6. Capital Accumulation and Economic Growth

Based on Mankiw, Chapter 9: *Capital Accumulation as a Source of Growth*

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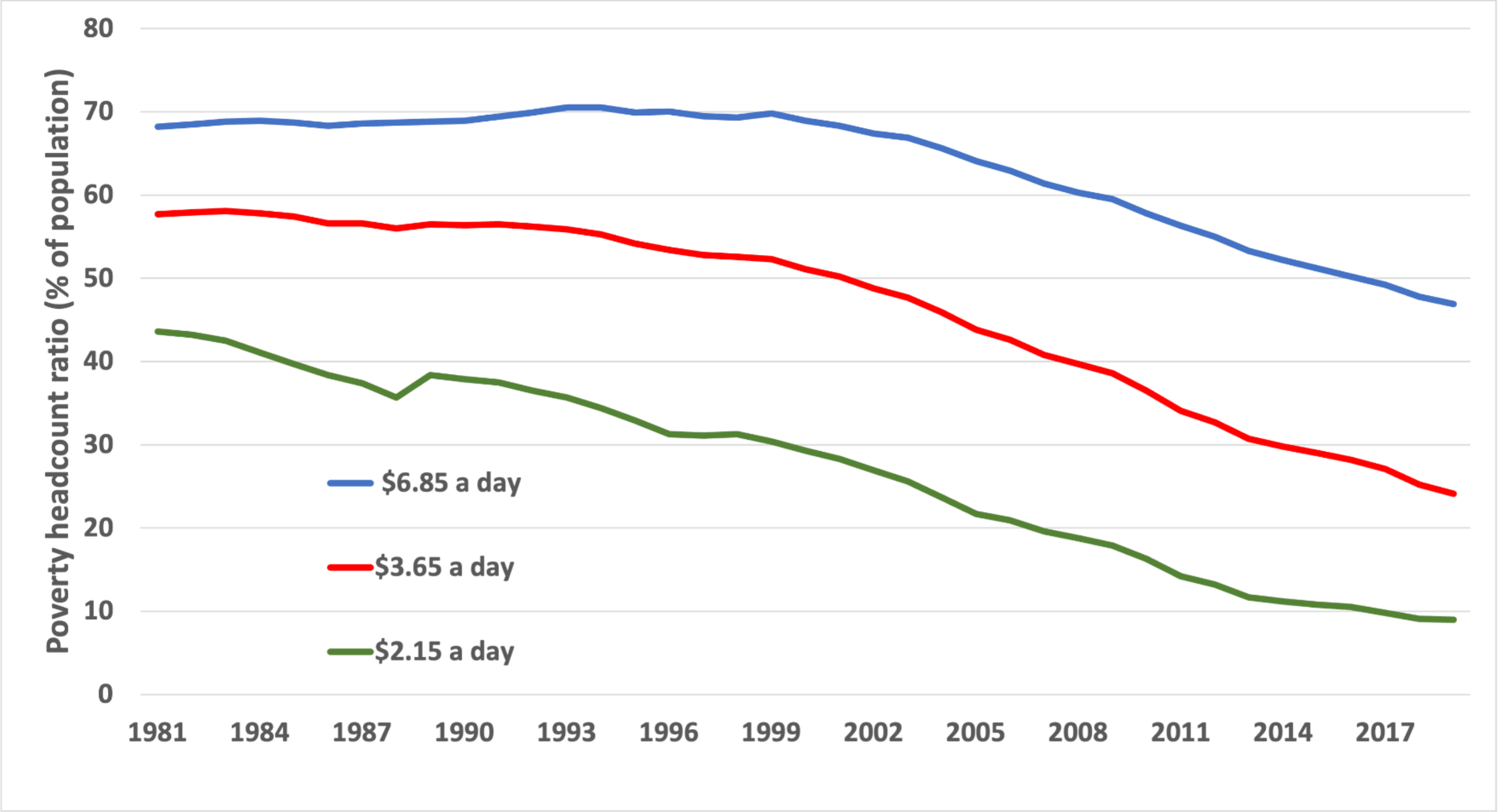
Why growth matters (1 of 2)

Economic growth raises living standards and reduces poverty . . .



Source: World Bank, Poverty and Inequality Platform

Why growth matters (2 of 2)

Anything that affects the long-run rate of economic growth—even by a tiny amount—will have huge effects on living standards in the long run.

Annual growth rate of income per capita	Increase in standard of living after 25 years	Increase in standard of living after 50 years	Increase in standard of living after 100 years
2.0%	64.0%	169.2%	624.5%
2.5%	85.4%	243.7%	1,081.4%

The lessons of growth theory

. . . can make a positive difference in the lives of hundreds of millions of people.

These lessons help us

- understand why poor countries are poor
- design policies that can help them grow
- learn how our own growth rate is affected by shocks and our government's policies

The Solow model

The Solow model

- It is named after Robert Solow, who won the 1987 Nobel Prize for contributions to the study of economic growth.
- It is
  - widely used in policymaking
  - a benchmark against which most recent growth theories are compared
- It looks at the determinants of economic growth and the standard of living in the long run.

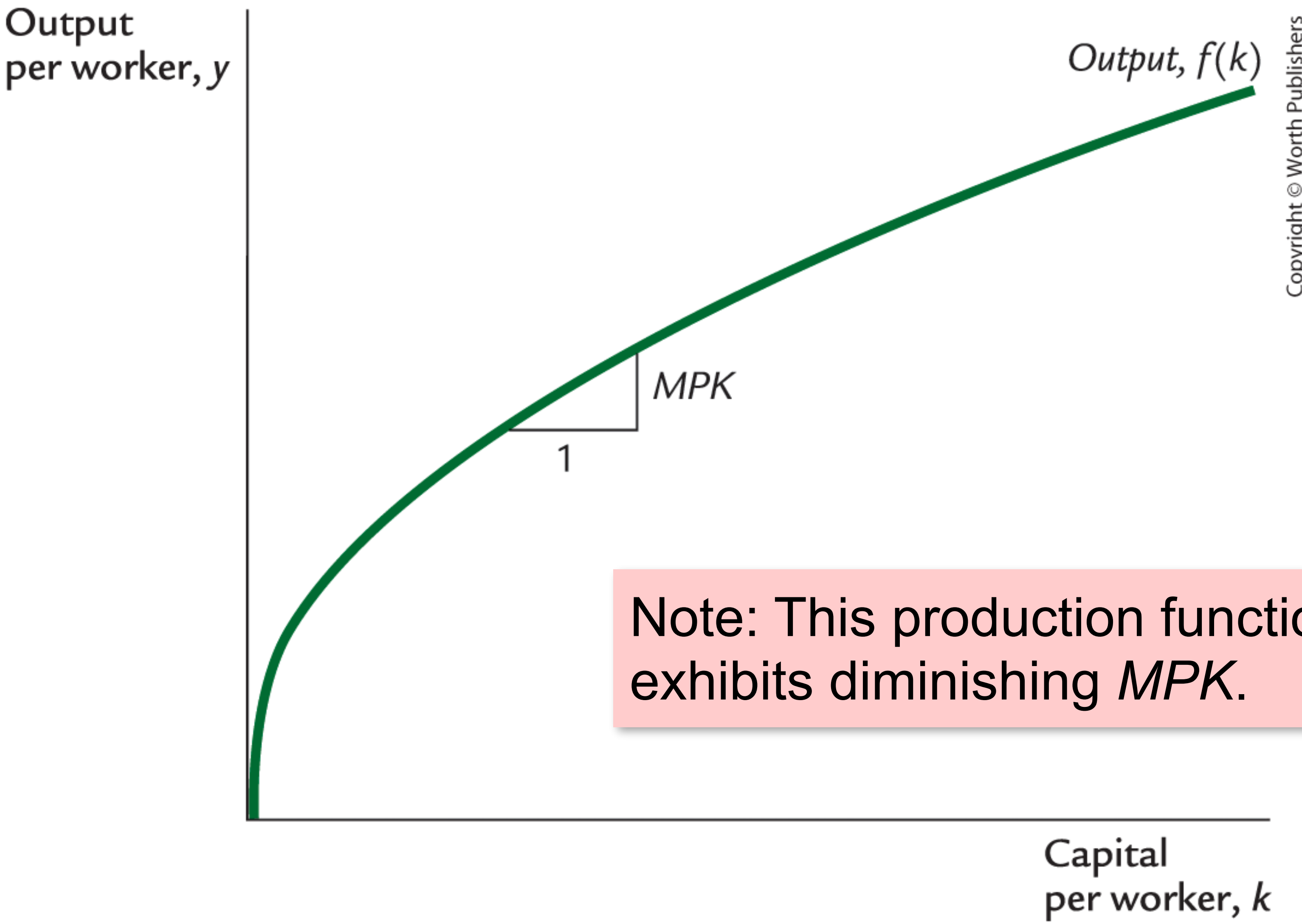
How the Solow model is different from previous models

1. We're back to a closed economy
2. ***K*** is no longer fixed: Investment causes it to grow, depreciation causes it to shrink.
3. ***L*** is no longer fixed: population growth (next week)
4. The consumption function is simpler
5. No ***G*** or ***T*** (only to simplify presentation; we can still do fiscal policy experiments)
6. Cosmetic differences

The production function (1 of 2)

- In aggregate terms: $\boldsymbol{Y = F(K, L)}$
- Define: $\boldsymbol{y = Y/L}$ = output per worker
  $\boldsymbol{k = K/L}$ = capital per worker
- Assume constant returns to scale:
  $\boldsymbol{zY = F(zK, zL)}$ for any $\boldsymbol{z} > 0$
- Pick $\boldsymbol{z = 1/L}$. Then
  - $\boldsymbol{Y/L = F(K/L, 1)}$
  - $\boldsymbol{y = F(k, 1)}$
  - $\boldsymbol{y = f(k)}$, where $\boldsymbol{f(k) = F(k, 1)}$

The production function (2 of 2)



Note: This production function exhibits diminishing *MPK*.

