7. Population Growth and Technological Progress

Based on Mankiw, Chapters 10: *Population Growth and Technological Progress* and 11: *Growth Empirics and Policy*

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Population growth

The Solow model with population growth

- Assume that the population and labor force grow at rate $\boldsymbol{n}$ (exogenous):

$$\frac{\Delta L}{L} = n$$

- Example: Suppose L = 1,000 in year 1 and the population is growing at 2% per year (n = 0.02).
- Then $\Delta L = n\, L = 0.02 \times 1{,}000 = 20$

  $\Rightarrow L = 1{,}020$ in year 2

  $\Rightarrow L = 1.02 \times 1{,}020 = 1{,}040$ in year 3

  and so on

Break-even investment

- $(\boldsymbol{\delta} + \boldsymbol{n})\boldsymbol{k}$ = **break-even investment**, the amount of investment necessary to keep $\boldsymbol{k}$ constant
- Break-even investment includes:
  - $\boldsymbol{\delta}\ \boldsymbol{k}$ to replace capital as it wears out
  - $\boldsymbol{n}\ \boldsymbol{k}$ to equip new workers with capital

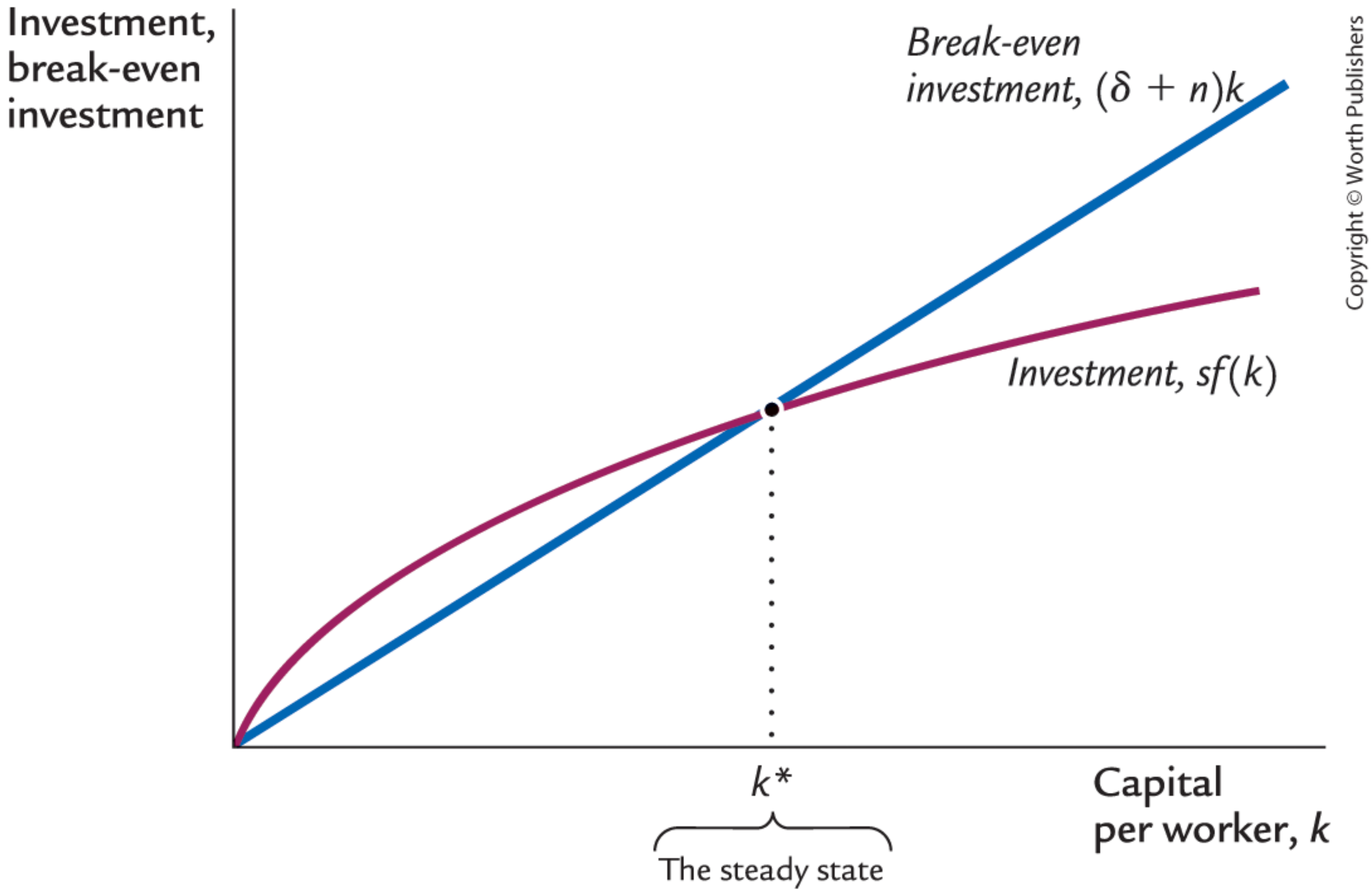
    (Otherwise, $\boldsymbol{k}$ would fall as the existing capital stock is spread more thinly over a larger population of workers.)

The law of motion for *k* with population growth

- With population growth, the law of motion for ***k*** is:

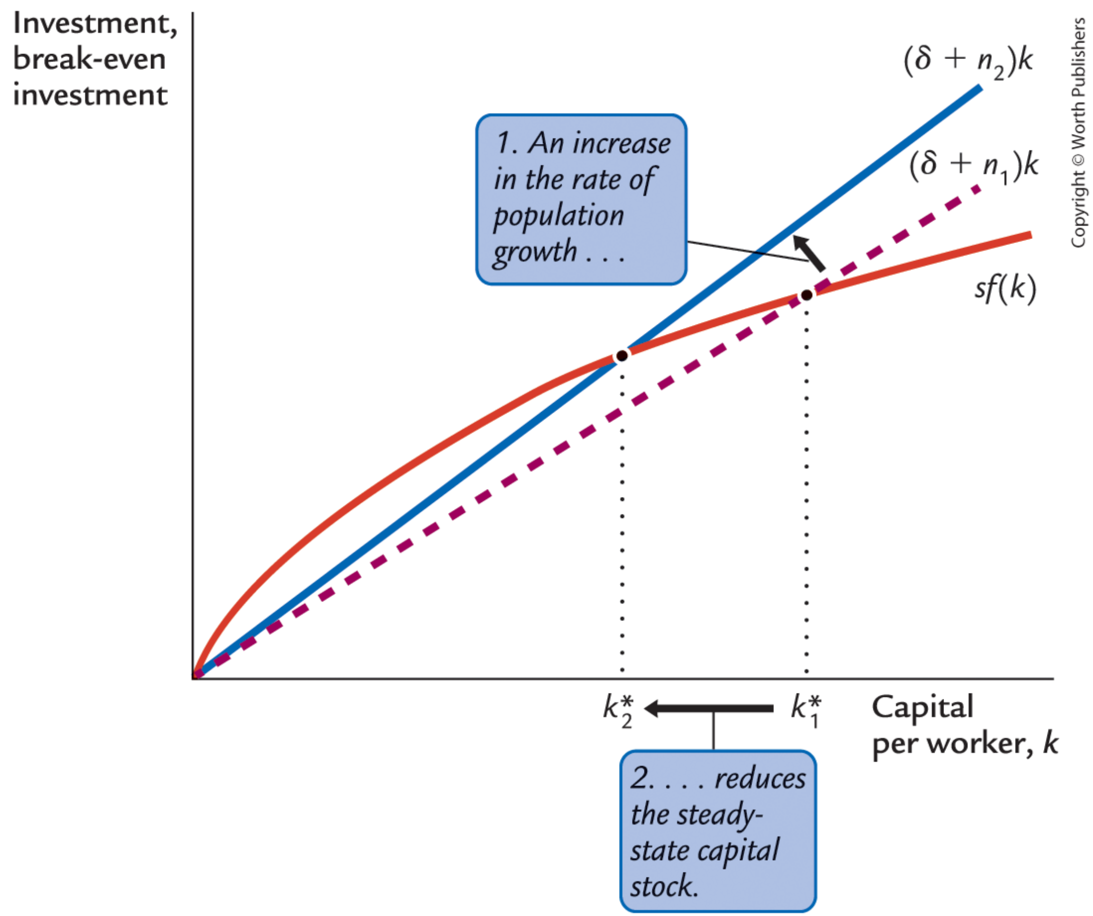
$$\Delta \boldsymbol{k} = \underbrace{\boldsymbol{s\,f(k)}}_{\text{actual investment}} - \underbrace{(\delta + \boldsymbol{n})\,\boldsymbol{k}}_{\text{break-even investment}}$$

The diagram of the Solow model with population growth



The impact of population growth

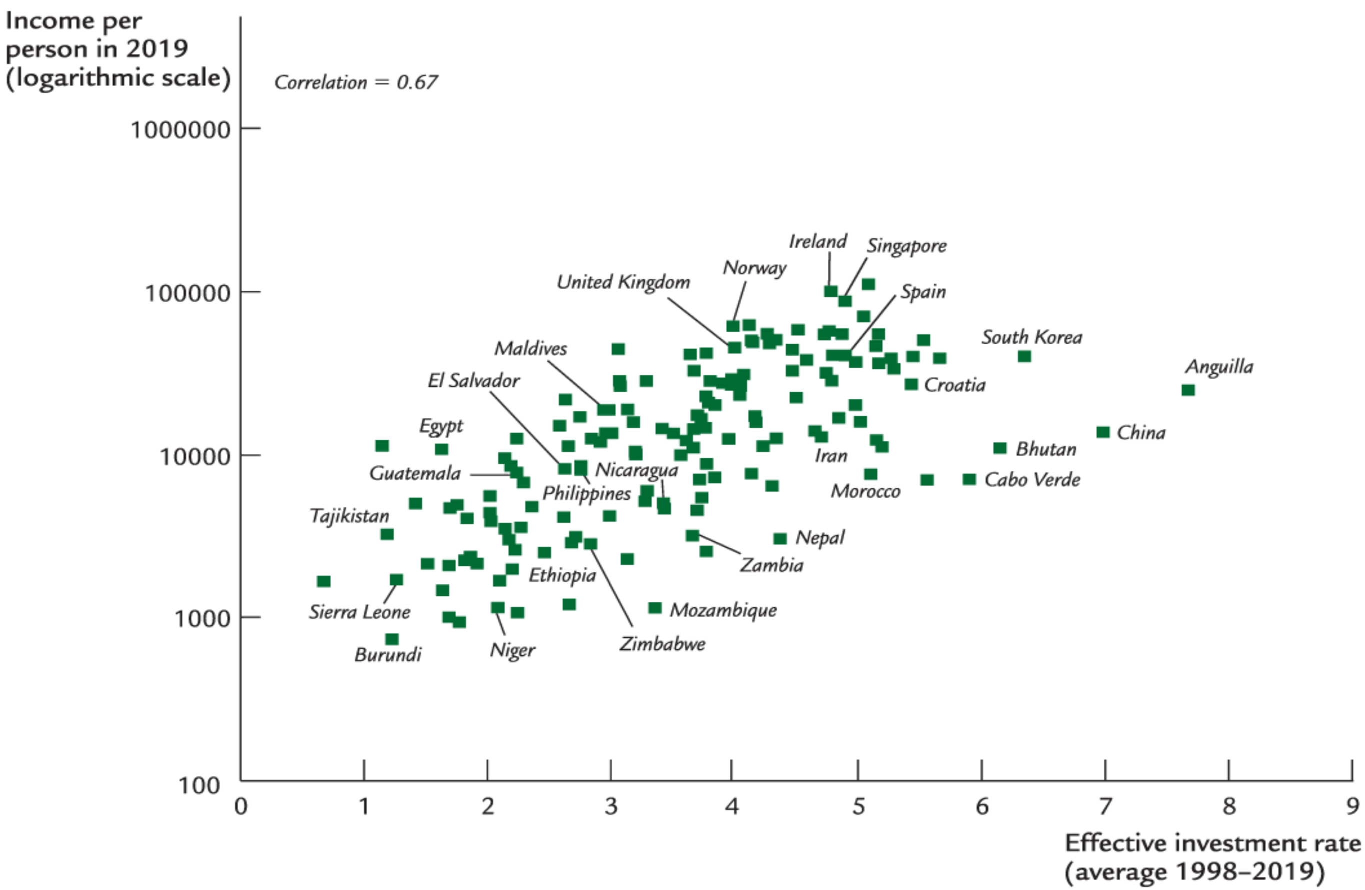
- An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.



Does this hold up in the data?

- The Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.
- Are the data consistent with this prediction?

International evidence on investment rates and income per person



The Golden Rule with population growth

- To find the Golden Rule capital stock, express $\boldsymbol{c}^*$ in terms of $\boldsymbol{k}^*$:

$$\begin{aligned} c^* &= y^* - i^* \\ &= f(k^*) - (\delta + n)k^* \end{aligned}$$

c^* is maximized when

$$\text{MPK} = \delta + n$$

Or, equivalently,

$$\text{MPK} - \delta = n$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.

