# 3. National Income

Based on Mankiw, Chapters 3 & 4: *National Income: How It Is Earned* & *National Income: How It Is Spent*

**Attila Gyetvai** | University of Florida, Department of Economics

# Outline of model

A closed economy, market-clearing model

- Aggregate supply
  - factor markets (supply, demand, price)
  - determination of output/income
- Aggregate demand
  - determinants of ***C***, ***I***, and ***G***
- Equilibrium
  - goods market
  - loanable funds market

# Aggregate supply

# Factors of production

***K*** = capital:
tools, machines, and structures used in production

***L*** = labor:
the physical and mental efforts of workers

# The production function: $Y = F(K, L)$

- Shows how much output (***Y***) the economy can produce from ***K*** units of capital and ***L*** units of labor
- Reflects the economy's level of technology
- Reflects how the existing technology transforms capital and labor into output
- Technological improvements result in more output from the same quantity of inputs
- Exhibits constant returns to scale

# Returns to scale: A review

Initially $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor $z$:

$K_2 = zK_1$ and $L_2 = zL_1$

(for example: if $z = 1.2$, then all inputs are increased by 20%)

What happens to output, $Y_2 = F(K_2, L_2)$?

- If **constant returns to scale**, $Y_2 = zY_1$
- If **increasing returns to scale**, $Y_2 > zY_1$
- If **decreasing returns to scale**, $Y_2 < zY_1$

# Returns to scale: Example 1

$$F(K,L) = \sqrt{KL}$$

$$F(zK,zL) = \sqrt{(zK)(zL)}$$

$$= \sqrt{z^2 KL}$$

$$= \sqrt{z^2}\sqrt{KL}$$

$$= z\sqrt{KL}$$

$$= zF(K,L)$$

*constant returns to scale for any* $z > 0$

# Returns to scale: Example 2

$$F(K,L) = K^2 + L^2$$

$$F(zK,zL) = (zK)^2 + (zL)^2$$

$$= z^2\left(K^2 + L^2\right)$$

$$= z^2F(K,L)$$

*increasing returns to scale for any* $z > 1$

# NOW YOU TRY

## Returns to scale

Determine whether each of these production functions has constant, decreasing, or increasing returns to scale:

(a) $F(K,L) = \dfrac{K^2}{L}$

(b) $F(K,L) = K + L$

## NOW YOU TRY

## Answers, part (a)

$$F(K,L) = \frac{K^2}{L}$$

$$F(zK,zL) = \frac{(zK)^2}{zL} = \frac{z^2K^2}{zL} = z\frac{K^2}{L}$$

$$= zF(K,L)$$

*constant returns to scale for any* $z > 0$

## NOW YOU TRY

## Answers, part (b)

$$F(K,L) = K + L$$

$$F(zK,zL) = zK + zL$$

$$= z(K + L)$$

$$= zF(K + L)$$

*constant returns to scale for any* $z > 0$

# Assumptions

1. Technology is fixed.
2. The economy's supplies of capital and labor are fixed at:

$$K = \overline{K} \text{ and } L = \overline{L}.$$

# Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

$$\overline{Y} = F\left(\overline{K}, \overline{L}\right).$$

# The distribution of national income

determined by **factor prices**, the prices per unit firms pay for the factors of production

- wage = price of ***L***
- **rental rate** = price of ***K***

# Notation

$W$ = nominal wage
$R$ = nominal rental rate
$P$ = price of output
$W/P$ = real wage
(measured in units of output)
$R/P$ = real rental rate

# How factor prices are determined

- *Quick pause: supply/demand of factors vs. final goods!*
- Factor prices are determined by supply and demand in factor markets.
- Recall that the supply of each factor is fixed.
- What about demand?

# Demand for labor

- Assume that markets are competitive: Each firm takes ***W***, ***R***, and ***P*** as given.
- Basic idea: A firm hires each unit of labor if the cost does not exceed the benefit.
  - cost = real wage
  - benefit = marginal product of labor

# Marginal product of labor (MPL)

**Definition:**
The extra output the firm can produce using an additional unit of labor (holding other inputs fixed):

$$MPL = F(K, L+1) - F(K, L)$$

## NOW YOU TRY

### Compute and graph *MPL*

a. Determine ***MPL*** at each value of ***L***.

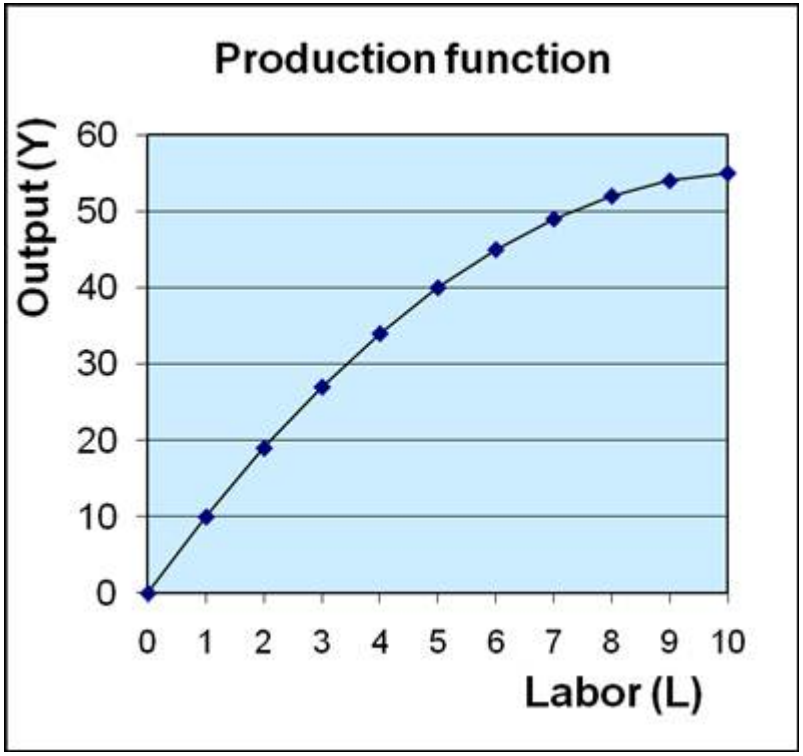
b. Graph the production function.

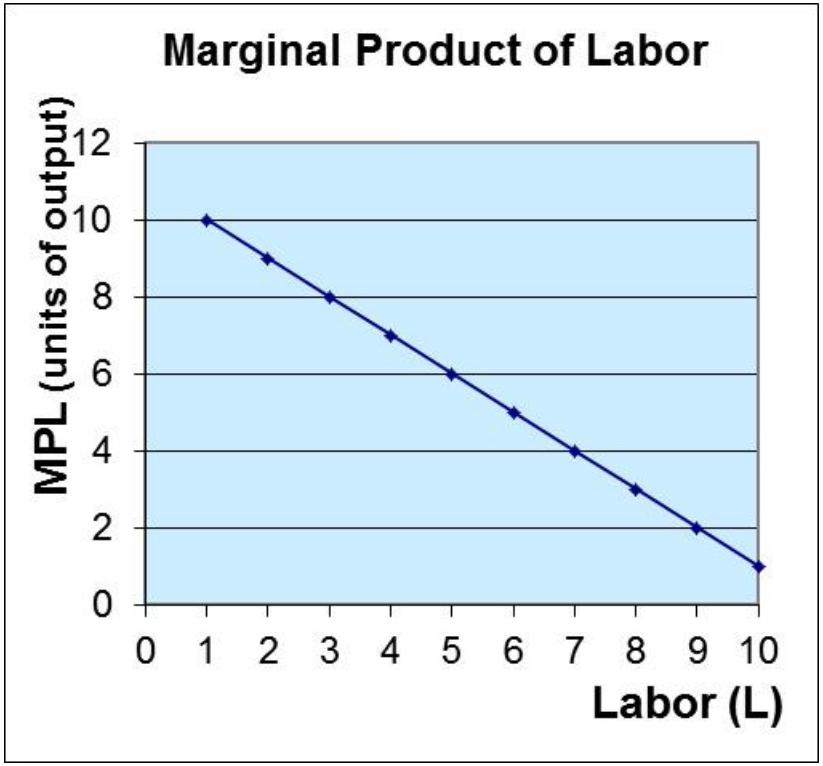
c. Graph the ***MPL*** curve with ***MPL*** on the vertical axis and ***L*** on the horizontal axis.

| ***L*** | ***Y*** | ***MPL*** |
|---|---|---|
| 0 | 0 | n.a. |
| 1 | 10 | |
| 2 | 19 | |
| 3 | 27 | |
| 4 | 34 | |
| 5 | 40 | |
| 6 | 45 | |
| 7 | 49 | |
| 8 | 52 | |
| 9 | 54 | |
| 10 | 55 | |

