

3. National Income

Based on Mankiw, Chapters 3 & 4: *National Income: How It Is Earned & National Income: How It Is Spent*

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Outline of model

- A closed economy, market-clearing model
- Aggregate supply
 - factor markets (supply, demand, price)
 - determination of output/income
- Aggregate demand
 - determinants of C , I , and G
- Equilibrium
 - goods market
 - loanable funds market

Aggregate supply

Aggregate demand

Equilibrium

Factors of production

- K : capital
 - tools, machines, and structures used in production
- L : labor
 - the physical and mental efforts of workers

The production function: $Y = F(K, L)$

- Shows how much output Y the economy can produce from:
 - K units of capital
 - L units of labor
- Reflects the economy's level of technology
- Reflects how existing technology transforms capital and labor into output
- Technological improvements result in more output from same level of inputs

Returns to scale

- We start in period 1 with $Y_1 = F(K_1, L_1)$
- Scale all inputs by the same factor $z > 0$:
 - $K_2 = zK_1$ and $L_2 = zL_1$
 - E.g., if $z = 1.2$, then all inputs are increased by 20%
- What happens to output, $Y_2 = F(K_2, L_2)$?
 - $Y_2 = zY_1$ for any $z > 0$: ???
 - $Y_2 > zY_1$ for any $z > 1$: ???
 - $Y_2 < zY_1$ for any $z > 1$: ???

Returns to scale

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 - E.g., if $z = 1.2$, then all inputs are increased by 20%
- What happens to output, $Y_2 = F(K_2, L_2)$?
 - $Y_2 = zY_1$ for any $z > 0$: *constant returns to scale*
 - $Y_2 > zY_1$ for any $z > 1$: *increasing returns to scale*
 - $Y_2 < zY_1$ for any $z > 1$: *decreasing returns to scale*

Returns to scale: Example 1

$$F(K, L) = \sqrt{KL}$$

$$F(zK, zL) = ???$$

Returns to scale: Example 1

$$F(K, L) = \sqrt{KL}$$

$$F(zK, zL) = \sqrt{(zK)(zL)}$$

$$= \sqrt{z^2 KL}$$

$$= z\sqrt{KL}$$

$$= zF(K, L)$$

- This production function has constant returns to scale for any $z > 0$

Returns to scale: Example 2

$$F(K, L) = K^2 + L^2$$

$$F(zK, zL) = ???$$

Returns to scale: Example 2

$$F(K, L) = K^2 + L^2$$

$$F(zK, zL) = (zK)^2 + (zL)^2$$

$$= z^2(K^2 + L^2)$$

$$= z^2F(K, L)$$

- This production function has increasing returns to scale for any $z > 1$

NOW YOU TRY

- Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

a. $F(K, L) = \frac{K^2}{L}$

b. $F(K, L) = K + L$

NOW YOU TRY

a. $F(K, L) = \frac{K^2}{L}$

$$F(zK, zL) = \frac{(zK)^2}{zL}$$

$$= \frac{z^2 K^2}{zL}$$

$$= z \frac{K^2}{L}$$

$$= zF(K, L)$$

- This production function has constant returns to scale for any $z > 0$

NOW YOU TRY

b. $F(K, L) = K + L$

$$\begin{aligned}F(zK, zL) &= zK + zL \\&= z(K + L) \\&= zF(K, L)\end{aligned}$$

- This production function has constant returns to scale for any $z > 0$

Assumptions

1. Technology $F(\cdot)$ is fixed over time
2. The economy's supplies of capital and labor are fixed: $K = \bar{K}$ and $L = \bar{L}$

Determining GDP

- Output is determined by the fixed factor supplies and the fixed technology

$$\bar{Y} = F(\bar{K}, \bar{L})$$

Factor prices

- The per unit prices that firms pay for the factors of production
 - Wage: price of L
 - Rental rate: price of K

Notation

- W : nominal wage
- R : nominal rental rate
- P : price of output
- W/P : real wage (measured in units of output)
- R/P : real rental rate

How factor prices are determined

- Quick pause: supply/demand of *factors* vs. *final goods!*
- Factor prices are determined by supply and demand in factor markets
- Recall that the supply of each factor is fixed
- What about demand?

Demand for labor

- Assume that markets are competitive: each firm takes W , R , and P as given
- Idea: a firm hires each unit of labor if the cost does not exceed the benefit
 - Cost: ???
 - Benefit: ???

Demand for labor

- Assume that markets are competitive: each firm takes W , R , and P as given
- Idea: a firm hires each unit of labor if the cost does not exceed the benefit
 - Cost: real wage
 - Benefit: marginal product of labor (MPL)

Marginal product of labor (*MPL*)

- Definition: the extra output firms can produce using an additional unit of labor
 - ... holding other inputs fixed

$$MPL = F(K, L + 1) - F(K, L)$$

or

$$MPL = \frac{\partial F(K, L)}{\partial L}$$

NOW YOU TRY

- a. Determine MPL at each value of L
- b. Graph the production function
- c. Graph the MPL curve with MPL on the vertical axis and L on the horizontal axis

L	Y	MPL
0	0	n.a.
1	10	
2	19	
3	27	
4	34	
5	40	
6	45	
7	49	
8	52	
9	54	
10	55	

NOW YOU TRY

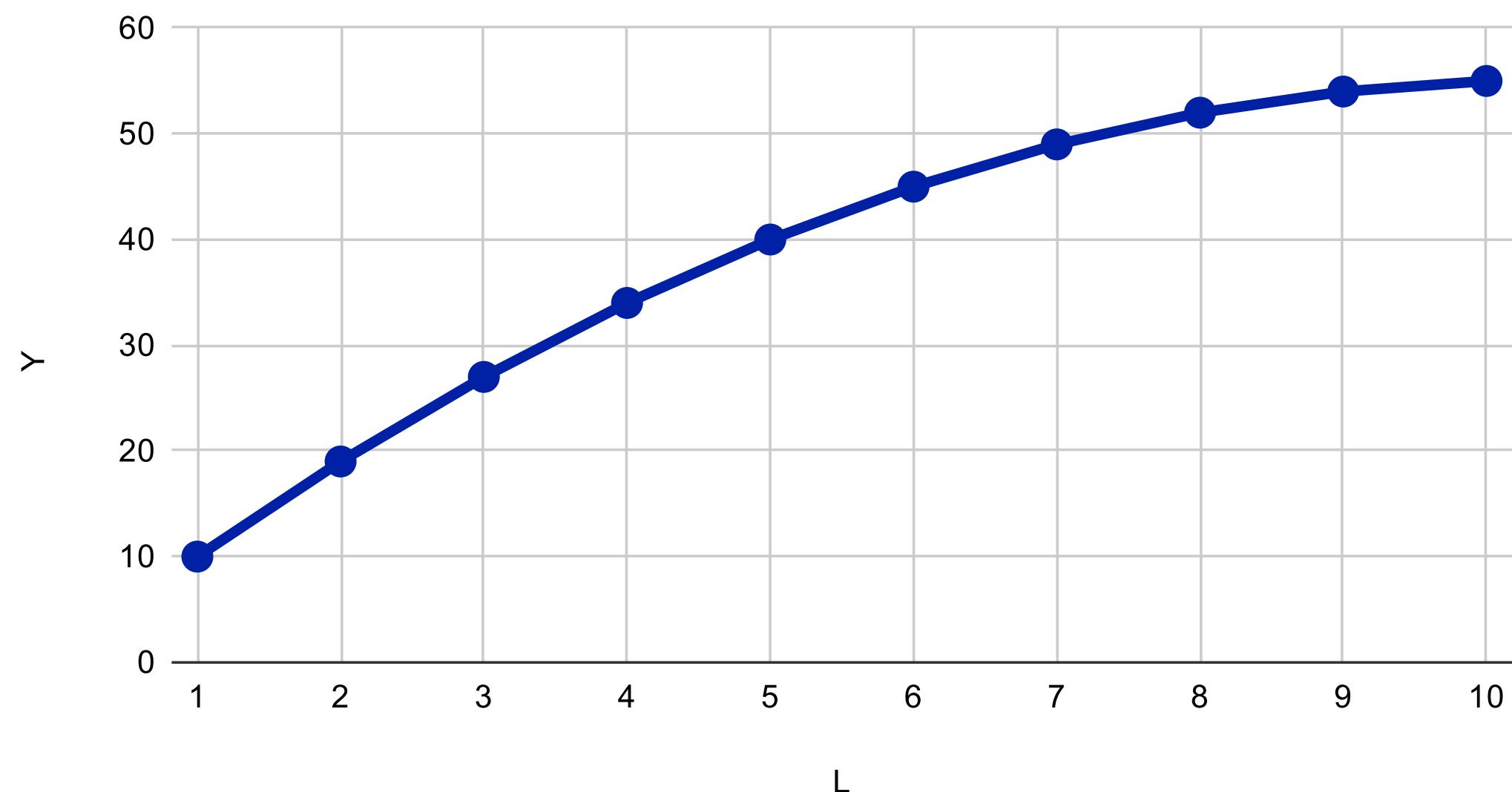
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L	Y	MPL
0	0	n.a.
1	10	10
2	19	9
3	27	8
4	34	7
5	40	6
6	45	5
7	49	4
8	52	3
9	54	2
10	55	1