The national income identity

- $\boldsymbol{Y} = \boldsymbol{C} + \boldsymbol{I}$ (remember, no $\boldsymbol{G}$ or $\boldsymbol{NX}$)
- In "per worker" terms:

$$\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{i},$$

where $\boldsymbol{c} = \boldsymbol{C}/\boldsymbol{L}$ and $\boldsymbol{i} = \boldsymbol{I}/\boldsymbol{L}$.

The consumption function

- ***s*** = the saving rate, the fraction of income that is saved (***s*** is an exogenous parameter)

Note: ***s*** is the *only* lowercase variable that is *not equal to* its uppercase version divided by ***L****.*

- Consumption function: ***c*** = (**1** – ***s***)***y*** *(per worker)*

Saving and investment

- Saving (per worker) $= \boldsymbol{y} - \boldsymbol{c}$
  $= \boldsymbol{y} - (\mathbf{1} - \boldsymbol{s})\boldsymbol{y}$
  $= \boldsymbol{sy}$
- National income identity per worker is $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{i}$

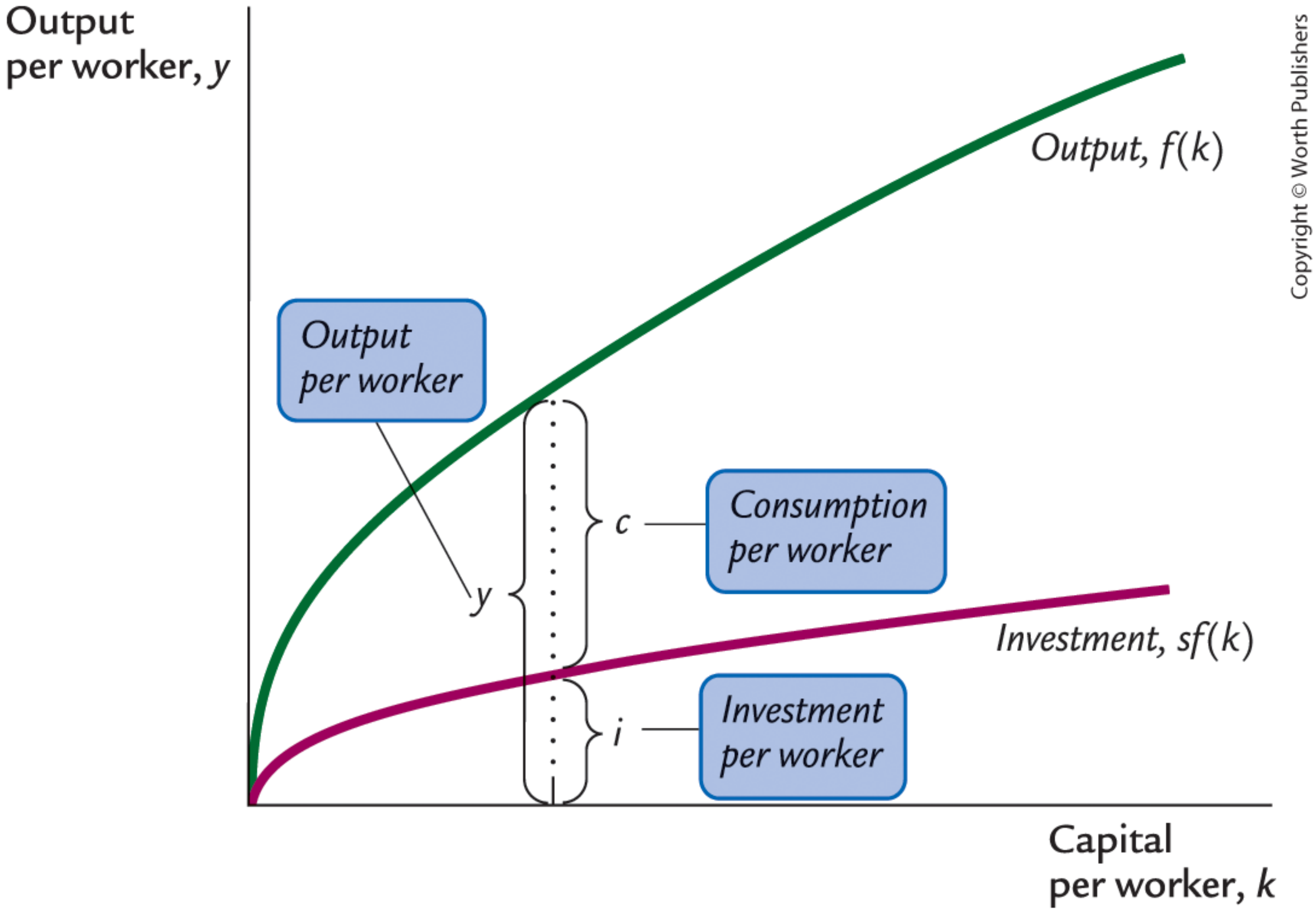
  Rearrange to get $\boldsymbol{i} = \boldsymbol{y} - \boldsymbol{c} = \boldsymbol{sy}$

  (investment = saving, as in the loanable funds model!)

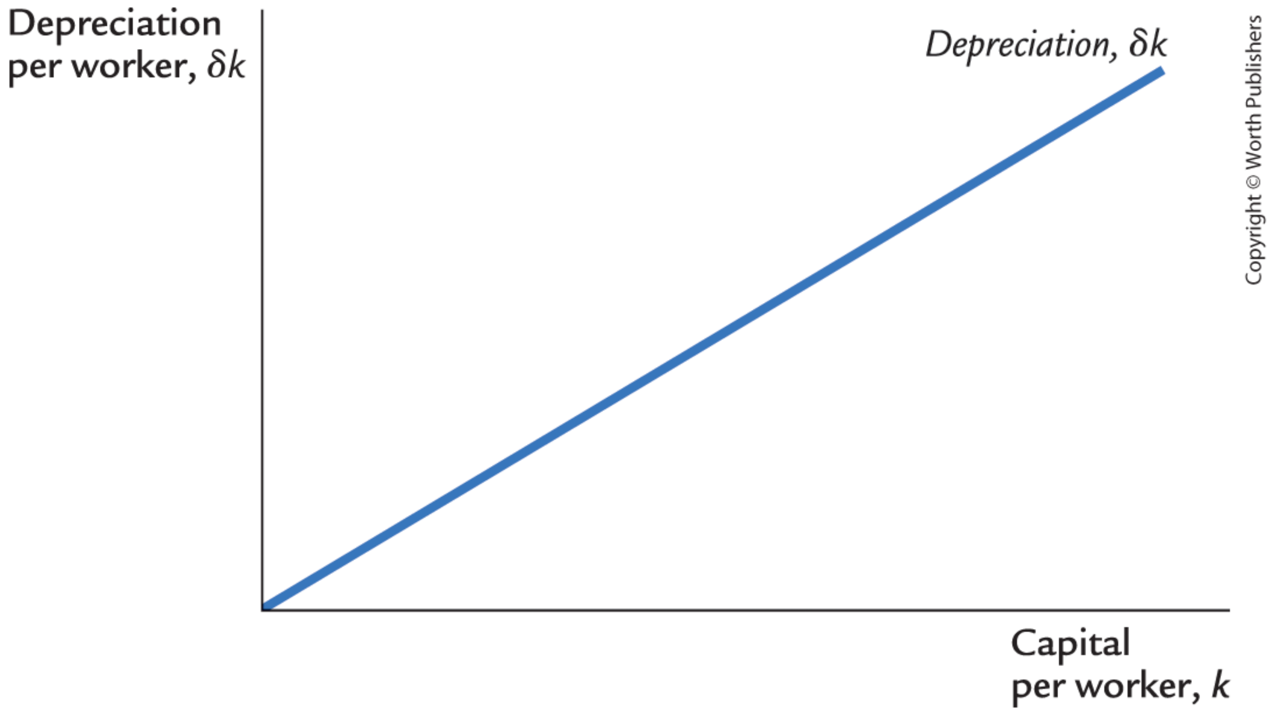
- Using the results above,

$$\boldsymbol{i} = \boldsymbol{sy} = \boldsymbol{sf}(\boldsymbol{k})$$

Output, consumption, and investment



Depreciation



δ = the rate of depreciation

k = the fraction of the capital stock that wears out each period

Capital accumulation

The basic idea: Investment increases the capital stock; depreciation reduces it.

change in capital stock = investment − depreciation

$$\Delta \boldsymbol{k} = \boldsymbol{i} - \delta \boldsymbol{k}$$

Since $\boldsymbol{i} = \boldsymbol{sf}(\boldsymbol{k})$, this becomes:

$$\boxed{\Delta \boldsymbol{k} = \boldsymbol{sf}(\boldsymbol{k}) - \delta \boldsymbol{k}}$$