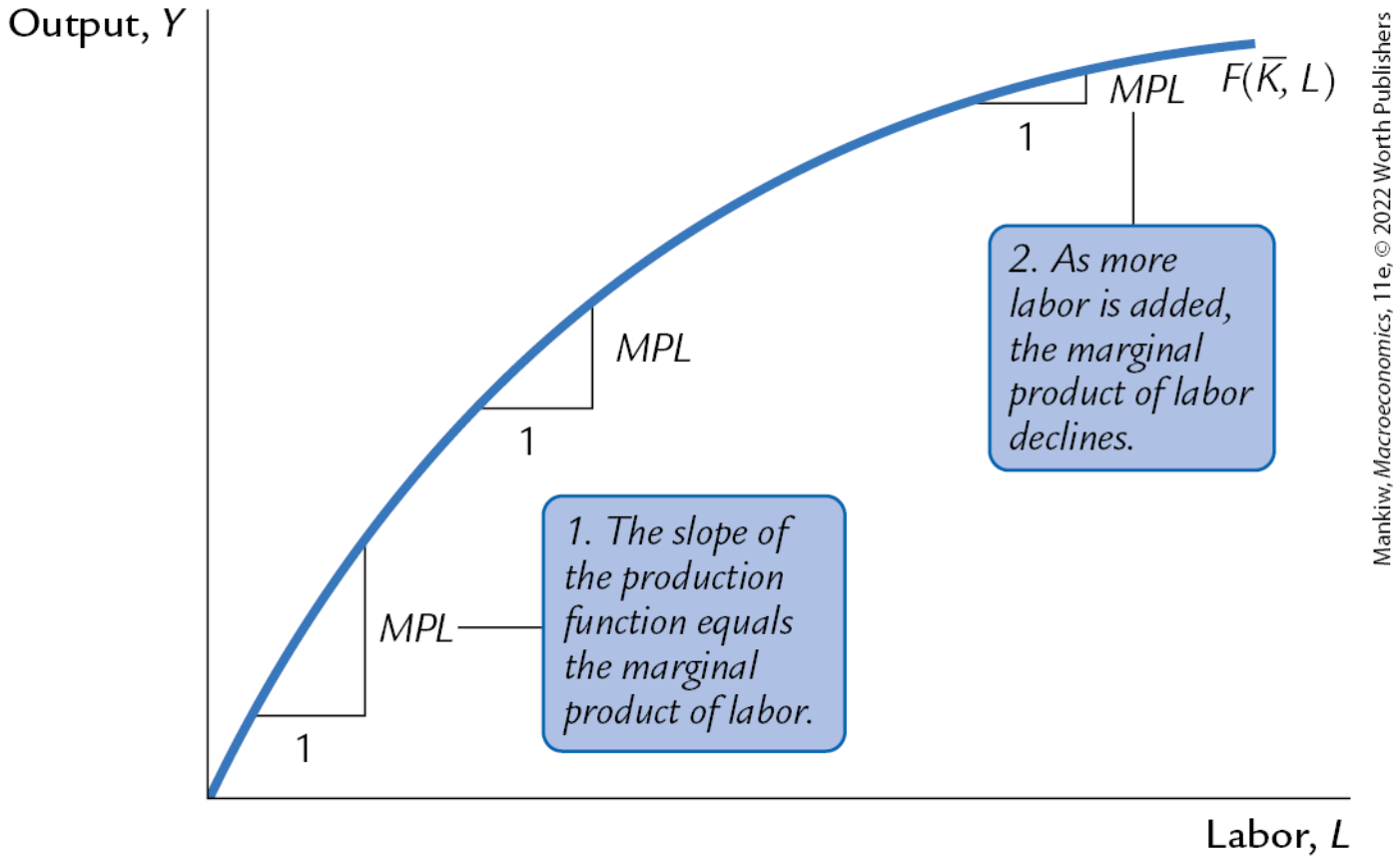
# NOW YOU TRY

## Compute and graph *MPL, Answers*



# *MPL* and the production function



# Diminishing marginal returns

- As one input is increased (holding other inputs constant), its marginal product falls.
- Intuition:
  If ***L*** increases while holding ***K*** fixed, machines per worker fall, worker productivity falls.

## NOW YOU TRY

## Identifying diminishing returns

Which of these production functions have diminishing marginal returns to labor?

(a) $F(K,L) = 2K + 15L$

(b) $F(K,L) = \sqrt{KL}$

(c) $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

# NOW YOU TRY

## Identifying diminishing returns, answers

Which of these production functions have diminishing marginal returns to labor?

(a) $F(K,L) = 2K + 15L$

No, ***MPL*** = 15 for all ***L***

(b) $F(K,L) = \sqrt{KL}$

Yes, ***MPL*** falls as ***L*** rises

(c) $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

Yes, ***MPL*** falls as ***L*** rises

## NOW YOU TRY

## *MPL* and labor demand

Suppose $W/P = 6$.

- If $L = 3$, should the firm hire more or less labor? Why?
- If $L = 7$, should the firm hire more or less labor? Why?

| *L* | *Y* | *MPL* |
|---|---|---|
| 0 | 0 | n.a. |
| 1 | 10 | 10 |
| 2 | 19 | 9 |
| 3 | 27 | 8 |
| 4 | 34 | 7 |
| 5 | 40 | 6 |
| 6 | 45 | 5 |
| 7 | 49 | 4 |
| 8 | 52 | 3 |
| 9 | 54 | 2 |
| 10 | 55 | 1 |

# NOW YOU TRY

## *MPL* and labor demand, answers

Suppose ***W/P*** = 6.

- If ***L*** = 3, should the firm hire more or less labor? Why?

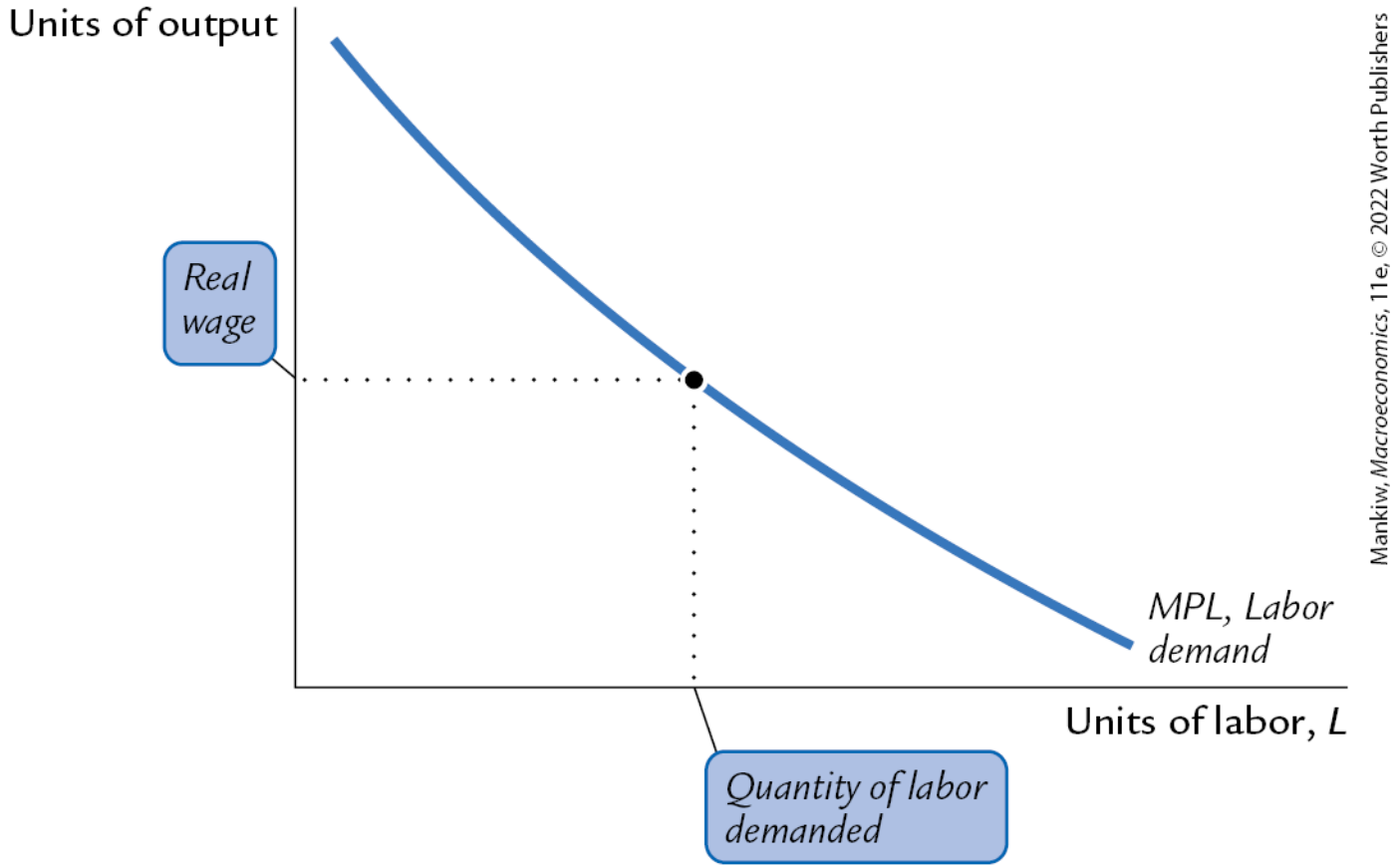
  Answer: More because the benefit of the fourth worker (***MPL*** = 7) exceeds its cost (***W/P*** = 6)

- If ***L*** = 7, should the firm hire more or less labor? Why?

