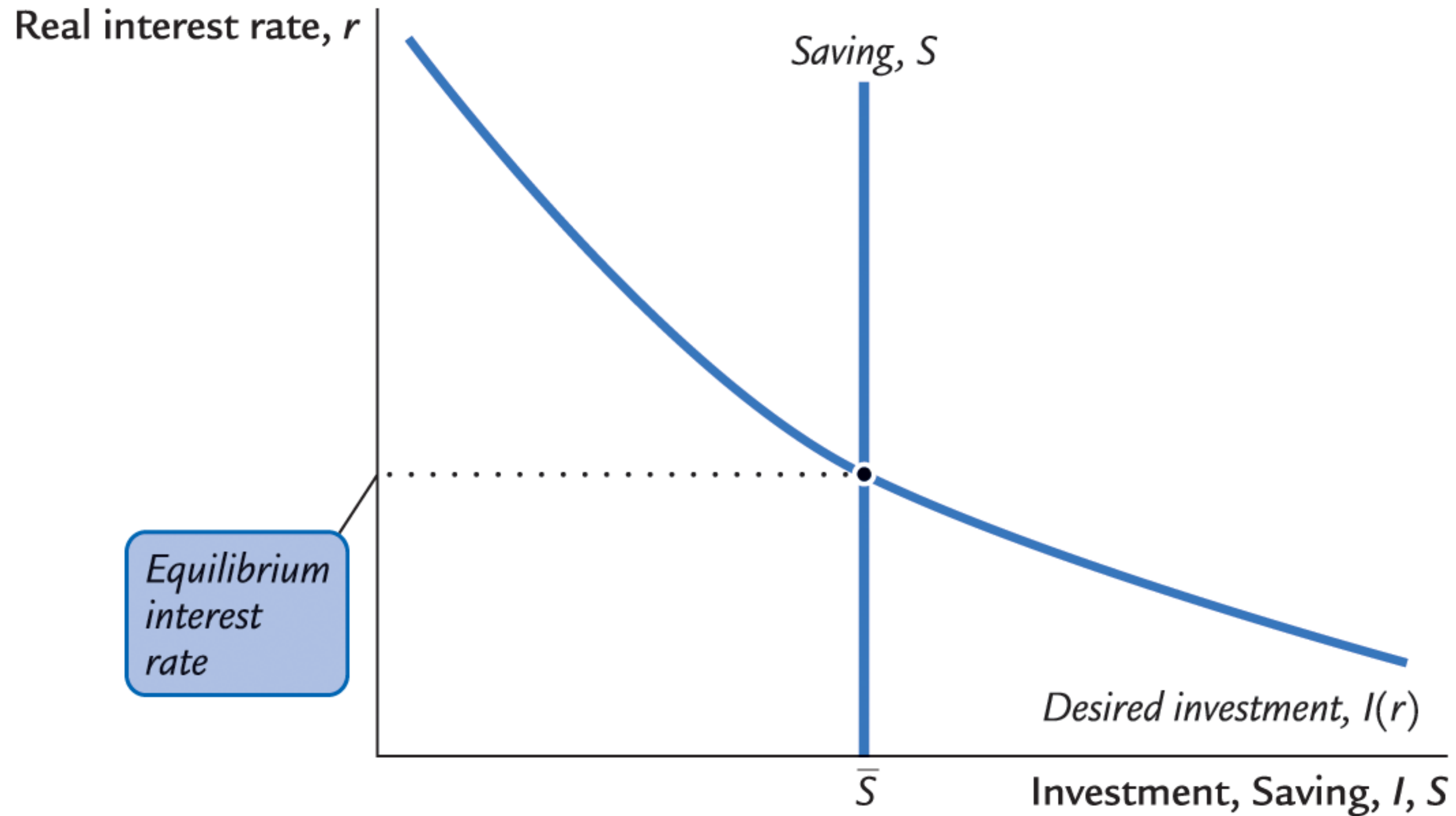
# U.S. federal gov't debt



Sources: Federal Reserve Bank of St. Louis; U.S. Office of Management and Budget via FRED®  
Shaded areas indicate U.S. recessions.

[fred.stlouisfed.org](https://fred.stlouisfed.org)

# Loanable funds equilibrium





# The special role of $r$

- $r$  adjusts to equilibrate the goods and loanable funds markets simultaneously
  - If the loanable funds market is in equilibrium, then
$$Y - C - G = I$$
  - Add  $(C + G)$  to both sides to get
$$Y = C + I + G \text{ (goods market equilibrium)}$$
- Thus: equilibrium in loanable funds market  $\Leftrightarrow$  equilibrium in goods market

# Recap: Mastering models

- To master a model, be sure to know:
  1. Which of its variables are endogenous and which are exogenous
  2. For each curve in the diagram, know:
    - 2.1. definition
    - 2.2. intuition for slope
    - 2.3. all the things that can shift the curve
  3. Use the model to analyze the effects of each item in 2.3.
- What are these in the loanable funds model?