The law of motion for capital (*k*)

$$\Delta k = s f(k) - \delta k$$

- The Solow model's central equation
- Determines behavior of capital over time . . .
- . . . which, in turn, determines behavior of all the other endogenous variables because they all depend on ***k***.
- Example:

income per person: $y = f(k)$

consumption per person: $c = (1 - s)\, f(k)$

The steady state, part 1

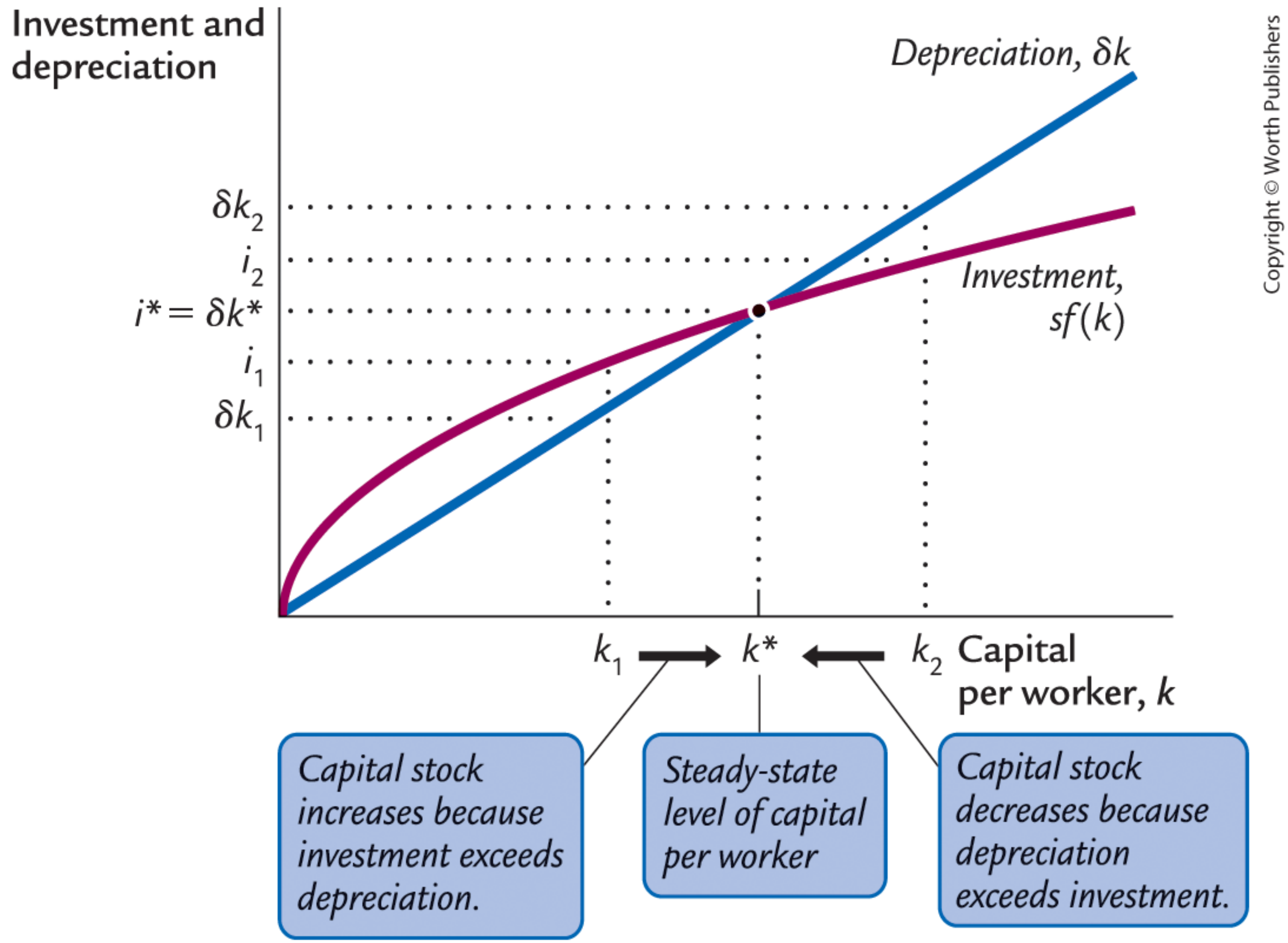
$$\boxed{\Delta \boldsymbol{k} = \boldsymbol{sf}(\boldsymbol{k}) - \delta \boldsymbol{k}}$$

If investment is just enough to cover depreciation
[$sf(k) = \delta k$],
then capital per worker will remain constant:

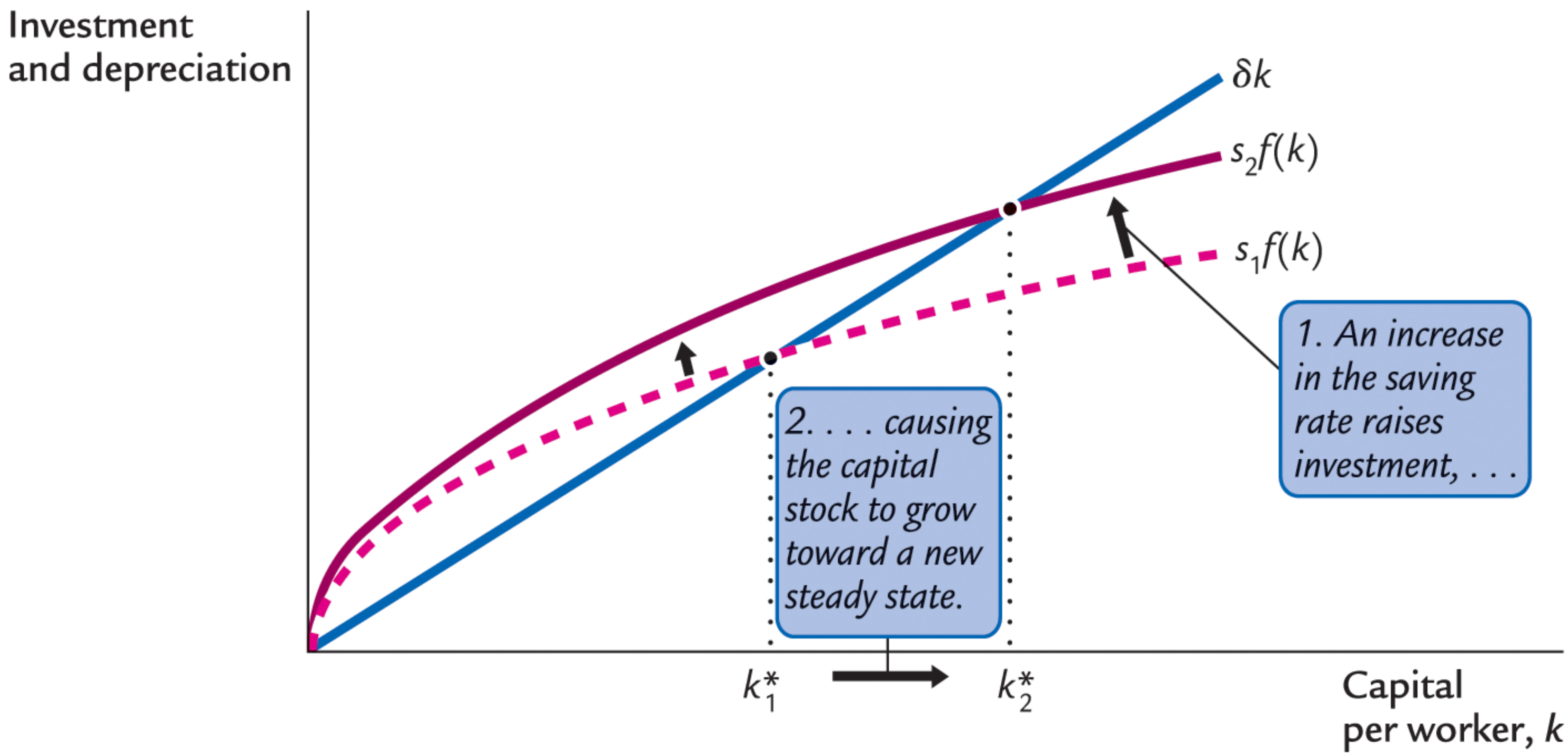
$$\Delta k = 0.$$

This occurs at one value of k, denoted k^*, called the ***steady-state capital stock***.