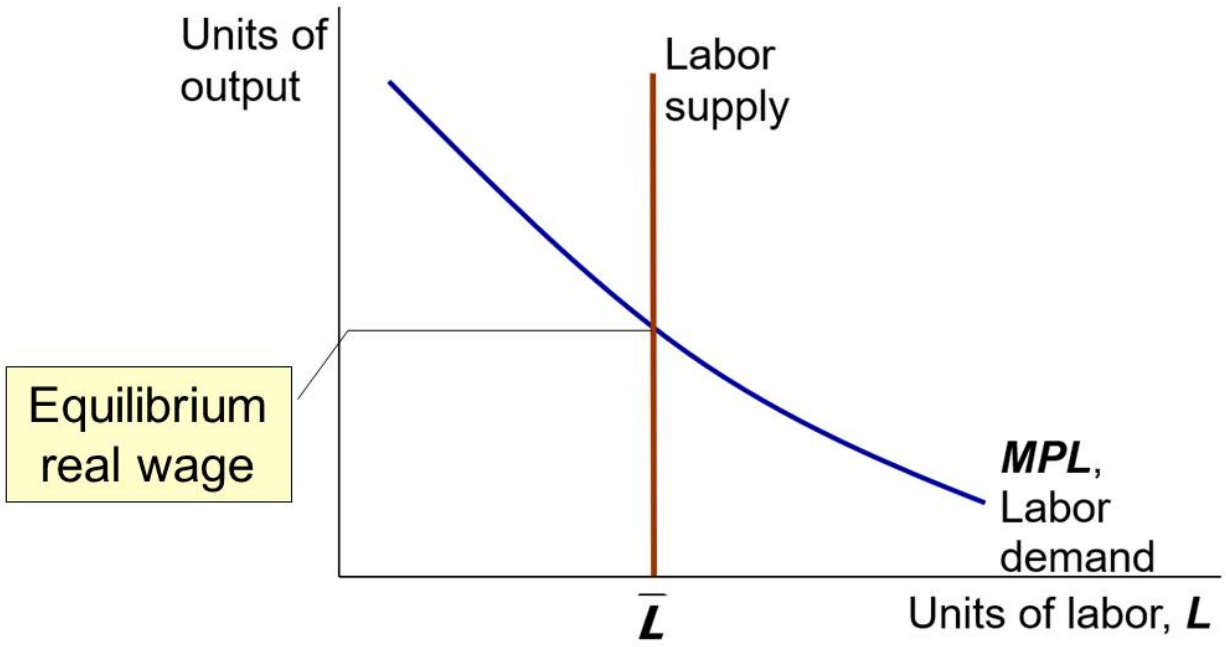
  Answer: Less because the seventh worker adds ***MPL*** = 4 units of output but costs the firm ***W/P*** = 6.

| ***L*** | ***Y*** | ***MPL*** |
|---|---|---|
| 0 | 0 | n.a. |
| 1 | 10 | 10 |
| 2 | 19 | 9 |
| 3 | 27 | 8 |
| 4 | 34 | 7 |
| 5 | 40 | 6 |
| 6 | 45 | 5 |
| 7 | 49 | 4 |
| 8 | 52 | 3 |
| 9 | 54 | 2 |
| 10 | 55 | 1 |

# *MPL* and the demand for labor



# The equilibrium real wage




The real wage adjusts to equate labor demand with supply.

# Determining the rental rate

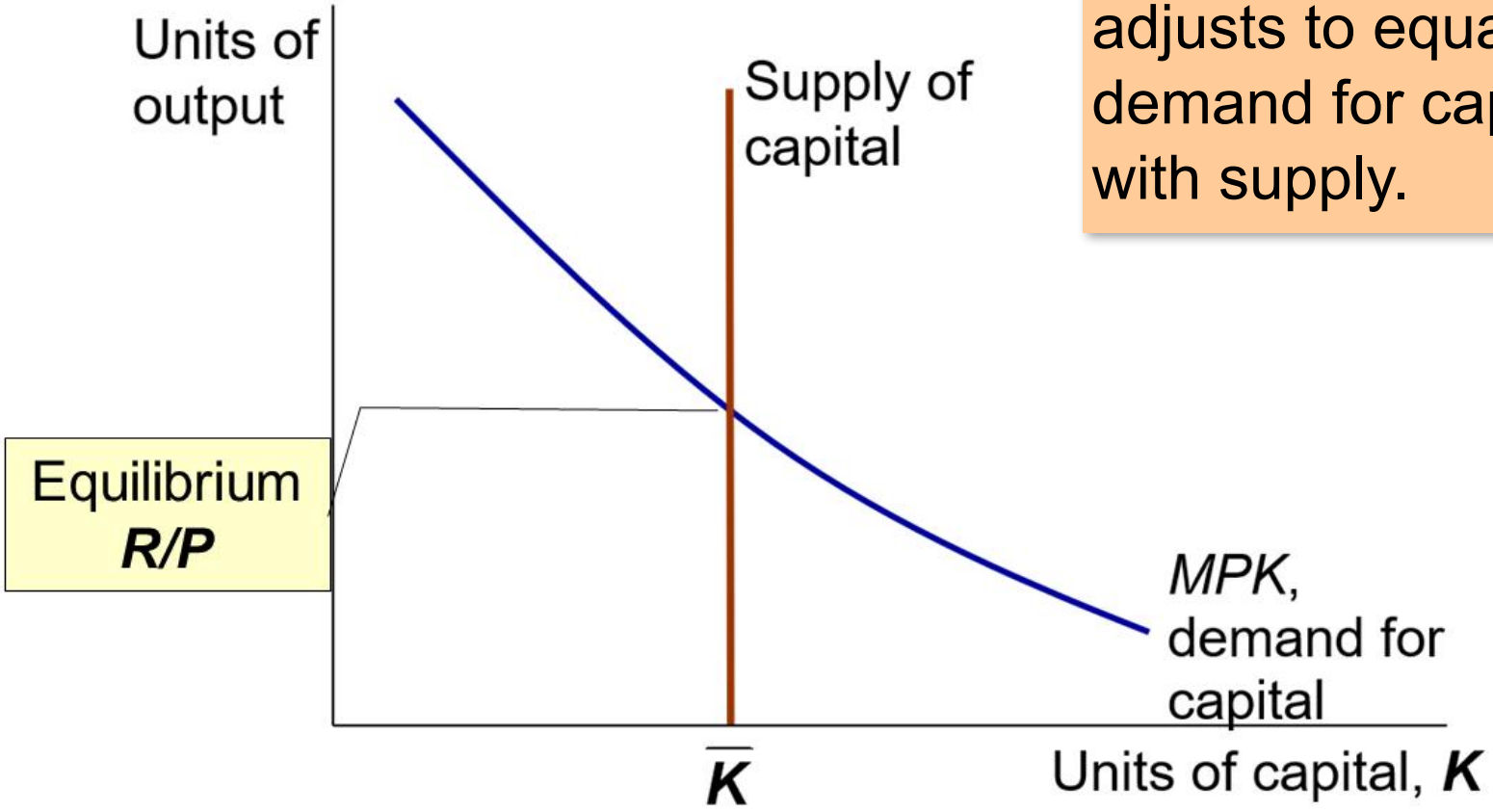
- We have just seen that ***MPL*** = ***W***/***P***.
- The same logic shows that ***MPK*** = ***R***/***P***:

  Diminishing returns to capital:
  ***MPK*** falls as ***K*** rises.

  The ***MPK*** curve is the firm's demand curve for renting capital.

  Firms maximize profits by choosing ***K*** such that ***MPK*** = ***R***/***P***.

# The equilibrium real rental rate




The real rental rate adjusts to equate demand for capital with supply.

# The neoclassical theory of distribution

- States that each factor input is paid its marginal product
- A good starting point for thinking about income distribution
- *Examples where this does not hold?*