NOW YOU TRY

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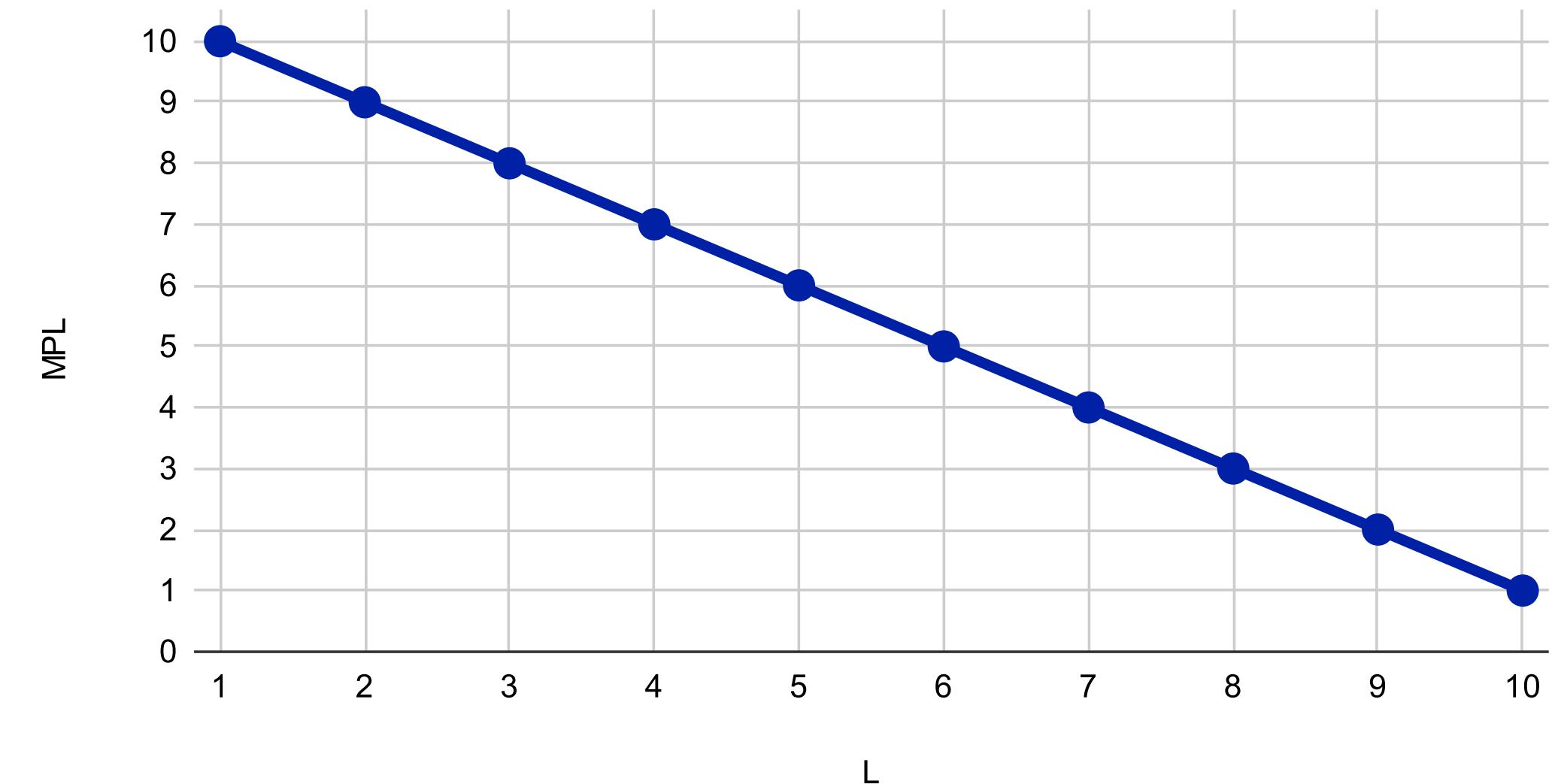
Production function



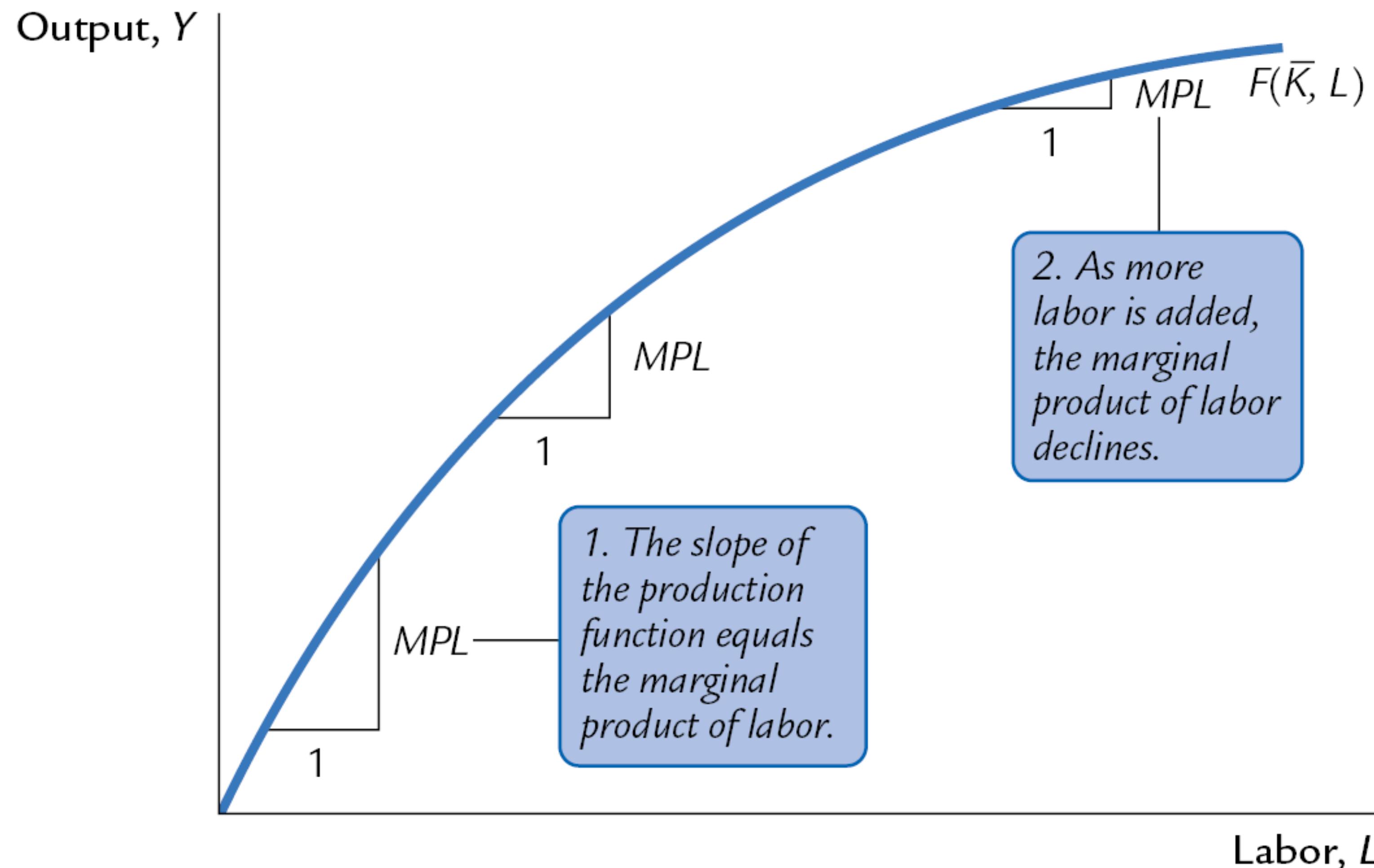
NOW YOU TRY

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Marginal product of labor



MPL and the production function



Diminishing marginal returns

- As an input is increased (holding other inputs fixed), its marginal product falls
- Intuition:
 - If L increases while holding K fixed, machines per worker fall, worker productivity falls

NOW YOU TRY

- Do these production functions have diminishing marginal returns to labor?
 - $F(K, L) = 2K + 15L$
 - $F(K, L) = \sqrt{KL}$
 - $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

NOW YOU TRY

- Do these production functions have diminishing marginal returns to labor?

a. $F(K, L) = 2K + 15L$

$MPL = 15 \implies \text{no}$, because it is constant as L increases

b. $F(K, L) = \sqrt{KL}$

$MPL = \frac{\sqrt{K}}{2\sqrt{L}} \implies \text{yes}$, because it is decreasing in L

c. $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

$MPL = \frac{15}{2\sqrt{L}} \implies \text{yes}$, because it is decreasing in L

NOW YOU TRY

- Suppose $W/P = 6$
- If $L = 3$, should the firm hire more or less labor? Why?
- If $L = 7$, should the firm hire more or less labor? Why?