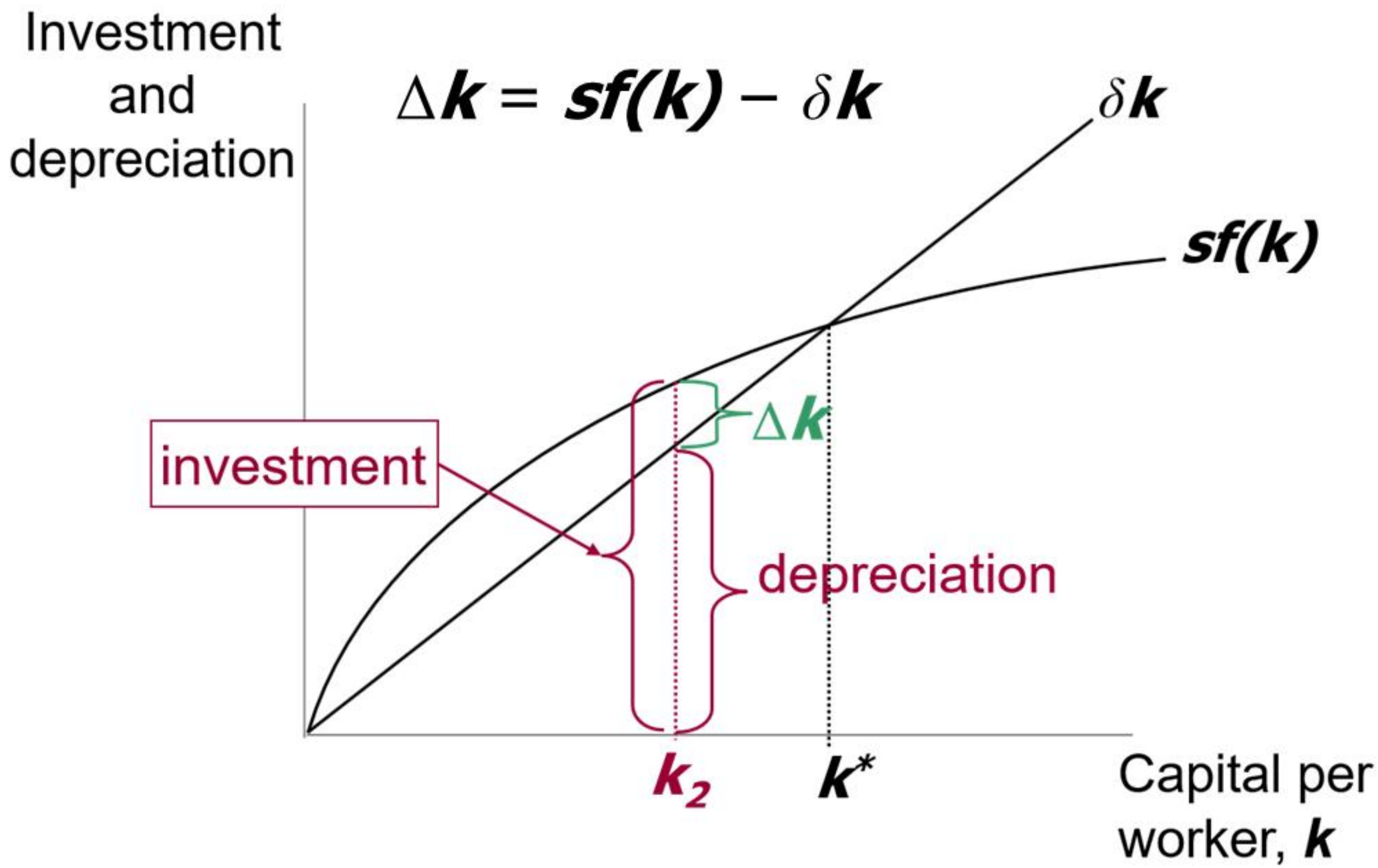
The steady state, part 2



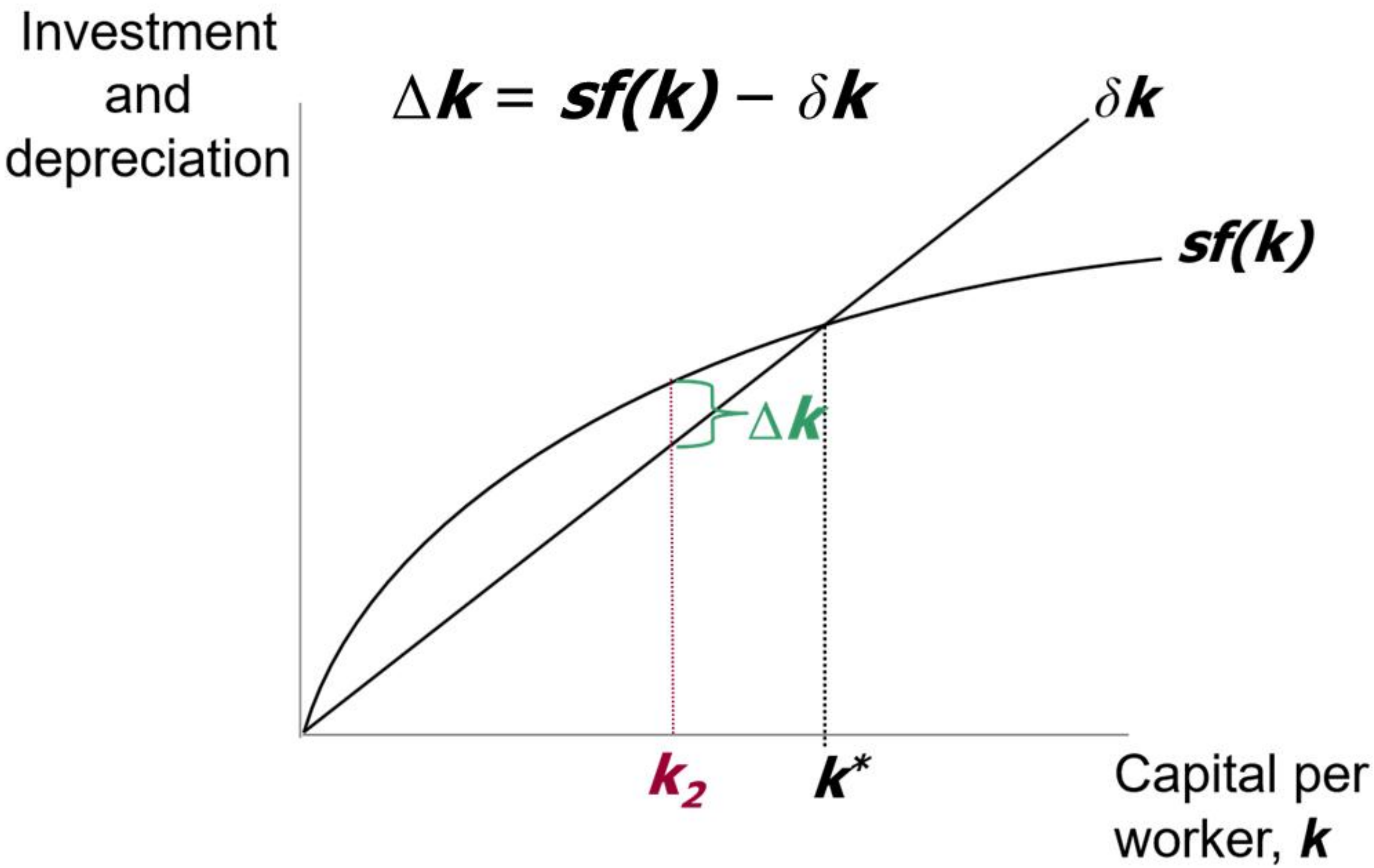
Moving toward the steady state, part 1



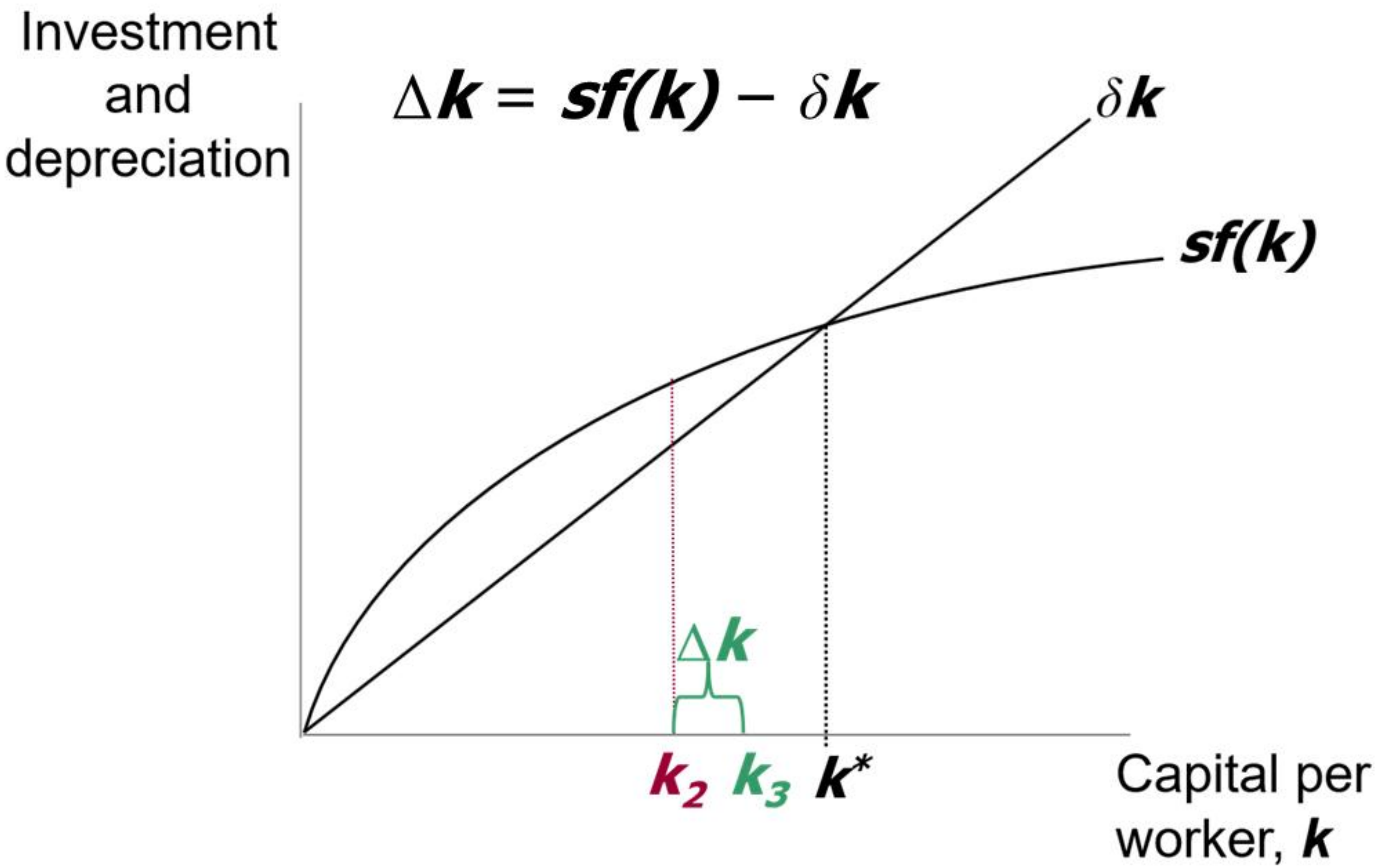
Moving toward the steady state, part 2



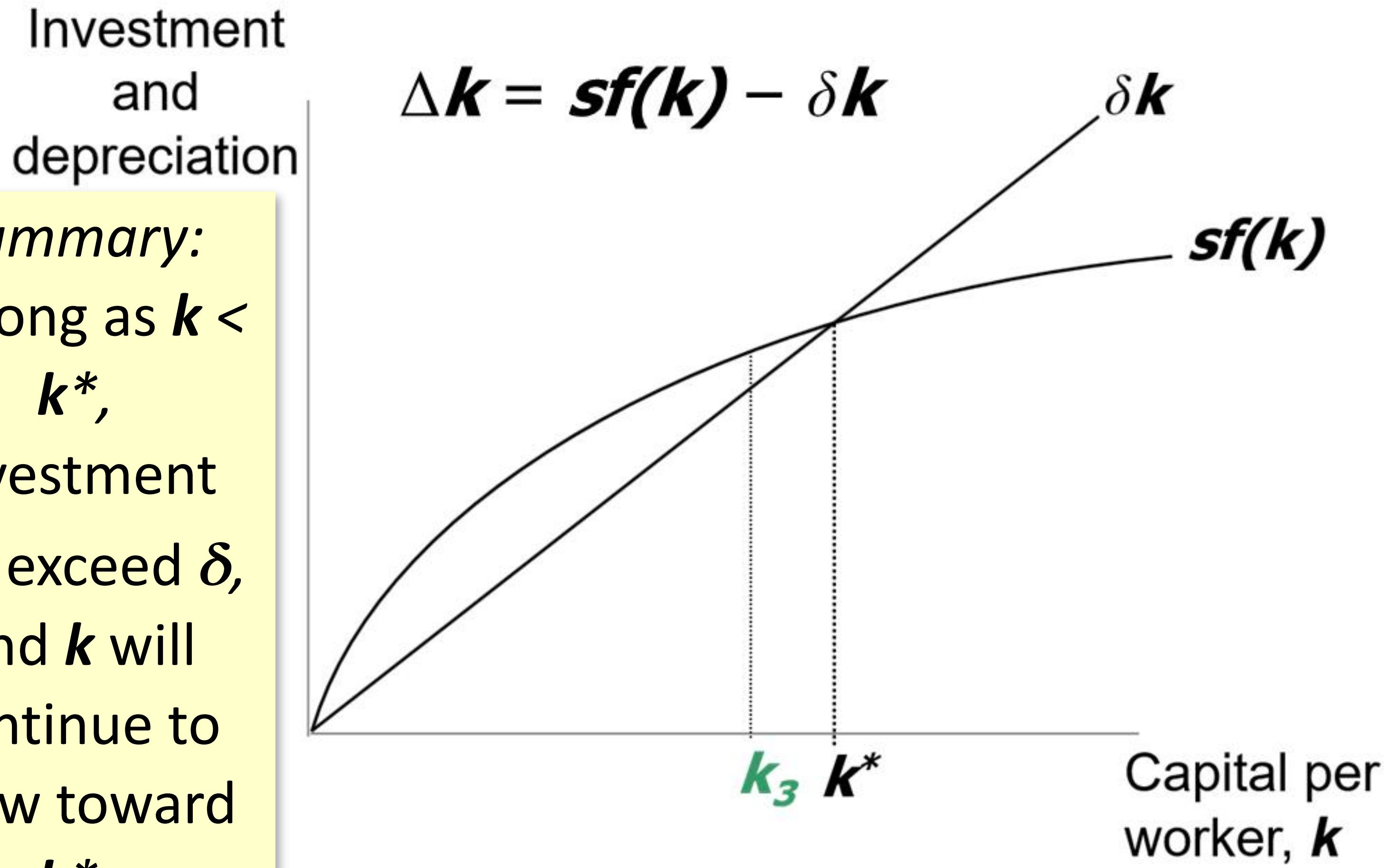
Moving toward the steady state, part 3



Moving toward the steady state, part 4



Moving toward the steady state, part 5



Summary:
As long as $k < k^*$,
investment will exceed δ,
and k will continue to grow toward k^*.

NOW YOU TRY
Approaching k^* from above

Draw the Solow model diagram, labeling the steady state $\boldsymbol{k^*}$.

On the horizontal axis, pick a value greater than $\boldsymbol{k^*}$ for the economy's initial capital stock. Label it $\boldsymbol{k_1}$.

Show what happens to $\boldsymbol{k}$ over time.
Does $\boldsymbol{k}$ move toward the steady state or away from it?

A numerical example, part 1

Production function (aggregate):

$$Y = F(K,L) = \sqrt{K \times L} = K^{1/2}L^{1/2}$$

To derive the per-worker production function, divide through by *L*:

$$\frac{Y}{L} = \frac{K^{1/2}L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2}$$

Then substitute $y = Y/L$ and $k = K/L$ to get

$$y = f(k) = k^{1/2}$$

A numerical example, part 2

Assume:

- $\boldsymbol{s} = 0.3$
- $\boldsymbol{\delta} = 0.1$
- initial value of $\boldsymbol{k} = 4.0$

Approaching the steady state: A numerical example