# How income is distributed to *L* and *K*

$$\text{Total labor income} = \frac{W}{P}\bar{L} = MPL \times \bar{L}$$

$$\text{Total capital income} = \frac{R}{P}\bar{K} = MPK \times \bar{K}$$

If the production function has constant returns to scale, then

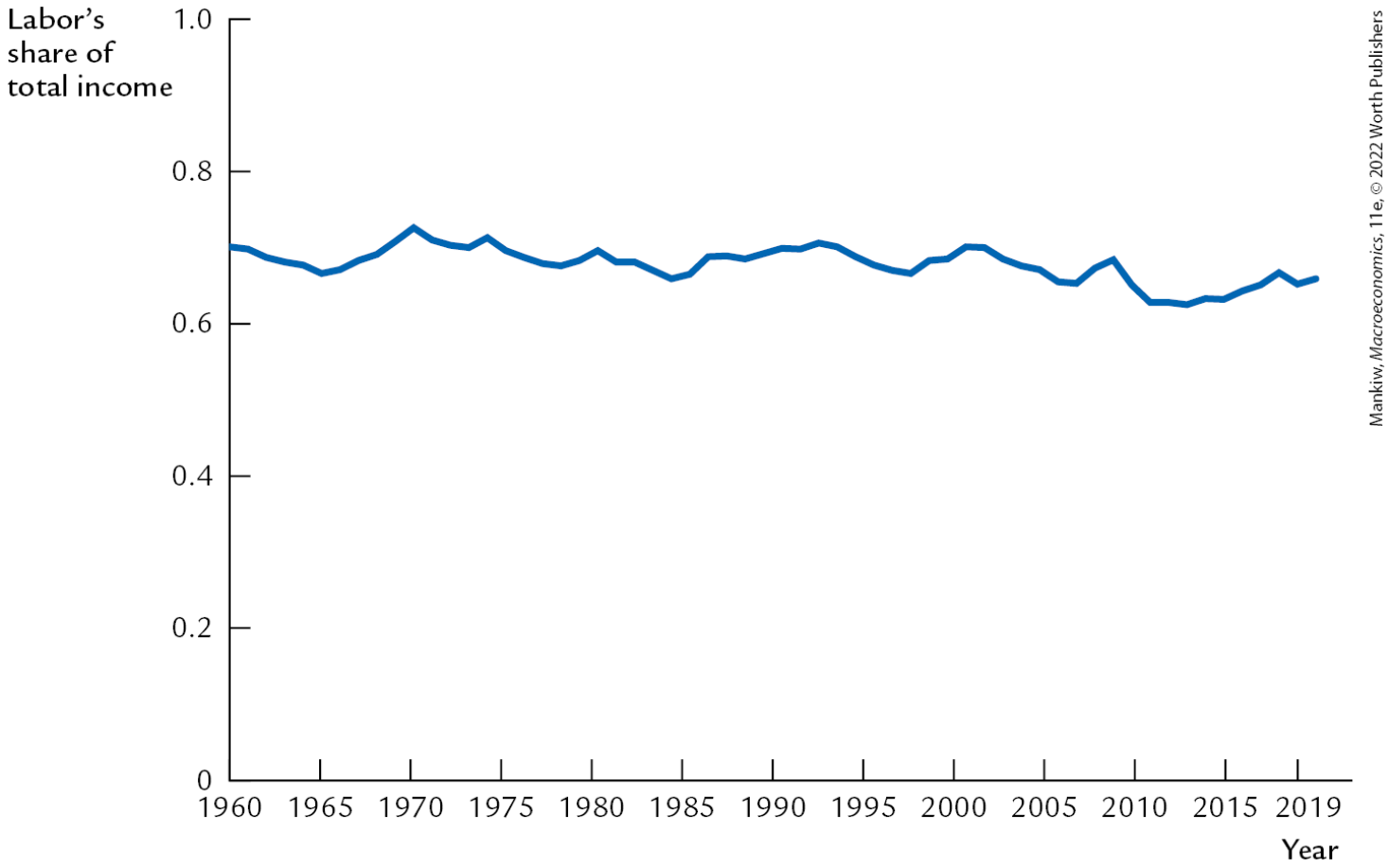
$$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K}$$

national income

labor income

capital income

# The ratio of labor income to total income in the United States, 1960–2019



# The Cobb–Douglas production function (1 of 2)

- The Cobb–Douglas production function has constant factor shares:

  $\alpha$ = capital's share of total income:

  capital income = $MPK \times K = \alpha Y$

  labor income = $MPL \times L = (1 - \alpha)Y$

- The Cobb–Douglas production function is:

$$Y = AK^{\alpha}L^{1-\alpha},$$

where $A$ represents the level of technology.

# The Cobb–Douglas production function (2 of 2)

Each factor's marginal product is proportional to its average product:

$$MPK = \alpha AK^{\alpha-1}L^{1-\alpha} = \frac{\alpha Y}{K}$$

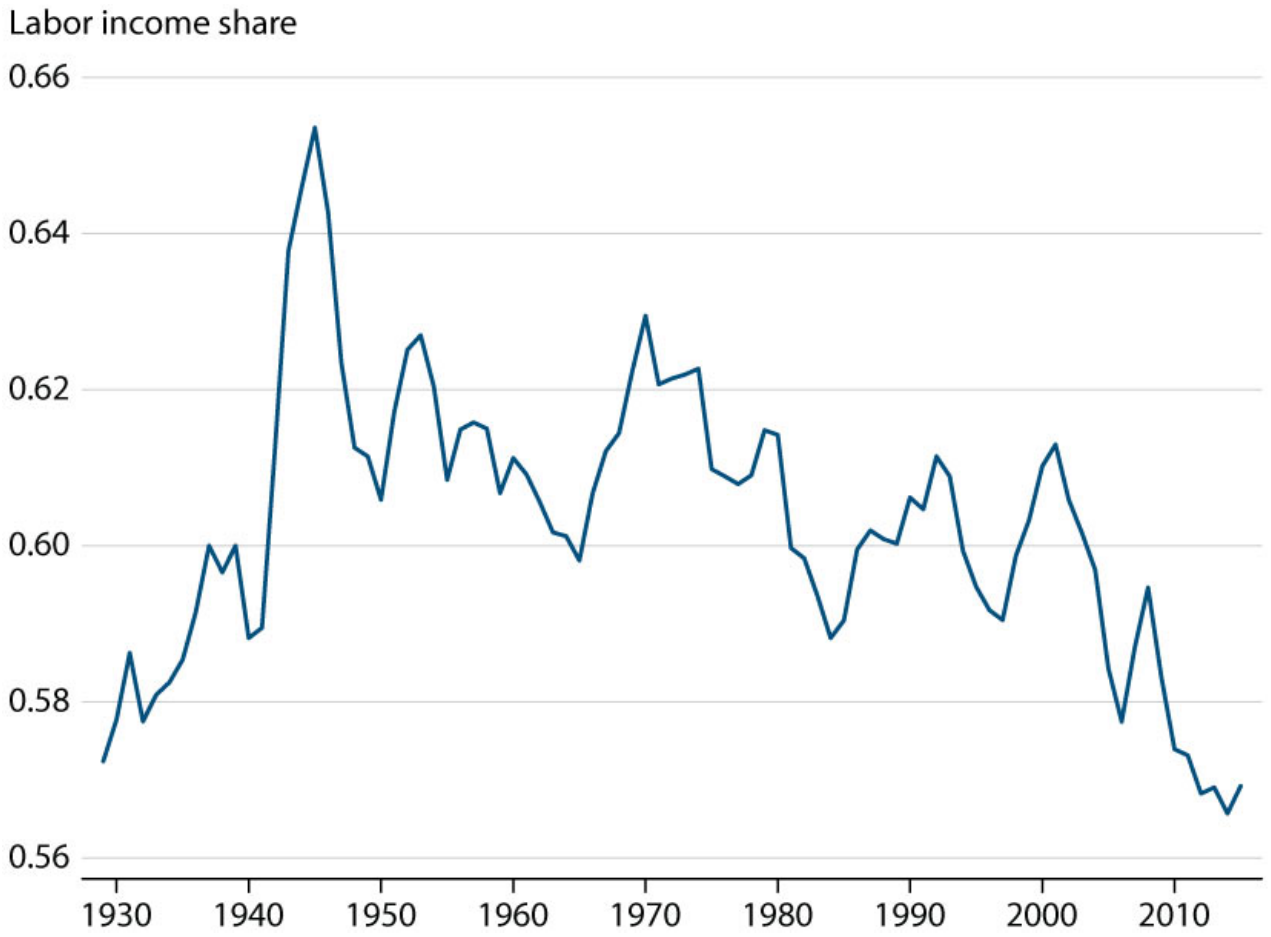
$$MPL = (1-\alpha)AK^{\alpha}L^{-\alpha} = \frac{(1-\alpha)Y}{L}$$

Therefore:

$$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K} = \frac{\alpha\bar{Y}}{\bar{K}}\bar{K} + \frac{(1-\alpha)\bar{Y}}{\bar{L}}\bar{L} = \alpha\bar{Y} + (1-\alpha)\bar{Y}$$

Constant labor share (= share of income accruing to labor)

# Declining labor share




Source: Aum and Shin (2020), https://www.stlouisfed.org/publications/review/2020/10/22/why-is-the-labor-share-declining

# Declining labor share and income inequality

- Lower labor share and higher capital share over time has exacerbated income disparity.
- Why?
  - Technological advances have reduced the role of labor and increase the role of capital (higher $\alpha$ in the production function).
  - Market power of firms has increased while worker power has decreased.

# Skill-biased technological change and inequality

- Technological progress increased the demand for skilled relative to unskilled workers (skill-biased technological change)
- Skills-biased technological change increased skilled workers' wages compared to unskilled ones
- Since the 1970s, technological progress continued, but educational advancements slowed
- Slower growth of skilled workers led to larger disparities in wages between skilled and unskilled workers, increasing inequality

# Globalization and inequality

- Trade dynamics and growth in international trade in the United States have increased the demand for skilled relative to unskilled labor.
- These changes have increased the skilled workers' wages compared to unskilled workers, exacerbating income inequality.
- International trade can exacerbate inequality, but most economists believe it benefits economies.