# Mastering the loanable funds model

1 of 2

- Things that shift the saving curve
  - Public saving
    - Fiscal policy: changes in  $G$  or  $T$
  - Private saving
    - Preferences
    - Tax laws: 401(k), IRA, replacing income tax with consumption tax

# CASE STUDY: The Reagan Deficits

1 of 2

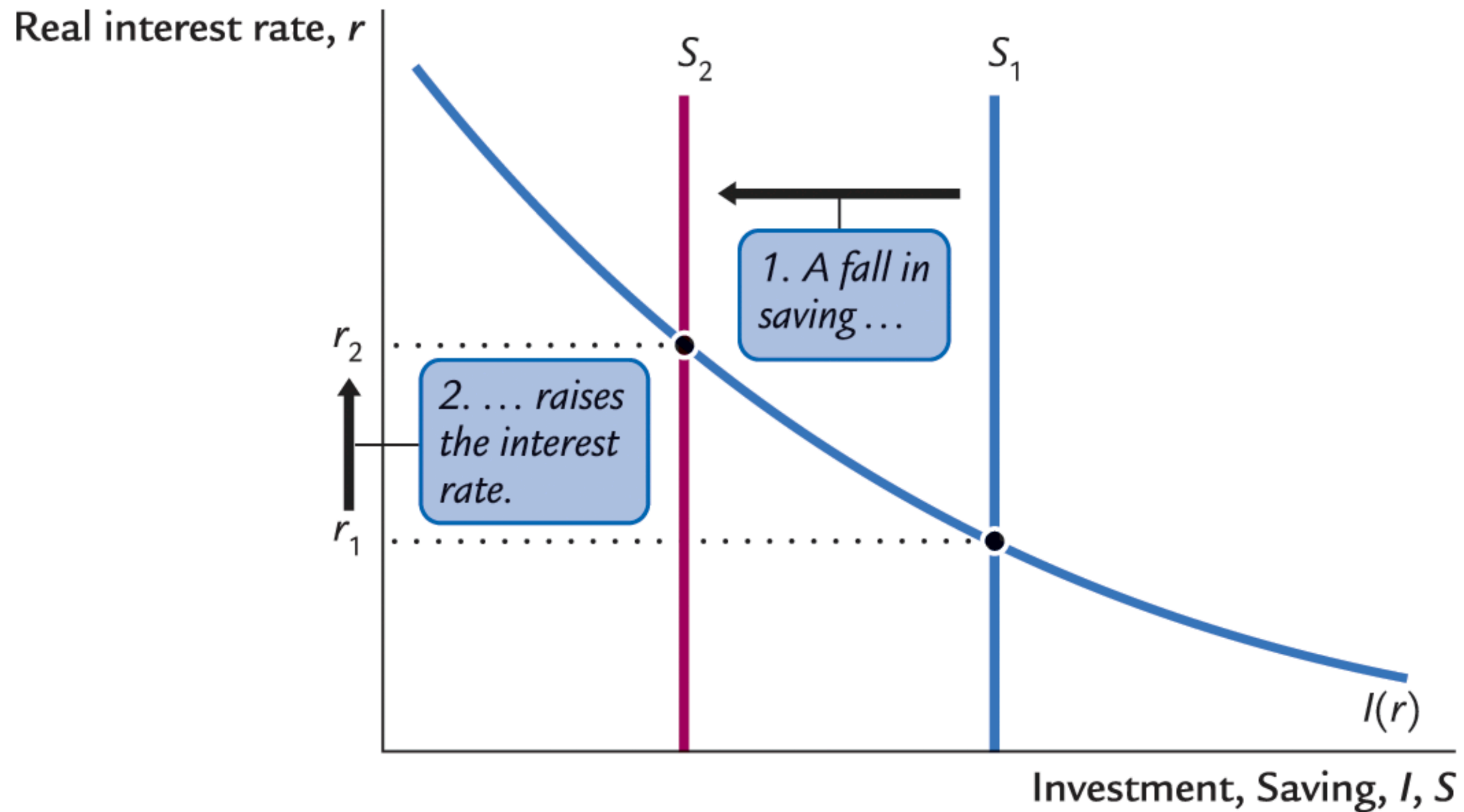
- Reagan policies during early 1980s
  - Increases in defense spending:  $\Delta G > 0$
  - Big tax cuts:  $\Delta T < 0$
- Both policies reduce national saving:

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}$$

$$\uparrow \bar{G} \implies \downarrow \bar{S} \quad \downarrow \bar{T} \implies \uparrow C \implies \downarrow \bar{S}$$