Alternative perspectives on population growth, part 1

- **The Malthusian model (1798)**
  - It predicts population growth will outstrip the Earth’s ability to produce food, leading to the impoverishment of humanity.
  - Since the time of Malthus, world population has increased sixfold, yet living standards are higher than ever.
  - Malthus neglected the effects of technological progress.

Alternative perspectives on population growth, part 2

- **The Kremerian model (1993)**
  - Posits that population growth contributes to economic growth.
  - More people ⇒ more scientists, engineers, and innovators

    ⇒ faster technological progress.
  - Evidence from very long historical periods shows that:
    - as world population growth rate increased, so did the rate of growth in living standards.
    - historically, regions with larger populations have enjoyed faster growth.

Technological progress

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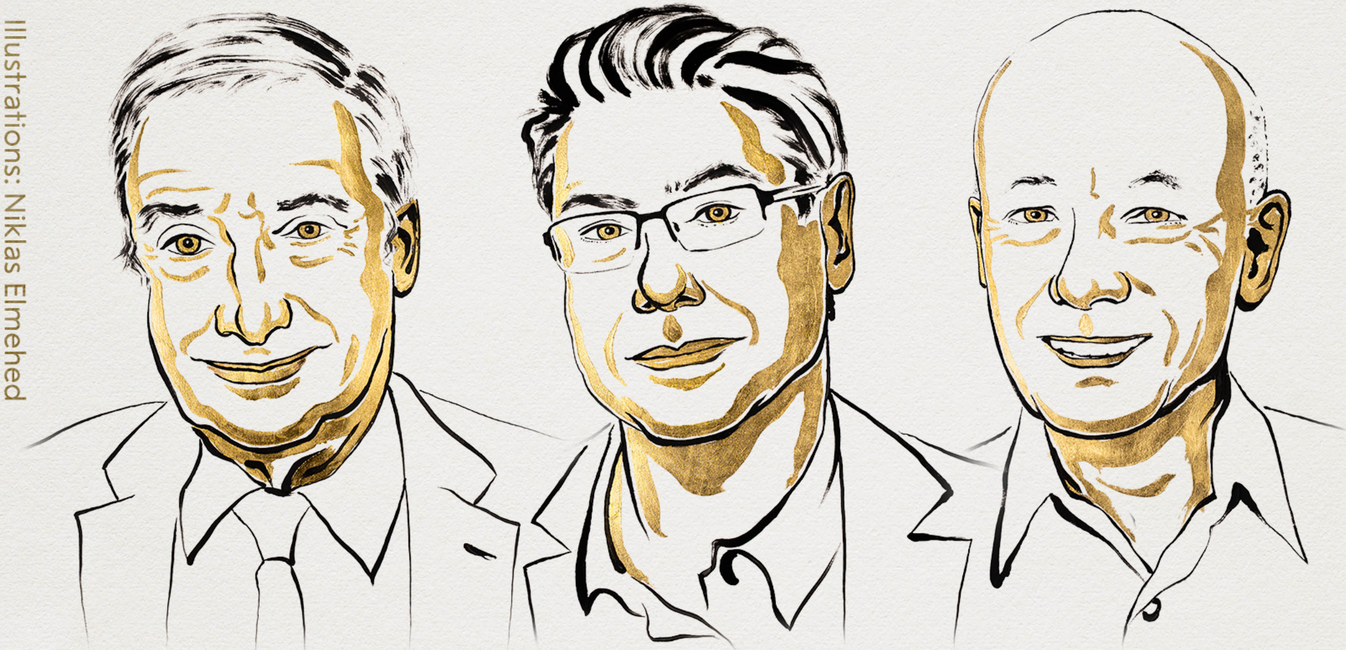

Illustrations: Niklas Elmehed

Joel Mokyr

"for having identified the prerequisites for sustained growth through technological progress"

Philippe Aghion

Peter Howitt

"for the theory of sustained growth through creative destruction"

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Adding technological progress to the Solow model

- In the Solow model,
  - the production technology is held constant.
  - income per capita is constant in the steady state.
- Neither point is true in the real world:
  - production technology is very different from the past
  - 1950–2025: U.S. real GDP per capita grew by a factor of over 4.

The Solow model with population growth and technological progress, part 1

- A new variable: $\boldsymbol{E}$ = labor efficiency
- We now write the production function as:

$$Y = F(K, L \times E),$$

- where $L \times E$ = number of effective workers.
  - Increases in labor efficiency have the same effect on output as increases in the labor force.

The Solow model with population growth and technological progress, part 2

- Notation:
  - $\boldsymbol{y} = \boldsymbol{Y} / (\boldsymbol{LE})$ = output per *effective worker*
  - $\boldsymbol{k} = \boldsymbol{K} / (\boldsymbol{LE})$ = capital per *effective worker*
- Production function per effective worker:
  - $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{k})$
- Saving and investment per effective worker:
  - $\boldsymbol{s}\,\boldsymbol{y} = \boldsymbol{s}\,\boldsymbol{f}(\boldsymbol{k})$

The Solow model with population growth and technological progress, part 3

- Assumption: Technological progress is **labor-augmenting**
  - It increases labor efficiency at the exogenous rate ***g***:

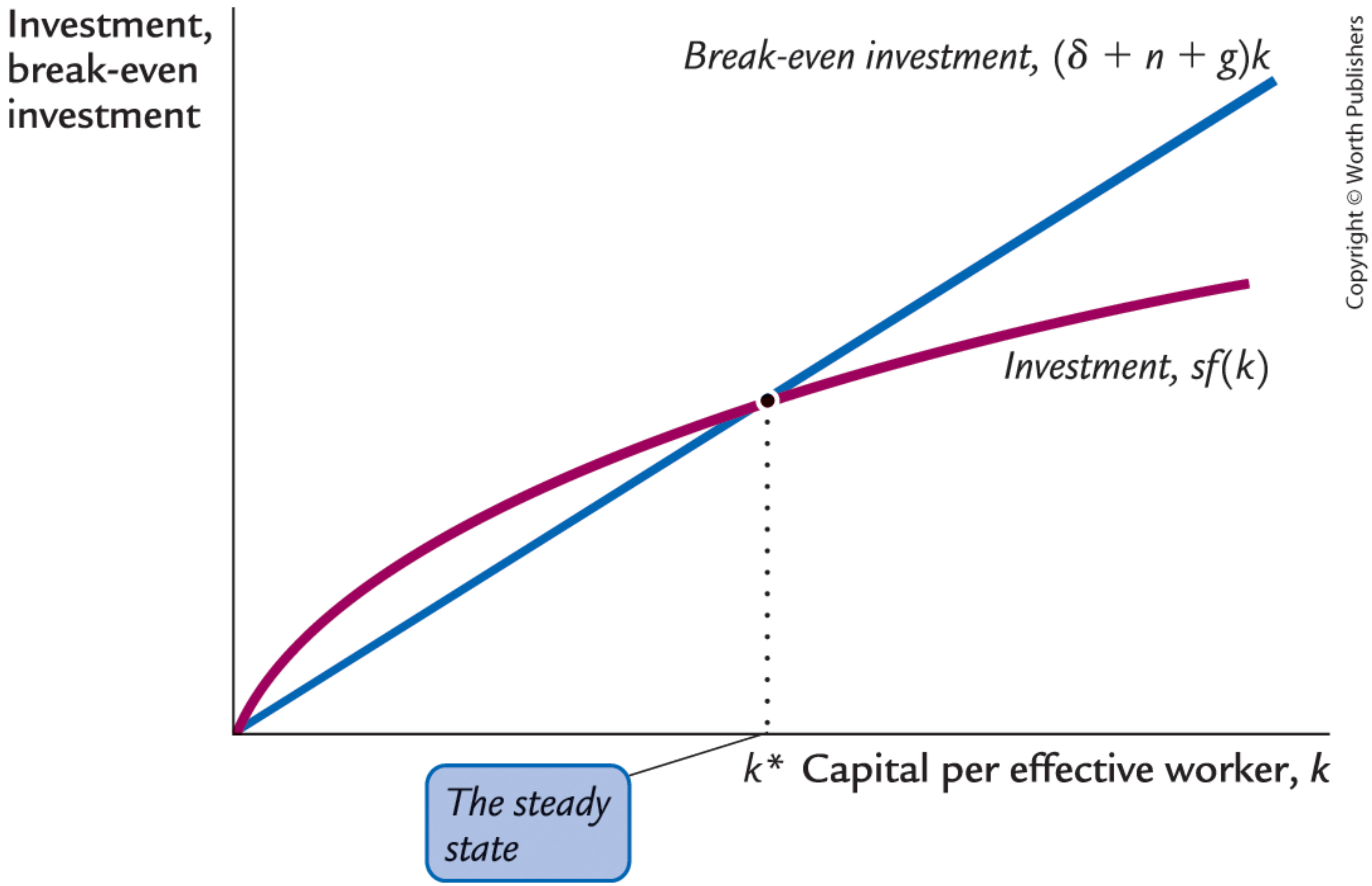
$$\boldsymbol{g = \frac{\Delta E}{E}}$$

The Solow model with population growth and technological progress, part 4

- $(\boldsymbol{\delta} + \boldsymbol{n} + \boldsymbol{g})\,\boldsymbol{k}$ = break-even investment:
  the amount of investment necessary to keep ***k*** constant.
- Consists of:
  - $\boldsymbol{\delta}\,\boldsymbol{k}$ to replace depreciating capital
  - $\boldsymbol{n}\,\boldsymbol{k}$ to provide capital for new workers
  - $\boldsymbol{g}\,\boldsymbol{k}$ to provide capital for the new “effective” workers created by technological progress

The Solow model with population growth and technological progress

$$\Delta k = s\,f(k) - (\delta + n + g)k$$



The Golden Rule with population growth and technological progress

- To find the Golden Rule capital stock, express c^* in terms of k^*:

$$c^* = y^* - i^*$$
$$= f(k^*) - (\delta + n + g)k^*$$

c^* is maximized when

$$MPK = \delta + n + g$$

Or, equivalently,

$$MPK - \delta = n + g$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate plus the rate of tech progress.

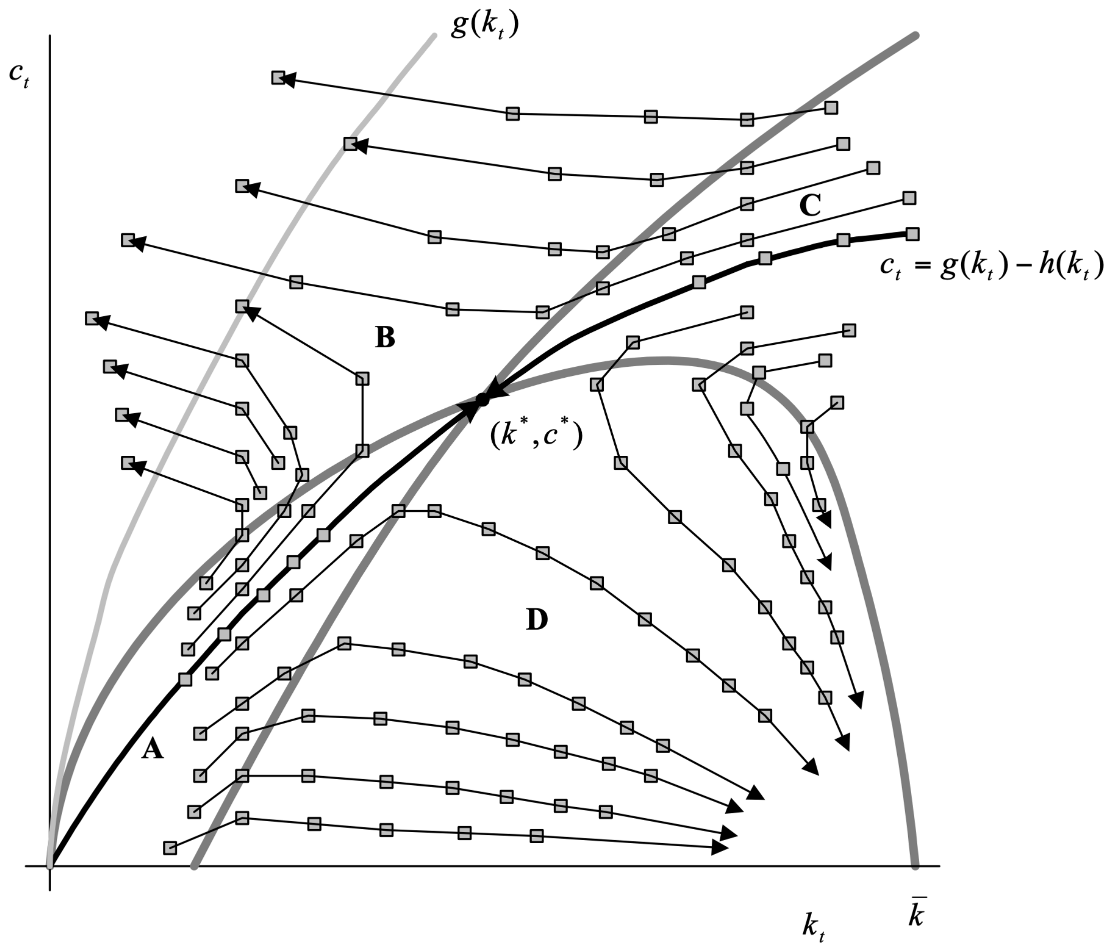
Steady-state growth rates in the Solow model with population growth and technological progress