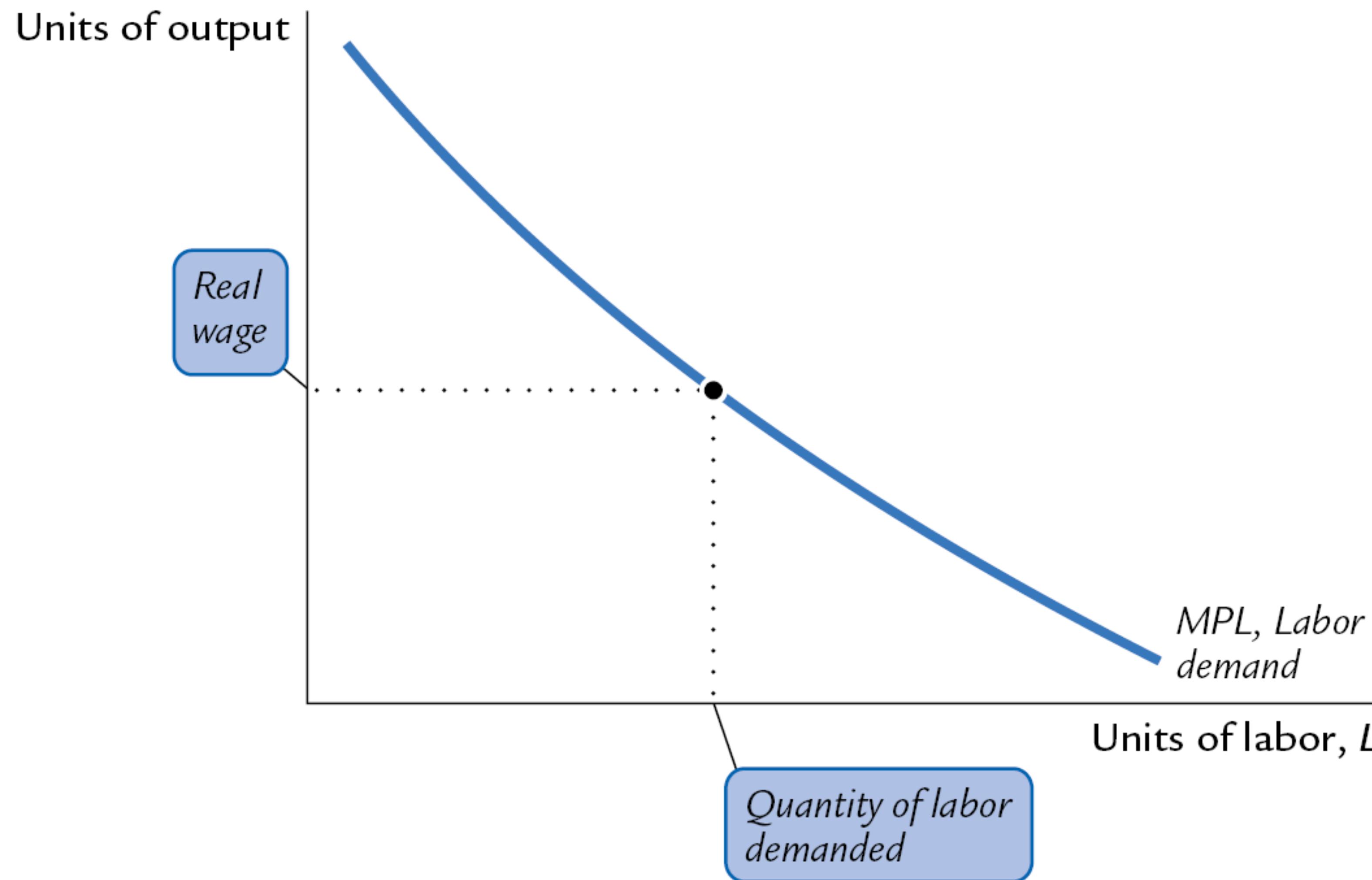
<i>L</i>	<i>Y</i>	<i>MPL</i>
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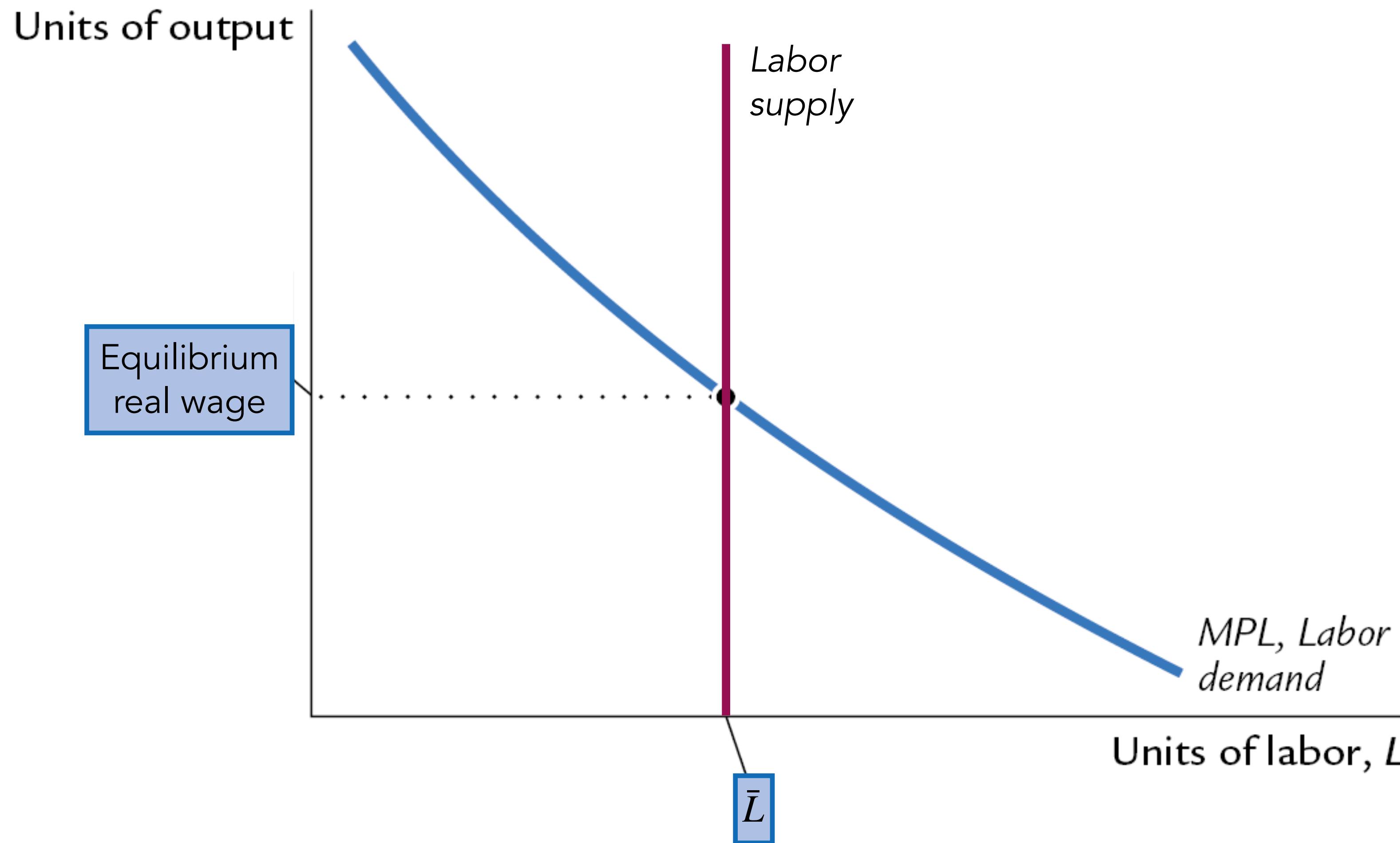
- Suppose $W/P = 6$
- If $L = 3$, should the firm hire more or less labor? Why?
 - **Answer:** More, because the benefit of the fourth worker ($MPL = 7$) exceeds its cost ($W/P = 6$)
- If $L = 7$, should the firm hire more or less labor? Why?
 - **Answer:** Less, because the seventh worker adds $MPL = 4$ units of output but costs the firm $W/P = 6$

L	Y	MPL
0	0	n.a.
1	10	10
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MPL and the demand for labor