# CASE STUDY: The Reagan Deficits

2 of 2

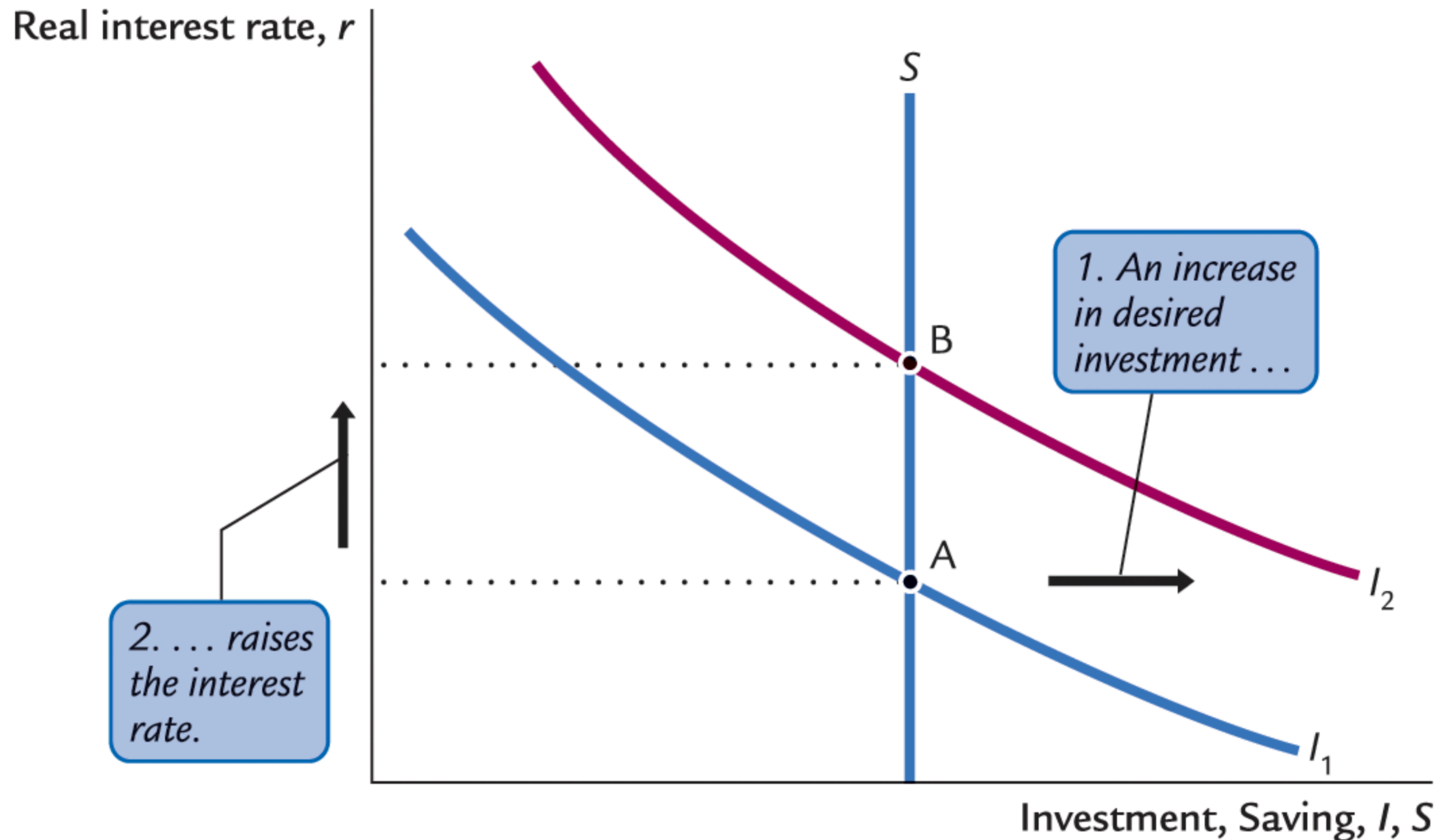


# Mastering the loanable funds model

2 of 2

- Things that shift the investment curve:
  - Technological innovations
    - ...to take advantage of innovations, firms must buy new investment goods
  - Tax laws that affect investment
    - E.g., investment tax credit