Variable	Symbol	Steady-State Growth Rate
Capital per effective worker	$k = K/(E \times L)$	0
Output per effective worker	$y = Y/(E \times L) = f(k)$	0
Output per worker	$Y/L = y \times E$	g
Total output	$Y = y \times (E \times L)$	$n + g$

Balanced growth

- The steady state exhibits **balanced growth**: Many variables grow at the same rate.
  - Prediction 1: ***Y***/***L*** and ***K***/***L*** grow at the same rate (***g***), so ***K***/***Y*** should be constant. This is roughly true in the real world.
  - Prediction 2: the real wage grows at the same rate as ***Y***/***L***, while the real rental price is constant. Also roughly true in the real world.
- Modern macro models describe balanced growth path equilibria

Balanced growth path: an example



Source: Craig Burnside's lecture notes (1st year PhD macro)

Topics in growth 1: Convergence

Convergence, part 1

- The Solow model predicts that, other things equal, low-income countries (with lower ***Y***/***L*** and ***K***/***L***) should grow faster than high-income ones.
- If true, then the income gap between high-income and low-income countries would shrink over time, causing living standards to *converge*.
- In the real world, many low-income countries do not grow faster than high-income ones. Does this mean the Solow model fails?

Convergence, part 2

- No, the Solow model does not fail because it predicts that, **other things equal**, poor countries (with lower ***Y***/***L*** and ***K***/***L***) should grow faster than rich ones.
  - In samples of countries with similar savings and population growth rates, income gaps shrink about 2 percent per year.
  - In larger samples, after controlling for differences in saving, population growth, and human capital, incomes converge by about 2 percent per year.

Convergence, part 3

- What the Solow model really predicts is **conditional convergence**: countries converge to their own steady states, which are determined by saving, population growth, and education.
- This prediction comes true in the real world.

Types of convergence

- Sigma-convergence: the dispersion of income shrinks across economies.
- Beta-convergence: poor countries catch up to rich ones. *How?*

Topics in growth 2: Endogenous growth

Endogenous growth theory

- Solow model:
  - Sustained growth in living standards is due to technological progress.
  - The rate of technological progress is exogenous.
- Endogenous growth theory:
  - In this set of models, the growth rate of productivity and living standards is endogenous.

The basic model, part 1

- Production function: $\boldsymbol{Y} = \boldsymbol{A}\,\boldsymbol{K}$
  where $\boldsymbol{A}$ is the amount of output for each unit of capital ($\boldsymbol{A}$ is exogenous and constant)
- Key difference between this model and Solow: ***MPK*** is constant here, diminishes in Solow
- Investment: $\boldsymbol{sY}$
- Depreciation: $\boldsymbol{\delta K}$
- Law of motion for total capital:
  - $\Delta \boldsymbol{K} = \boldsymbol{sY} - \boldsymbol{\delta K}$

The basic model, part 2

- $\Delta \boldsymbol{K} = \boldsymbol{sY} - \boldsymbol{\delta K}$
- Divide through by $\boldsymbol{K}$ and use $\boldsymbol{Y} = \boldsymbol{A}\,\boldsymbol{K}$ to get:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = sA - \delta$$

- If $\boldsymbol{s\,A} > \boldsymbol{\delta}$, then income will grow forever, and investment is the "engine of growth."
- Here, the permanent growth rate depends on $\boldsymbol{s}$. In the Solow model, it does not.

Does capital have diminishing returns?

- It depends on the definition of capital.
- If capital is narrowly defined (only plant and equipment), then yes.
- Advocates of endogenous growth theory argue that knowledge is a type of capital.
- If so, then constant return to capital is more plausible, and this model may be a good description of economic growth.

A two-sector model, part 1

- Two sectors:
  - Manufacturing firms produce goods.
  - R&D that increases labor efficiency in manufacturing.
- $\boldsymbol{u}$ = fraction of labor in research ($\boldsymbol{u}$ is exogenous)
- Manufacturing production function:
  - $\boldsymbol{Y} = \boldsymbol{F}\,[\boldsymbol{K}, (1 - \boldsymbol{u})\boldsymbol{E}\,\boldsymbol{L}]$
- R&D production function: $\Delta \boldsymbol{E} = \boldsymbol{g}(\boldsymbol{u})\boldsymbol{E}$
- Capital accumulation: $\Delta \boldsymbol{K} = \boldsymbol{s}\,\boldsymbol{Y} - \boldsymbol{\delta} \boldsymbol{K}$

A two-sector model, part 2

- In the steady state, manufacturing output per worker and the standard of living grow at rate $\Delta \boldsymbol{E} / \boldsymbol{E} = \boldsymbol{g}(\boldsymbol{u})$.
- Key variables:
  - ***s****:* affects the level of income but not its growth rate (same as in the Solow model)
  - ***u****:* affects level *and* growth rate of income

Economic growth as “creative destruction”

- Schumpeter (1942) coined term “creative destruction” to describe displacements resulting from technological progress:
  - The introduction of a new product is good for consumers but often bad for incumbent producers, who may be forced out of the market.
- Examples:
  - Luddites (1811–1812) destroyed machines that displaced skilled mill workers in England.
  - Walmart displaces many mom-and-pop stores.

