Calculus Refresher

Intermediate Macroeconomics (ECO 3203)

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You need to know some calculus for the course. The formal prerequisite is introductory calculus (MAC 2233 or higher). This document compiles the essential concepts we will use.

1 Rules of differentiation

Power rule: for any $\alpha \in \mathbb{R}$, $\alpha \neq 0$,

$$\frac{d}{dx}x^{\alpha} = \alpha x^{\alpha-1}$$

Logarithm:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

Linearity: for any function f and g and any $\alpha, \beta \in \mathbb{R}$,

$$(\alpha f + \beta g)' = \alpha f' + \beta g'$$

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Product rule:

$$(fg)' = f'g + fg'$$

Quotient rule:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Chain rule: for any function $h(x) = f(g(x))$,

$$h'(x) = f'(g(x)) \cdot g'(x)$$

2 Growth rates

Suppose we have three variables, X, Y, and $Z = XY$. These variables change over time, but we will omit the time index for ease of notation. Let Z' denote next period's Z, so that $Z' = Z + \Delta Z = Z\left(1 + \frac{\Delta Z}{Z}\right)$. We call the last term, $\frac{\Delta Z}{Z}$, the growth rate of Z.

Product rule:

$$\frac{\Delta Z}{Z} \approx \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$$

Quotient rule:

$$\frac{\Delta X}{X} \approx \frac{\Delta Z}{Z} - \frac{\Delta Y}{Y}$$

Now suppose we have a fourth variable, $W = X^\alpha$. Power rule:

$$\frac{\Delta W}{W} \approx \alpha \frac{\Delta X}{X}$$

2.1 Proof of the product rule for growth rates

We can write Z' as

$$\begin{aligned}
Z' &= X'Y' \\
&= X\left(1 + \frac{\Delta X}{X}\right) \cdot Y\left(1 + \frac{\Delta Y}{Y}\right) \\
&= \underbrace{XY}_{=Z}\left(1 + \frac{\Delta X}{X} + \frac{\Delta Y}{Y} + \frac{\Delta X}{X}\frac{\Delta Y}{Y}\right) \\
\frac{Z'}{Z} &= 1 + \frac{\Delta X}{X} + \frac{\Delta Y}{Y} + \frac{\Delta X}{X}\frac{\Delta Y}{Y} \\
\frac{Z' - Z}{Z} &= \frac{\Delta X}{X} + \frac{\Delta Y}{Y} + \frac{\Delta X}{X}\frac{\Delta Y}{Y} \\
\frac{\Delta Z}{Z} &= \frac{\Delta X}{X} + \frac{\Delta Y}{Y} + \frac{\Delta X}{X}\frac{\Delta Y}{Y}
\end{aligned}$$

The cross term, $\frac{\Delta X}{X}\frac{\Delta Y}{Y}$, is approximately zero for small changes. Therefore,

$$\frac{\Delta Z}{Z} \approx \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$$

As an example, assume that X grows by 1 percent and Y grows by 2 percent. According to the product rule, the growth rate of Z is approximately 3 percent (its exact growth rate is 3.02 percent).