$$\text{Assumptions}: \boldsymbol{y} = \sqrt{\boldsymbol{k}}; \boldsymbol{s} = 0.3; \delta = 0.1; \text{initial } \boldsymbol{k} = 4.0$$

Year	*k*	*y*	*c*	*i*	δk	Δk
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
5	4.768	2.184	1.529	0.655	0.477	0.178
10	5.602	2.367	1.657	0.710	0.560	0.150
25	7.321	2.706	1.894	0.812	0.732	0.080
100	8.962	2.994	2.096	0.898	0.896	0.002
∞	9.000	3.000	2.100	0.900	0.900	0.000

NOW YOU TRY
Solve for the steady state

Continue to assume

$\boldsymbol{s} = 0.3$, $\boldsymbol{\delta} = 0.1$, and $\boldsymbol{y} = \boldsymbol{k}^{1/2}$

Use the equation of motion

$$\Delta \boldsymbol{k} = \boldsymbol{s}\, \boldsymbol{f}(\boldsymbol{k}) - \boldsymbol{\delta k}$$

to solve for the steady-state values of ***k***, ***y***, and ***c***.

NOW YOU TRY
Solve for the steady state, answers

$\Delta \boldsymbol{k} = 0$ definition of steady state

$\boldsymbol{s}\,\boldsymbol{f}(\boldsymbol{k}^*) = \delta \boldsymbol{k}^*$ eq'n of motion with $\boldsymbol{k}$

$0.3\sqrt{\boldsymbol{k}^*} = 0.1\boldsymbol{k}^*$ using assumed values

$$3 = \frac{\boldsymbol{k}^*}{\sqrt{\boldsymbol{k}^*}} = \sqrt{\boldsymbol{k}^*}$$

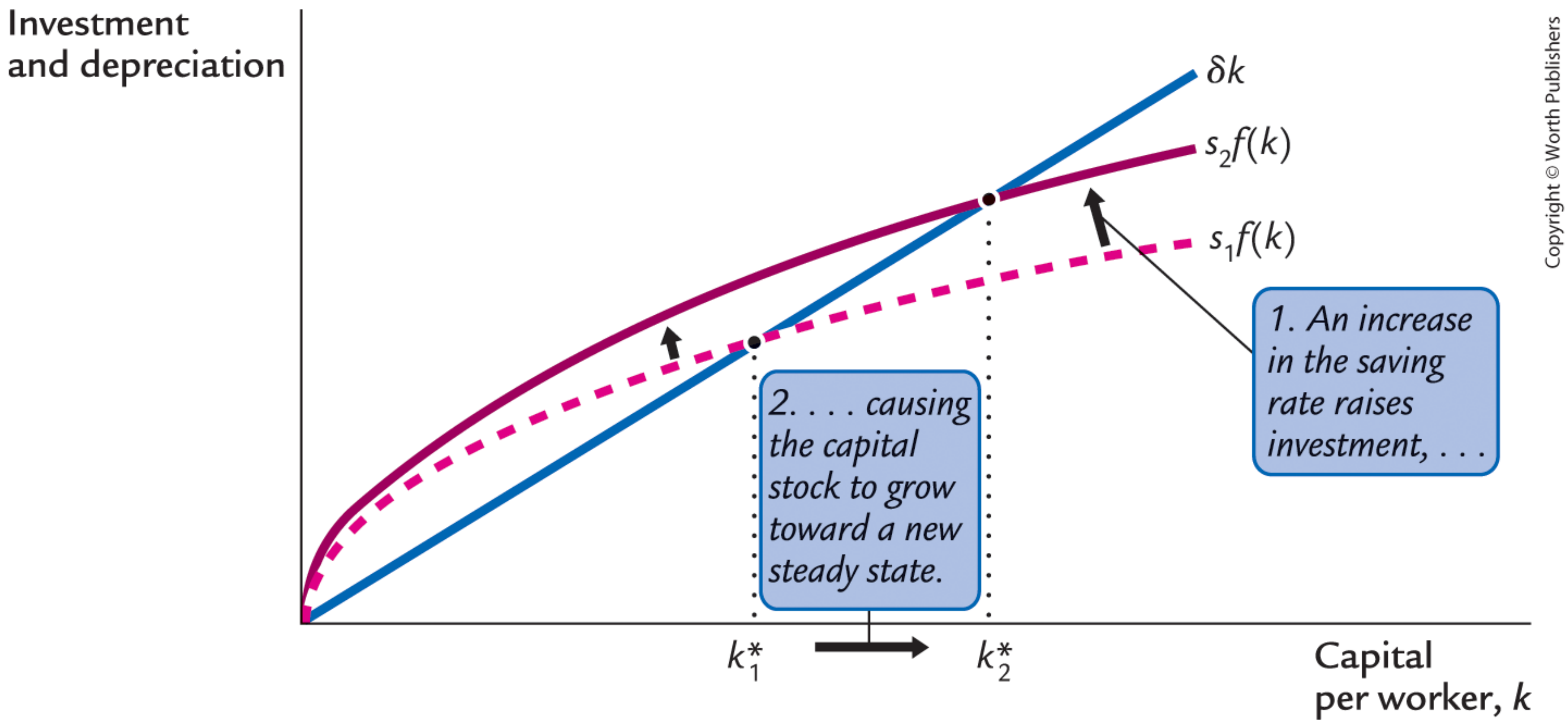
Solve to get: $\boldsymbol{k}^* = 9$ and $\boldsymbol{y}^* = \sqrt{k^*} = 3$

Finally, $\boldsymbol{c}^* = (1 - \boldsymbol{s})\boldsymbol{y}^* = 0.7 \times 3 = 2.1$

An increase in the saving rate

An increase in the saving rate raises investment . . .