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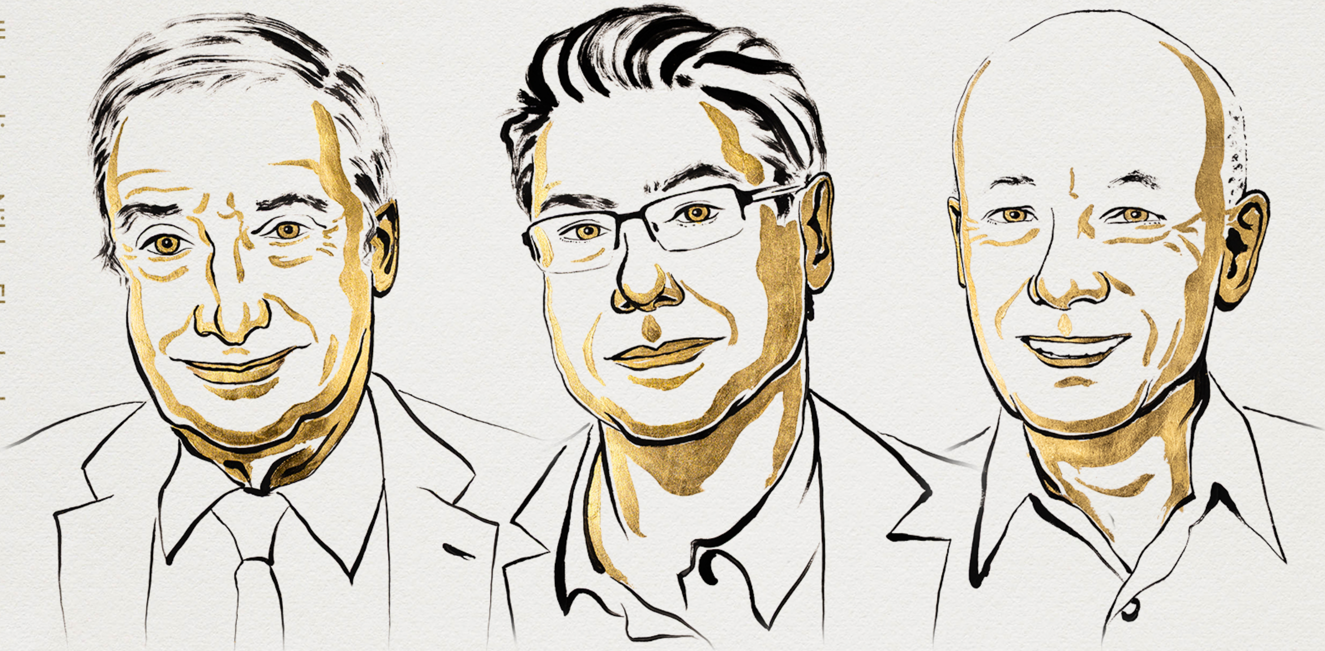
Illustrations: Niklas Elmehed



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Econometrica, Vol. 60, No. 2 (March, 1992), 323–351

A MODEL OF GROWTH THROUGH CREATIVE DESTRUCTION

BY PHILIPPE AGHION AND PETER HOWITT[1]

A model of endogenous growth is developed in which vertical innovations, generated by a competitive research sector, constitute the underlying source of growth. Equilibrium is determined by a forward-looking difference equation, according to which the amount of research in any period depends upon the expected amount of research next period. One source of this intertemporal relationship is creative destruction. That is, the prospect of more future research discourages current research by threatening to destroy the rents created by current research. The paper analyzes the positive and normative properties of stationary equilibria, in which research employment is constant and GNP follows a random walk with drift, although under some circumstances cyclical equilibria also exist. Both the average growth rate and the variance of the growth rate are increasing functions of the size of innovations, the size of the skilled labor force, and the productivity of research as measured by a parameter indicating the effect of research on the Poisson arrival rate of innovations; and decreasing functions of the rate of time preference of the representative individual. Under laissez faire the economy's growth rate may be more or less than optimal because, in addition to the appropriability and intertemporal spillover effects of other endogenous growth models, which tend to make growth slower than optimal, the model also has effects that work in the opposite direction. In particular, the fact that private research firms do not internalize the destruction of rents generated by their innovations introduces a business-stealing effect similar to that found in the partial-equilibrium patent race literature. When we endogenize the size of innovations we find that business stealing also makes innovations too small.

KEYWORDS: Endogenous growth, innovations, creative destruction.

Topics in growth 3: Growth accounting & productivity slowdown

Growth accounting

- Production function $\boldsymbol{Y} = \boldsymbol{AF}(\boldsymbol{K},\boldsymbol{L})$,
  - where $\boldsymbol{A}$ is *total factor productivity* (TFP).
- Can break down growth in output by:

$$\begin{array}{c}\text{Growth in}\\\text{Output}\end{array} = \begin{array}{c}\text{Contribution}\\\text{of Capital}\end{array} + \begin{array}{c}\text{Contribution}\\\text{of Labor}\end{array} + \begin{array}{c}\text{Growth in}\\\text{TFP}\end{array}$$

- With a Cobb-Douglas production function, the above decomposition becomes:

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- With a Cobb-Douglas production function, the above decomposition becomes:

$$\frac{\Delta Y}{Y} = \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} + \frac{\Delta A}{A},$$

  - where $\frac{\Delta A}{A}$ is the *Solow residual*.

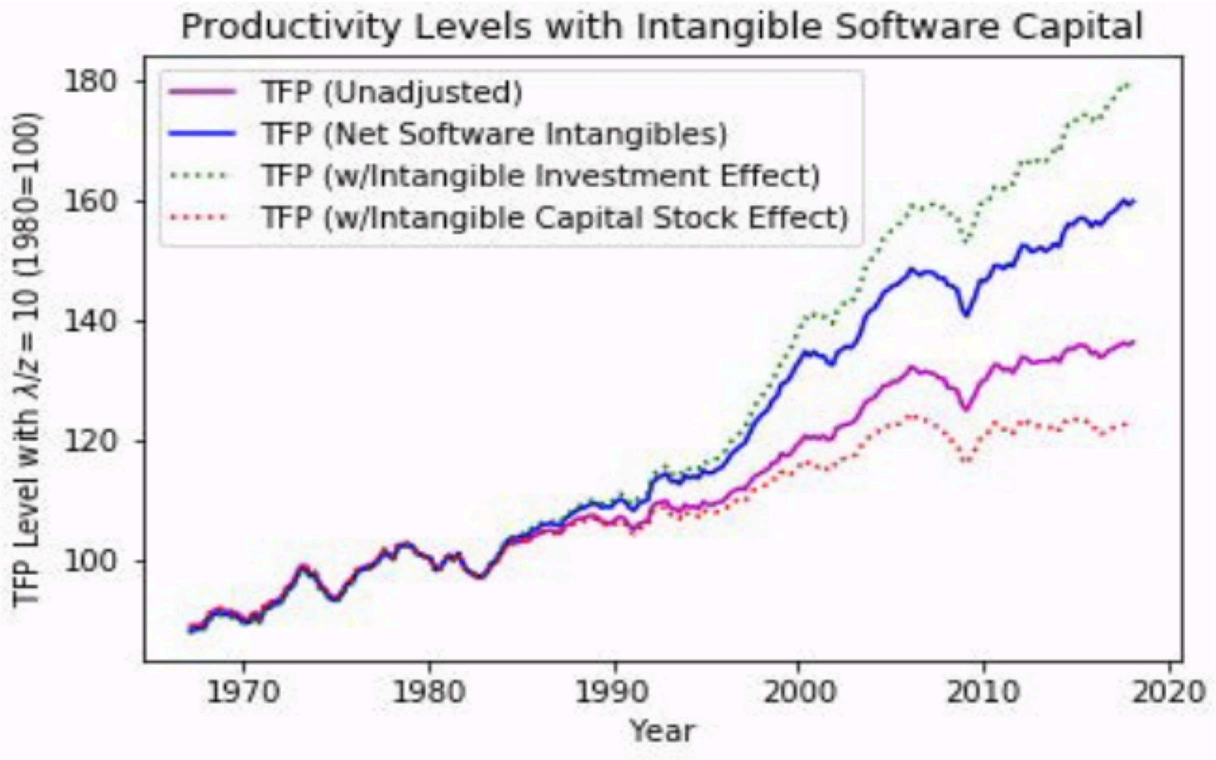
Accounting for economic growth in the United States

Years	Output Growth $\Delta Y/Y$	=	Capital $\alpha\Delta K/K$	+	Labor $(1-\alpha)\Delta L/L$	+	Total Factor Productivity $\Delta A/A$
					(Average percentage increase per year)		
1948–2022	3.4		1.3		1.0		1.1
1948–1973	4.2		1.3		1.0		1.9
1973–2022	2.9		1.3		1.0		0.6

Data from: U.S. Department of Labor. Data are for the nonfarm business sector. Parts may not add to total due to rounding.

