Collection

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Contents

1 Bayes' Theorem

Axiom 1 (Conditional probability):

$$P(A \cap B) = P(A)P(B|A) = P(B \cap A) = P(B)P(A|B) \tag{1}$$

Axiom 2:

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap \neg B) \\ &= P(B)P(A|B) + P(\neg B)P(A|\neg B) \end{aligned} \tag{2}$$

Bayes' Theorem:

$$\boldsymbol{P(A|B) \overset{(1)}{=} \frac{P(A)\,P(B|A)}{P(B)}} \tag{3}$$

$$\boldsymbol{\overset{(2)}{=} \frac{P(A)\,P(B|A)}{P(A)\,P(B|A) + P(\neg A)\,P(B|\neg A)}} \tag{4}$$

Chain rule:

$$P(A_1 \cap \cdots \cap A_n) = P(A_1)\,P(A_2|A_1)\,P(A_3|A_1, A_2)\ldots P(A_n|A_1 \ldots A_{n-1}) \tag{5}$$

2 Desnity & cumulative distributions

2.1 Probability density (mass) function (PDF)

For a random variable X with PDF f holds:

$$\boldsymbol{P(a \,<\, X \,<\, b) = \int_a^b f(x)dx} \tag{6}$$

2.2 Cumulative distribution function (CDF)

For a random variable X with PDF f, the CDF, F, is defined as:

$$\boldsymbol{F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx} \tag{7}$$

Therefore:

$$\boldsymbol{F(b) - F(a) = P(a < X \leq b)}$$

2.3 Independence

Definition 1 (Indepence 1)

$$P(A \cap B) = P(A)\,P(B)$$
$$P(\cap A_i) = \prod_i P(A_i)$$

Follows from the definition of conditional probability (1)*:*

$$P(A \cap B) = P(A)\,P(B|A) = P(A)\,P(B),\; P(B|A) = P(B)$$

Definition 2 (Conditional Indepence)

$$P(A \cap B|C) = P(A|C)\,P(B|C)$$

Corollary 1 (Indepence & Correlation)
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