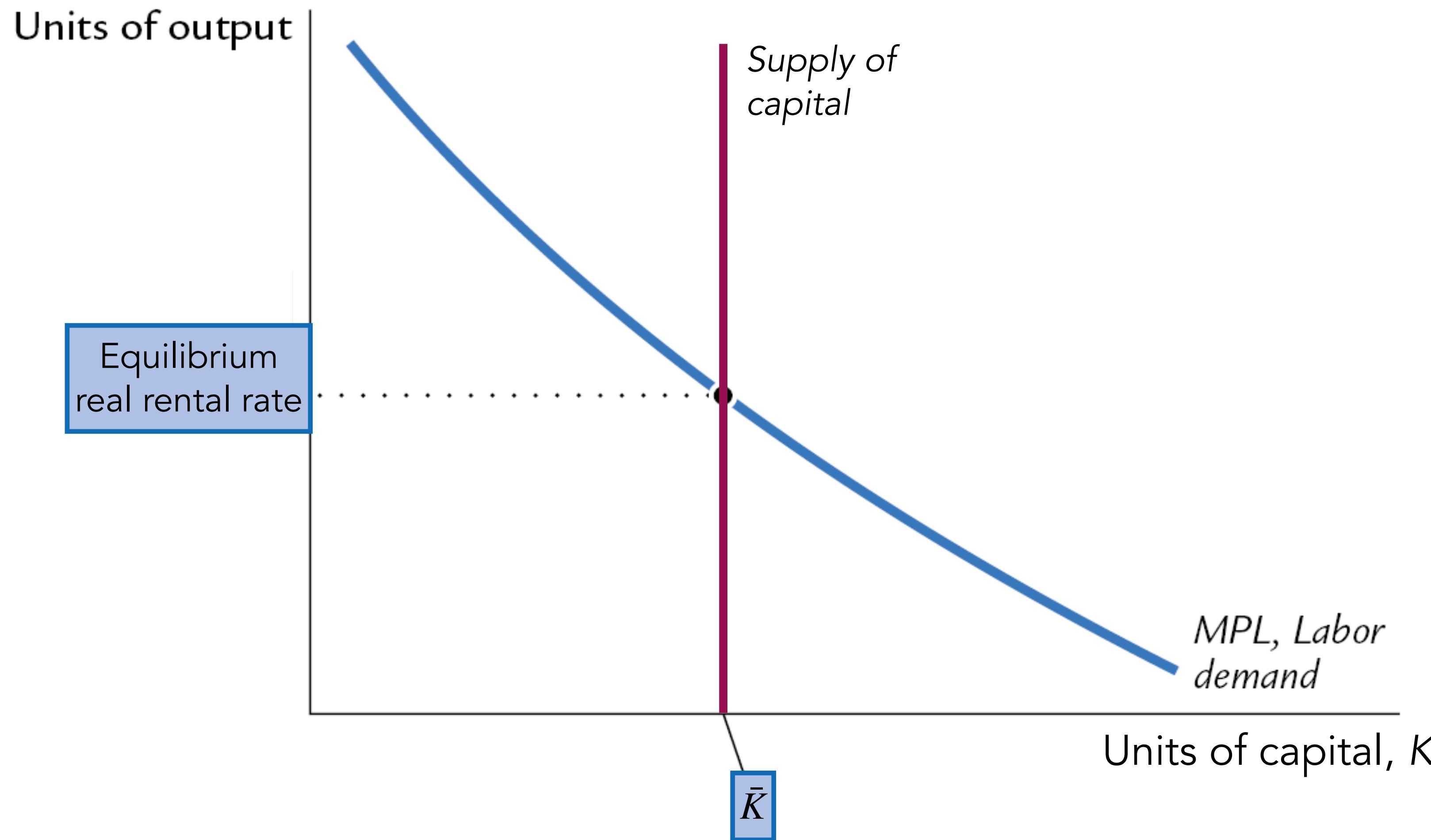
The equilibrium real wage



Determining the rental rate

- We have just seen that $MPL = W/P$
- The same logic shows that $MPK = R/P$
 - Diminishing returns to capital: MPK falls as K rises
 - The MPK curve is the firm's demand curve for renting capital
 - Firms maximize profits by choosing K^* such that $MPK = R/P$

The equilibrium real rental rate



Neoclassical theory of distribution

- States that each factor input is paid its marginal product
- A good starting point for thinking about income distribution
- Examples where this does not hold? *Class discussion*

How income is distributed to L and K

- Total labor income: $\frac{W}{P} \bar{L} = MPL \times \bar{L}$
- Total capital income: $\frac{R}{P} \bar{K} = MPK \times \bar{K}$
- If the production function has constant returns to scale, then

$$\bar{Y} = \underbrace{MPK \times \bar{K}}_{\text{capital income}} + \underbrace{MPL \times \bar{L}}_{\text{labor income}}$$

Cobb–Douglas production function

- Cobb–Douglas production function:

$$Y = AK^\alpha L^{1-\alpha} \text{ where } A \text{ represents the level of technology}$$

- The Cobb–Douglas production function has constant factor shares
 - α : capital's share of total income. Why?

Cobb–Douglas production function

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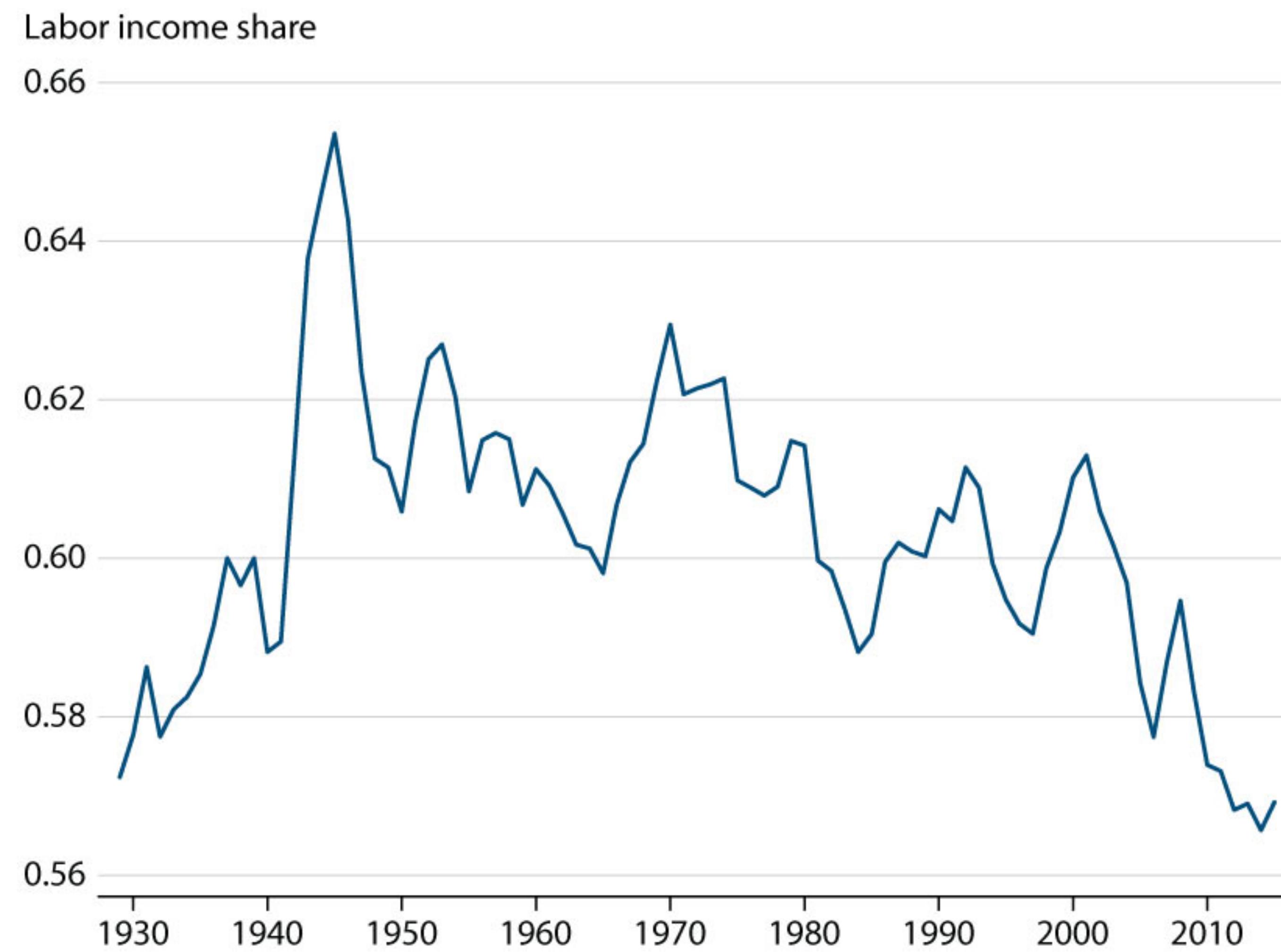
- α : capital's share of total income. Why?

$$MPK = A\alpha K^{\alpha-1} L^{1-\alpha} = \frac{\alpha Y}{K} \text{ and } MPL = AK^\alpha(1 - \alpha)L^{-\alpha} = \frac{(1 - \alpha)Y}{L}$$

$$\implies \bar{Y} = MPK \times \bar{K} + MPL \times \bar{L} = \alpha \bar{Y} + (1 - \alpha) \bar{Y}$$

- Does this hold true in the data?