# Additional sources of income inequality

- Educational slowdown
- Falling marriage rates among higher-educated workers (assortative matching)

# Aggregate demand

# Demand for goods and services (1 of 2)

- Demand side
  - Recall the determinants of the four components of GDP:
  - ***C—consumption***
  - ***I—investment***
  - ***G—government expenditures***
  - ***NX—net exports***

# Demand for goods and services (2 of 2)

Components of aggregate demand:

***C*** = consumer demand for goods and services

***I*** = demand for investment goods

***G*** = government demand for goods and services

(closed economy: no ***NX***)

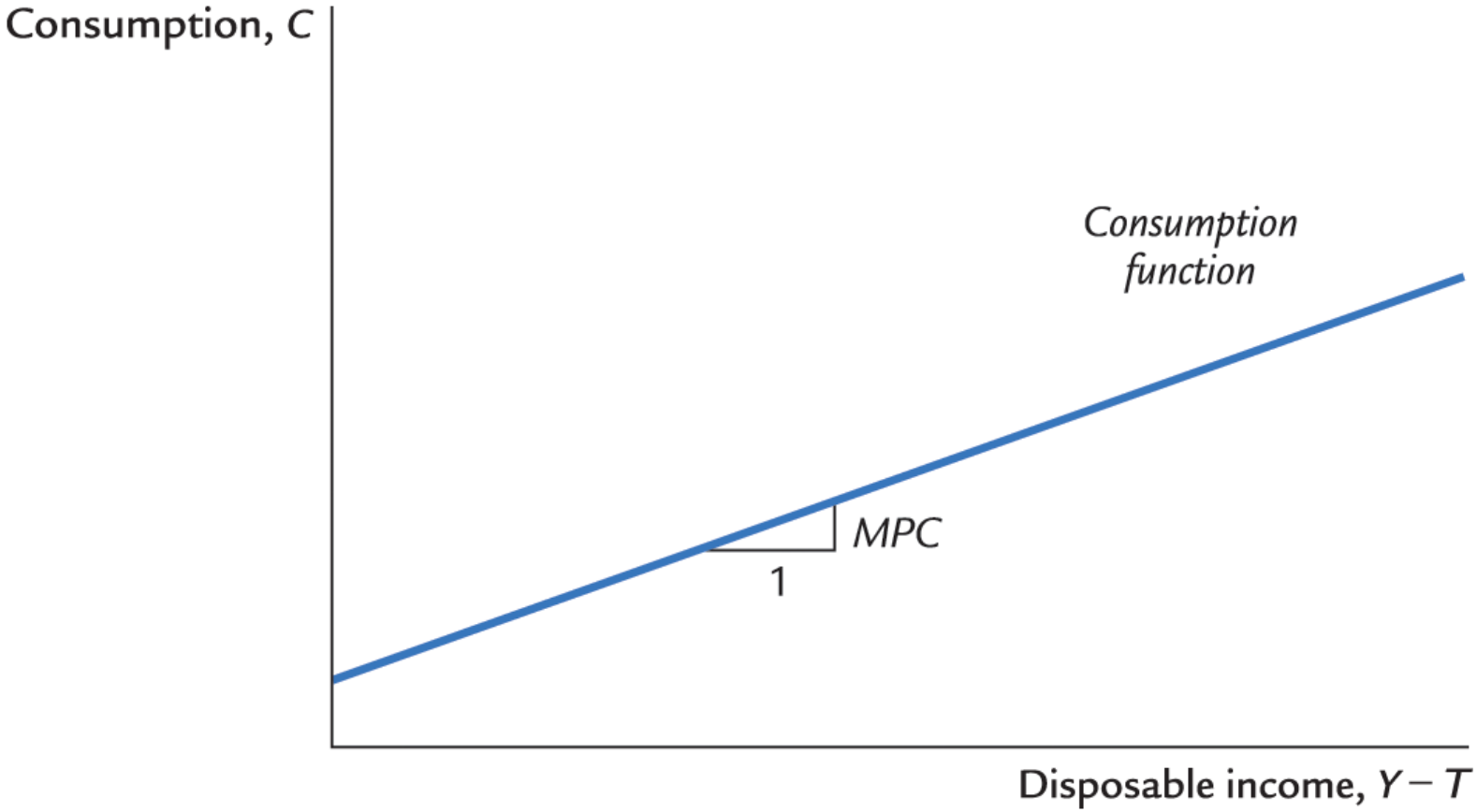
# Consumption, *C* (1 of 2)

- Households receive income from their labor and from owing capital.
- They pay taxes to the government.
- They decide how much of their after-tax income to consume and save.
- How much to consume and save (and how much to work) are the typical decisions households make in many economic models.

# Consumption, *C* (2 of 2)

- **Disposable income** is total income minus total taxes: ***Y*** – ***T.***
- Consumption function: ***C*** = ***C*** (***Y*** – ***T***)
- Definition: **marginal propensity to consume (MPC)** is the change in ***C*** when disposable income increases by one dollar.

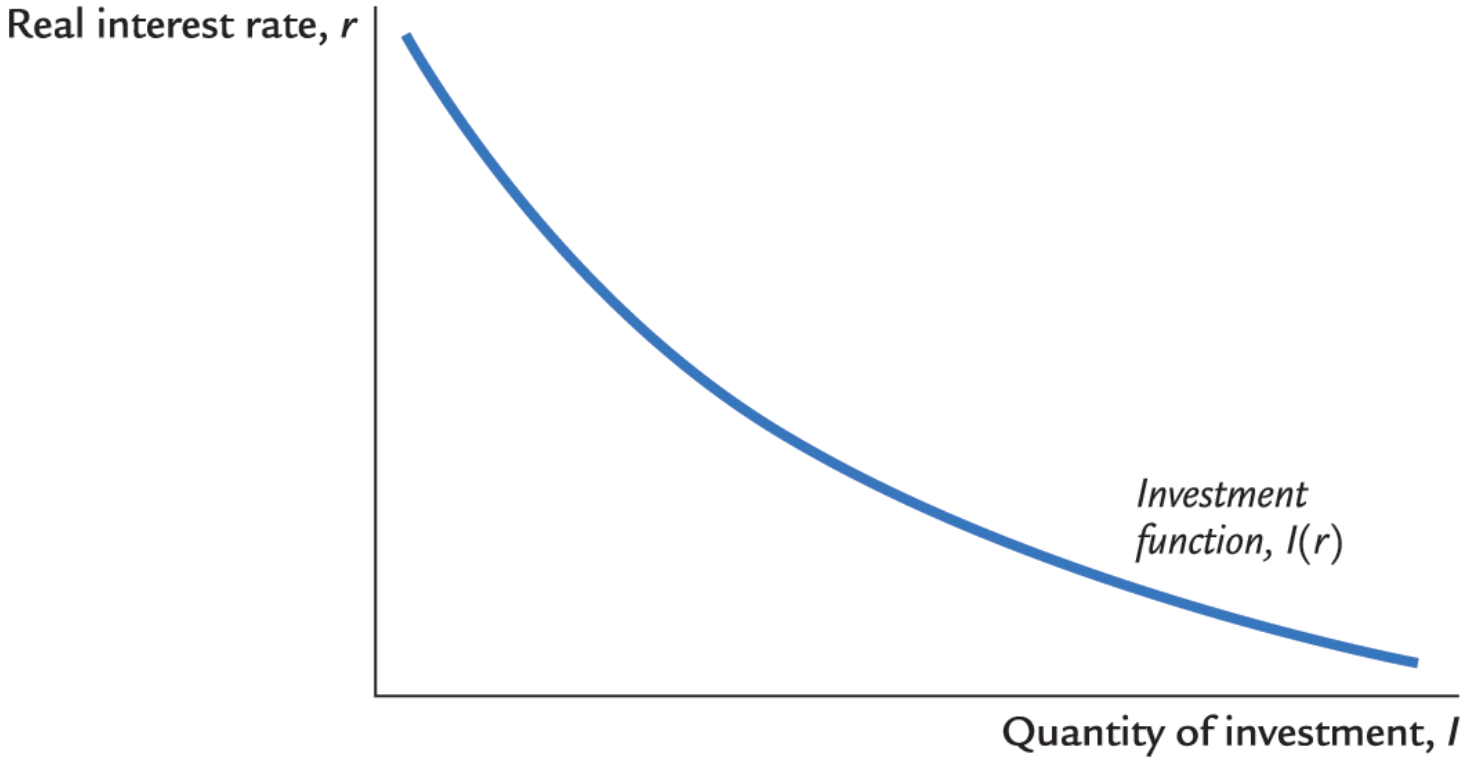
# The consumption function



# Investment, *I*

- The investment function is ***I*** = ***I*** (***r***), where ***r*** denotes the **real interest rate**, the nominal interest rate corrected for inflation.
- The real interest rate is:
  - the cost of borrowing
  - the opportunity cost of using one's own funds to finance investment spending
- So, ***I*** depends negatively on ***r***:
  - Here, ***r*** is the real interest rate, not the rental rate of capital (***R***).

# The investment function



# Government spending, *G*

- ***G*** = government spending on goods and services
- ***G*** excludes transfer payments (for example, Social Security benefits, unemployment insurance benefits)
- Assume that government spending and total taxes are exogenous:

$$G = \overline{G} \text{ and } T = \overline{T}$$

# Equilibrium

# The market for goods and services

Aggregate demand: $C\left(\overline{Y}-\overline{T}\right)+I\left(r\right)+\overline{G}$

Aggregate supply: $\overline{Y}=F\left(\overline{K},\overline{L}\right)$

Equilibrium: $\overline{Y}=C\left(\overline{Y}-\overline{T}\right)+I\left(r\right)+\overline{G}$

The real interest rate adjusts to equate demand with supply.