What is causing the productivity slowdown?

- Measurement problems



Source: Brynjolfsson, Rock, and Syverson (2021). "The Productivity J-Curve: How Intangibles Complement General Purpose Technologies."

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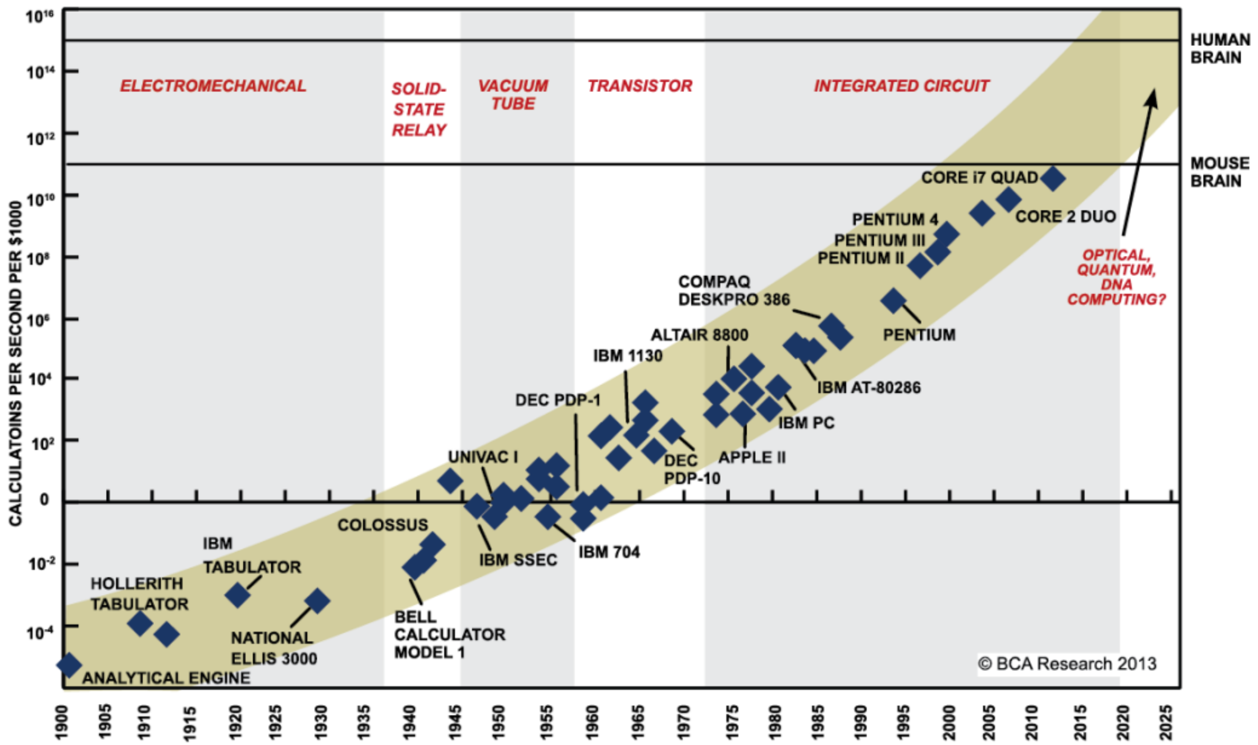
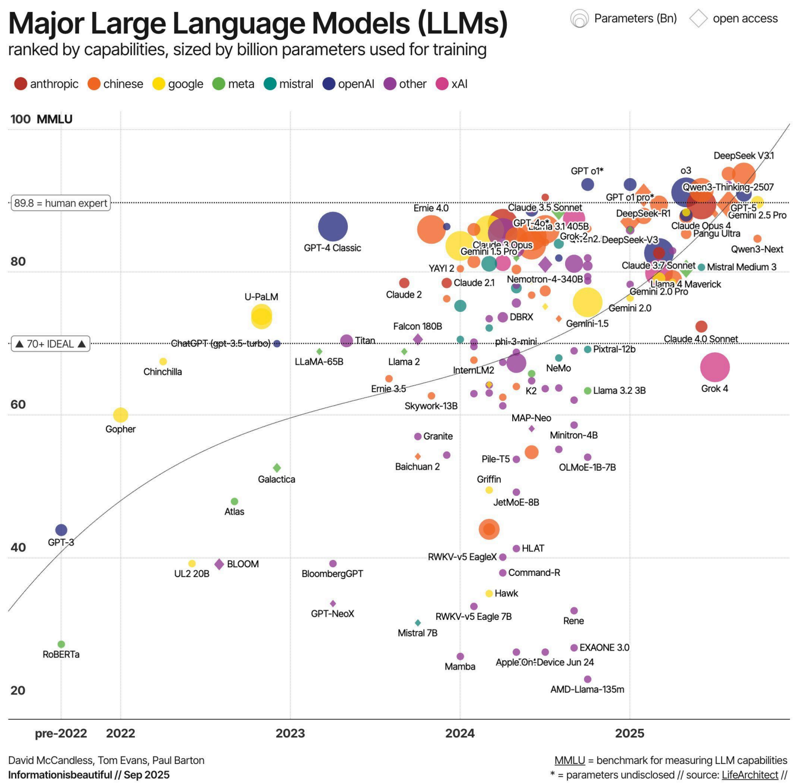


SOURCE: RAY KURZWEIL, "THE SINGULARITY IS NEAR: WHEN HUMANS TRANSCEND BIOLOGY", P.67, *THE VIKING PRESS*, 2006. DATAPOINTS BETWEEN 2000 AND 2012 REPRESENT BCA ESTIMATES.

What is causing the productivity slowdown?

- Measurement problems
- The nature of growth
  - Long-run capital growth: Moore's law
  - Short-run capital growth: S-curves



[Link]

What is causing the productivity slowdown?