Declining labor share in the US



Labor share and income inequality

- Lower labor share over time exacerbates income disparity
- Why?
 - Technological advances reduce the role of labor in production
 - Market power of firms increases while worker power decreases

Skill-biased technological change

- Technological progress increased the relative demand for skilled labor
- Since the '70s, technological progress continues but education is stagnant
- Therefore, skill-biased technological change increases wage inequality
 - Growing demand & stagnant supply of skilled workers
 - Skilled workers' wages grow compared to unskilled ones
 - Over time, wage inequality exacerbates income inequality

Globalization and inequality

- US trade growth also increases the relative demand for skilled labor
- Therefore, inequality grows
- However, trade is still beneficial on aggregate (more on this later)

Further sources of inequality

- Educational slowdown
- Falling marriage rates among higher-educated workers (assortative matching)

Aggregate supply
Aggregate demand
Equilibrium

Demand for goods and services

1 of 2

- Recall the determinants of the four components of GDP
 - C : consumption
 - I : investment
 - G : government expenditures
 - NX : net exports

Demand for goods and services

2 of 2

- Components of aggregate demand:
 - C : consumer demand for goods and services
 - I : demand for investment goods
 - G : government demand for goods and services
 - closed economy: no NX

Consumption, C

1 of 2

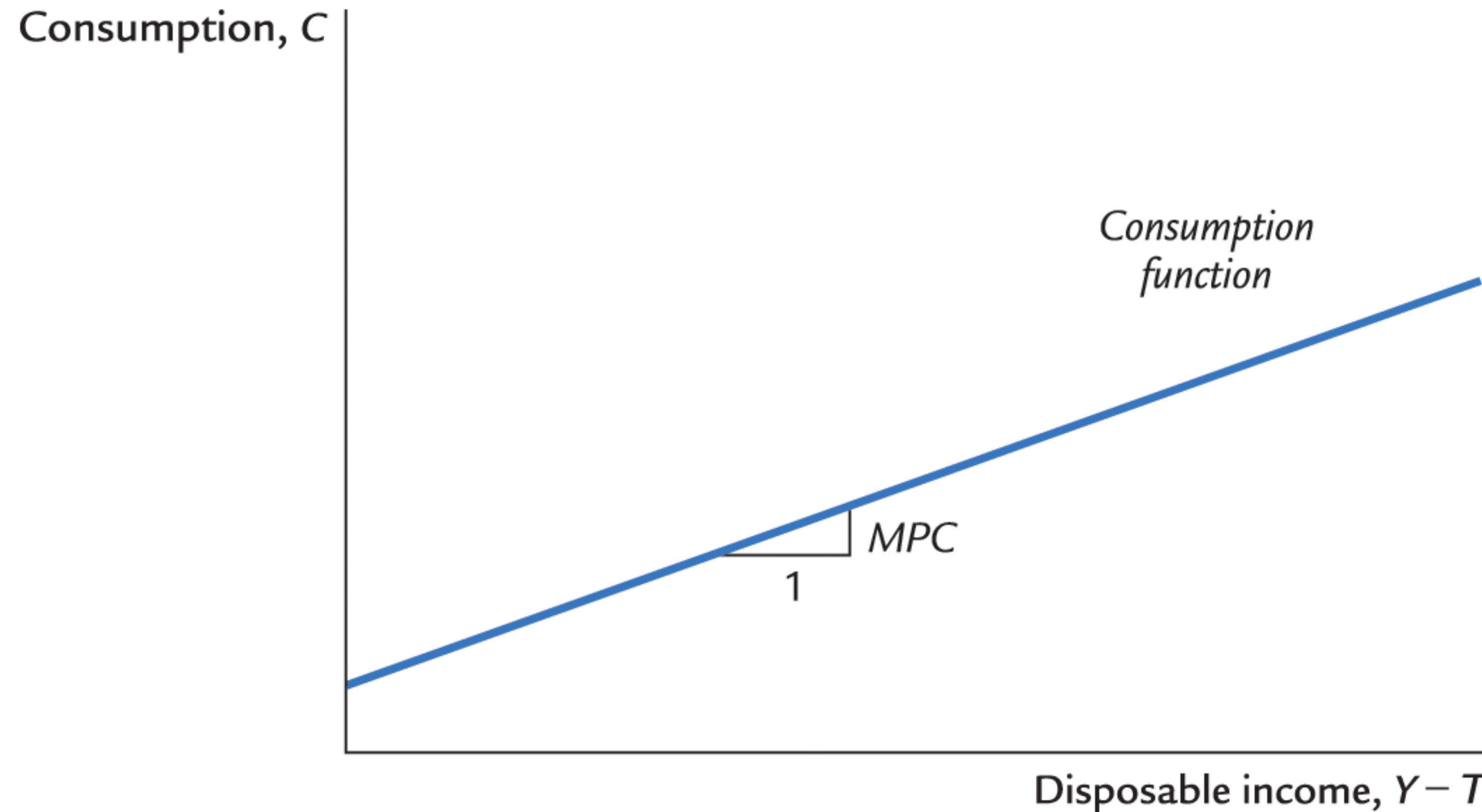
- Households receive income from their labor and from owing capital
- They pay taxes to the government
- They decide how much of their after-tax income to consume and save
- Consumption/savings are the typical decisions in our economic models

Consumption, C

2 of 2

- *Disposable income* is total income minus total taxes: $Y - T$
- Consumption function: $C = C(Y - T)$
- *Marginal propensity to consume (MPC)*
 - The change in consumption C when disposable income $Y - T$ increases by one dollar

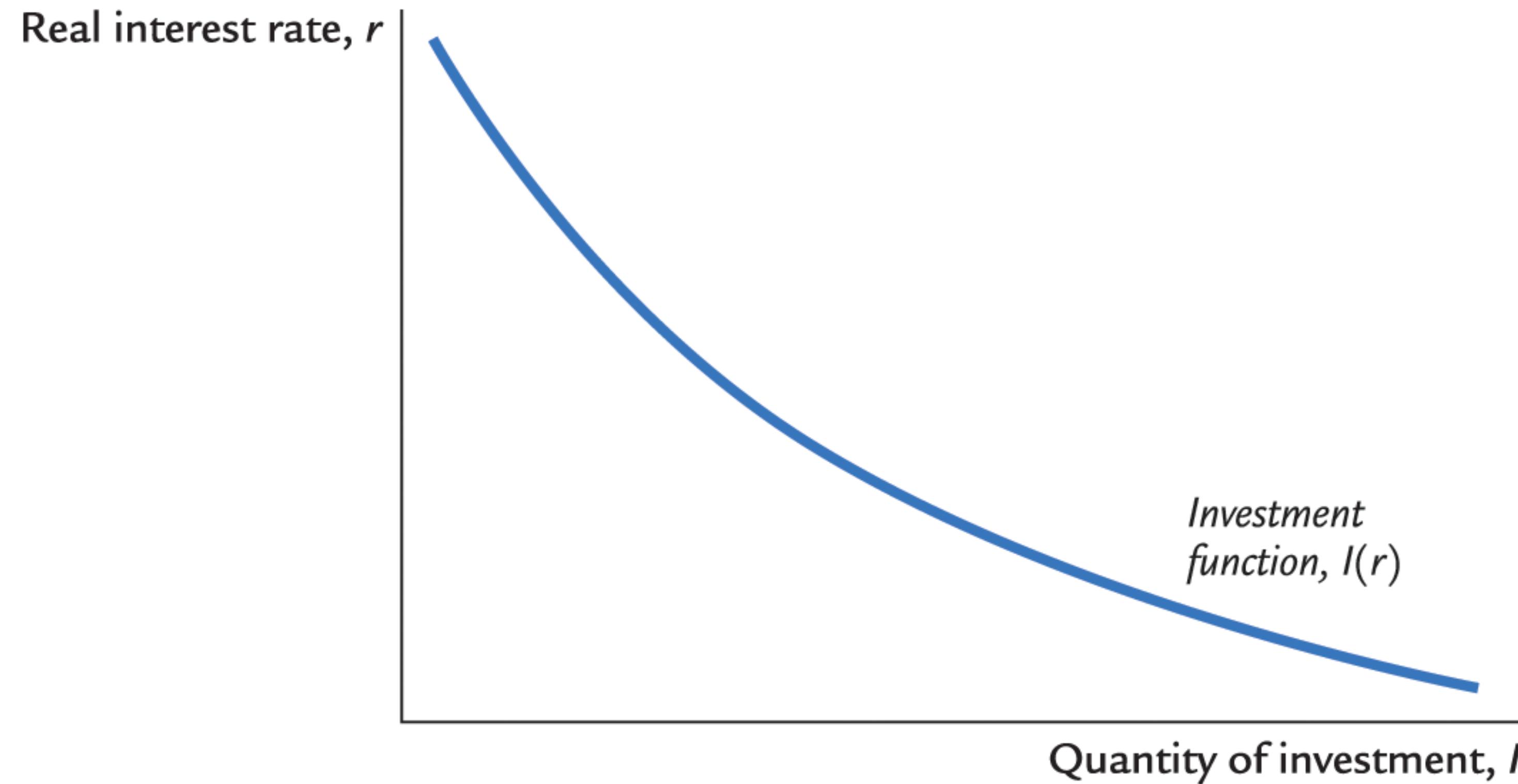
The consumption function



Investment, I

- Investment function: $I = I(r)$
 - r denotes the real interest rate, the nominal interest rate corrected for inflation
- The real interest rate is
 - ... the cost of borrowing
 - ... the opportunity cost of using one's own funds to finance investment spending
- So, I depends negatively on r
 - Note that r is the real interest rate, not the rental rate of capital R