. . . causing ***k*** to grow toward a new steady state:

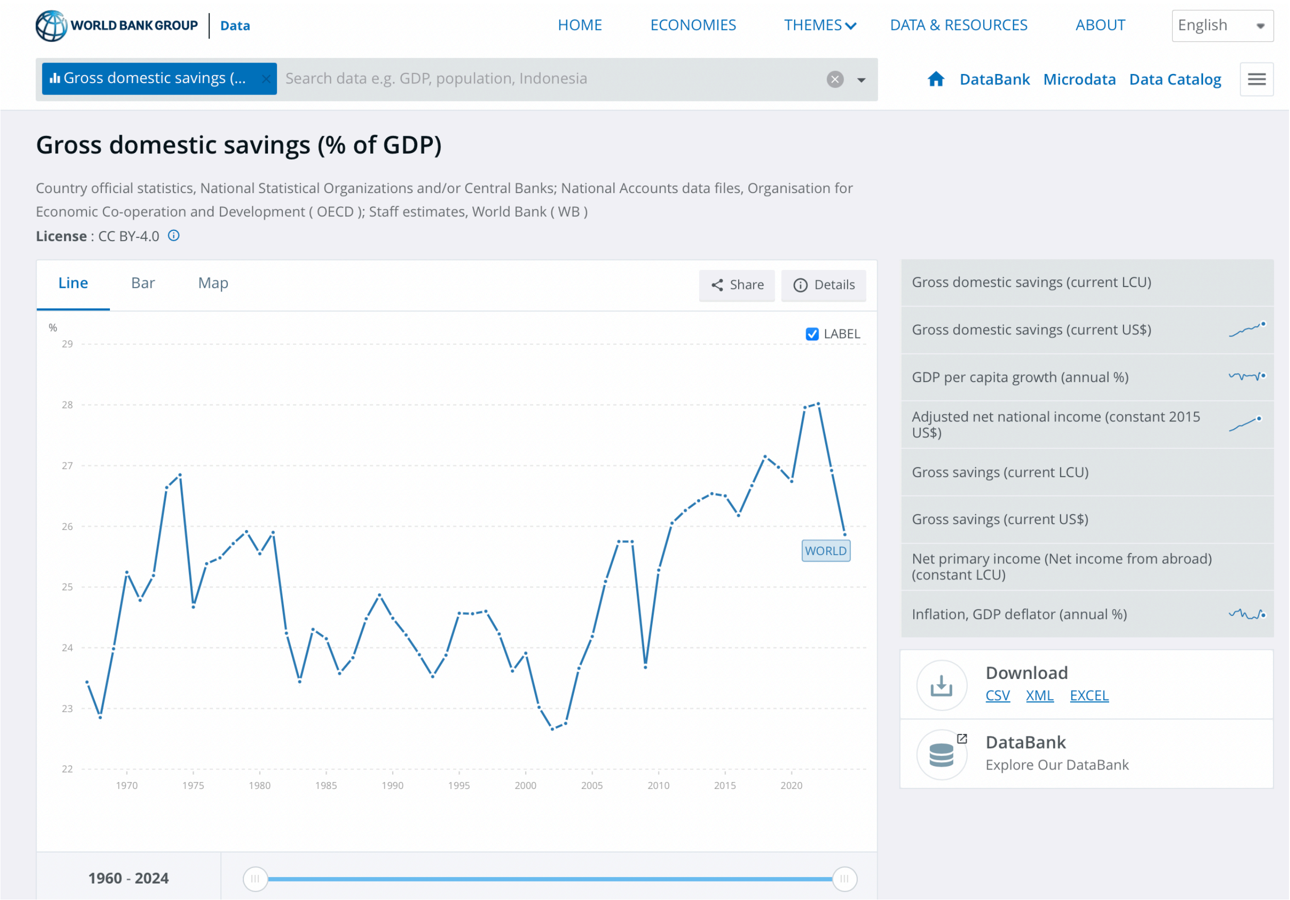


Does this hold up in the data?

- The Solow model predicts that countries with higher savings rates will have higher output in the long run
- Let’s test this empirically!

Quick & dirty empirical “analysis”

- Step 1: download data on savings rates and output



Quick & dirty empirical "analysis"

- Step 1: download data on savings rates and output
- Step 2: code up the "analysis"

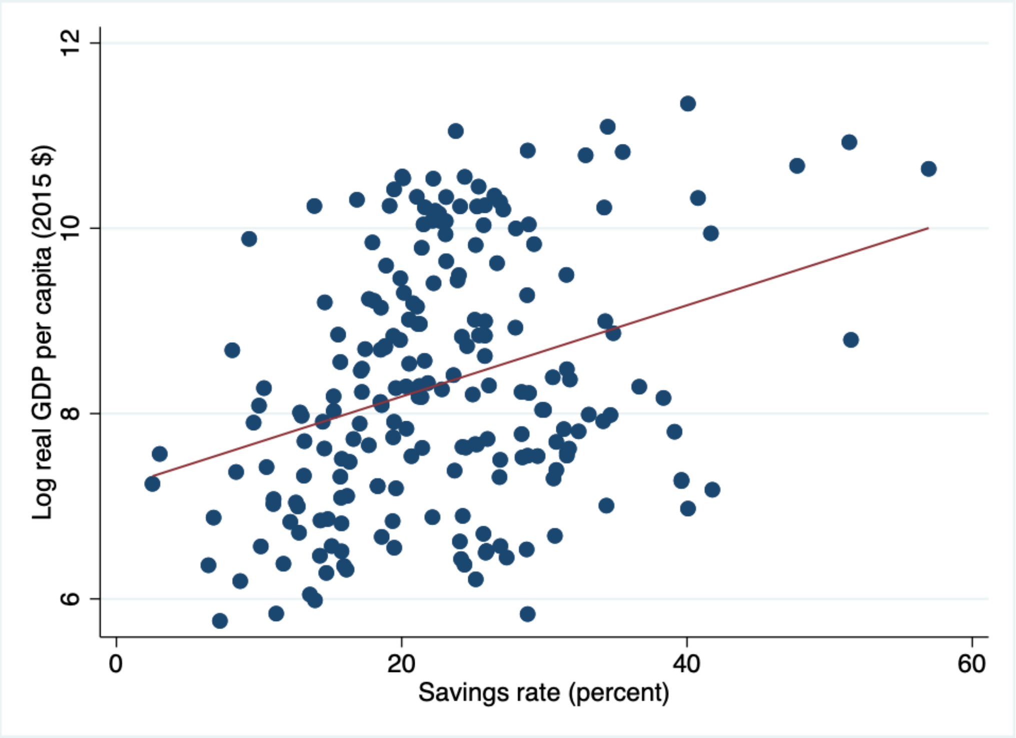
```
import delimited using "savings.csv", varnames(1) clear
reshape long v, i(countrycode) j(year)
rename v savings_rate
save "savings.dta", replace

import delimited using "rgdppc.csv", varnames(1) clear
reshape long v, i(countrycode) j(year)
save "/Users/attila/Downloads/rgdppc.dta", replace
rename v rgdp_pc
merge 1:1 countrycode year using "savings.dta", nogen

generate lrgdp_pc = log(rgdp_pc)
collapse (mean) lrgdp_pc savings, by(countrycode)
twoway (scatter lrgdp_pc savings if savings>0 & savings<100) (lfit lrgdp_pc savings if savings>0 & savings<100)
```

Quick & dirty empirical "analysis"

- Step 1: download data on savings rates and output
- Step 2: code up the "analysis"
- Step 3: evaluate the results



The Golden Rule: Introduction

- Different values of s lead to different steady states. How do we know which is the "best" steady state?
- The "best" steady state has the highest possible consumption per person: $c^* = (1 - s)\, f(k^*)$.
- An increase in s
  - leads to higher k^* and y^*, which raises c^*
  - reduces consumption's share of income $(1 - s)$, which lowers c^*.
- So, how do we find the s and k^* that maximize c^*?

The Golden Rule capital stock, part 1

k^*_{gold} = the **Golden Rule level of capital**, the steady-state value of k that maximizes consumption

To find it, first express c^* in terms of k^*:

$$
\begin{aligned}
c^* &= y^* - i^* \\
&= f(k^*) - i^* \\
&= f(k^*) - \delta k^*
\end{aligned}
$$