- Measurement problems
- The nature of growth
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  - Short-run capital growth: S-curves
- Complementarity between capital and labor inputs
  - It takes time for people to utilize new technology
  - “Steam machines/computers/robots/AI will take our jobs!”
  - Recall creative destruction & distributional impacts

What is causing the productivity slowdown?

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- Complementarity between capital and labor inputs
  - It takes time for people to utilize new technology
  - "Steam machines/computers/robots/AI will take our jobs!"
  - Recall creative destruction & distributional impacts
- (Possibly) improper growth policy

Topics in growth 4: Policy issues

Policy issues of investment and growth

- Allocative investment policies
- Growth policies
  1. Policies affecting the savings rate
  2. Policies affecting population growth & labor efficiency
  3. Policies affecting technological progress

Allocating the economy's investment

Two viewpoints:

1. Equalize tax treatment of all types of capital in all industries and let the market allocate investment to the type with the highest marginal product.
2. Industrial policy:
   Government should actively encourage investment in capital of certain types or in certain industries.

The debate surrounding industrial policy

- Pros:
  - Investment has *positive externalities* that private investors don’t *internalize.*
  - Industrial policy can incentivize private businesses to take a long-term view.
- Cons:
  - The government may not have the ability to “pick winners” (choose industries with the highest return to capital or biggest externalities).
  - Politics (for example, campaign contributions) rather than economics may influence which industries get preferential treatment.
- **Very** active research (e.g., The Industrial Policy Group)

Growth policies: the savings rate, part 1

- Use the Golden Rule to determine whether the U.S. saving rate and capital stock are too high, too low, or about right.
  - If $(MPK - \delta) > (n + g)$, the U.S. economy is below the Golden Rule steady state and should increase s.
  - If $(MPK - \delta) < (n + g)$, the U.S. economy is above the Golden Rule steady state and should reduce s.

Growth policies: the savings rate, part 2

To estimate ($MPK - \delta$), use three facts about the U.S. economy:

1. $k = 2.5\,y$
   The capital stock is about 2.5 times one year's GDP.
2. $\delta k = 0.1\,y$
   About 10% of GDP is used to replace depreciating capital.
3. $MPK \times k = 0.3\,y$
   Capital income is about 30% of GDP.

Growth policies: the savings rate, part 3

1. $\boldsymbol{k} = 2.5\ \boldsymbol{y}$
2. $\boldsymbol{\delta k} = 0.1\ \boldsymbol{y}$
3. $\boldsymbol{MPK} \times \boldsymbol{k} = 0.3\ \boldsymbol{y}$

To determine $\boldsymbol{\delta}$, divide **2** by **1**:

$$\frac{\delta \boldsymbol{k}}{\boldsymbol{k}} = \frac{0.1\boldsymbol{y}}{2.5\,\boldsymbol{y}} \Rightarrow \delta = \frac{0.1}{2.5} = 0.04$$

Growth policies: the savings rate, part 4

1. $\boldsymbol{k} = 2.5\ \boldsymbol{y}$
2. $\boldsymbol{\delta k} = 0.1\ \boldsymbol{y}$
3. $\boldsymbol{MPK} \times \boldsymbol{k} = 0.3\ \boldsymbol{y}$

To determine ***MPK***, divide **3** by **1**:

$$\frac{\boldsymbol{MPK} \times \boldsymbol{k}}{\boldsymbol{k}} = \frac{0.3\,\boldsymbol{y}}{2.5\,\boldsymbol{y}} \Rightarrow \boldsymbol{MPK} = \frac{0.3}{2.5} = 0.12$$

Hence, $\boldsymbol{MPK} - \boldsymbol{\delta} = 0.12 - 0.04 = \underline{0.08}$

Growth policies: the savings rate, part 5

- From the last slide: $MPK - \delta = 0.08$
- U.S. real GDP grows an average of 3% per year, so $n + g = 0.03$
- Thus,

$$MPK - \delta = 0.08 > 0.03 = n + g$$

- Conclusion:

Growth policies: the savings rate, part 5

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- U.S. real GDP grows an average of 3% per year, so $\boldsymbol{n + g} = 0.03$
- Thus,

$$\boldsymbol{MPK - \delta} = 0.08 > 0.03 = \boldsymbol{n + g}$$

- Conclusion:

The United States is below the Golden Rule steady state:
Increasing the U.S. saving rate would increase consumption per capita in the long run.

Growth policies: the savings rate, part 6

- Reduce the government budget deficit (or increase the budget surplus).
- Increase incentives for private saving:
  - Expand tax incentives for IRAs (individual retirement accounts) and other retirement savings accounts.

CLASS DISCUSSION
Growth policies: population growth and labor efficiency

What can policymakers do to change the rate of population growth?

How can policymakers create incentives to improve labor efficiency?

Growth policies: technological progress, part 1

- Much research is done by firms seeking profits.
- Firms profit from research:
  - Patents create a stream of monopoly profits.
  - There is extra profit in being first on the market with a new product.
- Innovation produces externalities that reduce the cost of subsequent innovation.
- How much R&D do firms do?
  - Positive externalities suggest not enough.
  - But there is much duplication of R&D effort among competing firms.
- Estimates: social return to R&D ≥ 40 percent per year.

Growth policies: technological progress, part 2

- Patent laws: encourage innovation by granting *temporary monopolies* to inventors of new products.
- Tax incentives for R&D and Capital Purchases (CapEx)
  - When you hear a corporation paid zero taxes, it is often because the company "reinvested" profits into R&D and CapEx.
- Industrial policy: encourages specific industries that are key for rapid technological progress (subject to the preceding concerns).
- Funding research & international knowledge flow
  - Philippe Aghion: born in France → PhD at Harvard → MIT → UCL → Harvard → LSE & INSEAD
  - Peter Howitt: born in Canada → PhD at Northwestern → Western → Ohio State → Brown
  - Joel Mokyr: born in the Netherlands → PhD at Yale → Northwestern

SUMMARY, PART 1

- Key results from the Solow model with population growth and technological progress:
  - The steady-state growth rate of income per person depends solely on the exogenous rate of technological progress.
  - The steady-state growth rate of total income depends solely on the exogenous rates of technological progress and population growth.
- Endogenous growth theory: Models that
  - examine the determinants of the rate of technological progress, which Solow takes as given.
  - explain decisions that determine the creation of knowledge through R&D.

SUMMARY, PART 2

- Recent studies have found that international variation in standards of living is attributed to capital accumulation and the efficiency with which capital is used.
- Growth accounting decomposes growth into its sources to measure technological progress called total factor productivity. In the United States, growth accounting shows a pronounced slowdown starting in the 1970s.
- Policymakers have various ways of increasing economics growth.