The investment function



Government spending, G

- G : government spending on goods and services
- G excludes transfer payments
 - E.g., Social Security benefits, unemployment insurance benefits
- Assume that government spending and total taxes are exogenous:
 $G = \bar{G}$ and $T = \bar{T}$

Aggregate supply
Aggregate demand
Equilibrium

The market for goods and services

Aggregate demand: $C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

Aggregate supply: $\bar{Y} = F(\bar{K}, \bar{L})$

Equilibrium: $\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$

- The real interest rate adjusts to equate demand with supply

The loanable funds market

- A simple supply–demand model of the financial system
- One asset: “loanable funds”
 - demand for funds: investment
 - supply of funds: saving
 - “price” of funds: real interest rate

Demand for funds: investment

- The demand for loanable funds
 - ... comes from investment
 - Firms borrow to finance spending on plants and equipment, new office buildings, etc.
 - Consumers borrow to buy new houses
 - ...depends negatively on r
 - r is the “price” of loanable funds (cost of borrowing)

Supply of funds: saving

- The supply of loanable funds comes from saving
 - Households use their savings to make bank deposits and purchase assets
 - These funds become available to firms to borrow and finance investment spending
 - The government may also contribute to saving if it does not spend all the tax revenue

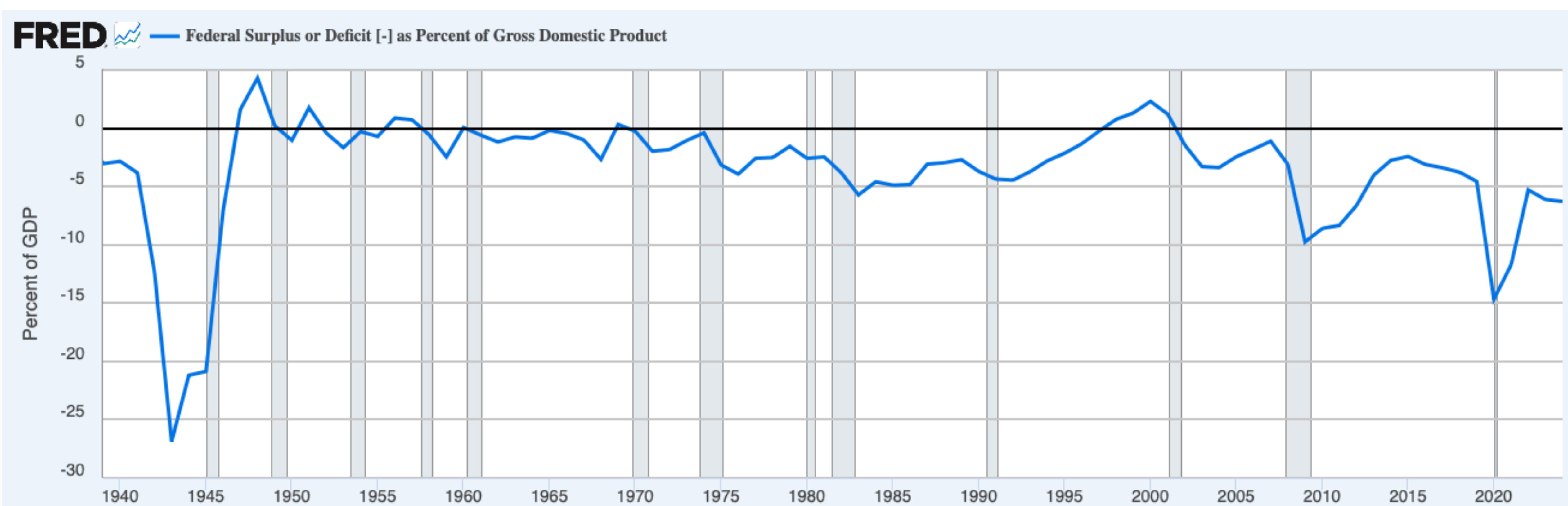
Types of saving

- *Private saving:* $(Y - T) - C$
- *Public saving:* $T - G$
- *National saving:* $S = \text{private saving} + \text{public saving}$
$$= (Y - T) - C + T - G$$
$$= Y - C - G$$
$$= I$$

Budget surpluses and deficits

- *Budget surplus:* $T > G$
 - Surplus: $T - G > 0$
- *Budget deficit:* $T < G$
 - Deficit: $T - G < 0$ (often expressed as $|T - G|$)
- *Balanced budget:* $T = G$
- The U.S. government finances its deficit by issuing Treasury bonds
 - I.e., borrowing from households and firms

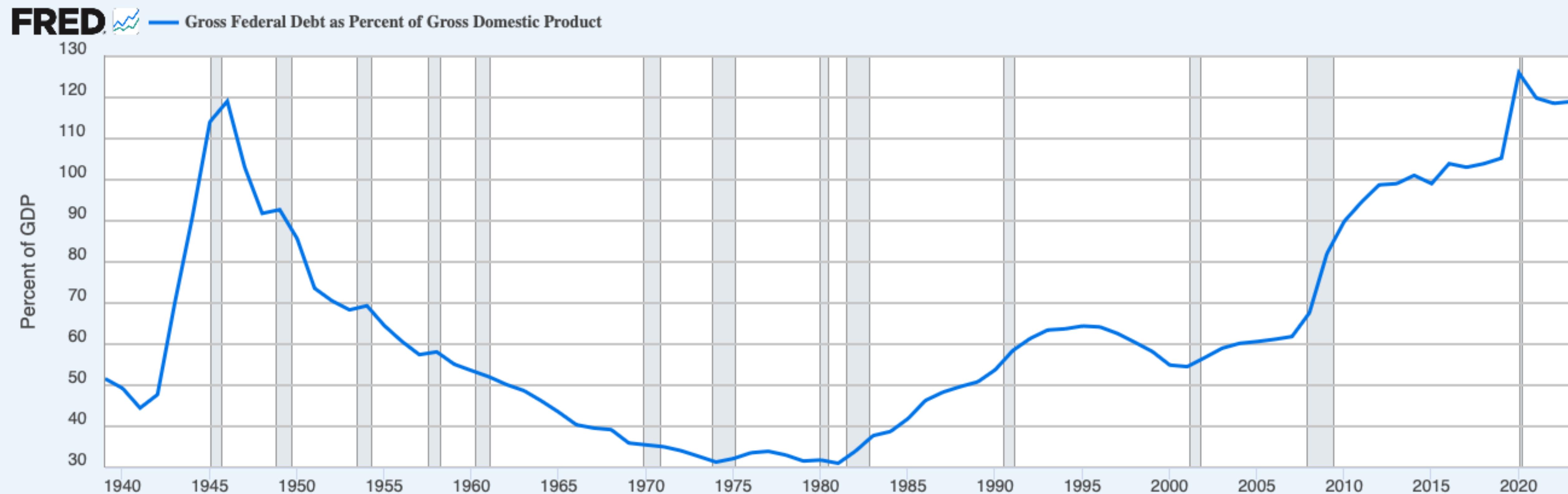
U.S. federal gov't surplus/deficit



Sources: Federal Reserve Bank of St. Louis; U.S. Office of Management and Budget via FRED®
Shaded areas indicate U.S. recessions.