# The loanable funds market

- A simple supply–demand model of the financial system.
- One asset: “loanable funds”
  - demand for funds: investment
  - supply of funds: saving
  - “price” of funds: real interest rate

# Demand for funds: Investment

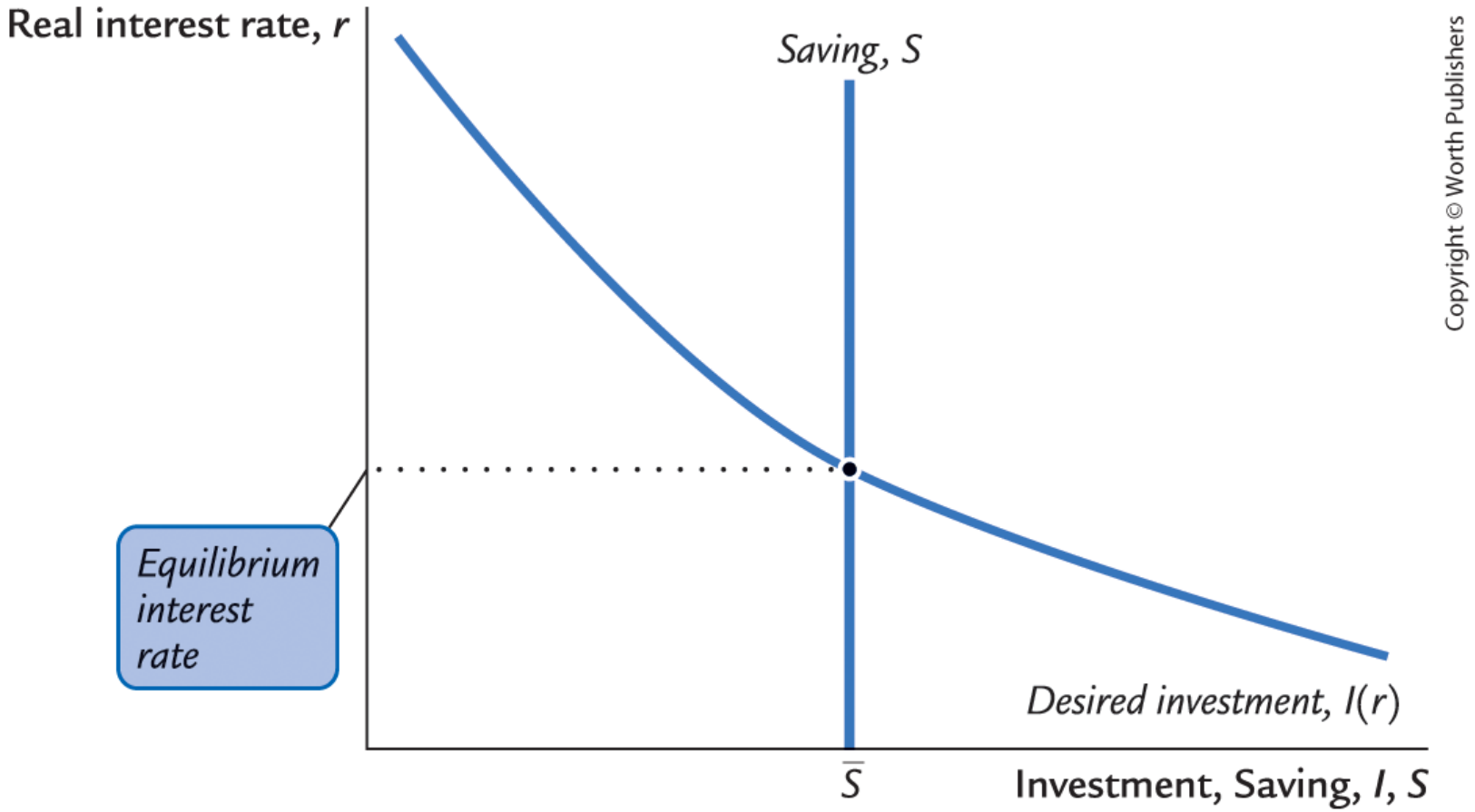
The demand for loanable funds:

- comes from investment:
  Firms borrow to finance spending on plants and equipment, new office buildings, etc. Consumers borrow to buy new houses.
- depends negatively on ***r***:
  ***r*** is the "price" of loanable funds (cost of borrowing).

# Supply of funds: Saving

- The supply of loanable funds comes from saving:
  - Households use their savings to make bank deposits and purchase bonds and other assets. These funds become available to firms to borrow and finance investment spending.
  - The government may also contribute to saving if it does not spend all the tax revenue it receives.

# Loanable funds demand curve



# Types of saving

**Private saving** = $(Y - T) - C$

**Public saving** = $T - G$

**National saving**, $S$

= private saving + public saving

= $(Y - T) - C + T - G$

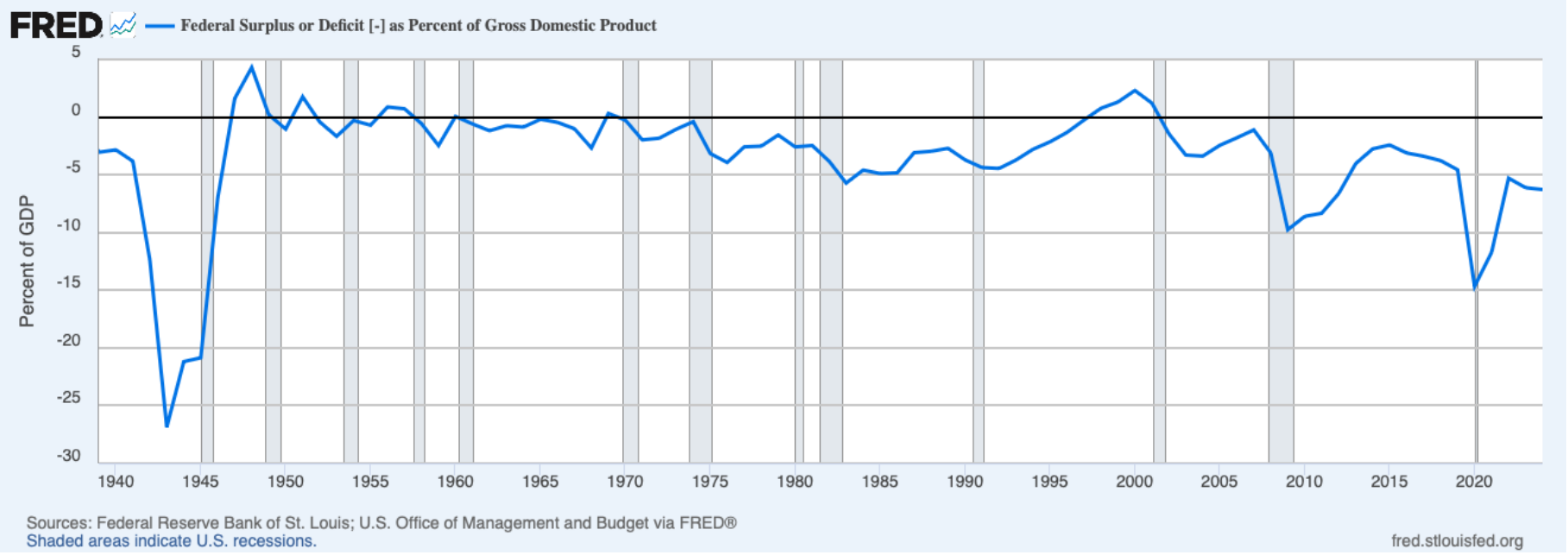
= $Y - C - G$

*What is S equal to?*

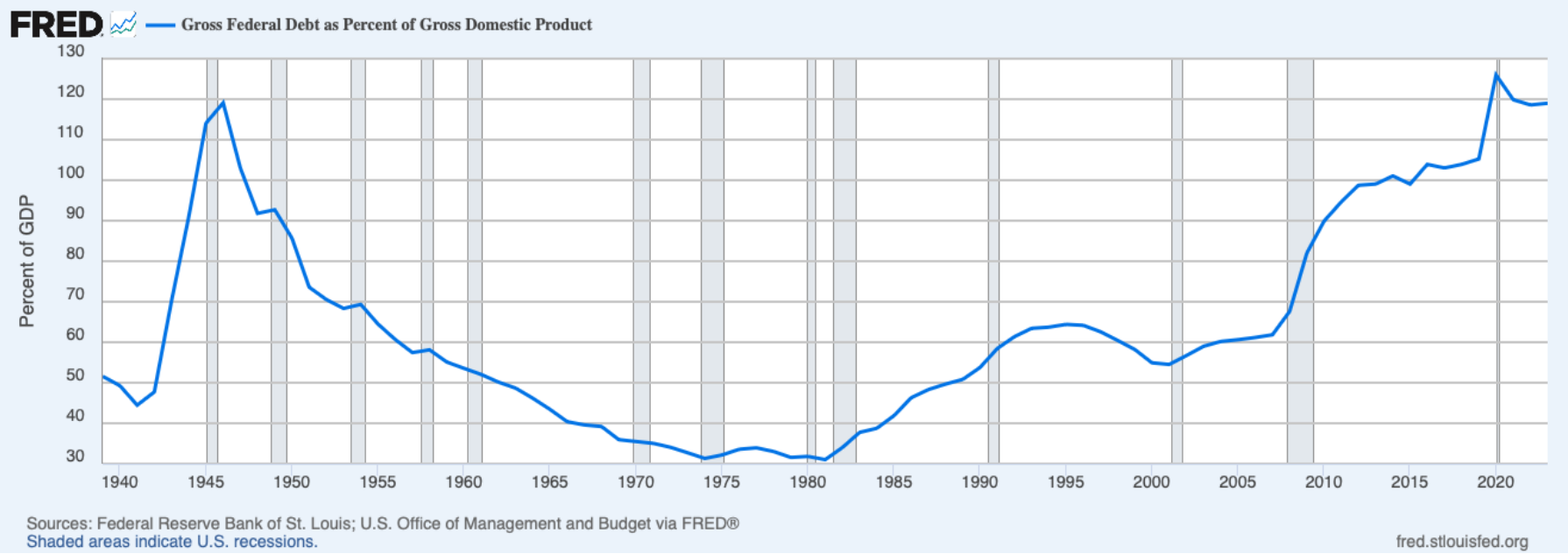
# Budget surpluses and deficits

- If ***T*** > ***G***, **budget surplus** = (***T*** – ***G***)
  = public saving.
- If ***T*** < ***G***, **budget deficit** = (***G*** – ***T***) and public saving is negative.
- If ***T*** = ***G***, **balanced budget**, public saving = 0.
- The U.S. government finances its deficit by issuing Treasury bonds—that is, borrowing.