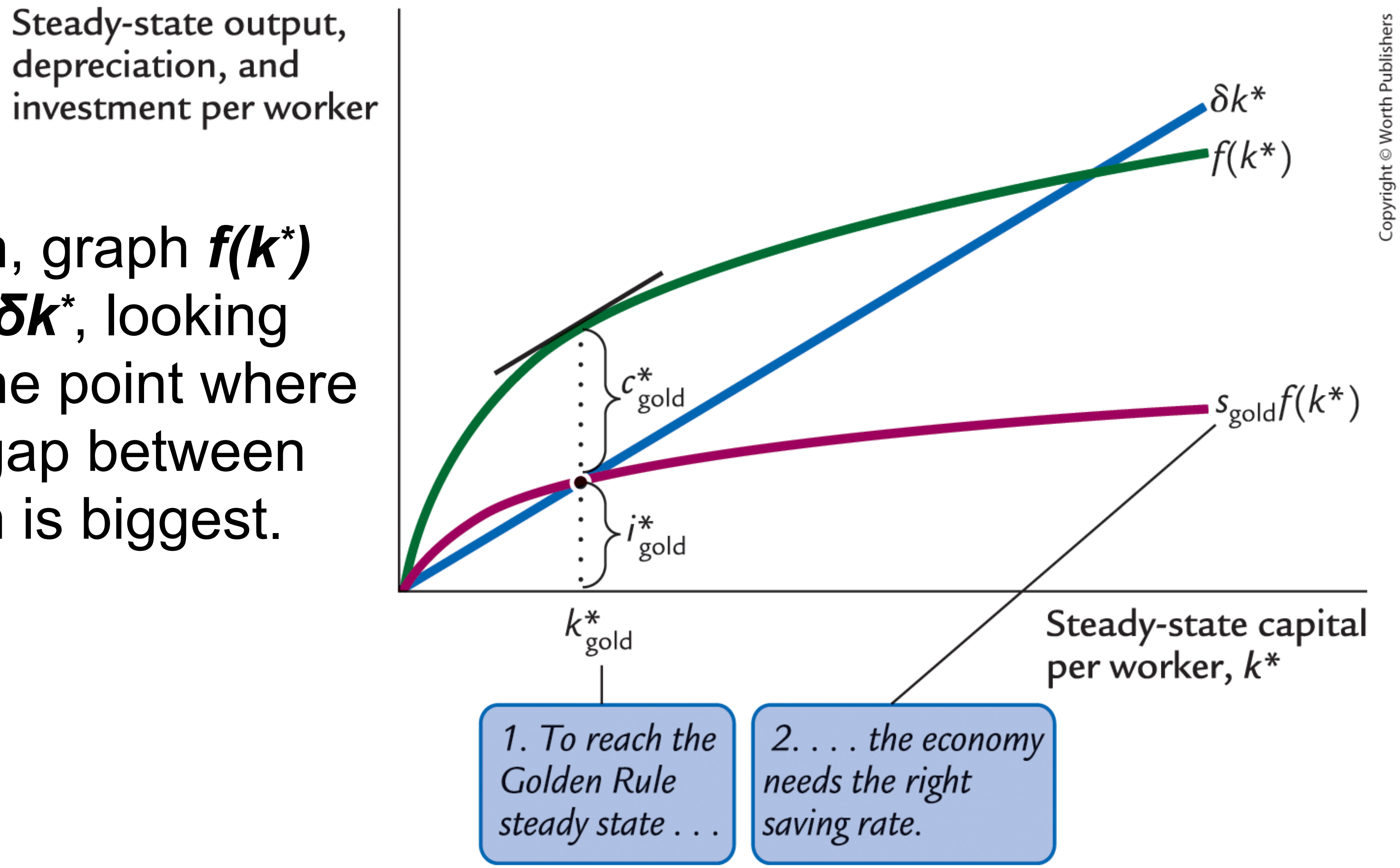
In the steady state:

$$i^* = \delta k^*$$

because $\Delta k = 0$.

The Golden Rule capital stock, part 2

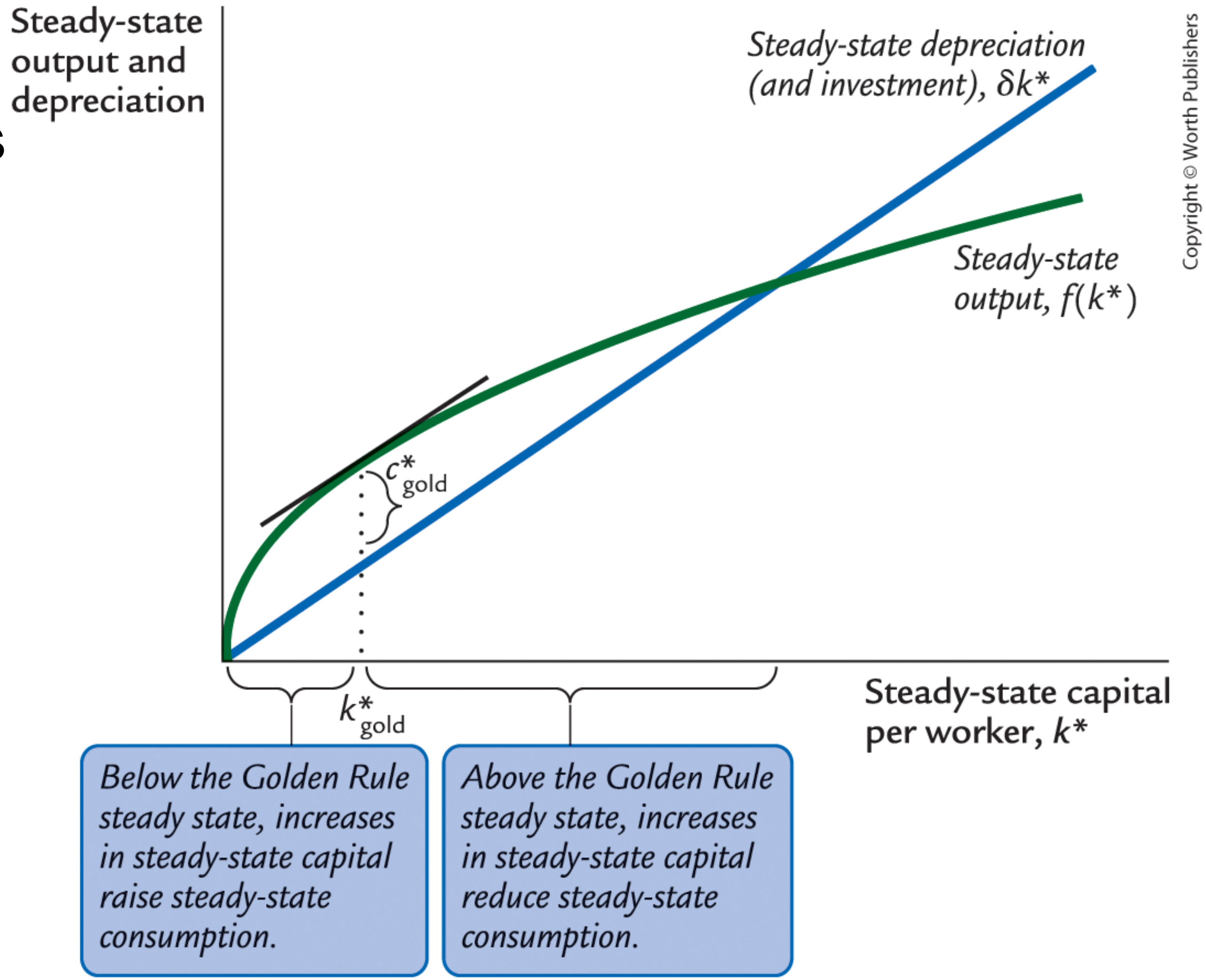
Then, graph ***f(k*)*** and ***δk****, looking for the point where the gap between them is biggest.



The Golden Rule capital stock, part 3

$c^* = f(k^*) - \delta k^*$ is biggest where the slope of the production function equals the slope of the depreciation line:

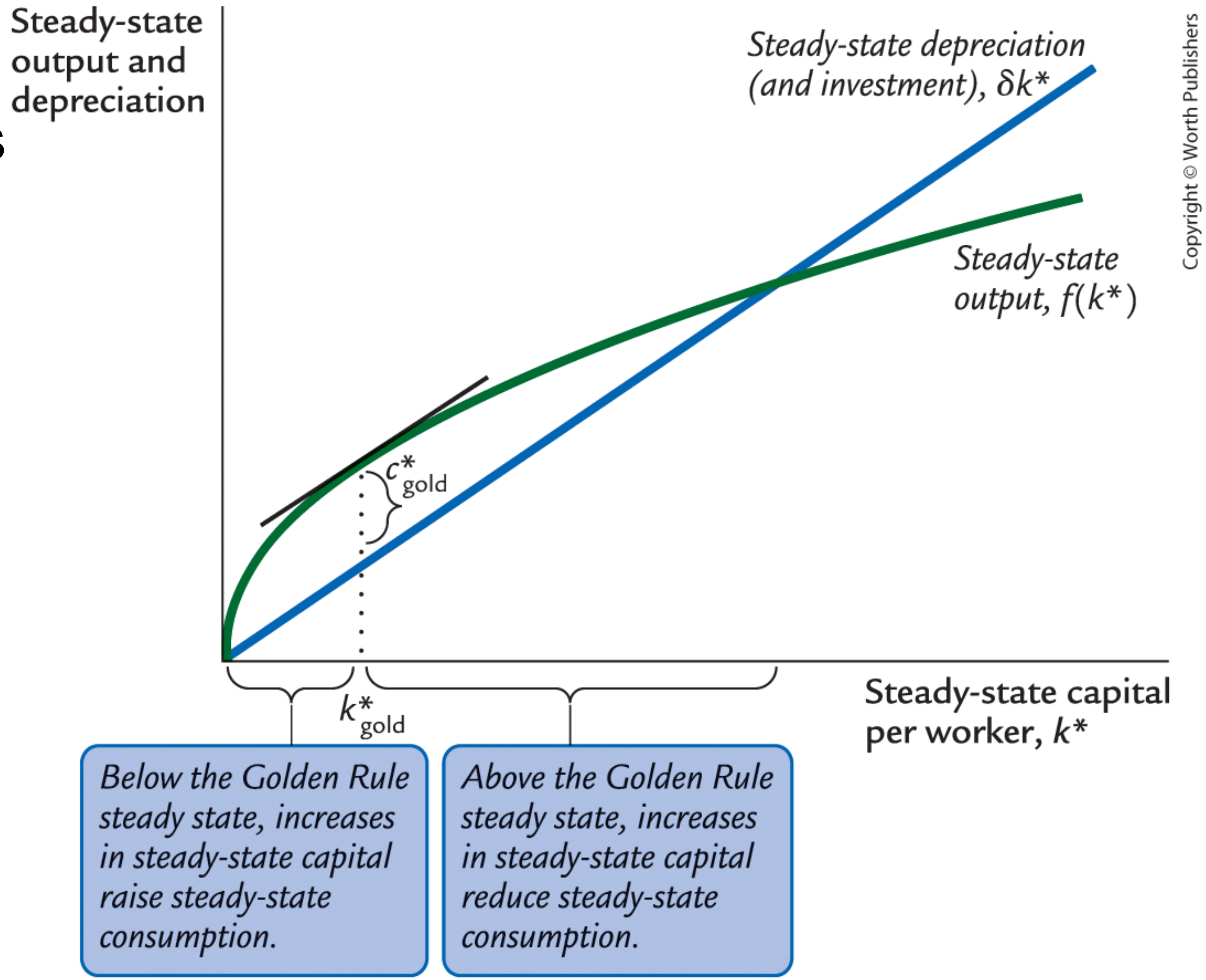
???