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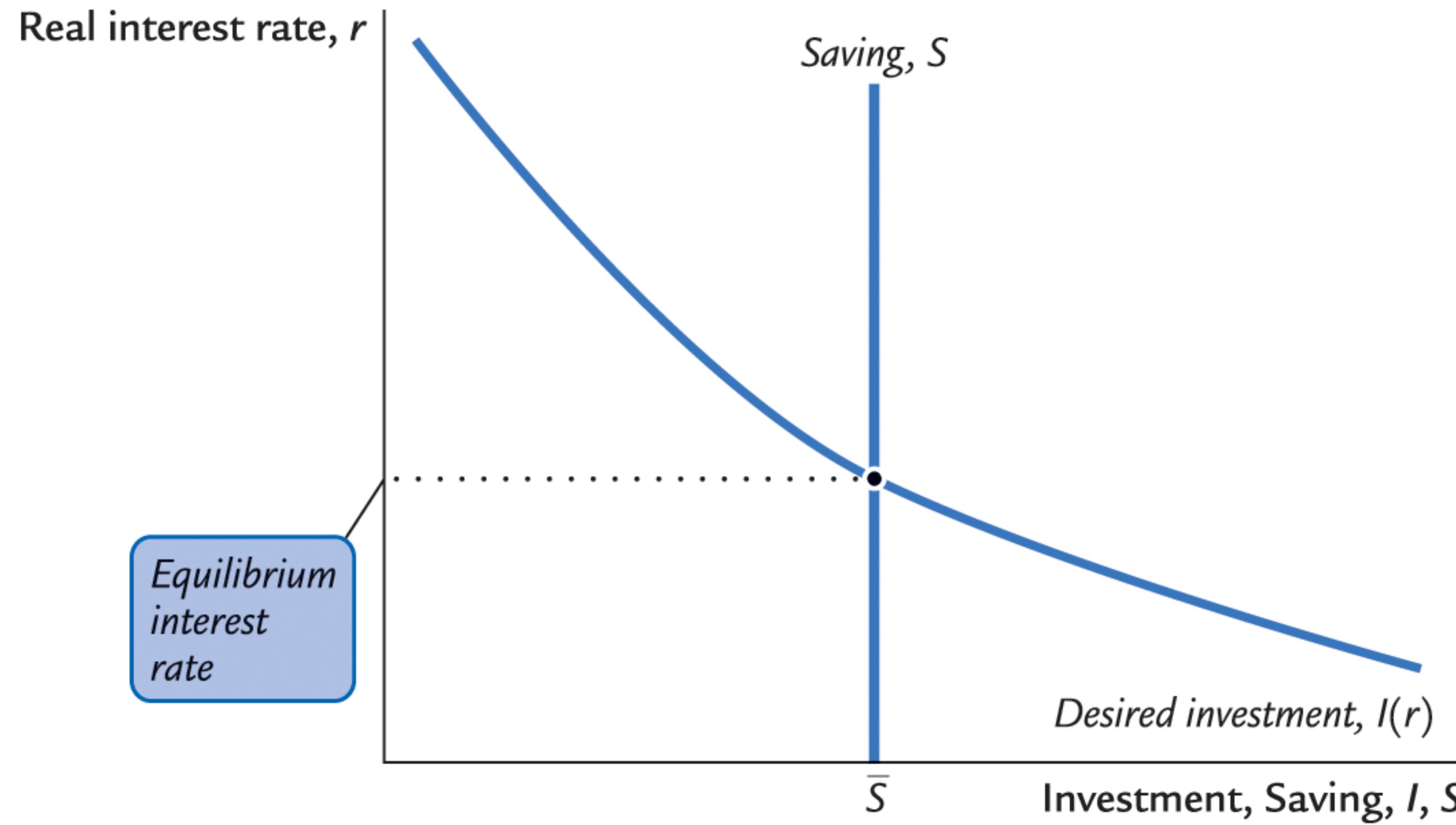
U.S. federal gov't debt



Sources: Federal Reserve Bank of St. Louis; U.S. Office of Management and Budget via FRED®
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Loanable funds equilibrium



The special role of r

- r adjusts to equilibrate the goods and loanable funds markets simultaneously

- If the loanable funds market is in equilibrium, then

$$Y - C - G = I$$

- Add $(C + G)$ to both sides to get

$$Y = C + I + G \text{ (goods market equilibrium)}$$

- Thus: equilibrium in loanable funds market \Leftrightarrow equilibrium in goods market

Recap: Mastering models

- To master a model, be sure to know:
 1. Which of its variables are endogenous and which are exogenous
 2. For each curve in the diagram, know:
 - 2.1. definition
 - 2.2. intuition for slope
 - 2.3. all the things that can shift the curve
 3. Use the model to analyze the effects of each item in 2.3.
- What are these in the loanable funds model?

Mastering the loanable funds model

1 of 2

- Things that shift the saving curve
 - Public saving
 - Fiscal policy: changes in G or T
 - Private saving
 - Preferences
 - Tax laws: 401(k), IRA, replacing income tax with consumption tax

CASE STUDY: The Reagan Deficits

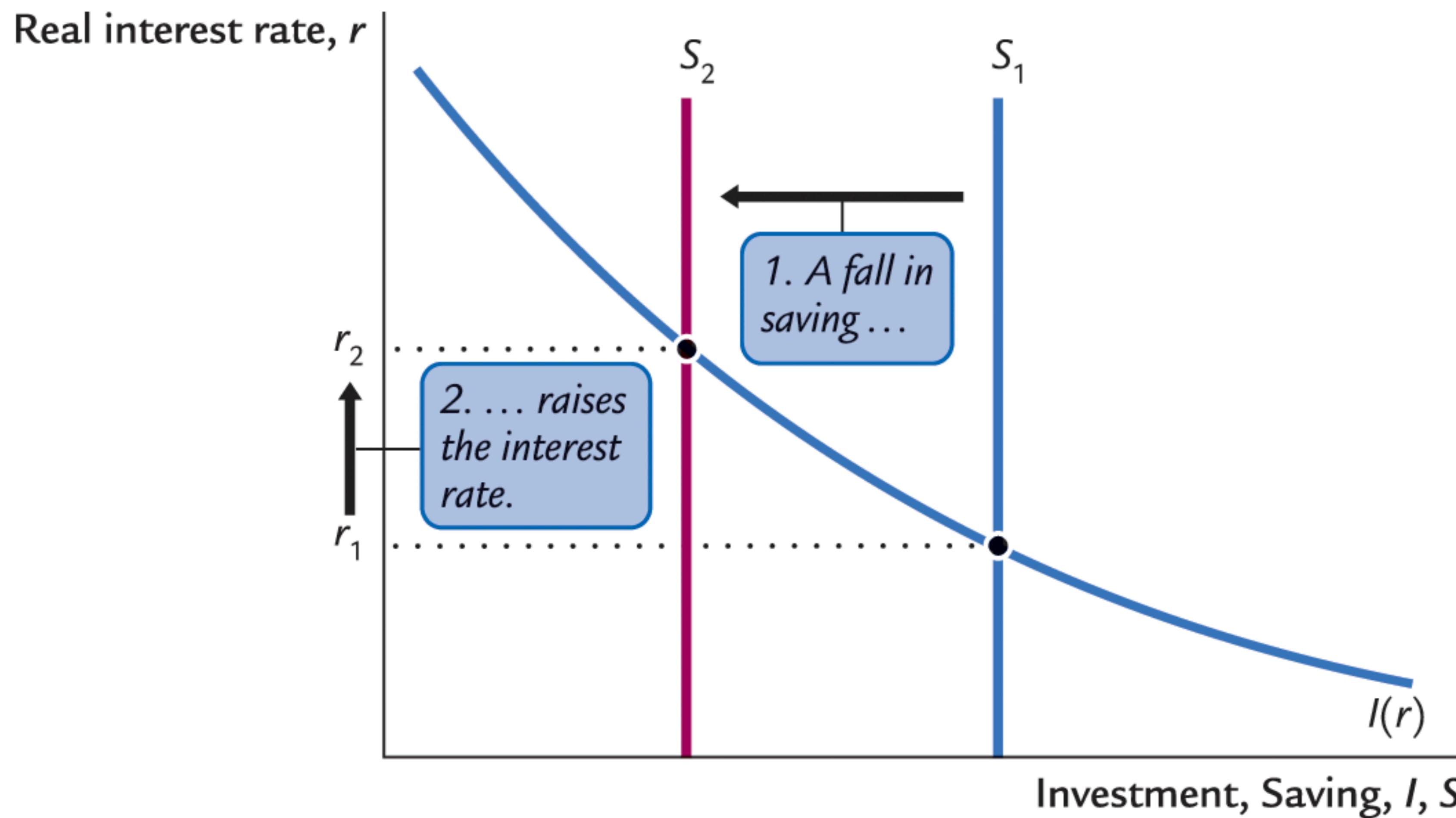
1 of 2

- Reagan policies during early 1980s
 - Increases in defense spending: $\Delta G > 0$
 - Big tax cuts: $\Delta T < 0$
- Both policies reduce national saving:

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}$$
$$\uparrow \bar{G} \implies \downarrow \bar{S} \quad \downarrow \bar{T} \implies \uparrow C \implies \downarrow \bar{S}$$