# U.S. federal government surplus/deficit, 1939–2024

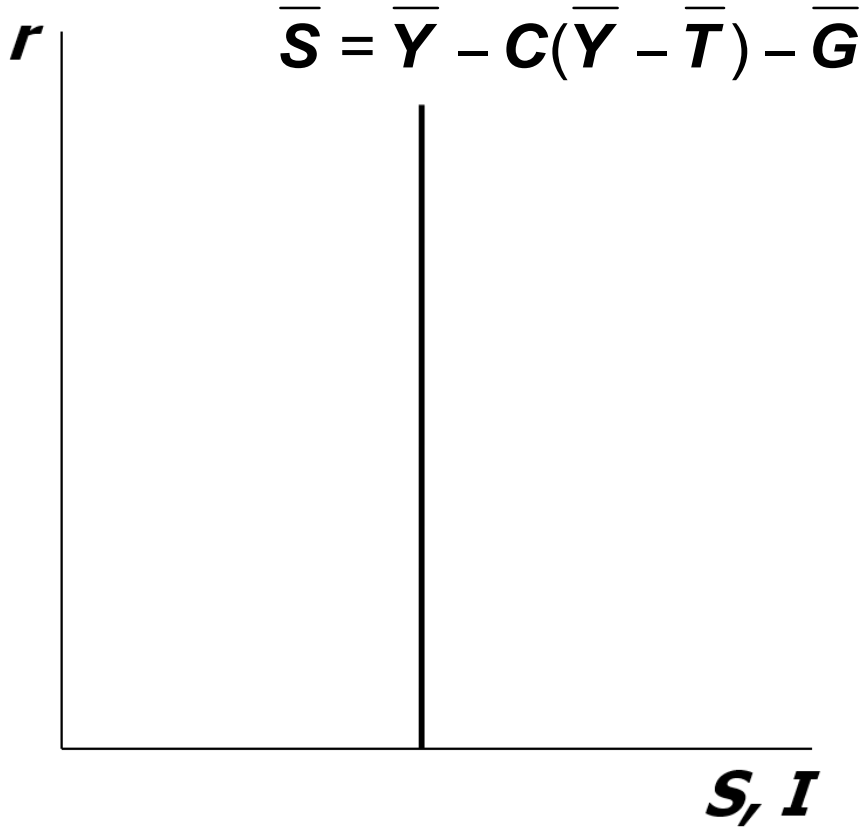


# U.S. federal government debt, 1939–2023

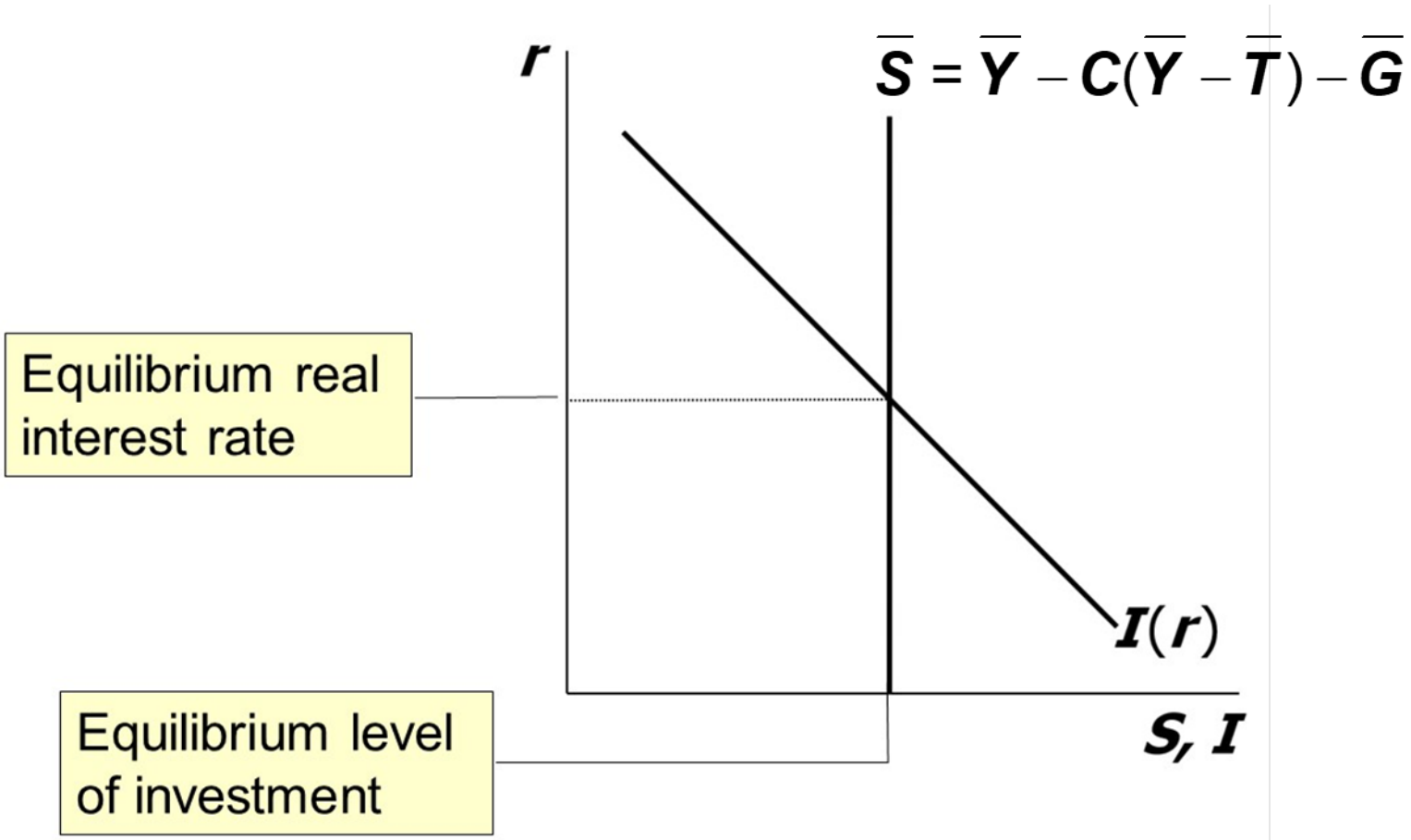


# Loanable funds supply curve

By assumption, national saving does not depend on $r$, so the supply curve is vertical.



# Loanable funds market equilibrium



# The special role of *r*

***r*** adjusts to equilibrate the goods market *and* the loanable funds market simultaneously:

If the loanable funds market is in equilibrium, then

$Y - C - G = I$

Add ($C + G$) to both sides to get

$Y = C + I + G$ *(goods market equilibrium)*

Thus:

Equilibrium in loanable funds market ⇔ Equilibrium in goods market

# *Recap:* Mastering models

To master a model, be sure to know:

1. Which of its variables are endogenous and which are exogenous.
2. For each curve in the diagram, know:
   a. definition
   b. intuition for slope
   c. all the things that can shift the curve
3. Use the model to analyze the effects of each item in 2c.

# Mastering the loanable funds model (1 of 2)

Things that shift the saving curve:

- public saving
  - fiscal policy: changes in ***G*** or ***T***
- private saving
  - preferences
  - tax laws that affect saving
    - 401(k)
    - IRA
    - replace income tax with consumption tax

# CASE STUDY: The Reagan Deficits (1 of 2)

- Reagan policies during early 1980s:
  - increases in defense spending: $\Delta \boldsymbol{G} > 0$
  - big tax cuts: $\Delta \boldsymbol{T} < 0$
- Both policies reduce national saving:

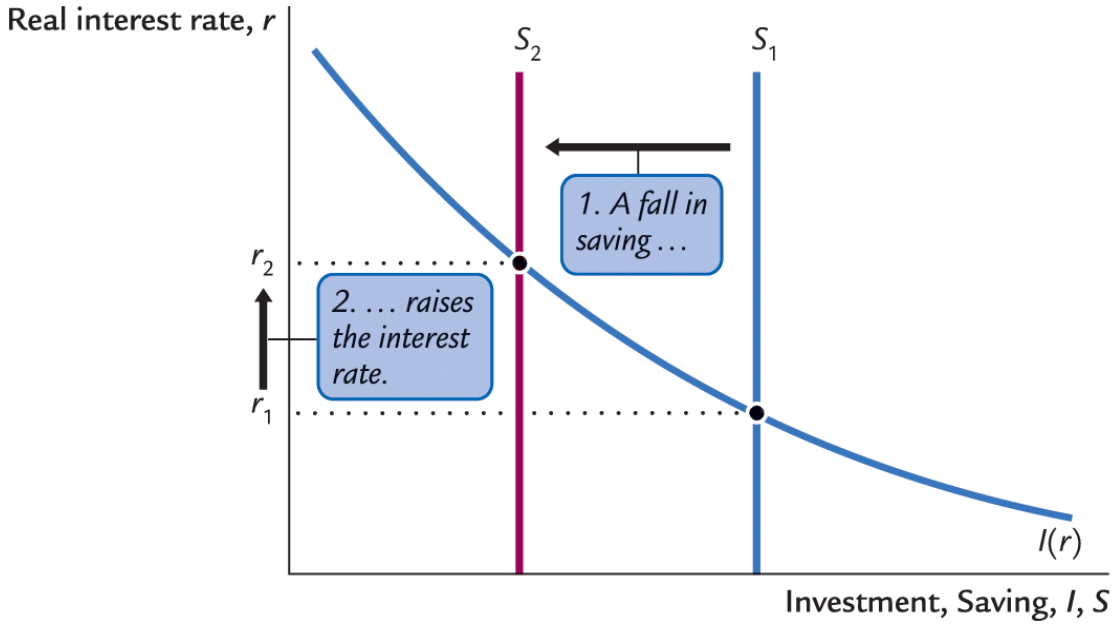
$$\overline{\boldsymbol{S}} = \overline{\boldsymbol{Y}} - \boldsymbol{C}\left(\overline{\boldsymbol{Y}} - \overline{\boldsymbol{T}}\right) - \overline{\boldsymbol{G}}$$

$$\uparrow \overline{\boldsymbol{G}} \Rightarrow \downarrow \overline{\boldsymbol{S}} \qquad \downarrow \overline{\boldsymbol{T}} \Rightarrow \uparrow \boldsymbol{C} \Rightarrow \downarrow \overline{\boldsymbol{S}}$$

# CASE STUDY: The Reagan Deficits (2 of 2)

Saving decreases from $S_1$ to $S_2$:

Results in the real interest rate increasing.



# Mastering the loanable funds model (2 of 2)

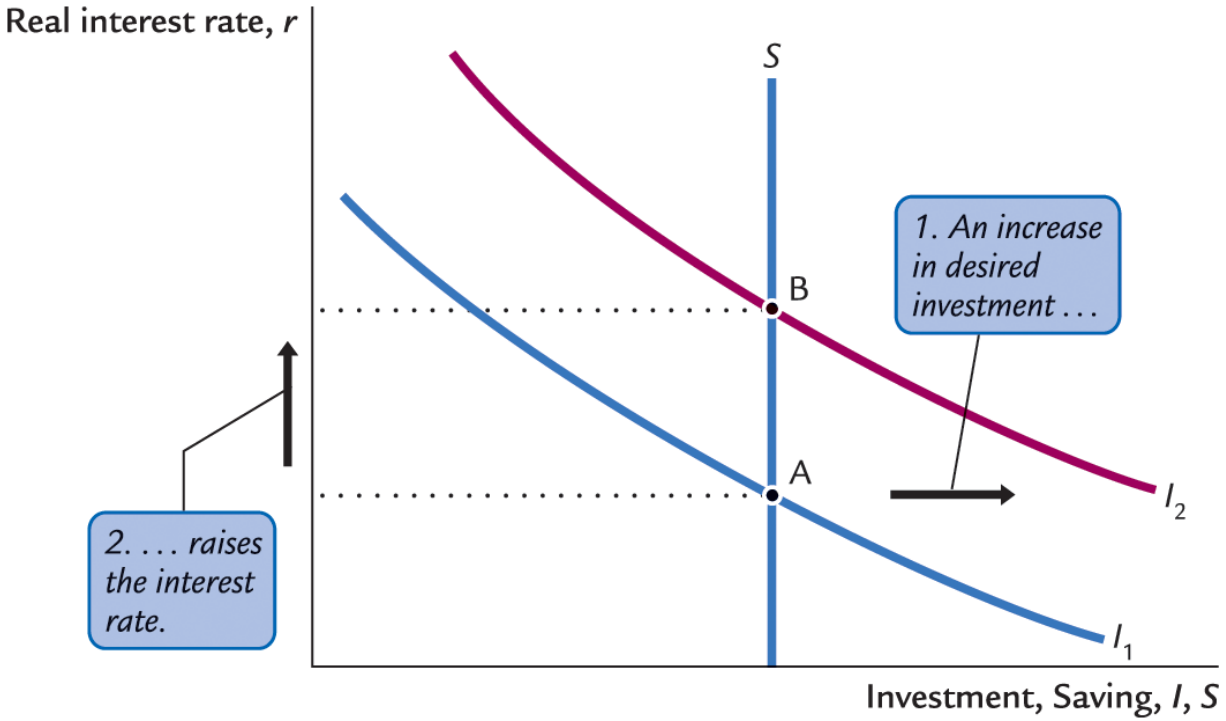
Things that shift the investment curve:

- some technological innovations
  - to take advantage of some innovations, firms must buy new investment goods
- tax laws that affect investment
  - example: investment tax credit

# An increase in investment demand

Investment Demand increases from $I_1$ to $I_2$:

Results in an increase in *r*. No change in equilibrium investment.



# NOW YOU TRY

- Suppose that new legislation requires employers to match 401(k) contributions.
- What happens to the savings and investment curves?
- What happens to the interest rate and the level of savings/ investment?

# Saving and the interest rate

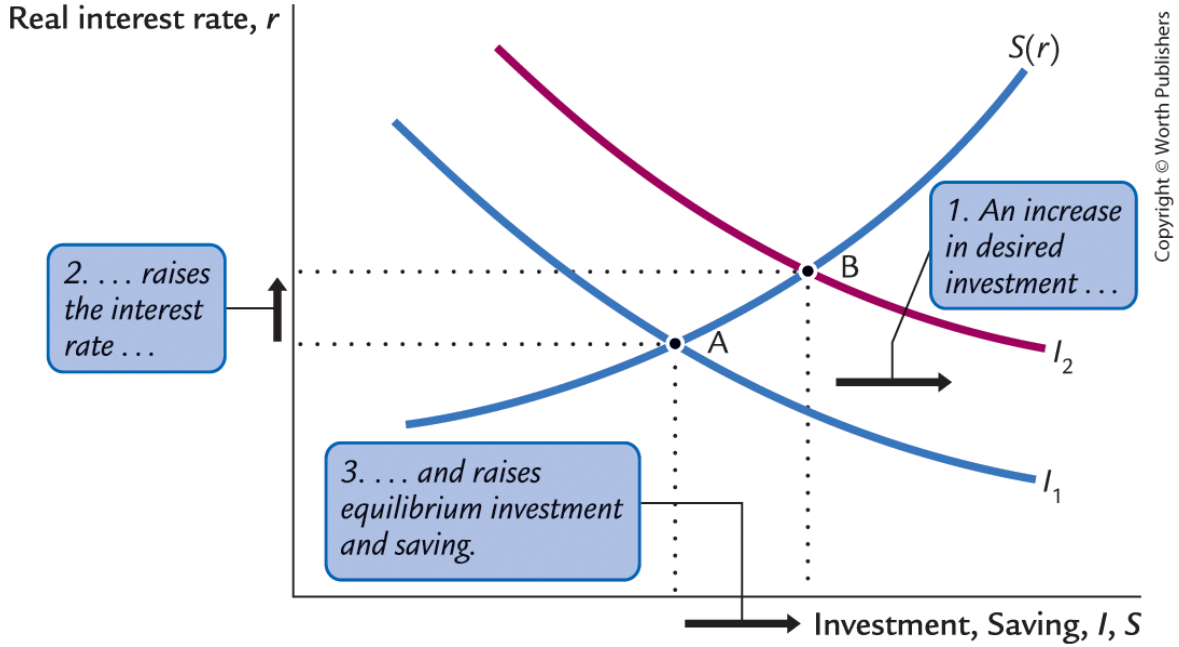
- So far, we assumed that saving does not depend on the interest rate
- Why might saving depend on ***r***?
- How would the results of an increase in investment demand be different?
  - Would ***r*** rise as much?
  - Would the equilibrium value of ***I*** change?

# An increase in investment demand when saving depends on *r*

Investment Demand increases

From $I_1$ to $I_2$:

Results in an increase in *r* and an increase in equilibrium investment.



# SUMMARY, PART 1

- Total output is determined by:
  - the economy's quantities of capital and labor
  - the level of technology
- Competitive firms hire each factor until its marginal product equals its price.
- If the production function has constant returns to scale, then labor income plus capital income equals total income (output).

# SUMMARY, PART 2

- Increasing inequality among workers can be explained by numerous factors.
- Declining labor share, education slowdown among skilled workers, the rise of globalization and various cultural changes.

# SUMMARY, PART 3

- A closed economy's output is used for consumption, investment, and government spending.
- The real interest rate adjusts to equate the demand for and supply of:
  - goods and services.
  - loanable funds.

# SUMMARY, PART 4

- A decrease in national saving causes the interest rate to rise and investment to fall.
- An increase in investment demand causes the interest rate to rise but does not affect the equilibrium level of investment if the supply of loanable funds is fixed.