# An increase in investment demand



## NOW YOU TRY

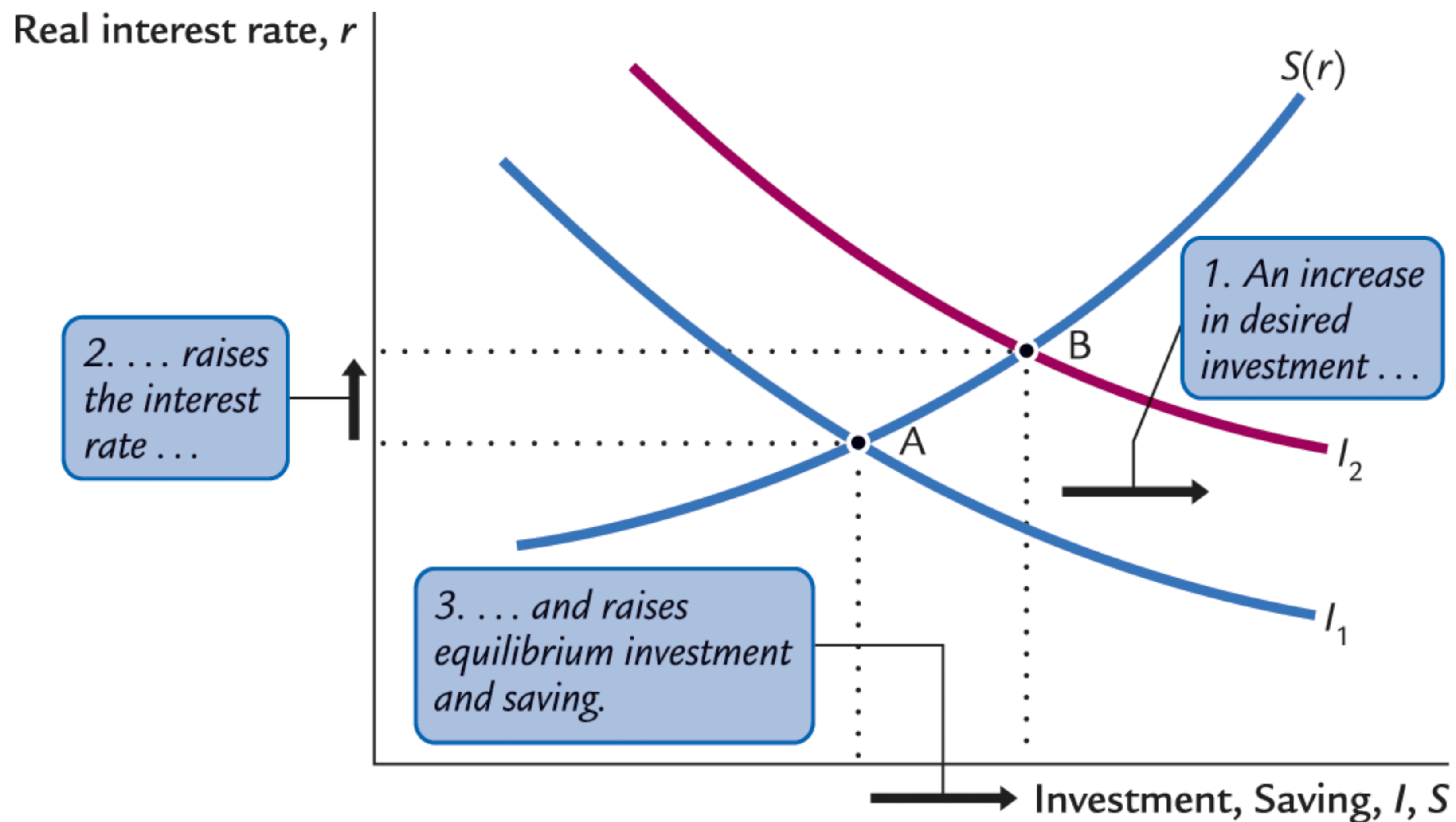
- New legislation requires employers to match 401(k) contributions
- What happens to the savings and investment curves?
- What happens to the interest rate and the level of savings/investment?



# Saving and the interest rate

- So far, we assumed that saving does not depend on the interest rate
- Why might saving depend on  $r$ ?
- How would the results of an increase in investment demand be different?
  - Would  $r$  rise as much?
  - Would the equilibrium value of  $I$  change?

# When saving depends on $r$



# SUMMARY

1 of 4

- Total output is determined by
  - ... the economy's quantities of capital and labor
  - ... the level of technology
- Competitive firms hire each factor until its marginal product equals its price
- With constant returns to scale, labor income + capital income = total output

# SUMMARY

2 of 4

- Increasing inequality among workers can be explained by numerous factors
  - Declining labor share
  - Education slowdown among skilled workers
  - Rise of globalization
  - Various cultural changes

# SUMMARY

3 of 4

- In a closed economy, output is used for
  - ... consumption
  - ... investment
  - ... government spending
- The real interest rate adjusts to equate the demand for and supply of
  - ... goods and services
  - ... loanable funds

# SUMMARY

4 of 4

- A decrease in saving causes
  - ... the interest rate to rise
  - ... investment to fall
- An increase in investment demand causes
  - ... the interest rate to rise
  - ... but does not affect the level of investment if the supply of loanable funds is fixed