CASE STUDY: The Reagan Deficits

2 of 2

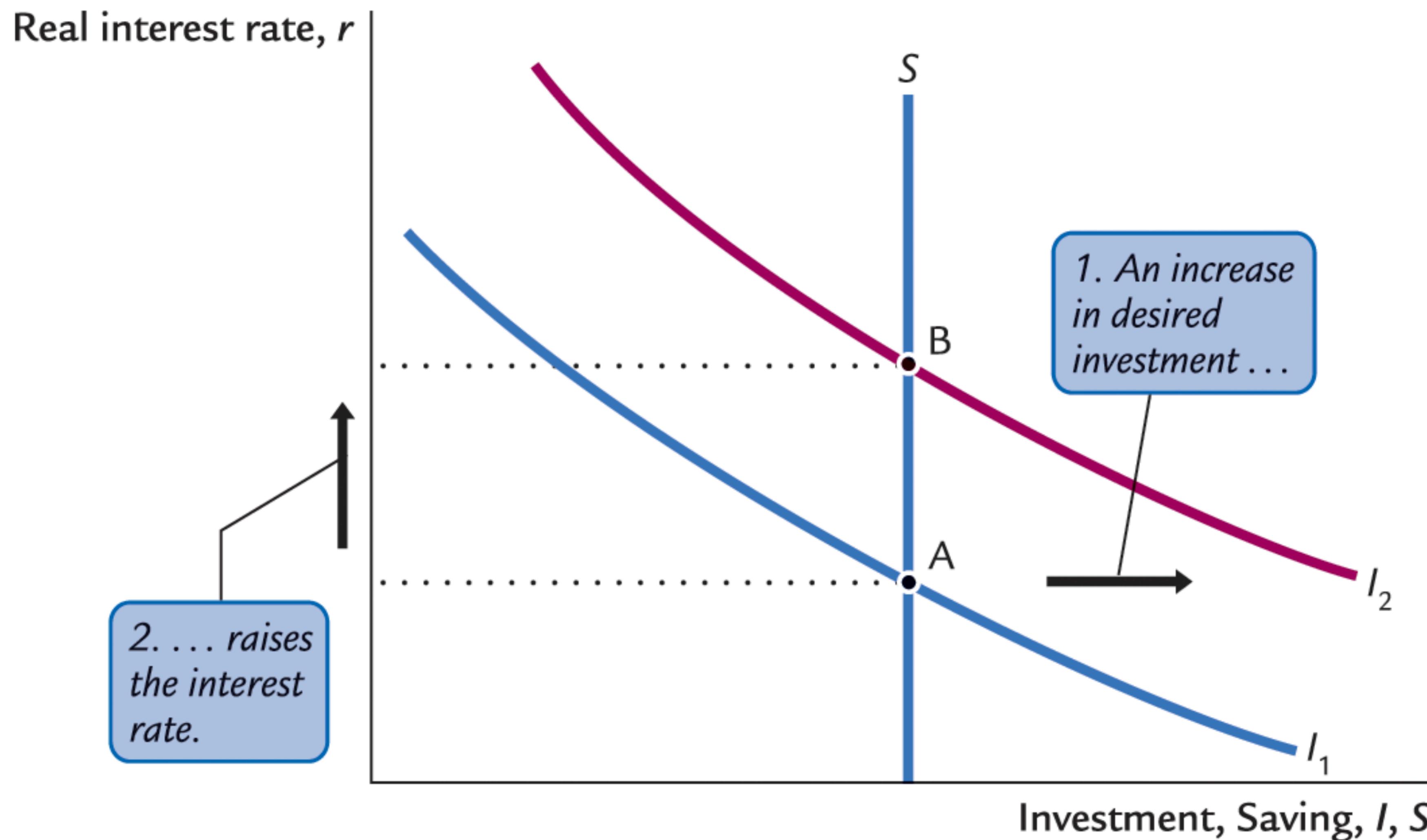


Mastering the loanable funds model

2 of 2

- Things that shift the investment curve:
 - Technological innovations
 - ...to take advantage of innovations, firms must buy new investment goods
 - Tax laws that affect investment
 - E.g., investment tax credit

An increase in investment demand



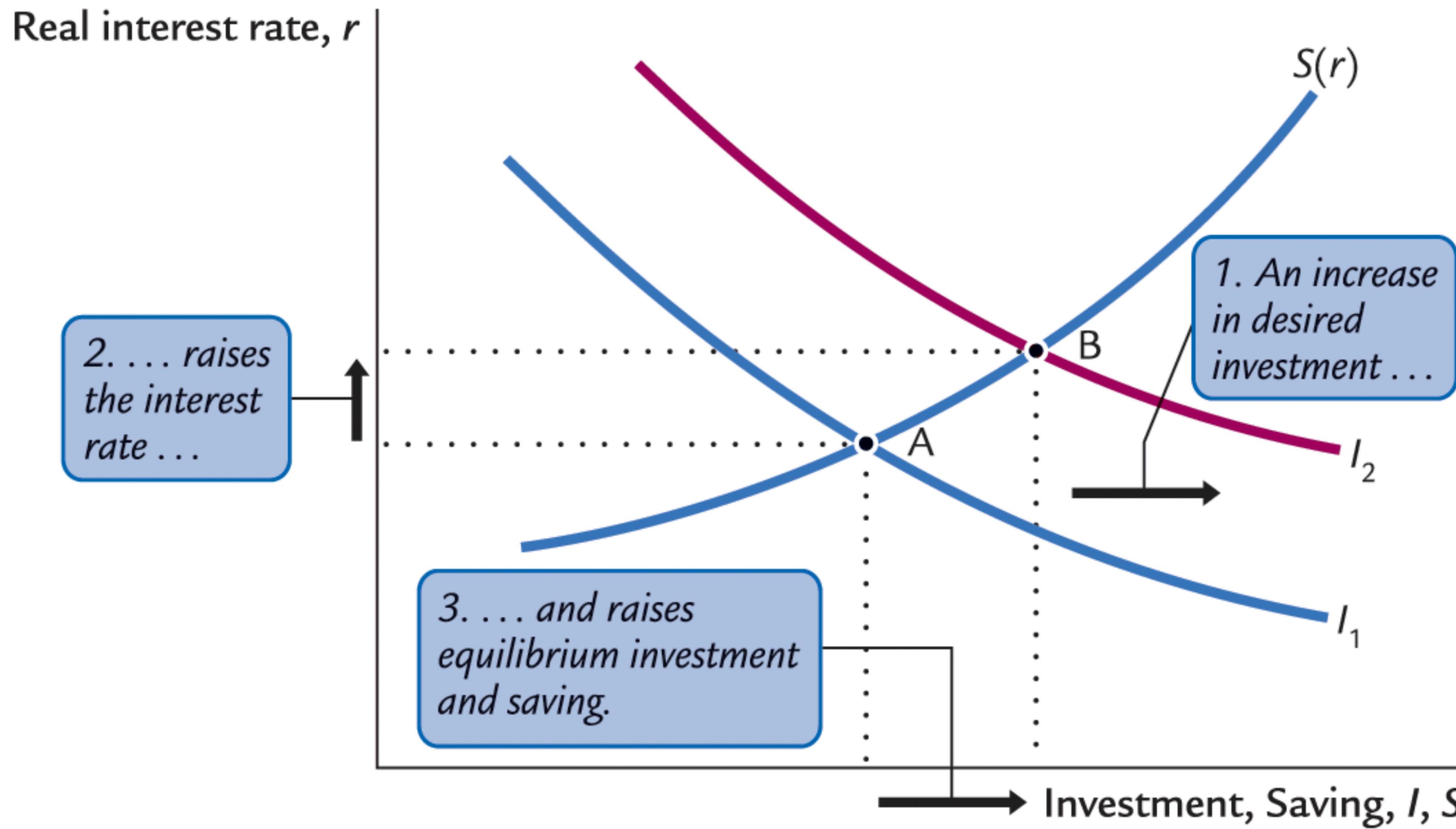
NOW YOU TRY

- New legislation requires employers to match 401(k) contributions
- What happens to the savings and investment curves?
- What happens to the interest rate and the level of savings/investment?

Saving and the interest rate

- So far, we assumed that saving does not depend on the interest rate
- Why might saving depend on r ?
- How would the results of an increase in investment demand be different?
 - Would r rise as much?
 - Would the equilibrium value of I change?

When saving depends on r



SUMMARY

1 of 4

- Total output is determined by
 - ... the economy's quantities of capital and labor
 - ... the level of technology
- Competitive firms hire each factor until its marginal product equals its price
- With constant returns to scale, labor income + capital income = total output

SUMMARY

2 of 4

- Increasing inequality among workers can be explained by numerous factors
 - Declining labor share
 - Education slowdown among skilled workers
 - Rise of globalization
 - Various cultural changes

SUMMARY

3 of 4

- In a closed economy, output is used for
 - ... consumption
 - ... investment
 - ... government spending
- The real interest rate adjusts to equate the demand for and supply of
 - ... goods and services
 - ... loanable funds

SUMMARY

4 of 4

- A decrease in saving causes
 - ... the interest rate to rise
 - ... investment to fall
- An increase in investment demand causes
 - ... the interest rate to rise
 - ... but does not affect the level of investment if the supply of loanable funds is fixed