The Golden Rule capital stock, part 3

$c^* = f(k^*) - \delta k^*$ is biggest where the slope of the production function equals the slope of the depreciation line:

$$MPK = \delta$$



NOW YOU TRY
The Golden Rule

Continue to assume

$\boldsymbol{\delta} = 0.1$ and $\boldsymbol{y} = \boldsymbol{k}^{1/2}$

What is the Golden Rule level of capital? Output? Investment? Consumption? Savings rate?

NOW YOU TRY
The Golden Rule, solution

$$
\begin{aligned}
MPK^*_{\text{gold}} &= \delta \\
\frac{1}{2}{k^*_{\text{gold}}}^{-\frac{1}{2}} &= 0.1 \\
k^*_{\text{gold}} &= 0.2^{-2} = 25 \\
y^*_{\text{gold}} &= 25^{\frac{1}{2}} = 5 \\
i^*_{\text{gold}} &= \delta k^*_{\text{gold}} = 0.1 \times 25 = 2.5 \\
c^*_{\text{gold}} &= y^*_{\text{gold}} - i^*_{\text{gold}} = 5 - 2.5 = 2.5 \\
s^*_{\text{gold}} &= i^*_{\text{gold}}/y^*_{\text{gold}} = 2.5/5 = 50\%
\end{aligned}
$$

The transition to the Golden Rule steady state

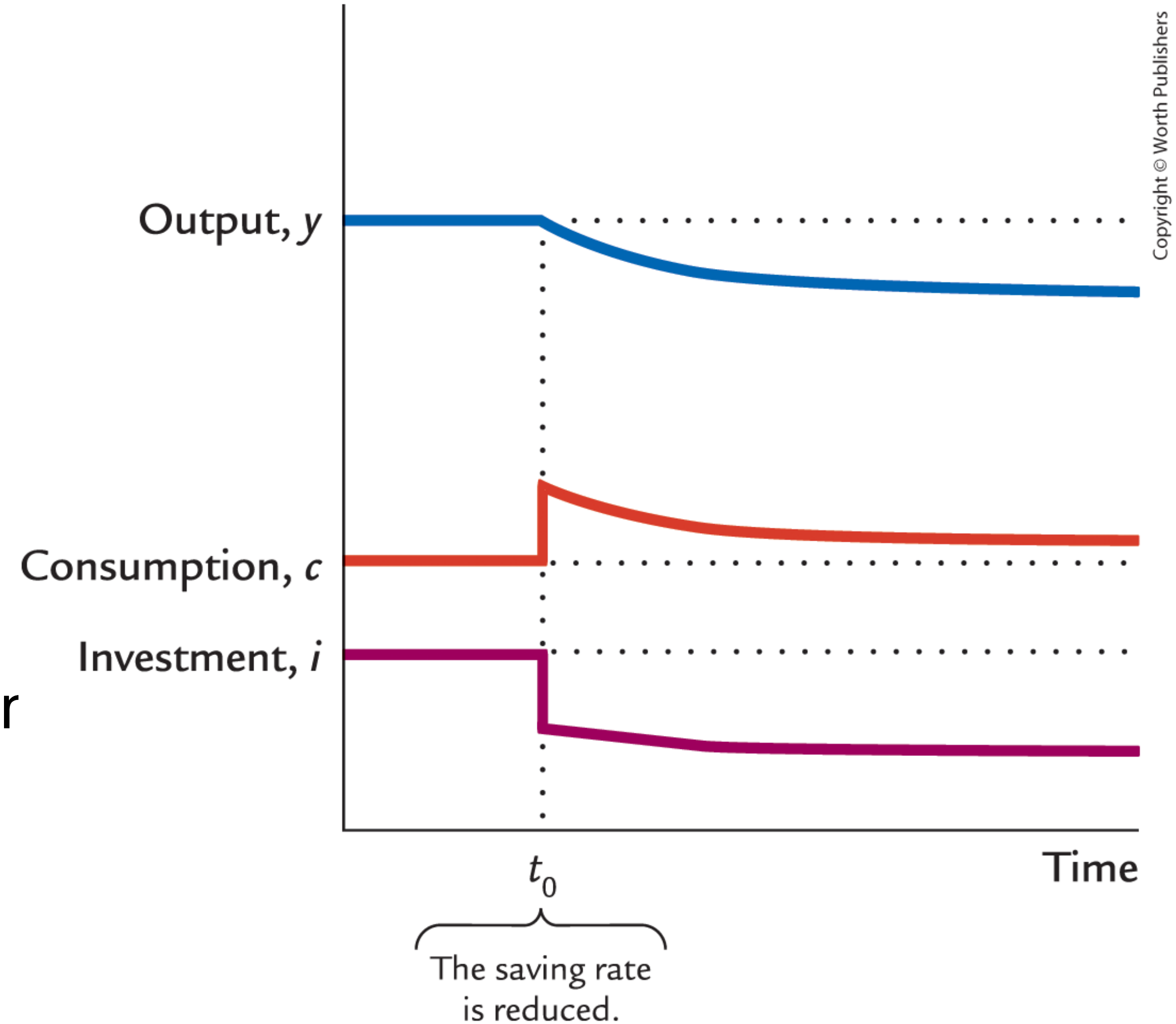
- The economy does not have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust ***s***.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $k^* > k^*_{gold}$

then increasing c^* requires a fall in s.

In the transition to the Golden Rule, consumption is higher at all points ir time.

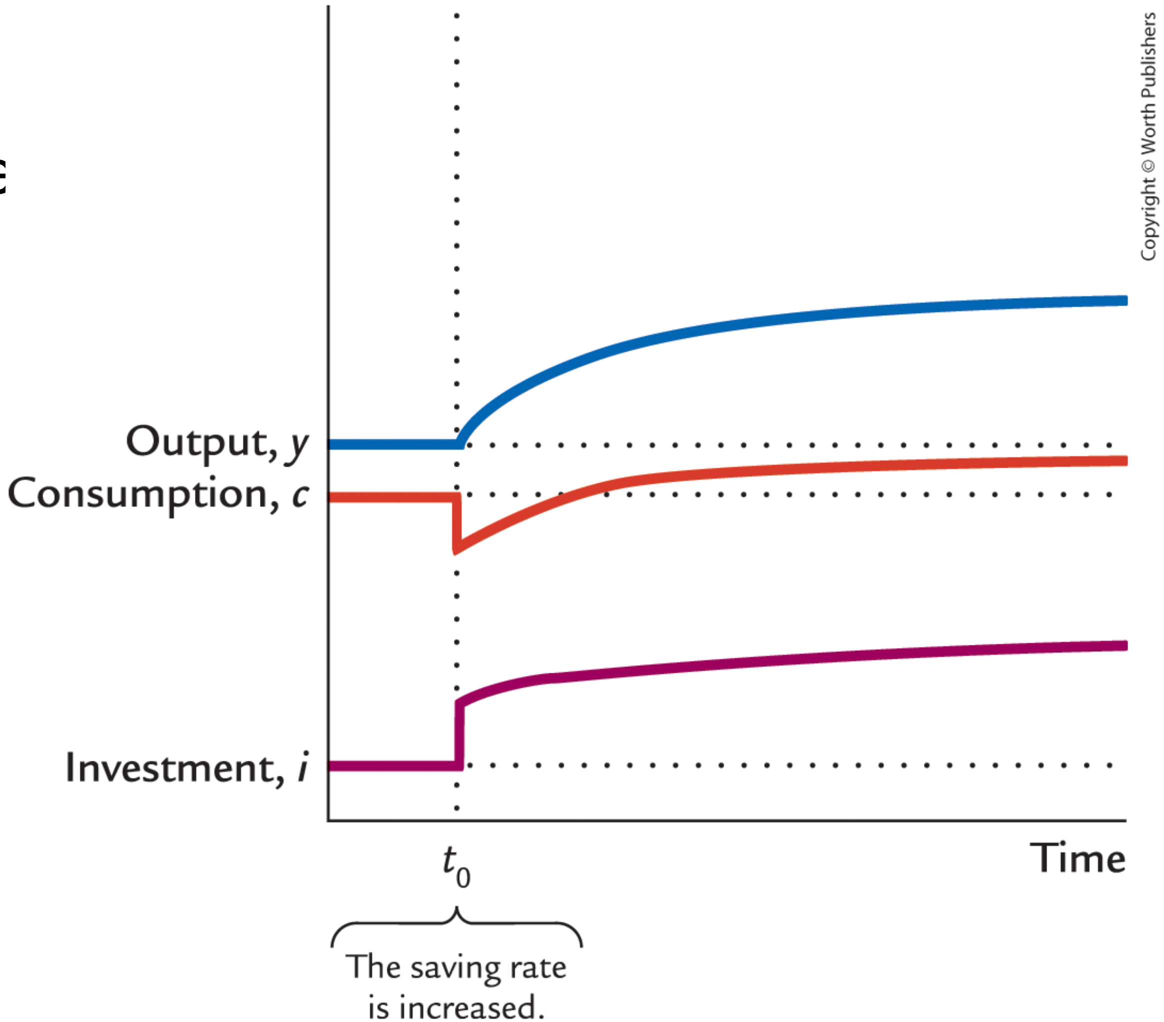


Starting with too little capital

If $k^* < k^*_{gold}$

then increasing c^* requires an increase in s.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



SUMMARY, PART 1

- The Solow growth model shows that, in the long run, a country's standard of living depends:
  - positively on its saving rate
- An increase in the saving rate leads to:
  - higher output in the long run
  - faster growth temporarily
  - but not faster steady-state growth

SUMMARY, PART 2

- If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.
  If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.